United States
Department of
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Forest Service

**United States
Department of
Agriculture**

Research Paper
PSW- 163

Optimum Use of Air Tankers in Initial Attack: selection, basing, and transfer rules

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Publisher:

Pacific Southwest Forest and Range Experiment Station
P.O. Box 245
Berkeley, California 94701

December 1982

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IN BRIEF . . .

Greulich, Francis E.; O'Regan, William G. **Optimum use of air tankers in initial attack: selection, basing, and transfer rules.** Res. Paper PSW-163. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 1982. 8 p.

Retrieval Terms: air tanker model, initial attack, resource allocation

The air tanker is an extremely effective fire suppression resource when used during the early stages of fire growth, but it is also expensive. Cost-effective allocation of this resource relies on two interrelated decisions: selection of particular air tankers for the season, and their assignment to home bases. Also, the possibility exists of daily reassignment of air tankers in anticipation of fire occurrence.

This paper reports a study to develop a mathematical model that would assist fire managers in assigning air tankers throughout the fire season.

The solution to this resource allocation problem is a mixed integer linear program consisting of two types of variables—integer and continuous. Integer variables represent the assignment of particular air tankers to specific home bases. Continuous variables define the probabilities of assigning specific air tankers to specific air bases contingent upon fire conditions for a given day.

Fire information that controls daily air tanker transfer and allocation decisions is viewed as a Markov process. The data may be derived from any number of relevant sources, such as National Fire Danger Rating values, lightning counts, and the number of on-going fires.

Air tanker output for a specific assignment may be specified by the model user. These output estimates will be based in part on the given fire-day conditions. The objective of the model, therefore, is to maximize the output of the entire air tanker system over the fire season.

This seasonal output of the air tanker system is limited by several constraints, foremost of which is the annual budget. The cost of contracting and operating a fleet of air tankers must not exceed available funds. Steady state conditions are also imposed. Included among these conditions are normalization and non-negativity constraints. The restrictive character of these steady state constraints lessens as length of fire season increases. Other constraints may be imposed to further control budget and air tanker allocations.

The computerized model was tested by using the air tanker system of District 1, California Division (now Department) of Forestry. This district, headquartered in Santa Rosa, includes all of northern coastal California. Three air bases serve as home bases for air tankers, from which initial attack sorties may be made on fire: Rhonerville, in Humboldt County; Ukiah, in Mendocino County; and Sonoma County, at Santa Rosa. The model uses a fourth air base to represent release from standby condition. The air tanker system consists of five aircraft available for contract: two F7F's and three TBM's. These aircraft have different costs and outputs associated with their use, and differ in their possible home base assignments. The expected period of air tanker use is 107 days, from July 1 through October 15.

The model was tested at nine annual budget levels, of which one level—for $152,000—was examined in detail. The results verified the mathematical structure of the model. They also showed that the computer programs needed to generate the data structure, solve a mixed integer linear program, and interpret the solution can be written and executed. The results also suggest the adequacy of the model in describing and solving a real-world situation. The model permits a more general and realistic portrayal of the decision process within an air tanker program than has been heretofore possible.

Chemical retardants dropped from air tankers can be effective in fire suppression—especially during the early stages. But the maintenance and operation of aircraft are expensive. To use air tankers efficiently, fire managers must anticipate each day the likely patterns of fire occurrence, and must transfer aircraft to forward bases accordingly. Before the fire season starts, they must decide on appropriate aircraft to contract and where to base these air tankers. Their decisions depend upon several factors, including expectations about fire occurrence patterns throughout the season, transfer and use rules, and budget levels for the air tanker program.

This paper reports the development of a mixed integer linear program model to assist the manager of a wildfire suppression program in seasonal and daily assignment of air tankers. It describes the application of the model in District 1 of the California Division (now Department) of Forestry. The model is a more generalized form of a procedure given by Manne (1960).

MATHEMATICAL MODEL

Early in any given day during the fire season, the manager assesses the status of the air tanker fleet. The air tankers are assigned to air bases for that given day, as determined by several known factors, including

- Present location of each aircraft
- Type of fire-day
- Probability distribution of projected fire-day conditions
- Transfer costs, use costs, and productivity for each aircraft, airbase, and fire-day condition.

Aircraft assignment is made according to a transfer/use rule which maximizes expected output subject to such constraints as budget and aircraft availability.

Payment is based on the level of daily use, which includes the cost of transferring an air tanker between bases and the per diem cost of overnight stays away from its home base.

At the beginning of the fire season, air tankers are contracted and assigned to home bases. Air tanker assignments and the amount of funds allocated to the program imply specific daily transfer/use decisions. Both the seasonal and daily decisions should be so made that the total cost of operating the air tanker program for the season does not exceed budgeted funds, and expected output of the air tanker program remains as high as feasible for anticipated expenditures. A mixed integer linear program (MILP) model provides the framework for analysis and solution of the problem.

Variables

The model uses two different types of variables—integer and continuous. Variable values of interest are those assumed when the objective function attains its highest value over the feasible range as delimited by the set of constraints on air tanker activity. From these variable values are obtained the optimal decision rules for the specified model. (See the Glossary for definitions of all variables.)

A binary variable, D(I,JH), assigning air tanker I to airbase JH as its home base, is defined as equal to one if affirmative. In all other cases it is zero. The range of the index, I, is from 1 through IMAX. These variables represent the seasonal allocation decision. If D(I,JH) is zero for all air bases {JH} for any given air tanker I, it implies that air tanker I will not be used during the coming fire season. The set of air bases {JH} considered feasible home bases for air tanker I will generally depend on air tanker I, because rarely are no restrictions encountered on the assignment of aircraft to home bases.

A continuous variable, X(J1,J2,K2;I,JH), is defined over the closed interval [0,1]. It may be interpreted as the probability that air tanker I, which has home base JH, will be at J1 on the morning of some randomly selected day, that fire-day condition K2 is observed early in the day, and that the air tanker is then sent to airbase J2. It is possible, of course, that J2=J1. Constraints discussed later permit the probabilistic interpretation of this variable. The daily transfer/use decision rules are then easily derived from the optimum values of X(J1,J2,K2; I,JH):

$$X(J2 \mid J1,K2;I,JH) = \frac{X(J1,J2,K2;I,JH)}{\sum_{J2} X(J1,J2,K2;I,JH)} \qquad (1)$$

These values are the conditional probabilities of sending air tanker I, which has home base JH, to air base J2, given that the air tanker is currently at air base J1 (stationed there from the previous day), and that fire condition K2 has been observed. In almost all cases this value will be either zero or one. This situation is attributed to the model structure and greatly facilitates application of the derived decision rules.

Objective

The manager then selects those air tanker-home base combinations that, in conjunction with optimal expected daily decisions about air tanker usage, will be the most cost-effective. On any randomly selected day during the fire season the air tankers under contract are at specific air bases. Each air tanker I has an expected output, Q(I,J2,K2) which is sharply related to its assigned air base, J2, and the observed fire-day condition, K2. The objective, then, is to maximize expected output, Z, of all air tankers on any randomly selected day during the fire season:

$$Z = \sum_{I} \sum_{JH} \sum_{J1} \sum_{J2} \sum_{K2} X(J1,J2,K2;I,JH)Q(I,J2,K2) \qquad (2)$$

That is, the expected output for each air tanker-home base combination (I,JH) is obtained by summing the output function Q(I,J2,K2) over the probability space given by the frequency function X(J1,J2,K2;I,JH).

Constraints

The constraints define both the external limits on total system activity and the internally imposed restrictions on subsystem activity. For the basic mathematical model, the following constraints have been identified as the minimal necessary set.

An air tanker under contract can be assigned to only one home base. To enforce this restriction, a constraint is given for each air tanker I:

$$\sum_{JH} D(I,JH) \leq 1 \qquad (3)$$

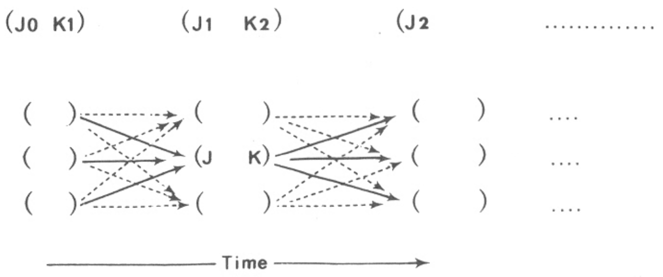
That is, the binary variable D(I,JH), when summed over the set of potential home bases {JH} for air tanker I, must be less than or equal to one.

It is assumed that the time series of fire-day conditions can be adequately described by a first-order Markov process. The probability, $g(K2 \mid K1)$, of observing fire-day condition K2 on any day depends only on the previous day's condition, K1. On any randomly selected day, designated as day 2 of a 2-day sequence, the state of the process for any specified air tanker-home base combination (I,JH) is given as (J1,K2). The index J1 designates the air base assignment prior to transfer. The index K2 is the fire-day condition observed on that same day. The system may be illustrated as follows:

(J1,K2)
(1,1)
(1,2)
(1,3)
(2,1)
(2,2)
(2,3)

In the above example, two air bases and three fire-day conditions give six possible states for the air tanker process. On the basis of the given state of the process, a decision (J2) is made to assign the air tanker to a particular location for that day.

Time is then brought into the diagram:



The arrows (both dotted and solid) represent the possible transitions from the states on one day to the states on the next. Attached to each arrow is a probability (to be calculated) of making any given transition. From this diagram, the steady state condition can be written. The probability of entering any arbitrary state, denoted as (J,K) and indicated by the converging solid lines, must equal the probability of leaving the same state, indicated by the solid diverging lines. Converging and diverging probabilities may be written as, respectively:

$$\sum_{K1} \left[g(K2=K \mid K1) \cdot \sum_{J0} X(J0,J1=J,K1;I,JH) \right]$$

and

$$\sum_{J2} X(J1=J,J2,K2=K;I,JH)$$

In order for steady state conditions to hold, the probability of entering a particular state must equal the probability of leaving. This condition must hold for every possible state; namely, the arbitrary state (J,K):

$$\sum_{J2} X(J1=J,J2,K2=K;I,JH) = \sum_{K1} \left[g(K2=K \mid K1) \cdot \sum_{J0} X(J0,J1=J,K1;I,JH) \right] \qquad (4)$$

For each (I,JH) combination, an equation normalizing the probability space must also be given:

$$\sum_{J1} \sum_{J2} \sum_{K2} X(J1,J2,K2;I,JH) = D(I,JH) \qquad (5)$$

The binary variable D(I,JH) is either zero or one. If it is zero, the corresponding frequency function is zero throughout, the steady state conditions (eq. 4) are satisfied, and the contribution to the ATS output is zero. Non-negativity constraints on the continuous variables and a value of one for the binary variable in equation 5 permit the probabilistic interpretation which is given to the set of variables, X(J1,J2,K2;I,JH).

A budget, B, is allocated to air tanker use over the fire season. The expected number of days in the fire season, S, in conjunction with a guaranteed payment for each day that the air tanker is expected to be available, establishes a fixed cost. F(I,JH), for the season. An expected daily cost, C(I,JH,J1,J2,K2), is also associated with the air tanker-home base combination (I,JH) when the air tanker is sent from air base J1 to air base J2 and fire-day condition K2 prevails. It is assumed that total fire season costs will not exceed total budgeted expenditures:

$$S\left[\sum_{I} \sum_{JH} \sum_{J1} \sum_{J2} \sum_{K2} X(J1,J2,K2;I,JH)\ C\ (I,JH,J1,J2,K2)\right] \tag{6}$$

$$+\sum_{I} \sum_{JH} D(I,JH)\ F\ (I,JH) \leqslant B$$

The summation of the cost function C(I,JH,J1,J2K2) over the probability space given by the frequency function X(J1,J2,K2;I,JH) for each air tanker-home base combination gives the expected ATS cost for any randomly selected day. Multiplying this term by the expected number of days in the season, S, and adding to it the applicable fixed seasonal costs, yields the expected expenditure for the season. This expected expenditure must be less than or equal to available budgeted funds, B.

In application, this constraint will not hold exactly because only expected values can be calculated at the beginning of the season. Since the actual air tanker system cost over the season will certainly differ from the expected cost, the problem arises of anticipating and adjusting for any wide deviation from the expected level of expenditure. The variance of expected cost could be calculated in order to give the manager some indication of potential problems. Another possibility is recomputing the results during the season by using an expenditures-adjusted current budget.

Integer and non-negativity constraints are also imposed:

$$D(I,JH) = \{0,1\} \tag{7}$$

$$X(J1,J2,K2;I,JH) \geqslant 0 \tag{8}$$

MODIFICATION AND EXTENSION

The basic model can be modified by adding constraints or extending its range of applicability through appropriate re-specification of model elements. Although it is impossible to list all possible constraints or to speculate on the outer limits of applicability, the following examples should give some insight to the range of possibilities.

The basic model can be modified substantially by defining additional constraints, such as those which restrict the allocation of funds or air tankers.

In some situations a minimum payment, B(I,JH), must be guaranteed for air tanker flight time during the season. Such a restriction may be stated in the following form:

$$S \sum_{J1} \sum_{J2} \sum_{K2} A(I,JH,J1,J2,K2)\ X(J1,J2,K2;I,JH) \geqslant B(I,JH) \cdot D(I,JH) \tag{9}$$

in which A(I,JH,J1,J2,K2) is the payment for air tanker flight time under the given conditions.

The problem of air base set-up costs can be addressed. Assume that an amount W has been allocated to cover these expenditures. If airport JH is selected as a home base, then an expenditure F(JH) is incurred. Define the binary integer variable D(JH) such that:

$$D(JH) = \begin{cases} 1, \text{ if airport JH is used} \\ 0, \text{ otherwise} \end{cases}$$

The following constraints are added to the program:

$$\sum_{I} D(I,JH) \leqslant D(JH) \cdot IMAX, \quad \text{for all JH} \tag{10}$$

$$D(JH) = \{0,1\}, \quad \text{for all JH} \tag{11}$$

$$\sum_{JH} F(JH) \cdot D(JH) \leqslant W \tag{12}$$

The allocation of an air tanker I to a particular air base J a proportion, r, of those days that fire-day condition index K is observed can be obtained by adding the constraint:

$$\frac{\sum_{J1} X(J1,J2 = J,K2 = K;I,JH)}{\sum_{J1} \sum_{J2} X(J1,J2,K2 = K;I,JH)} \geqslant r \tag{13}$$

The expected number, $\overline{V}$, of air tankers working at base J, given fire-day condition K, can be controlled by adding a constraint of the following type:

$$\sum_{I} \sum_{JH} \left[\frac{\sum_{J1} X(J1,J2 = J,K2 = K;I,JH)}{\sum_{J1} \sum_{J2} X\ (J1,J2,K2 = K;I,JH)}\right] \leqslant (\geqslant)\ V^* \tag{14}$$

or, after summing out terms,

$$\sum_{I} \sum_{JH} \left[\frac{X\ (J2 = J,K2 = K;I,JH)}{X\ (K2 = K;I,\ JH)}\right] \leqslant (\geqslant) V^* \tag{15}$$

In order to put into linear form, substitute:

$$X\ (K2 = K;I,JH) = g(K2 = K) \tag{16}$$

This result follows since the probability of observing fire-day conditions K does not depend on either I or JH and is equal to the steady state value which may be obtained from $g(K2 \mid K1)$. Significantly, variance of the number of air tankers allocated to air base J on fire-day condition K is low. $\overline{V}$ is the value of the left-hand side of equation 14 at the solution point. Where the constraint is active, then $\overline{V} = V^*$. On any day with fire condition index K the number, V, of air tankers working at base J is observed. V is not necessarily equal to $\bar{V}$, but:

$$\text{Var.}\{V\} = \sum_{I} \sum_{JH} \left[X(J \mid K{:}I,JH) - X^2(J \mid K{:}I,JH) \right] \quad (17)$$

is low, since $X(J \mid K{:}I,JH)$ is in most cases either zero or one.

Integer variables permit a variety of possible constraints to be included (Dantzig 1960). For example, if it is desired to have at least one air tanker with air base JH as its home base, the following constraint should be added:

$$\sum_{I} D(I,JH) \geqslant 1 \quad (18)$$

The addition of such constraints usually reduces computer turnaround.

The scope of applicability can be extended by redefining the variable and constant elements of the basic model.

In developing the basic model, we assumed that a first-order Markov process adequately describes the flow of information on which the transfer/use decision is made. If, in fact, a higher order Markov process is more appropriate, such respecification is possible within the context of the basic MILP model.

Related to this respecification is the possibility of more than one transfer/use decision point during the day. In the current model the transfer/use decision is made once daily in the early morning hours. Futher information may normally arrive during the day and serve as the basis for a second transfer/use decision point in the afternoon. Adding this second decision point is possible within the basic model design.

The output values related to air tanker assignment and use at different air bases as specified in the objective function may be variously defined and estimated. These estimates may be relatively straightforward or the result of more sophisticated simulation techniques. The air tanker output may be defined only for the initial attack period on fires or, if secondary air tanker tasks are considered relatively important, for the entire period of uncontrolled burning.

The simulation techniques discussed above might include a fire behavior model, allowances for air tanker downtime, maneuverability of individual air tanker types, and any other factors which significantly affect output.

APPLICATION OF THE MODEL

The model was used to analyze the air tanker system of District 1, California Division (now Department) of Forestry (CDF) (*table 1*). This district, headquartered in Santa Rosa, includes all of northern coastal California. Three air bases serve as home bases for air tankers, from which initial attack sorties may be made on fires: (a) Rohnerville, in Humboldt County; (b) Ukiah, in Mendocino County; and (c) Sonoma County, at Santa Rosa. The model uses a fourth air base to represent release from standby. Zero cost and output are associated with assignment to this fourth air base.

Five air tankers are available for contract within the district: two F7F's and three TBM's. These aircraft have different costs and outputs associated with their use. One of the F7F's may be assigned to either Ukiah or Sonoma County. The other F7F may be assigned only to Rohnerville; the first TMB may be assigned only to Ukiah; the last two TBM's can be assigned only to the Sonoma County air base.

The expected period of air tanker use is 107 days, from July 1 through October 15, when the air tankers contracted for the season may be transferred between bases for use on initial attack fires.

Historically, these air tanker initial attack (ATIA) fires have been strongly correlated with the brush burning index classes for CDF fire danger rating areas 120 and 175. The daily brush burning index class for each of these two areas might then be considered as one possible basis for allocating air tankers.

Associated with each air tanker is a fixed cost for the season (*table 1*). When an air tanker is contracted, this fixed cost guarantees availability of the aircraft through the nominal length of the fire season.

A per diem cost of $20 is charged for every night an air tanker spends away from its home base. Air tankers are assumed to spend the night at the base to which they are assigned for the day. Returning the aircraft to its home base after a work day at another base is not considered, although it could be considered by adding two more dummy bases to the model. This cost is added to the transfer cost between bases. Costs and output were derived by using the data and procedures described by Greulich and O'Regan (1975).

Once an air tanker arrives at the air base where it will spend the next 24 hours, it becomes available for initial attack on any fire within the jurisdiction of that base. The number of ATIA fires and consequent cost and output associated with air tanker use may be predicted on the basis of the fire-day condition.

The fire-day condition is based on the brush burning index from CDF fire danger rating areas 120 and 175. The five burning index classes run from 1 (low) to 5 (extreme). The fire-day condition is based on combinations of the two burning index classes from areas 120 and 175. Of 25 possible combinations 18 were observed and subsequently used to constitute the fire-day condition index. These two fire danger rating areas were selected as the basis for the fire-day condition index

Table 1—*Fixed cost for availability of air tankers through expected fire season length*

Air tanker	(I)	Home base (JH)		Fixed cost (dollars)
F7F	(1)	Rohnerville	(1)	16,576
F7F	(2)	Ukiah or Sonoma County	(2) or (3)	16,576
TBM	(3)	Ukiah	(2)	11,396
TBM	(4)	Sonoma County	(3)	11,396
TBM	(5)	Sonoma County	(3)	11,396

Table 2—*Probabilities calculated from 9 years of five-season data for predicting tomorrow's fire-day (burning) condition indexes, given today's indexes, for assignment of air tankers*[1]

Today's fire-day condition (burning) indexes	Corresponding brush burning index Classes in CDF FDR areas[2]: 120	175	Tomorrow's fire-day condition (burning) index: 1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
1	1	1	0.5000	0.4000					0.1000											
2	1	2	0.0843	0.3012	0.1807	0.0121			0.2048	0.1446	0.0361	0.0121		0.0241						
3	1	3	0.0370	0.1605	0.1729	0.0617			0.1358	0.2963	0.0864	0.0124		0.0370						
4	1	4		0.0238	0.2143	0.2857	0.0238		0.0714	0.1191	0.2143	0.0476								
5	1	5			0.1111	0.2222				0.2222	0.4445									
6	2	1							0.2000	0.4000				0.2000	0.2000					
7	2	2	0.0337	0.1461	0.0674	0.0112		0.0225	0.2809	0.2697	0.0674		0.0225	0.0562	0.0112		0.0112			
8	2	3	0.0115	0.0402	0.1092	0.0287	0.0057		0.1034	0.3046	0.1839	0.0230	0.0115	0.0805	0.0460	0.0345		0.0058	0.0115	
9	2	4	0.0086	0.0173	0.0948	0.0862	0.0172	0.0086	0.0345	0.2414	0.2328	0.1035		0.0517	0.0345	0.0603		0.0086		
10	2	5		0.0513	0.0769	0.1026	0.1282		0.0257	0.1282	0.2051	0.1795		0.0513	0.0256	0.0256				
11	3	2						0.1429	0.1428	0.2857			0.1429	0.1428			0.1429			
12	3	3		0.0755	0.0189				0.0566	0.2075	0.1321	0.0943	0.0377	0.2076	0.1321	0.0377				
13	3	4		0.0606		0.0606		0.0303	0.0606	0.1212	0.1819	0.0303		0.1212	0.1515	0.1212				0.0606
14	3	5		0.0370	0.0370					0.0370	0.2593	0.1852		0.0371	0.1482	0.2222			0.0370	
15	4	2												0.5000	0.5000					
16	4	3												0.5000		0.5000				
17	4	4							0.3333	0.3334		0.3333								
18	4	5												0.5000	0.5000					
	Associated steady state probabilities[3]		0.0400	0.1035	0.0999	0.0525	0.0113	0.0063	0.1131	0.2178	0.1451	0.0488	0.0088	0.0664	0.0414	0.0338	0.0025	0.0025	0.0038	0.0025

[1]Example of how values are applied: If today's fire-day condition is 8 (brush burning index classes: 2 in Area 120; 3 in Area 175), probability that tomorrow's fire-day condition will be the same is .3046, or about one chance in three; probability that it will be worse in both areas is .0920 (sum of probabilities in columns 13, 14, 17, and 18 for line 8).

[2]CDF = California Division (now Department) of Forestry; FDR = Fire Danger Rating (indexes range from 1, or low, to 5, or extreme).

[3]Probability that any randomly selected day during fire season will have a given fire-day condition (for example, probability of observing brush burning index class 4 in Area 120 is .0113 (sum of columns 15-18 in this line).
These values also represent the expected proportions of the total number of days in the fire season that the designated fire-day condition will be observed (for the previous example, slightly more than 1 percent of the days during a typical fire season will be characterized by a brush burning index of 4 in Area 120).

because they correlate well with fire starts and because together they give adequate coverage of CDF District 1. Area 120 is contained almost entirely within Humboldt County, covering a central band about 12 miles wide extending from east of Eureka to the Mendocino County line. Area 175 covers the lower half of Lake County and the northeast half of Napa County in a band 25 miles wide extending from above Clear Lake to below Lake Berryessa.

Nine years of fire season data (1961-1969) were used to estimate the transition matrix elements for the first-order Markov process from which the steady state values may then be calculated (table 2). Differences between the steady state entries of the table and the values appearing in table 5 in an earlier report (Greulich and O'Regan 1975) are due to additional years incorporated into the data base.

RESULTS

An increase in budget level is matched by a corresponding increase in aircraft output (*figure 1*). The curve AA' is the convex envelope of cost-output points associated with the annual decision to contract all available air tankers (five) and to assign the second F7F to the Sonoma County air base as home base. The curve BB' is the convex cost-output envelope associated with the annual decision to contract all air tankers except the Ukiah-based TBM, with the second F7F assigned to the Sonoma County air base. The existence of fixed costs and integer decision variables give rise to the pronounced "lumpiness" of the cost-output frontier (solid line). Little variation is noted in the average cost over the range of output considered here (*table 3*). The marginal cost does, however, increase quite rapidly at the higher levels of output. Significantly, this increase is slow until a large percentage of maximum possible output has been achieved for any given set of air tanker home base configurations. Transfer activity now appears to be more economical than indicated in a previous study (Greulich and O'Regan 1975).

The optimal decision structure under an annual budget of $152,000 was examined in more detail. Of this annual budget, $67,340 was used to pay associated fixed availability costs of the air tankers. All five air tankers were contracted and the second F7F was stationed at Sonoma County. The balance of $84,660 was expended on transfer flight time ($1355), per diem costs ($4215), and flight time on fires ($79,090).

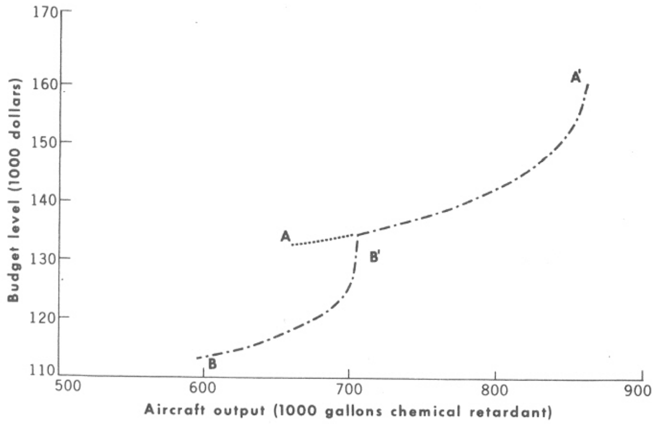


Figure 1—Cost-effective production frontier for California Division of Forestry District 1 air tankers and air bases used for initial attack.

Optimal transfer/use pattern (*table 4*) shows the air base to which each air tanker should be sent based on the fire day condition and its present air base location. The three TBM's spend every day at the Sonoma County air base regardless of the fire-day condition. The first F7F is at the Sonoma County air base under every fire day condition except for the first, when it is sent to Rohnerville. The second F7F is at Rohnerville 14 percent of the time when the first day condition is 1. For the balance of those days and for all other days of fire-day conditions other than 1, it is stationed at Sonoma County air base.

All of the air tankers, if possible, should have the Sonoma County air base as their home base. This result is a direct consequence of the particular cost-output structure used in the model. The procedure used to generate this data is open to review and modification (Greulich and O'Regan 1975). *Table 5* gives daily and seasonal costs and outputs, by airplane.

A fire season is essentially a series of days with random fire conditions. At the beginning of the season, a plan is devised that is optimal for an average season. For the season actually encountered, the plan will not necessarily meet budgetary and other constraints. Variances of expected payments could be computed to give some indication of the probabilities of failures to meet budgetary and other constraints. As an operational response to this essential randomness of the process, the model can be rerun at certain times, say halfway through and three-quarters of the way through the season. At each run, one would work with the remaining budget and the remaining season to devise new transfer use rules to come close to meeting budgetary and other constraints.

Table 3—*Cost-output relationships for nine different budget levels.*

Budget (dollars/year, 1000's)	Output (gal./year, 1000's)	Average cost per gal (dollars)	Marginal cost per gal (dollars)
161.37	857.391	0.188	3.288
152.00	846.411	.180	.649
147.00	825.510	.178	.187
142.00	788.836	.180	.101
137.00	739.108	.185	.101
130.00	703.711	.185	2.143
125.00	697.869	.179	.735
120.00	674.978	.178	.101
115.00	625.275	.184	.100

CONCLUSIONS

The results of this application of the model suggest that

- The basic model as described has the correct mathematical structure.
- The computer programs needed to generate the data structure, solve a large MILP, and interpret the solution can be written and executed.
- A first-order Markov process in at least one real-world situation seems to describe adequately the arrival of fire day condition information.

Dimensionality, though still a problem, is not insurmountable, if the analyst gives careful attention to problem specification, is prepared to use ancillary information, and is willing at times to sacrifice some detail.

The model allows a more general and realistic portrayal of the decision process within an air tanker program than has been heretofore possible and the important interrelationship between annual and daily decisions has been brought within its

Table 4—*Optimal decisions on assignment of air tankers, given a $152,000 budget*

Air tanker and seasonally assigned home base	Daily air base assignment, given fire-day condition index indicated
F7F—Rohnerville	In Sonoma County—under all fire-day conditions, except when index is one, in which case, air tanker transfered to Rohnerville.
F7F—Sonoma County	In Sonoma County—under all fire-day conditions, except when index is one. If index is one, decide whether to transfer air tanker to Rohnerville, and in such a way that it is expected to be sent there about 14 percent of the time, and kept in Sonoma County rest of the time. If index is one and air tanker is at Rohnerville, keep it there until index changes, at which time, transfer air tanker to Sonoma County.[1]
TBM—Ukiah	In Sonoma County—under all fire-day conditions (this air tanker had to use Ukiah as its home base; if that constraint lifted, Sonoma County would probably be selected as home base).
TBM, TBM—Sonoma County	In Sonoma County, for these two air tankers—under all fire-day conditions.

[1]Decision process is illustrated by use of a spin-wheel pointer that has 14 percent of the wheel circumference painted blue, and 86 percent painted red. If pointer stops on blue portion, send air tanker to Rohnerville; if it stops on red, tanker remains in Sonoma County.

Table 5—*Expected cost and output per day by air tanker under optimal seasonal configuration and daily transfer rules with $152,000 budget and 107-day season.*

Configuration		Expected cost per day[1]	Expected seasonal cost[1]	Expected output per day	Expected seasonal output
Air tanker	Home base				
		Dollars	*Dollars*	*Gallons*	*Gallons*
F7F	Rohnerville	190.63	20,397	1846.00	197,522
F7F	Sonoma County	156.77	16,775	1831.13	195,931
TBM	Ukiah	161.27	17,256	1411.08	150,986
TBM	Sonoma County	141.27	15,116	1411.08	150,986
TBM	Sonoma County	141.27	15,116	1411.08	150,986

[1]No fixed cost included.

structure. Daily transfer/use decisions are based on fire day condition information described as a Markov process. The system state includes not only fire day condition information but also the current location of air tankers.

Although the current model gives a better representation than was previously possible, it still has many limitations. Static modeling fails to recognize initial conditions or to allow possible changes in the decision rules during the season. Nonlinearity, which may be especially important in the daily output function, has not been recognized. Despite such limitations, the model possesses a good measure of realism and elegance in its structure and has much to offer the air tanker manager.

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Greulich, Francis E.; O'Regan, William G. **Allocation model for air tanker initial attack in firefighting.** Res. Note PSW-301. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 1975. 8 p.

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GLOSSARY

A(I,JH,J1,J2,K2)	Expect payment for flight time during one day
B	Total annual budget for air tanker operations
B(I,JH)	Guaranteed annual minimum payment for flight time
C(I,JH,J1,J2,K2)	Expect payment for the air tanker through one day
D(JH)	Integer variable: 1 if used as a home base; 0 otherwise
D(I,JH)	Integer variable: 1 if the air tanker uses the base as home base; 0 otherwise
F(JH)	Home base set-up cost
F(I,JH)	Guaranteed, fixed, seasonal payment for air tanker availability
g(K2)	Probability that a randomly selected day has the indicated fire day condition
g(K2 | K1)	Conditional probability of the current fire day condition given the fire day condition of the previous day
I	Index for the air tankers in the system. I = 1 . . . IMAX
J	Index number for a particular air base in the system
J1	Air base at which an air tanker is stationed on the morning when a transfer/use decision is to be made
J2	Air base to which an air tanker is transferred following a transfer/use decision
JH	Home base number (JH always occurs with I as (I,JH). For example (1,3) indicates that aircraft 1 has base 3 as its home base)
K	Fire day condition for a particular day
K1	Fire day condition on the day preceding the transfer/use decision
K2	Fire day condition on the day of the transfer/use decision
Q(I,J2,K2)	Expected output of air tanker I transferred to J2 on a day of type K2
r	A fraction, less than one
S	Expected number of days in the fire season
V	Number of air tankers assigned to a particular air base on any given day
$\bar{V}$	Expected number of air tankers assigned to a particular base on any given day
V*	Limit on the number of air tankers assigned to a particular base on any given day
W	Total budget available to cover home base set-up costs
X(J1,J2,K2;I,JH)	Joint probability that air tanker I, which has home base JH, will be at air base J1 on same randomly selected day, that fire condition K2 is observed early the next day and air tanker I is sent to base J2
X(J2 | J1,K2;I,JH)	Conditional probability; transfer rules for the air tanker I
X(K2;I,JH)	Fire day condition probability; equivalent to g(K2)
Z	Expected total daily output of the air tanker system

Greulich, Francis E.; O'Regan, William G. **Optimum use of air tankers in initial attack: selection, basing, and transfer rules.** Res. Paper PSW-163. Berkeley, CA: Pacific Southwest Forest and Range Experiment Station, Forest Service, U.S. Department of Agriculture; 1982. 8 p.

Fire managers face two interrelated problems in deciding the most efficient use of air tankers: where best to base them, and how best to reallocate them each day in anticipation of fire occurrence. A computerized model based on a mixed integer linear program can help in assigning air tankers throughout the fire season. The model was tested using information from California Division (now Department) of Forestry District 1, which in 1967 maintained a fleet of five aircraft and three air bases. The results confirmed the soundness of the model's mathematical structure and demonstrated that a computer program can be written to interpret the solution to this resource allocation problem.

Retrieval Terms: air tanker model, initial attack, resource allocation