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P.O. BOX 245, BERKELEY, CALIFORNIA 94701

ALLOCATION MODEL FOR AIR TANKER INITIAL ATTACK IN FIREFIGHTING

Francis E. Greulich William G. O'Regan

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Timely and appropriate use of air tankers in firefighting can bring high returns, but their misuse can be expensive when measured in operating and other costs. An allocation model has been developed for identifying superior strategies for air tanker initial attack, and for choosing an optimum set of allocations among airbases. Data are presented for a representative initial attack allocation area and strategies are computed for alternative budget levels. Although designed for determining day-to-day operating rules, the model is flexible enough to be applied to policy questions concerning number and type of aircraft and location of airports.

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In the modern firefighting arsenal, few suppression techniques are as effective in initial attack or as expensive in operation as the air tanker.[1] The return on the appropriate and timely use of the air tanker is high. Correspondingly, the penalty for its mismanagement is severe when measured in operating and opportunity costs.

A significant condition that prevents the best possible expected use of the air tanker is the stochastic nature of fire occurrence. This condition, combined with the restrictions of an annual operating budget, makes it difficult to deploy aircraft among bases on a daily basis in a way that maximizes their firefighting value over the season.

The derivation of operating rules by applying operations research techniques is therefore likely to make the system more effective. Even when such rules are not considered to be directly applicable, it is often possible to gain insight into the possible implications of major structural changes in the system by comparing results obtained from a model, applied with and without the change assumption.

This note describes an allocation model which first identifies superior strategies for air tanker initial attack, and then selects, for a given budget level, the optimum allocation of aircraft among airbases. The model was tested by applying it to a field situation. Results are reported, limitations of the model are considered, and areas for potential modification are examined.

The allocation model seeks to use available information to a fuller extent in planning air tanker operations in firefighting than is now practiced. Designed primarily to determine day-to-day operating rules, the model is flexible enough for application to many important policy issues as well.

AIR TANKER ALLOCATION MODEL

Only the primary role of the air tanker—dropping chemical retardants on or in advance of a fire during its early stages—is considered in this analysis. An initial attack sortie is defined here as the act of dropping a full retardant load in the vicinity of a fire that is in the air tanker initial attack (ATIA) category. This category is defined on the basis of ground-force travel time requirements peculiar to each protection area. The air tanker initial attack period is defined here to be 1 hour, beginning with the dispatch of the first air tanker.

The number of initial attack sorties that aircraft i (where i runs from 1 through I) with flight speed s_i (miles/hr) and ground service time t_i (hr/sortie) could make on a fire burning at a distance d (miles) from an airbase is denoted $Q_i(d)$ (sorties/fire) and is calculated as follows:

$$T_i = (2d/s_i) + t_i \tag{1}$$

in which T_i (hr/sortie) is the aircraft cycle time. Then

$$q_1' = 1/T_i \tag{2}$$
$$q_1 = \text{INTEGER PART } \{q_1'\} \tag{3}$$
$$q_2' = (s_i/d)(1 - q_1 T_i) \tag{4}$$
$$q_2 = \begin{cases} 1 : \text{if } q_2' \geq 1 \\ 0 : \text{otherwise} \end{cases} \tag{5}$$

Finally, then

$$Q_i(d) = q_1 + q_2 \tag{6}$$

The cost of making these sorties, $C_i(d)$ (dollars/fire)—when R_i (dollars/hr) is the contracted flight time pay rate—is calculated as

$$C_i(d) = Q_i(d) \cdot R_i \cdot (2d/s_i) \tag{7}$$

In this particular formulation, we do not consider retardant and ground service costs.

Equally spaced concentric circles are inscribed around each airbase j (where j runs from 1 through J) within the mutually exclusive zones of influence, out to but not beyond the planned initial attack zone limit *(fig. 1)*. The number of such concentric bands around airbase j is designated as N_j. The distance between perimeters is denoted as Δ. The distance to the center of the nth band is denoted as d_n:

$$d_n = (n - ½)\Delta \tag{8}$$

Let u_{jn} be the number of ATIA fires occurring in band n, airport j, during some time period. Then

$$f_{jn} = \frac{u_{jn}}{\sum_{n=1}^{N_j} u_{jn}} \tag{9}$$

in which f_{jn} is the relative frequency with which a fire occurring in the jurisdiction of airbase j is in band n. All fires occurring in band n are assumed to be at distance d_n from the airbase.

The expected output from aircraft i with retardant capacity G_i (gal/sortie) stationed at airbase j—given that a randomly located fire qualifying for air tanker

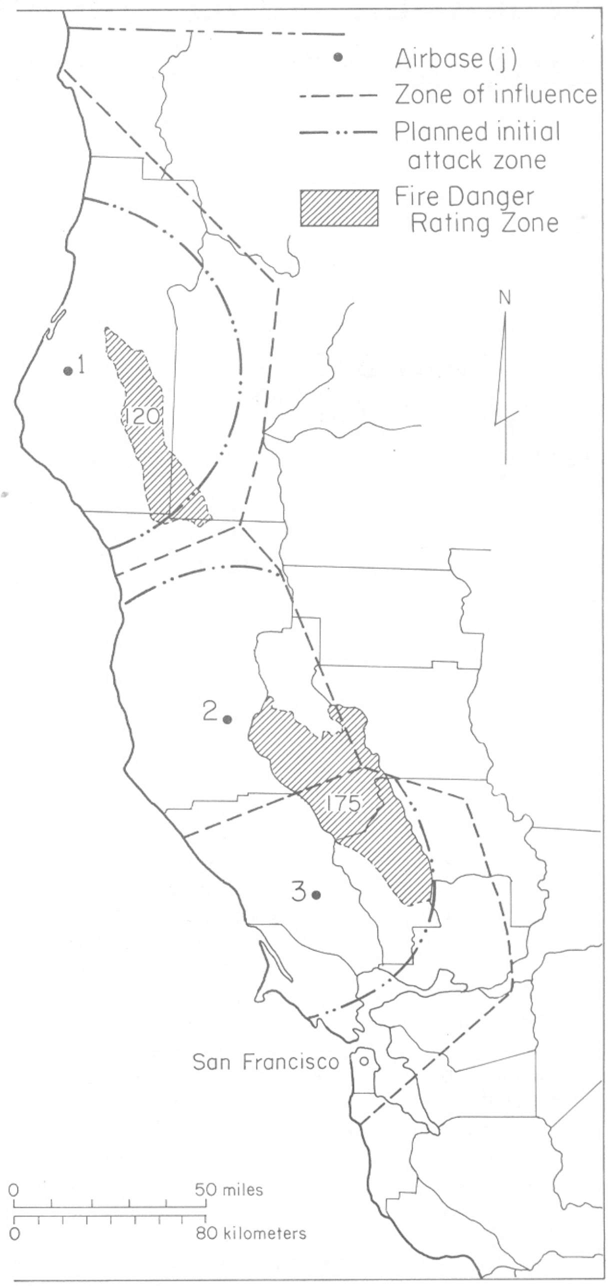

Figure 1—District 1 of the California Division of Forestry has three airbases: (1) Rohnerville, in Humboldt County; (2) Ukiah, in Mendocino County; and (3) Sonoma County, at Santa Rosa.

initial attack occurs—is Q_{ij} (gal/fire):

$$Q_{ij} = \sum_{n=1}^{N_j} f_{jn} \cdot Q_i(d_n) \cdot G_i \qquad (10)$$

The expected cost C_{ij} (dollars/fire) is

$$C_{ij} = \sum_{n=1}^{N_j} f_{jn} \cdot C_i(d_n) \qquad (11)$$

For each airbase a fire danger rating area is selected. The daily burning index class for each of these areas will be used to predict fire frequency within the related airbase zone of influence. These fire danger rating areas are selected on the basis of their value in predicting fire occurrence.

Define $h_j\,(m|k)$ as the probability at airbase j that m fires will occur on a day with burning index class k.

Let the ranges of m and k be given by equations (12) and (13):

$$0 \leq m \leq M, \text{ integer} \qquad (12)$$
$$1 \leq k \leq 5, \text{ integer} \qquad (13)$$

in which M is the largest number of fires historically observed on any day at any airbase, and 5 is the largest integer in the burning index class system.

Using historical data, evaluate $h_j\,(m|k)$ for all airbases, all m and all k.

The expected number of ATIA fires during 1 day for airbase j, given burning index class k, is determined as

$$\overline{m}_j(k) = \sum_{m=0}^{M} m \cdot h_j(m|k) \qquad (14)$$

The expected output and cost per day for air tanker i stationed at airbase j, when the burning index class is k, are respectively

$$Q_{ijk} = Q_{ij} \cdot \overline{m}_j(k) \qquad (15)$$

and

$$C_{ijk} = C_{ij} \cdot \overline{m}_j(k) + 2P_{ij} \qquad (16)$$

in which P_{ij} (dollars/day) is the one-way cost of transferring aircraft i from its home base to airbase j.

The empirical probability of all possible combinations of burning index classes is obtained from historical data for each airbase. For example:

$$g(1,1,\ldots 1) = \frac{\text{Number of days with ratings } (1,1,\ldots 1)}{\text{Total number of days}} \qquad (17)$$

In general,

$$g(K) = \frac{\text{Number of days with combination K of ratings}}{\text{Total number of days}} \qquad (18)$$

K = a vector that takes on all possible combinations of ratings, $(k_1, k_2, \ldots k_J)$. (19)

Unless the possibility of simultaneously occurring initial attack fires at an airbase is considered, a bias will exist in the estimates of Q_{ijk} and C_{ijk}. The severity of this bias is directly related to the distribution $h_j\,(m|k)$. The more ATIA fires that occur during a typical 11-hour working day, the more probable it will be that at least during some of the time, fire initial attacks will overlap. If such fires do overlap, then this simplified procedure for calculating expected cost and output leads to an overestimation of the actual values.

In studying 3 years of data for three airbases used to test the model, we found—at least in these particular cases—that the overlap in fires was not sufficient to affect the results significantly. The maximum number of observed initial attack fires on any day never exceeded six, and only rarely was it more than three.

A transfer rule (L) assigns each aircraft to an airbase. The transfer rule selected on a given day is decided on the basis of the observed values of the k_j's for that day. Where k_j represents the burning index at airbase j, then $K = (k_1, k_2, \ldots k_J)$. Upon completion of the day's activity, all aircraft that are out return to their home bases. For each K there are J^I possible transfer rules. A decision variable X_{ij} (K) is defined for each K, all i and j such that X_{ij} (K) is 1 if aircraft i is sent to airbase j and is equal to zero otherwise. Then a transfer rule may be written:

$$L(p,K) = \begin{bmatrix} X_{11}(K) & X_{12}(K) & X_{13}(K) & \ldots & X_{1J}(K) \\ X_{21}(K) & X_{22}(K) & X_{23}(K) & \ldots & \\ \vdots & & & & \\ X_{I1}(K) & \ldots & & & X_{IJ}(K) \end{bmatrix} \qquad (20)$$

for each p, K; in which p is an index running from 1 to J^I. The matrix element for a particular transfer rule is denoted $L_{ij}(p,K)$.

The decision on the best strategy for K' days, given an allocation of dollars to those days so identified, is independent of the best strategy for K'' days. Therefore, taking each K, one at a time, all feasible transfer rules $L(p,K)$, p=1, J^I, are examined, and only the dominant transfer rules $L^*(p,K)$, p=1, p_K are retained. Each feasible transfer rule has an expected season's cost, C(p,K), and an expected season's output, Q(p,K) if selected and followed:

$$Q(p,K) = S \cdot g(K) \sum_j \sum_i Q_{ijk} \cdot L_{ij}(p,K) \qquad (21)$$

and

$$C(p,K) = S \cdot g(K) \sum_j \sum_i C_{ijk} \cdot L_{ij}(p,K) \qquad (22)$$

in which S is the total number of days in the annual air tanker use season. It is from among all of these possible cost/output points $p = 1, J^I$ that the dominant rules must be selected for each K.

The set of dominant strategies in this formulation is identified by (a) first ordering all strategies on day K by increasing cost; (b) starting with the lowest-cost strategy and making sequential comparisons of output until a strategy is found with a higher output; and (c) using this higher cost-output strategy for further comparison in repetition of step (b). This procedure is followed until the set of all possible strategies has been exhausted. Those strategies selected for comparison represent the set of dominant strategies. The dimensions of the problem have been reduced significantly by following this procedure. A further reduction could be achieved by selecting strategies that are represented by extreme points of the convex envelope formed by line segments connecting any two points in the original set of dominant strategies *(fig. 2)*.

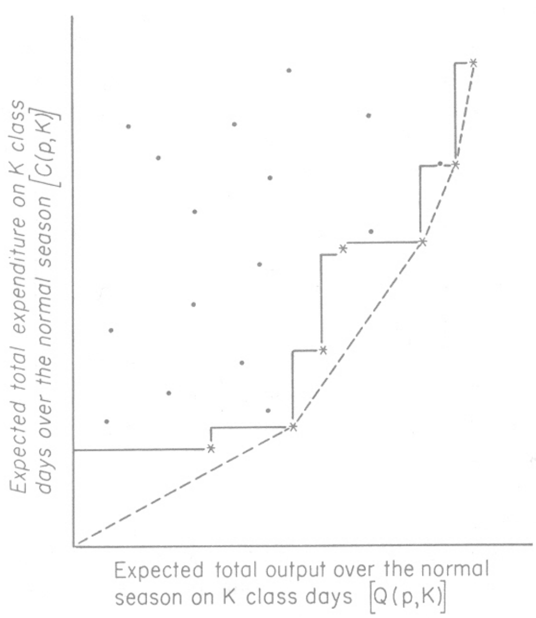

Figure 2—Strategies for a K class day: points indicate the finite set of simple feasible strategies; asterisks indicate the finite set of dominant strategies; and the dashed line is the locus of dominant compound strategies.

Finally a decision variable Y(p,K) is defined to be the fraction of days with fire index class vector K that transfer rule L*(p,K) is to be used with associated seasonal expected cost C*(p,K) and output Q*(p,K).

The problem is then placed into a linear programming format:

$$\text{MAX } Z = \sum_K \sum_p Q^*(p,k) \cdot Y(p,K) \qquad (23)$$

subject to

$$\sum_K \sum_p C^*(p,K) \cdot Y(p,K) \leq B \qquad (24)$$

$$\sum_p Y(p,K) = 1 \qquad \text{all } K \qquad (25)$$

$$Y(p,K) \geq 0 \qquad \text{all } K, p=1, p_K \qquad (26)$$

in which B is the budgeted money for flight time for air tanker initial attack.

APPLICATION OF MODEL

The model was applied to data from District 1 of the California Division of Forestry (CDF), headquartered at Santa Rosa. District 1 covers all of northern coastal California. Within the District are three airbases from which air tankers operate when making initial attack sorties on fires: (1) Rohnerville, Humboldt County (Fortuna); (2) Ukiah, Mendocino County; and (3) Sonoma County Airport (Santa Rosa). Within the model there is provided a fourth airbase with zero cost and output; its use represents the release of aircraft from standby status.

The District is accordingly partitioned into airbase zones of influence and planned initial attack zones *(fig. 1)*. Fires that occur on CDF-protected lands during the season when air tankers can be used, and are more than 15 minutes' travel time from the nearest ground station, are automatically subject to initial attack by aircraft stationed at that airbase, flight conditions permitting.

During the 1967 fire season, five aircraft were contracted for District 1: an F7F at Rohnerville; an F7F and a TBM at Ukiah, and two TBM's at the Sonoma County Airbase. The aircraft varied in flight speed, retardant-carrying capacity, and service time *(table 1)*.

The transfer costs *(table 2)* of those five aircraft were calculated on the basis of these flight time pay rates: \$316.80 for the F7F at Rohnerville; \$306.90 for the F7F at Ukiah, and, \$222.75 for each of the TBMs.

Two fire danger rating areas of the CDF, 175 and 120 *(fig. 1)*, were identified as having brush burning

Table 1—*Flight speed, chemical retardant-carrying capacity, and service time of contracted aircraft, California Division of Forestry*[1]

Aircraft(i)	Flight speed (S_i)	Retardant capacity (G_i)	Service time (t_i)
	Mph	*Gal*	*Hr*
F7F (i-1,2)	200	800	0.25
TBM (i-3,4,5)	160	600	.20

[1]Sources: Adams, Darius. 1966. Instructions for construction of the dispatch guide. Calif. Div. For., Sacramento. Reinecker, H. P. and C. B. Phillips. 1960. Fighting forest fires with air tankers, 1958-59. 149 p., illus. Calif. Div. For., Sacramento.

Table 2—*One-way transfer costs for each aircraft from its home base to any other airbase in system*

	Transfer cost, aircraft (i)				
Airbase(j)	F7F (1)	F7F (2)	TBM (3)	TBM (4)	TBM (5)
			Dollars		
Rohnerville(1)	0.00	171.87	155.93	224.98	224.98
Ukiah(2)	177.41	.00	.00	72.80	72.80
Santa Rosa(3)	255.98	78.57	72.80	.00	.00

indices strongly correlated with the historical fire loads of the affected zones.

All fires occurring between June 1 and December 31 over a 3-year period (1963-1965) were sorted to separate out those fires that fell within the category of ATIA fires. These fires were then correlated with both burning index classes and air distance from the airbase *(tables 3, 4)*.

Five years of data (1961-65) between June 1 and October 15 were used to generate the joint distribution of burning index classes *(table 5)*.

Using these data, we tested the ATIA model at six different budget levels. The data were processed by the linear programming code M3-LP on an IBM 7090-94 computer. The cost-output curve generated shows the results *(fig. 3)*. The output of one typical computer run is given and corresponds to a budget level of $118,000 *(table 6)*.

An examination of the optimal transfer pattern under a budget of $118,000 shows these characteristics. There are days—representing about 30 percent of the total season—when no transfer activity is indicated. These days are those identified by the brush burning index class vector (k_1,k_2,k_3) as (1,1,1), (1,2,2), (2,2,2), (3,2,2) or (4,3,3). At the other extreme all of the aircraft are transferred to airbase 3 (Santa Rosa) on 28 percent of the days. These days are characterized by a burning index class of 4 or 5 in fire danger rating area 175. Airbase 4, which is a dummy with zero cost and output, is never used. At this

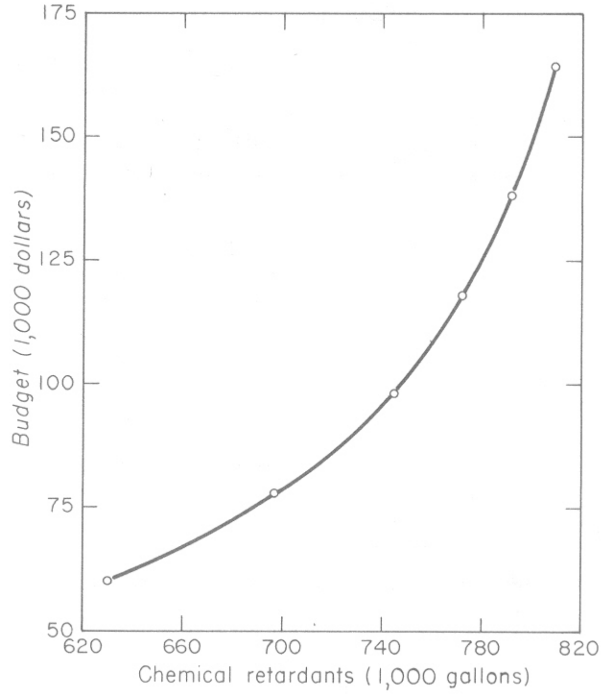

Figure 3—Maximum expected air tanker output as a function of level of budget expenditure, ranging from no aircraft transfers to maximum aircraft transfers.

Table 3—*Expected number of air tanker initial attack fires occurring between 0800 hours and 1900 hours, based on the burning index class for each airbase*[1]

Burning index (k_j)	Expected number of fires, airbase (j)		
	Rohnerville (1)	Ukiah (2)	Sonoma County (3)
1	0.3058	0.0364	0.0424
2	.4595	.4773	.5455
3	.7209	.7303	.8652
4	1.0000	.8286	1.3857
5	(2)	1.3182	1.7273

[1]For airbase 1 the burning index class is from fire danger area 120 of the California Division of Forestry. For airbases 2 and 3 it is fire danger rating area 175 *(fig. 1)*.

[2]None during years studied.

Table 4—*The relative frequencies of the fire distances for each of the three airbases in District 1, California Division of Forestry.*

Concentric band number (n)	Air distance [1] (d_n)	Relative frequency of fire distance, airbase (j)		
		Rohnerville(1)	Ukiah(2)	Sonoma Co.(3)
	Miles			
1	2.5	0.0040	0.0432	0.0212
2	7.5	.1107	.0719	.0500
3	12.5	.0791	.0863	.1385
4	17.5	.1502	.1823	.1385
5	22.5	.1739	.1990	.1769
6	27.5	.0791	.1631	.1692
7	32.5	.1225	.1559	.1750
8	37.5	.0870	.0791	.1308
9	42.5	.0870	.0192	–
10	47.5	.0870	–	–
11	52.5	.0198	–	–

[1] Measured from airbases to midpoint of the concentric band.

Table 5—*Joint frequency distribution of brush burning index classes from two fire danger rating areas, California Division of Forestry* [1]

Brush burning index class, area 120	Relative frequency of brush burning index class, area 175 (k_2 and k_3)				
(k_1)	1	2	3	4	5
1	0.0467	0.1150	0.1133	0.0417	0.0083
2	.0033	.1317	.2367	.1383	.0317
3	.0000	.0083	.0650	.0283	.0233
4	.0000	.0017	.0017	.0033	.0017
5	.0000	.0000	.0000	.0000	.0000

[1] The correlation between k_2 and k_3 is 1 because they come from the same fire danger rating area.

budget level, there are no conditions under which aircraft will be released from standby status.

The two TBMs at Santa Rosa are never transferred. The F7F and TBM at Ukiah, however are sent occasionally to either Rohnerville or Santa Rosa and the F7F is the more frequently moved of the two aircraft. On the basis of the transfer patterns, the F7F is apparently the more cost-effective of the two aircraft types.

Over the entire range of transfer activity, the marginal and average costs per gallon of delivered retardant increase *(fig. 3).* At the point designated "no transfer activity" the marginal cost is about equal to the average cost at 9 cents per gallon. At a budget level of $118,000, the average cost is 15 cents while the marginal cost has climbed to 87 cents. At the point of maximum expected output, the average cost is 21 cents per gallon, while the marginal cost is a very high $3.64 per gallon.

The precipitous increase in marginal costs would seem to suggest the difficulty of extracting significant increases in air tanker output through a transfer system based on the expected fire load indicators used here. It is likely that an analysis by the decisionmaker equating cost product ratios of this fire control input and alternatives would result in a low level of transfer activity.

LIMITATIONS OF MODEL

Admittedly the model developed and applied in this report has greatly simplified the realities of an air tanker system. On some possible points of contention this model can be readily modified. On others, however, modification is difficult. The only recourse open to the decisionmaker is to keep in mind specific model limitations when interpreting the output.

Within the realm of easily attainable modifications are the following:

Table 6—*Transfer patterns for a budget of $118,000, indicating for each observed vector of burning index classes the destination airbase*[1] *for each aircraft, for District 1, California Division of Forestry*

Vector of burning index classes (k_1,k_2,k_3)	Pattern: F7F (1) from 1 to	F7F (2) from 2 to	TBM (3) from 2 to	TBM (4) from 3 to	TBM (5) from 3 to	Percent of days designated transfer pattern is used (100 • Y(p,K))
(1,1,1)	1	2	2	3	3	100
(1,2,2)	1	2	2	3	3	100
(1,3,3)	2	3	2	3	3	16
(1,3,3)	3	3	2	3	3	84
(1,4,4)	3	3	3	3	3	100
(1,5,5)	3	3	3	3	3	100
(2,1,1)	1	1	1	3	3	100
(2,2,2)	1	2	2	3	3	100
(2,3,3)	2	3	2	3	3	100
(2,4,4)	3	3	3	3	3	100
(2,5,5)	3	3	3	3	3	100
(3,2,2)	1	2	2	3	3	100
(3,3,3)	1	3	2	3	3	100
(3,4,4)	3	3	3	3	3	100
(3,5,5)	3	3	3	3	3	100
(4,2,2)	1	1	1	3	3	100
(4,3,3)	1	2	2	3	3	100
(4,4,4)	3	3	3	3	3	100
(4,5,5)	3	3	3	3	3	100

[1]1=Rohnerville; 2=Ukiah; 3=Sonoma Co.

1. The addition of a down-for-maintenance factor for each aircraft.

2. The automatic release of aircraft from standby whenever the burning index falls below a specified level.

3. Within-season updating of transfer rules to conform to current system status.

4. The disallowance of certain transfer patterns.

5. The adjustment of retardant gallonage figures to reflect the relative effectiveness of each aircraft type in each area.[2]

6. The use of probability models to extrapolate occurrence probabilities of burning index classes so that transfer rules are obtained for all possible burning index class combinations—and not just those observed in the available historical data.

7. The collection and incorporation of more data of the type used here, or the incorporation of better fire load indicators.

Major drawbacks to the model that are not readily amenable to elimination or significant amelioration are:

1. Lack of recognition of air tanker demand beyond initial attack as it has been defined here.

2. The initial attack demand is only an approximation based on historical information about fire starts.

3. Lack of recognition of aircraft queuing delays.

4. Expansion of the model to include more airbases becomes progressively more difficult because of the increasing dimensional requirements.

5. No overnight holdovers of an aircraft at bases other than its home base are allowed.

CONCLUSIONS

The single most important benefit of a model of this type is the access by the decisionmaker to a formally stated, logically developed representation of the air tanker initial attack system. Some decisions formerly made on the basis of ill-defined verbal models could now be based on a model with stated assumptions giving quantitative responses to "what if" type questions. Such a model significantly improves the informational quality of one of the many inputs available to the decisionmaker.

A potentially valuable use of the model would be an evaluation of alternative aircraft types, their numbers, and their spatial distributions. For example, the transfer patterns observed in the study would seem to suggest that it may be more efficient to base the Ukiah F7F at Santa Rosa. A second run of the model using this spatial distribution of aircraft with revised cost figures would quickly show the relative worth of this change.

NOTES

[1]Anon. 1965. *An evaluation of air tanker operations in north coastal California during 1964.* Fire Control Exp. 11, 21 p., illus. Calif. Div. For., Sacramento. Pierce, Monte K. *Organization and management of air tanker operation.* (Paper presented at the Nat. Forest Fire Seminar on Aircraft Management, Oct. 26, 1970.)

[2]Maloney, James. 1973. A summary of the development and application of an optimizing air tanker allocation model. Can. For. Serv. Pub. 1324, 29 p.

The Authors

FRANCIS E. GREULICH has been a biometrician with the Station's biometrics unit, at Berkeley, since 1974. He was graduated from Oregon State University (B.S., in forestry, 1965; B.S., in mathematics, 1965). **WILLIAM G. O'REGAN** is Station biometrician. He is also a lecturer in the College of Natural Resources, University of California, Berkeley, where he earned B.S. (1949) and Ph.D. degrees (1962) in agricultural economics. He has been a member of the Station staff since 1957.

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