Optimal Economic Selection of Road Design Standards for Timber Harvesting Operations—A Corrected Analytical Model

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ABSTRACT. A theoretical development is given that corrects a long-accepted analytical model used in road system design. The model assumes that a forested area with uniform conditions of road construction is to be accessed by truck road for timber harvest. A discrete number of specified road design standards are available. Each design standard has known costs per unit length associated with road construction and use. The resulting total cost for any segment of the road when constructed to a given standard and used for timber haul depends fundamentally on the volume of timber transported over that segment. Transported volumes may vary along the length of the road, however, thus changing the associated costs. The emended economic decision rules for specifying road standards along the entire length of the road even as haul volumes change are developed. Examples are given. FOR. SCI. 43(4):589–594.

Additional Key Words: Forest engineering, logging engineering, road networks, economic optimization.

SINCE ITS PUBLICATION IN 1942, Professor Donald Matthews' textbook, *Cost Control in the Logging Industry*, has been a valued source of information to forest engineers. Researchers and practitioners alike continue to refer to its pages for ideas and direction. Recent research on road network design led to a review of the service standard determination of roads as developed and presented in Matthews' text. Serious flaws were encountered in his analytical development of a model applicable to current research in road network design. It is the purpose of this technical note to correctly develop this particular analytical model and show some applications of key results.

Road Design Standard Selection

In a 1901 presentation to the Western Society of Engineers, Bernard Fernow accentuated the decisive role of economics in the selection of logging road standards. The relative costs of construction and hauling were to be weighed in the selection of the road standard. The size of the tract (harvest volume) was a determining factor in this analysis. Lower standard, temporary roads were to be used as feeders to permanent roads of higher quality. Fernow draws on "financial calculations under German conditions"—an indication of early interest by European forest engineers in the question of selecting road standards based on economic criteria. In fact, Fernow could have drawn on an extensive body of rigorous economic theory developed by his contemporaries in European railroad engineering. A turn-of-the-century publication by Wilhelm Launhardt (1900-1902) gives some indication of the range and depth of this early work in the economic analysis and design of transportation systems. Several early English language forest engineering textbooks (Gayer 1908, Schenck 1912, Bryant 1913) also mention different road standards and factors relevant to their selection. Nowhere, however, in this early forest engineering literature is a formal economic model presented to provide guidance to the selection of the appropriate standard.

A more recent European perspective with regard to a number of harvesting layout design issues including road standard selection via an analytical model is given in a paper by Klemencic (1965). As with Fernow and Launhardt, Klemencic recognizes the time value of money and makes use of present value calculations. He separates one-time construction costs from those of maintenance and hauling which may continue over a number of years. Klemencic gives an equation for determining the economic transition point from a road of low standard to one of higher standard. Unfortunately, this formula is of only limited utility in the assessment of transportation networks where major central-

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ized landings are employed or more than two discrete road standards are available. A more general problem description is required under these conditions. For this purpose the appropriate description is provided by Matthews in his chapter on the economic service standard for roads.

Model Development

In order to unambiguously describe and quantitatively analyze the economic optimization problem the following variables are defined:

- j an index on road class from lowest standard to highest ($j = 1$: lowest, 2, . . ., J: highest); it is used as a variable subscript.
- r_j cost per unit length of a road built to standard j: dollars per kilometer (dollars per station[1]). Included here are all costs that are invariant with respect to the volume of timber hauled over a road segment but which are however directly proportional to the length of the segment; e.g., engineering, construction, and decommissioning costs.
- h_j cost per unit volume-distance over a road of standard j: dollars per m^3-kilometer (dollars per Mbf-station). Included here are all costs that are directly proportional to the volume of timber as well as to the distance hauled; e.g., hauling and road maintenance costs.
- v tributary volume of timber per unit length of road: m^3 per kilometer (Mbf per station).
- V_p total volume of timber which will be harvested above an arbitrary point p on the road and which will be hauled past that point: m^3 (Mbf).
- S length of connecting road between two designated points: kilometers (stations).
- Z_p total cost associated with the road system above an arbitrary point p: dollars.

Referring to Figure 1, a point ($P = 0$) is selected anywhere along a length of proposed haul road. A second point ($P = S$) a distance S down the road toward the mill from the first point is also selected. Then, under the assumption that a tributary timber volume, v, will be arriving with uniform intensity along the length of the segment, the total volume passing point S can be written as

$$V_s = V_0 + vS \qquad (1)$$

With continuing reference to Figure 1, the proximate, very small segment, ΔS, of road is now examined with respect to its associated costs. The contribution to total cost associated with this small finite segment may be written as

$$\Delta Z = [(V_s)(h\Delta S)] + \left[(v\Delta S\left(h\frac{\Delta S}{2} \right) \right] + [r\Delta S] \qquad (2)$$

The three sets of bracketed terms on the right hand side of Equation (2) are justified as follows. The first set yields the volume-distance related cost for all the timber volume above the point S that will pass over the small segment ΔS. The second gives the volume-distance related cost for the tributary timber volume accumulated over this same small segment. Note that, on average, this latter tributary volume must travel half the segment length. The third set of terms provides the distance-related cost associated with the small segment.

The terms of Equation (2) may then be rearranged to form the finite difference ratio

$$\frac{\Delta Z}{\Delta S} = V_s h + \frac{vh\Delta S}{2} + r \qquad (3)$$

and the limit taken

$$\lim_{\Delta S \to 0} \frac{\Delta Z}{\Delta S} = \frac{dZ}{dS} = V_s h + r \qquad (4)$$

Using Equation (1), substitute into Equation (4) and rearrange to obtain

$$dZ = (hV_0 + hvS + r)dS \qquad (5)$$

This variables separable form is integrated to obtain

$$Z = hV_0 S + \frac{1}{2} hvS^2 + rS + C \qquad (6)$$

where C is the constant of integration. Defining Z to be Z_0 at $P = 0$ yields

$$Z_s = hV_0 S + \frac{1}{2} hvS^2 + rS + Z_0 \qquad (7)$$

Having derived these general relationships, some corollary results may be immediately obtained.

If the tributary volume, v, along the road segment is 0, then minimizing the increase in total cost, $Z_S - Z_0$ [from Equation (7)] over the segment of length S only requires selection of the standard j from those available that yields the smallest value of $V_0 h_j + r_j$.

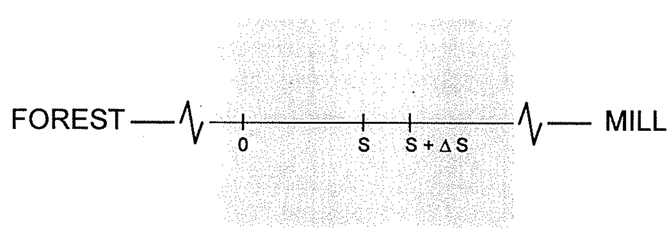

Figure 1. Haul road segment with a constant tributary volume.

[1] A horizontal distance of 100 ft (30.5 m).

[2] Tabulated factors for log scale conversion from board foot measure to cubic meters are given by Hartman et al. (1976).

As an approximation for small values of S all terms in Equation (7) other than those that are constant or linear in S may be ignored, yielding

$$Z_s \approx hV_0S + rS + Z_0 \tag{8}$$

Recall now that there was no restriction placed on the selection of the initial point ($P = 0$) except for the uniform tributary timber volume along the length of the following road segment. In Equation (8) the length of this segment has been made arbitrarily small effectively eliminating tributary volume considerations. Hence it may be concluded that at any arbitrarily selected point ($P = 0$) in the road system the construction standard j that yields the smallest value of $V_0h_j + r_j$ for that point should be applied. The optimal (economic) cost per unit length of road at that point is therefore determined as

$$\underset{j}{MIN}\{V_0h_j + r_j\} \tag{9}$$

From Equation (7) it is also seen that if the tributary volume, v, is greater than 0 then the total cost, Z_S, increases as the square of the distance, S. This nonlinear increase in cost is entirely due to the tributary volume of timber. Assume that at some arbitrary initial point ($P = 0$) the total cost is minimized by constructing to a standard j. This cost minimizing standard will be used for a distance, S, to be determined, at which point construction to the next higher standard $j + 1$ is justified by the increased haul volume. This switch to a higher standard will be made when the marginal cost [dZ/dS of Equation (5)] of using standard j just equals the marginal cost if a switch is made to standard $j + 1$; viz.

$$h_jV_0 + h_jvS + r_j = h_{j+1}V_0 + h_{j+1}vS + r_{j+1} \tag{10}$$

Isolating S in Equation (10), and appending to it the subscript j, yields

$$S_j = \frac{\Delta r_j + V_0\Delta h_j}{-v\Delta h_j} \tag{11}$$

where

$$\Delta r_j \equiv r_{j+1} - r_j \qquad \text{for } j = 1,\ldots,J-1 \tag{12a}$$

and

$$\Delta h_j \equiv h_{j+1} - h_j \qquad \text{for } j = 1,\ldots,J-1 \tag{12b}$$

Equation (11) is the principal result of this re-examination of Matthews' model. The foregoing analysis has yielded an outcome fundamentally different from that obtained by Matthews. The result is an emended economic decision rule that specifies the distance below the arbitrarily selected point ($P = 0$) that construction to standard j should continue. Having reached this distance from the selected starting point, the road should now be constructed to standard $j + 1$. The first and second examples of the next section will numerically illustrate the value of this analytical result.

A major assumption in obtaining this last result is that the tributary volume remains constant for the entire distance and that in general there are no volume discontinuities. A typical volume discontinuity might occur at a road junction where additional timber volume from another area joins the flow down to the mill. Volume discontinuities and how they can be addressed will be examined in the next section of this note.

The total timber volume, ${}^*V_j$, at which this transition from standard j to standard $j + 1$ occurs can easily be found. Following an argument similar to that used in the development of Equation (1) write

$$V_j = V_0 + vS_j \tag{13}$$

Substitute into this equation using Equation (11) so that

$$V_j = V_0 + v\left[\frac{\Delta r_j + V_0\Delta h_j}{-v\Delta h_j}\right] \tag{14}$$

and then simplify to obtain

$${}^*V_j = -\left[\frac{\Delta r_j}{\Delta h_j}\right] \qquad \text{for } j = 1,\ldots,J-1 \tag{15}$$

The asterisk on V in Equation (15) denotes a haul volume characterizing an optimal transition point in the road standard. At the point where this volume is reached in moving down the road toward the mill there should be a transition from the construction of road standard j to standard $j + 1$. This last equation agrees with a result given by Matthews (1942), who arrives at it from a distinctly different direction.

The maximum economic length, ${}^*S_j$, of any segment built to standard j assuming a constant tributary volume v can readily be found. Based on Equation (15) write

$${}^*V_{j-1} = -\left[\frac{\Delta r_{j-1}}{\Delta h_{j-1}}\right] \tag{16}$$

with ${}^*V_0 = \Delta r_0 / \Delta h_0 \equiv 0$ when $j = 1$. Substitution of ${}^*V_{j-1}$ into Equation (11) for V_0 and simplification yields

$${}^*S_j = \left[\frac{1}{v}\right]\left[\frac{\Delta r_{j-1}}{\Delta h_{j-1}} - \frac{\Delta r_j}{\Delta h_j}\right] \qquad \text{for } j = 1,\ldots,J-1 \tag{17}$$

The algebra may be verified by using Equations (15) and (16) to reduce Equation (17) to a form that by inspection is seen to be correct

$$^*S_j = \frac{^*V_j - ^*V_{j-1}}{v} \qquad (18)$$

The total cost of constructing and using a unit length of road built to standard j is

$$TC = Vh_j + r_j \qquad (19)$$

and the average cost per unit volume hauled over the unit length is then

$$AC = h_j + \frac{r_j}{V} \qquad (20)$$

The range of volume within which standard j is optimal vis-à-vis the alternative standards can be determined by Equation (15). Within this range the AC-curve for standard j is lower than that of any other available standard. When plotted against haul volume these AC-curves for alternative standards collectively define a least-cost frontier. This least-cost frontier moves downward to the right with increasing volume and higher road standard. An example of this least-cost frontier based on Matthews' data (Table 1) is shown in Figure 2. Lumpiness of the AC frontier is accentuated when the number of alternative road standards is reduced. When road classes 2, 3, and 4 are eliminated, the range of the two remaining classes, 1 and 5, must be extended to fill in the gap. The increased lumpiness of the AC frontier results in higher road system costs than would otherwise apply. From this figure it is also observed that the marginal cost penalty for mistaken road standard selection is particularly high for lower road classes.

Examples in Application

The preceding analytical results of the model are now illustrated in more tangible fashion. The three examples that follow give some indication of the range as well as the methods of application.

An Interior Mainline Road of Considerable Length

The problem description given by Matthews (1942) when addressing the determination of economic service standards for an interior mainline road is entirely appropriate for many

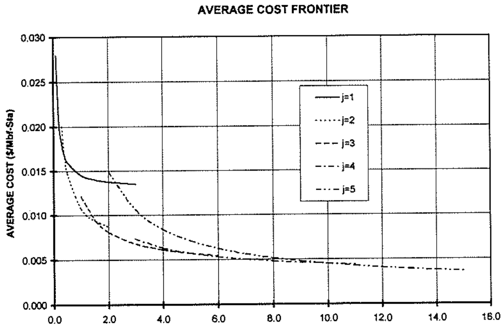

Figure 2. An average cost frontier curve based on road construction and use cost data provided in Matthews (1942).

Table 1. Cost by category for five road standards as defined by Matthews (1942).

Road class j	Cost per unit haul volume-distance h_j ($/mbf-sta)	Cost per unit road length r_j ($/sta)
1	0.0130	1.50
2	0.0067	4.00
3	0.0041	8.00
4	0.0033	12.00
5	0.0019	26.00

contemporary applications. The newly derived economic decision rules of this paper are found to result in significantly different construction decisions than those given by Matthews. Lower total costs for the construction and use of the road are obtained by the new decision rules.

In Matthews' example, a 200 station road is to be extended up the length of a narrow valley. There are no spur roads and a constant tributary timber volume of 100 thousand bf of timber per station of road will arrive at the road along its entire length. There are five different road standards in use by the company with associated costs as specified in Table 1. Based on these costs the entries of Table 2 are readily calculated using Equations (12) and (15) of the previous section. The optimal length of road to be built to each standard is then calculated using Equation (11) and is shown in Table 3. Following Matthews, the lowest road class is excluded from among the possible construction standards for this problem.

The results given by Matthews are listed in Table 4 for comparative purposes. Very significant differences are noted when comparing road segment lengths in this table with those of the previous table. In this example, only a slightly larger total cost is observed. Other examples, especially where fewer road classes are available, will result in larger cost differences. For example if only class 2 and 5 roads were available in this particular case and only 100 stations of road were to be extended up the valley, a comparison of the two rules yields a 17% cost difference ($3046 vs. $3550).

Road Standard Selection with Volume Discontinuities

Volume discontinuities are an often observed characteristic of logging road systems. Such discontinuities arise in many ways. Centralized landings, road junctions, and changing tributary volumes are common sources of a volume discontinuity. Most of these discontinuities, if deemed significant, are easily handled within the model.

Table 2. Optimal haul volume intervals for five road standards as defined by Matthews (1942).

Road class j	Finite differences: Cost per unit haul volume-distance Δr_j ($/mbf-sta)	Finite differences: Cost per unit road length Δh_j ($/sta)	Transition point volume $^*V_j$ (mbf)	Economic interval (mbf)
1	2.50	–0.0063	397	0–397
2	4.00	–0.0026	1,538	397–1,538
3	4.00	–0.0008	5,000	1,538–5,000
4	14.00	–0.0014	10,000	5,000–10,000
5	—	—		10,000+

Table 3. Optimal road segment length and cost for Matthews' example using corrected analytical solution.

Road class j	Optimal segment length $^*S_j$ (sta)	Total segment cost ($)
1	Following Matthews, assumed not to be an option in this problem	
2	15.38	140.76
3	34.62	740.97
4	50.00	1,837.50
5	bal. (=100.00)	5,450.00
Totals	200.00	8,169.23

NOTE: The tributary volume is 100 mbf per station in this example from Matthews.

Figure 3, Table 5, and the first two columns of Table 6 describe a logging road system with a variety of volume discontinuities. These discontinuities and their origins are as follows. At point E a major centralized landing is to be used. At point D future extension of the road system will provide access to a known volume of standing timber. At point A timber volume from an existing road system will be diverted over a planned connecting road segment. At point F the tributary volume changes. At points C and B, branch roads each carrying timber volumes come together. The third column of Table 6 is calculated from the volume information. Using the cost data of Table 1 and the calculated intervals in the final column of Table 2 the optimal road standard may be determined at those discrete points where it is anticipated that a change in standard may occur. These optimal standards have been determined and are listed in the fourth column of Table 6. Optimal road standard transition points are then obtained using Equation (11), and these are given in the final column.

Optimal Location—The Launhardt Road Junction Problem

A fundamental problem in road network design is the optimal economic location of the road junction point connecting three horizontal control points. A turn-of-the-century publication by Professor Launhardt (1900-1902) gives one of the first English language descriptions of this transportation problem. A complete analytical solution for this problem has recently been developed and will be utilized here (Greulich 1995). In Launhardt's junction problem, it is assumed that costs associated with construction and use of connecting road segments are

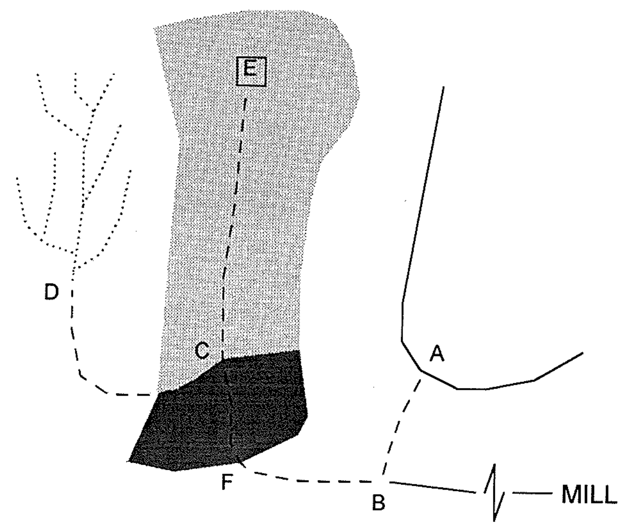

Figure 3. A hypothetical logging road system demonstrating a variety of haul volume discontinuities.

known. The road segment cost plainly depends on the standard selected. For logging roads the optimal economic standard is determined by the volume of material hauled over a segment, and this volume can vary with the network configuration. The following example portrays this application.

The three control point (CP) locations are specified using a Cartesian coordinate system: CP1:(5,10), CP2:(25,5), and CP3:(20,15). For this particular problem the unit of distance is stations, and the above coordinates are expressed in those units. Road classes and costs from Matthews (Table 1) are used. The first control point, CP1, fixes the proposed takeoff point on the existing road system. The second and third control points are centralized landings from which 4.0 mmbf and 1.5 mmbf respectively will be hauled. It is assumed that there is no tributary volume. The costs per unit distance of constructing and using the optimal economic road standards are calculated as follows. A road over which 4.0 mmbf will be hauled is best built to road class 3 (Table 2) and, based on its construction and use costs (Table 1), it will have a cost of $24.40/sta. For a road carrying 1.5 mmbf road class 2 is optimal at a cost of $14.05/sta. For the combined volume of 5.5 mmbf road class 4 is optimal and the cost is $30.15/sta. Having calculated these costs, the optimal network configuration can now be easily identified by a procedure furnished elsewhere (Greulich 1995). Coordinates of the optimal road junction location are thereby found to be (15.97, 9.84). The cost of constructing and using this optimized network will be $672.76, a savings of 8% over the next best network configuration.

Table 4. Optimal road segment length and cost as given in Matthews' example.

Road class j	Matthews' calculated segment length (sta)	Matthews' calculated segment cost ($)	Author's corrected segment cost ($)
1	Following Matthews, assumed not to be an option in this problem		
2	30	431.55	421.50
3	39	1,111.50	1,103.51
4	61	2,745.00	2,734.94
5	70	4,021.15	4,014.50
Totals	200	8,309.20	8,274.44

NOTE: The last column contains segment costs corrected by the author for a calculating error in Matthews' text but otherwise follows the procedure given in his book.

Table 5. Given distances and tributary volumes for labeled road segments.

Road segment			Tributary volume
From	To	Distance (sta)	(mbf/sta)
A	B	60	0
E	C	100	30
D	C	50	20
C	F	40	50
F	B	60	0
B	Mill	—	—

Table 6. Calculated optimal road standard at selected points based on the haul volume that will pass through each point.

Road system segment	Timber haul volume at beginning of segment (mmbf) Given	Calculated	Optimal standard at beginning of segment (Std.)	Calculated distance from beginning of segment where road standard should change to one of a higher quality (Std. @ sta)
A – B	9.5		4	
B – Mill		21.0	5	
C – F		9.5	4	5 @ 10
D – C	4.5		3	4 @ 25
E – C	1.0		2	3 @ 18
F – B		11.5	5	

Concluding Statement

An analytical model developed by Matthews for the determination of the economic service standard of a timber haul road has been corrected. As evidenced in the first example, taken from Matthews' textbook, the revised formulas yield significantly different results from those presented by Matthews. The two additional examples of this note serve to illustrate the application and general utility of the formulas.

A necessary precursor to good forest road network design is a firm understanding of the correct economic basis for road standard selection. Fundamental insight into the relationship between road network costs, various elements of the operational environment, and physical layout has been gained. The formal analytical model and its results presented here should stimulate additional advances in both theoretical and practical design optimization of harvest transportation systems.

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Appendix

On pages 171–173 of his 1942 text, Matthews develops a formula for determining the optimum length of road constructed to a given standard. There are two fundamental flaws in the analysis as it is presented. The comments that follow are summary in nature, and the reader is referred to Matthews for notation and details of the development.

The first error involves the calculation of the total savings that would result from road improvement. For that tributary volume accrued along any unit road length, the road improvement savings associated with that volume are not s but rather 1/2 s. Matthews sums the series

$$s+2s+3s+\ ...\ +ns=\left[\frac{(n)(n+1)(s)}{2}\right] \qquad \text{(A1)}$$

whereas the series should read

$$\frac{s}{2}+\frac{3s}{2}+\frac{5s}{2}+...+\frac{(2n-1)s}{2}=\left[\frac{n^2s}{2}\right] \qquad \text{(A2)}$$

The second error involves the application of a break-even formula when in fact direct marginal economic analysis is required. The road should be constructed to a given standard until for the next highest standard the marginal construction cost equals the marginal haul cost saving associated with the decision to go to the higher standard for the next unit length of road. Matthews writes

$$nr=\left[\frac{(n)(n+1)s}{2}\right] \qquad \text{(A3)}$$

That is,

$$r=\left(\frac{1}{2}\right)(n+1)s \qquad \text{(A4)}$$

whereas economic optimality requires that

$$\frac{\partial[nr]}{\partial n}=\frac{\partial\left[\frac{n^2s}{2}\right]}{\partial n} \qquad \text{(A5)}$$

That is

$$r=ns \qquad \text{(A6)}$$