Near-optimal location of two landings on flat, uniform terrain

Francis E. Greulich

Abstract: This paper presents a solution to the general transportation problem of optimally locating two facilities in the plane with the objective of minimizing expected straight-line travel cost from the facilities to demand points that are continuously distributed in probability across a complex polygonal region. Using this procedure, two landings can be optimally located on a harvest unit with respect to yarding costs; however, the cost of building roads to and hauling logs from the two landings must be optimized separately. In this respect, the present model does not optimize the trade-off between yarding and trucking of logs. Two examples are presented and analyzed. Future research opportunities are identified.

Résumé : Cet article présente une solution au problème général de transport qui consiste à localiser de façon optimale deux installations dans le plan avec l'objectif de minimiser le coût attendu du transport en ligne droite à partir de points de demande qui ont une distribution continue de probabilité à travers une région polygonale complexe. À l'aide de cette procédure, deux jetées peuvent être localisées de façon optimale dans une unité de récolte en fonction des coûts de débusquage. Cependant, le coût associé à la construction des chemins vers les deux jetées et au transport des billes à partir de ces jetées doit être optimisé séparément. À cet égard, le modèle actuel n'optimise pas le compromis entre le débusquage et le transport des billes par camion. Deux exemples sont présentés et analysés. Des occasions futures de recherche sont identifiées.

[Traduit par la Rédaction]

Introduction

The purpose of this paper is to extend the range of application of the traditional harvesting cost minimization model. The traditional model is based on fundamental concepts developed during the course of more than 150 years of transportation studies. Its earliest applications in forestry date to the late 1920s. This early transportation research has been described by Greulich (2003) but some more recent transportation research of relevance to the present work must be introduced. A brief review of this pertinent research will open the paper. Particular attention is given to a relatively unknown optimization procedure and its application to transportation systems. Modifications to this procedure, required for the current application, are then presented. Applications and verification of this modified procedure follow and constitute the body of the paper. A discussion of current model deficiencies and future research directions concludes the paper.

Previous work

A previous paper in the *Canadian Journal of Forest Research* presented an optimal location model for a single central landing (Greulich 1991). The harvesting conditions being modeled in the current paper are the same as those previously described. The primary focus of this paper is to extend the single landing location optimization process to two landings under similar modeling assumptions and harvesting unit conditions.

Included in this previous publication were reviews of related work both in and outside of forestry. A search through the recent transportation science literature suggests that there has been no interim advance in location optimization theory applicable to the above-described location problem in forestry. A recent paper by Valero Franco et al. (2008) examined the general location problem, cited old and current publications from the transportation science and operations research literature, and confirmed that treatment of the specific problem of interest to foresters has not progressed beyond that of the papers cited in 1991.

In this paper, the cost-minimizing simultaneous placement of two landings will be investigated. A concept, fundamental to the development that follows, is described in the seminal papers of Cooper (1963, 1964). Leon Cooper is generally acknowledged to be the originator of the alternate location and allocation algorithm, hereafter referred to as the location–allocation algorithm (LAA). Cooper's applications of the LAA primarily focused on discrete demand points. He did, however, address continuous demand regions in several publications (Cooper 1974; Katz and Cooper 1974, 1976; Cooper 1978*a*, 1978*b*). In their 1974 publication, Katz and Cooper showed that the expected value of the distance from a point to a random location described by any probability distribution defined over a planar region is a strictly convex function. Parenthetically, it is noted that this proof is more general and predates a corresponding proof for the uniform distribution taken over any polygonal region given by Greulich (1991). Cooper did not, however, extend the LAA to multifacility op-

Received 29 July 2011. Accepted 9 November 2011. Published at www.nrcresearchpress.com/cjfr on 11 January 2012.

F.E. Greulich. School of Forest Resources, University of Washington, Seattle, WA 98195-2100, USA.

E-mail for correspondence: greulich@u.washington.edu.

timization with continuous demand. It was Leamer (1968), and later Cavalier and Sherali (1986), who applied Cooper's algorithm to the optimal location of more than one facility serving a single convex polygonal region of uniform demand. The limitations of these procedures in the forestry yarding application have been previously discussed. These limitations, found too restrictive for the one-landing model, are now to be removed for a two-landing model.

In forestry, Gibson and Egging (1973) and Gibson and Rodenberg (1975) presented two of the earliest applications of LAA modeling for multiple-landing selection. In their procedure, a list of possible landing locations is specified by the analyst and the centroids of the areas to be yarded are calculated. The solution process looks for the optimal assignment of yarded-area centroids to a set of landings selected from among a limited number of candidate locations. This modeling approach is referred to as "site-selecting location–allocation" by Love et al. (1988). Other forestry researchers who, early on, used similar site-selecting LAA models were Dykstra and Riggs (1977) in harvest planning and Hodgson and Newstead (1978, 1983) in airtanker system analysis. The recent work by Chung et al. (2004) and Contreras and Chung (2007) showed the high level of development that has been attained in the application of site-selecting LAA modeling to timber harvest planning. Increased computational power has greatly expanded the number of candidate locations for landings that can be examined using these models. Increased computational speed also permits a finer division of the yarded area so that many criticisms against the use of centroids no longer apply.

A second type of model used in harvest planning is the "site-generating location–allocation" model (Love et al. 1988). The defining characteristic of these models in their harvest planning applications is the freedom to locate central landings at an infinity of points across an area or along a line rather than restricting their location to a finite set of candidate points. The earliest formal models of central landing location optimization were of this type done in continuous space (Matthews 1942). Peters (1978) later significantly improved this type of model by deriving and employing the analytically correct expected yarding distance formula. The landing optimization model of this present paper falls within this latter category of site-generating LAA models. Contemporary continuous space location optimization models in forestry can now apply computer-based numerical solution procedures rather than attempting to determine solutions through analytical or graphical methods.

The optimization process

This section provides a brief overview of the optimization procedure to establish the context for the applications that follow. It provides limited mathematical detail and the reader interested in additional mathematical development is invited to consult the cited references.

The optimization process used in this paper follows the conceptual outline of the LAA described by Cooper (1963, 1964). It is a two-step process that is initiated with user-specified trial locations for the two landings. In the first step, the two current landing locations are used to divide the total area to be yarded. The total area is divided into two mutually exclusive divisions such that any infinitesimal area within a given division is closer to its corresponding landing than the other. This is the area allocation step of Cooper's procedure. The second step is to relocate each landing within its newly defined division. This facility-location step of Cooper's procedure minimizes the expected cost of transporting turns to their respective landings given the current division of the total area between the two landings. This two-step process is repeated until there is no significant improvement (decrease) in the total expected transportation cost for the two landings.

Cooper's LAA is designed to find facility locations or centers (landings in this application) that simultaneously satisfy two conditions. First, the planar demand region (the area to be yarded) is optimally divided so that each center has allocated to it all potential demand locations closer to it than the other. Clearly, the perpendicular bisector of the line segment connecting the pair of centers serves to define these two divisions. Second, within each division, the associated center is located such that the expected transportation cost across all potential demand locations within the division is at a minimum.

A key assumption is that the transportation cost function is the same for each center; to wit, it depends fundamentally on the distance to the center and, most importantly, it does not otherwise depend on the specific center to which transportation is provided.

If both landing locations are to be optimally located, the two initial locations, provided by the user, must be different and belong to the convex hull of the polygonal region(s) that define the harvest unit. If one of the landing locations is fixed and the other is to be optimized, then the fixed landing location can be any point in the plane, but the initial point for the landing to be optimized must be different and belong to the convex hull. If neither landing location is to be optimized, then the two distinct points may be located anywhere on the plane.

The procedure used to implement Cooper's LAA employs rather basic mathematical techniques and a well-known optimization algorithm. Given two landing locations, the perpendicular bisector of the line segment connecting them is easily determined analytically to a degree of precision limited only by the computational precision of the computer. Likewise, the points at which this perpendicular bisector intersects line segments defining a polygonal region's boundary can be analytically determined with comparable precision. The areas of the two divisions can also then be delineated to machine precision using closed-form analytical relationships. This completes the area allocation step of the process.

Once the area has been optimally allocated between the two landings, the location of each landing within its division is optimized. This optimization procedure uses a modified Newton's method as described in a previous publication (Greulich 1991). The characteristics of this procedure are well known and thoroughly described in the previously cited literature. This location optimization step, by changing landing locations, improves (reduces) the expected transportation cost for each division but vitiates the division boundary-defining results of the preceding area allocation step. Hence, the area allocation step must now be repeated with these new landing locations.

Iterative application of this two-step process is continued until there is no significant decrease in the total expected transportation cost for the two landings. By the nature of the algorithm being applied, convergence to either a relative minimum or a saddle point is assured. Unfortunately, however, there is no guarantee that the landing locations that are found, even if not saddle points, will provide the lowest possible transportation cost. The landing locations found by the algorithm are only guaranteed to meet first-order local conditions for optimality; there may be multiple centers meeting these conditions, some of which may have higher expected costs than the globally optimal location. Convergence to specific landing locations, when there are multiple locations meeting the necessary conditions for optimality, is determined by the user-specified initial points. Good judgment and the use of alternative initial landing locations should provide a high level of user confidence that the landing location (s) with the lowest possible cost has (have) in fact been found.

Applications and verification

A two-landing optimization model was written in Fortran to demonstrate the optimization process and its practical feasibility. Verification of the model with its Fortran coding was conducted using a separate program.

An illustrative example

The optimization procedure suggested in this paper will be constructively illustrated with a relatively simple example. A polygonal region (Fig. 1) is defined by a sequence of vertices that are entered following a counterclockwise orientation. The starting point of the traversing set of generated line segments may be any one of the four vertices, e.g., starting with point (1,2), the following points are entered in sequence: (8,1), (4,3), and (5,9). The traverse then closes back on the beginning point (1,2). The linear measurement unit of this example and the following is the hectometre (hm).

The polygonal region in this example is not partitioned but consists of one area of uniform turn-building and yarding conditions; to wit, all turns are assumed to be uniformly and independently distributed in probability across this area. The partitioning of a region into separate areas with differing turn densities is easily accommodated and will be demonstrated with another example to follow.

Two landings are to be located and all turns found across the entire area will be yarded to these two landings. It is desired to determine the location of the two landings such that the expected cost of yarding all turns is minimized. It will be assumed in this example that the yarding cost for any turn is simply its distance to the landing. In general, it is assumed that the yarding cost can be approximated by a quadratic function of the yarding distance to a landing. The cost of harvesting the logs is assumed to be independent of all landing-related factors other than straight-line distance to the landing. A landing-independent wander factor may be incorporated if deemed appropriate. In this example and the following, the wander factor will be set to 1, i.e., the skidder follows a straight-line path between landing and turn location.

To start the optimization algorithm, the analyst specifies two different points as initial locations for the landings. If the location of both landings is to be optimized, then these two initial locations must both belong to the convex hull of the polygonal region(s), in this case the closed triangular region defined by the vertices (1,2), (8,1), and (5,9). One or both of the initial landing locations can be fixed in place by the analyst, i.e., not optimized. In this latter case, the fixed landing location(s) can be any point(s) on the plane. Any landing location to be optimized must be given an initial point belonging to the convex hull that is different from the initial point of the other landing. In this example, both landing locations are to be optimized and their starting locations for the algorithm have been user-specified as (4,7) and (6,3) for landings labeled A and B, respectively (Fig. 1). The cost equation coefficients are entered as 0.0, 1.0, and 0.0 for the constant, linear, and quadratic terms, respectively.

Once the initial locations have been user-specified, the LAA continues by allocating yarded areas to each landing. This area allocation process is done by dividing the polygonal region into two parts using the perpendicular bisector of the line connecting the two landing locations (Fig. 1). All turns across the harvest unit are to be taken to the nearest landing as determined by this area allocation process. In this example, the midpoint between the two landings is located at (5,5), and the perpendicular bisector is found to intersect the lines-of-traverse of the polygonal region at (4.2727,4.6364) and (1.8000,3.4000). Of the total area within the polygonal region, 13.5000 ha, the areas yarded to landings A and B are 4.9455 and 8.5545 ha, respectively. The corresponding expected yarding distances are 1.6873 and 2.6740 hm. At the end of this first area allocation step of the algorithm, the area-weighted average yarding distance is 2.3126 hm. Area is here serving as a proxy for turns, since there is only one turn density in this simple example and the density constant, turns per unit area, need not be treated explicitly.

The next step in the algorithm requires the optimal (re-)location of the two landings within the area divisions that have just been defined by the area allocation step. An illustratively informative procedure will be done prior to executing the location optimization step. This procedure involves translation followed by rotation of the polygonal region together with the two current landing locations. As a first step, the midpoint, (5,5), of the perpendicular bisector is made the origin of the coordinate system. Next, a counterclockwise rotation of 1.1071 radians (63.4349 degrees) places the perpendicular bisector on the y-axis and landing A on the negative x-axis (Fig. 2). With this reorientation of the polygonal region and landing locations, it is now easy to describe and carry out precalculations needed for the optimal location step of the algorithm. Parenthetically, it is noted that this translation–rotation operation is not needed for vertex projection onto the perpendicular bisector but it is done here for purposes of descriptive clarity.

Those turns falling within that portion of the total area falling to the left of the y-axis should be taken to landing A and those falling to the right of the y-axis to landing B. To use the location optimization algorithm, the coordinates for the polygonal region shown in Fig. 2 and listed in Table 1 are divided into two parts. This division is illustrated in Fig. 3 and listed in Table 1. Note that specification of these two separate parts of the total area is easily done. For example, identification of the area from which turns will be sent to

Fig. 1. Reentrant quadrilateral showing user-selected starting points for landings A and B and the area-allocating perpendicular bisector.

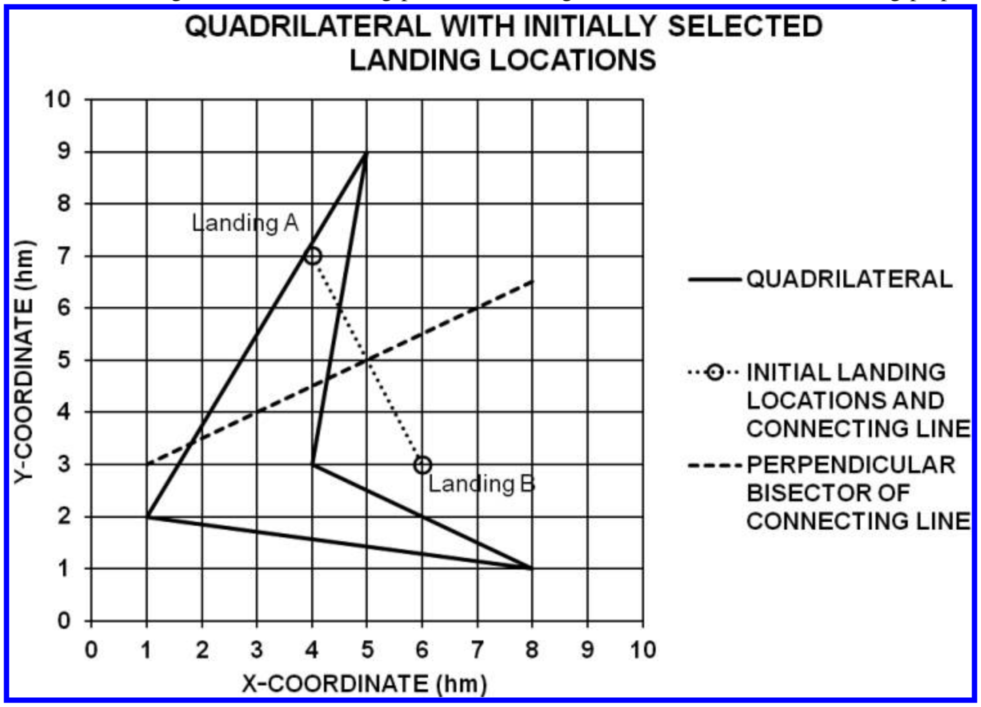
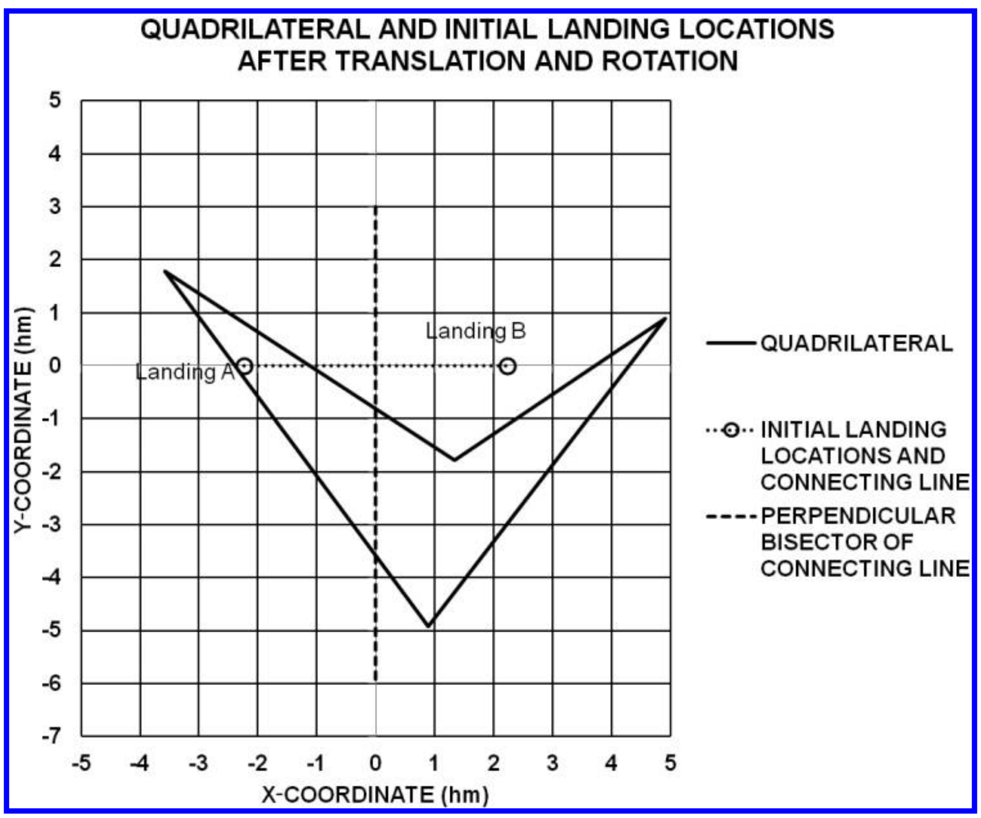


Fig. 2. Initial configuration of landings and allocated areas for the reentrant quadrilateral after translation and rotation prior to landing location optimization.



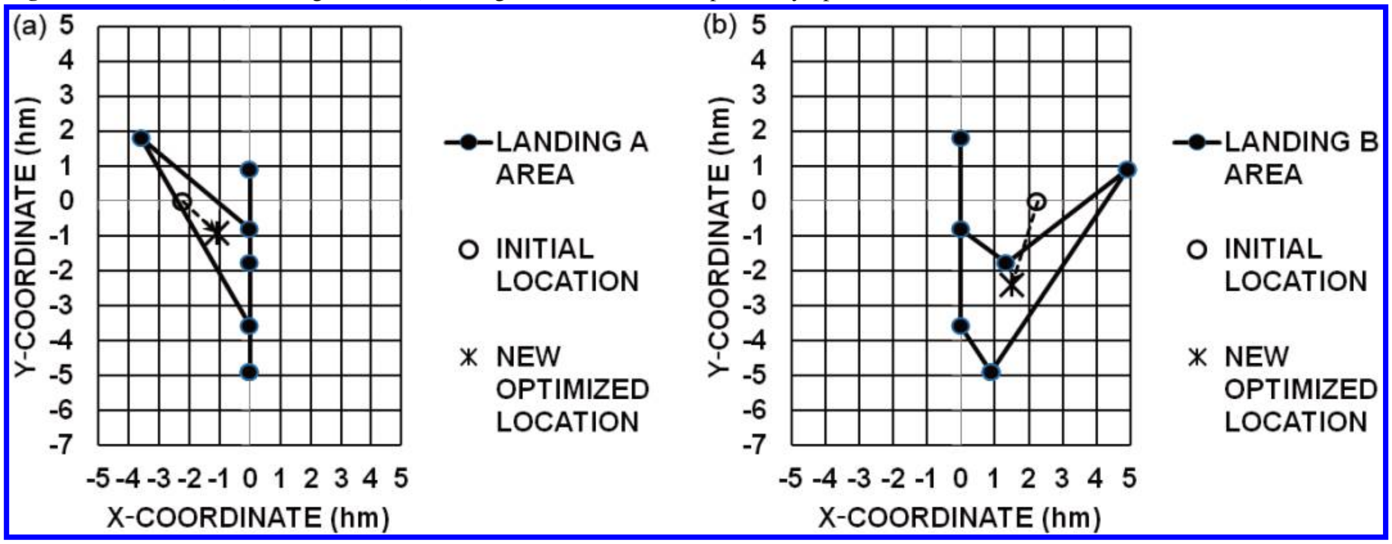
landing A proceeds as follows. Start with the original sequence of points around the polygon, including the intercepts between the perpendicular bisector and the traverse line segments (Table 1; Fig. 2 coordinates). All coordinate pairs in this sequence for which the *x*-coordinate is positive are projected onto the *y*-axis by setting the *x*-coordinate equal to zero. A similar procedure defines the area sent to landing B. These coordinates defining the two separately yarded areas are plotted in Fig. 3.

The location optimization algorithm given by Greulich (1991) can now be applied separately to these two areas using their redefined coordinates as listed in Table 1. Gorski et

Table 1. Coordinates of the reoriented quadrilateral and the two landing locations during the first location–allocation algorithm iteration.

	Figure 2 coordinates		Figure 3*a* coordinates		Figure 3*b* coordinates	
Sequence of points	*x*	*y*	*x*	*y*	*x*	*y*
1	0.894 4	–4.919 3	0.000 0	–4.919 3	0.894 4	–4.919 3
2	4.919 3	0.894 4	0.000 0	0.894 4	4.919 3	0.894 4
3	1.341 6	–1.788 9	0.000 0	–1.788 9	1.341 6	–1.788 9
Intercept	0.000 0	–0.813 1	0.000 0	–0.813 1	0.000 0	–0.813 1
4	–3.577 7	1.788 9	–3.577 7	1.788 9	0.000 0	1.788 9
Intercept	0.000 0	–3.577 7	0.000 0	–3.577 7	0.000 0	–3.577 7
1	0.894 4	–4.919 3	0.000 0	–4.919 3	0.894 4	–4.919 3
Landing locations before optimization			–2.236 1	0.000 0	2.236 1	0.000 0
Landing locations after optimization			–1.095 0	–0.906 2	1.485 6	–2.388 4

Fig. 3. Movement of (*a*) landing A and (*b*) landing B to their new, independently optimized locations.



al. (2007) have described and justified this general optimization procedure and mentioned its use for similar transportation problems in the context of the LAA. The initial landing locations and their newly optimized locations are listed in Table 1 and shown in Fig. 3. The areas yarded to landings A and B, respectively, have not changed in this step but the expected yarding distances to the newly optimized landing locations are 1.1895 and 1.4128 hm. The new weighted average yarding distance is 1.3310 hm, a substantial improvement over that calculated after the preceding area allocation step.

By inverting the rotation and translation process, these newly optimized locations and the polygonal region can be moved back to their original coordinate grid. The algorithm now continues by (re-)allocating the polygonal region using the perpendicular bisector of these two new locations. This alternating LAA cycle continues until the difference in the weighted average yarding distances calculated following each of the two steps meets the convergence criterion. For this example, and the stated initial coordinates, the optimal locations for landings A and B are found to be (3.7958,5.6648) and (3.4402,2.6481), respectively. Their yarded areas and expected yarding distances are 4.6609 ha and 1.0989 hm for landing A and 8.8391 ha and 1.4209 hm for landing B. For the total harvest unit, these locations yield a weighted average yarding distance of 1.3097 hm.

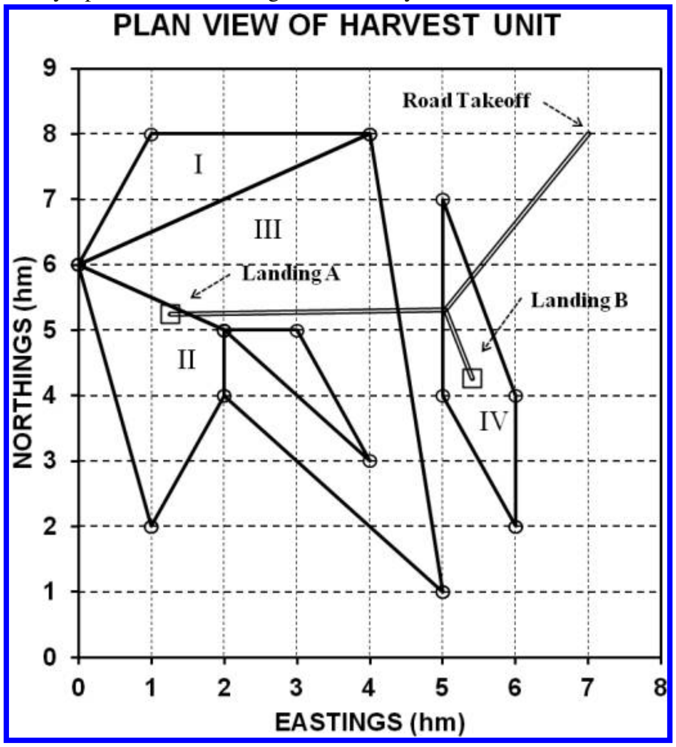
A more complex example

The example that follows presents operational conditions that are closer to those that might be encountered in practice when designing a harvest unit layout. While the unit is located on flat, uniform terrain, the example does have an uneven distribution of turns. Both landing locations will be optimized for the unit. The costs of spur road construction and use are not integrated into the model, but they will be addressed through a separate optimization model. In this section of the paper, the objective function is specified for the general case, the harvest unit described, the harvesting cost structure explained, modeling results presented, and a comparison made with single landing location optimization.

The general objective function

The objective function for the two-landing model is the same as that used to optimize the location of a single landing except that a second landing is now included and optimization of the truck road related costs are addressed in a separate

Fig. 4. Plan view of the harvest unit for the more complex example with the optimal landing locations shown along with the independently optimized connecting truck road system.



analysis. The notation, while more detailed, is consistent with that given by Greulich (1991). An index k is used to label the partitions within the harvest unit and the two landing locations are identified as A and B. The proportion of all turns on the harvest unit to be found within partition k is denoted as p_k. The number of turns on partition k is m_k so that $p_k = m_k/\Sigma m_k$. Every turn on every partition will be yarded to one of the two landings, but it is likely that there will be some partitions where turns are sent to both landings depending on which one is closer. Since turns are assumed to be uniformly distributed in probability over the area A_k of each partition, the expected proportion of the turns sent from partition k to landing A is given by $A_{kA}/(A_{kA} + A_{kB})$ with a corresponding form for landing B. The expected yarding distance for turns sent from partition k to landing A is denoted $E\{\rho_{kA}\}$ with a similar form for landing B. The expected square of the distance follows a similar format for each landing. A separate wander factor w_k may also be applied to each partition. Following Greulich (1991), the expected cost of yarding a randomly selected turn from those to be sent from partition k to landing A is calculated by an equation of the following form:

$$[1] \qquad E\{YC_{kA}\} = \beta_{0k} + \beta_{1k} w_k E\{\rho_{kA}\} + \beta_{2k} w_k^2 E\{\rho_{kA}^2\}$$

with a similar equation for landing B. The objective function is then given by

$$[2] \qquad \underset{\{x_A, y_A, x_B, y_B\}}{\text{MIN}} Z = \sum_{\forall k} p_k \left[\left(\frac{A_{kA}}{A_{kA}+A_{kB}} \right) E\{YC_{kA}\} + \left(\frac{A_{kB}}{A_{kA}+A_{kB}} \right) E\{YC_{kB}\} \right]$$

where the expected yarding cost from each partition to a landing is based on integrating over that portion of the partition spatially allocated to the specific landing. In this formulation of the objective function, the expected cost per yarded turn is to be minimized by selecting the coordinate location of landings A and B, here specified as (x_A, y_A) and (x_B, y_B), respectively.

Example harvest unit conditions

A plan view of the harvest unit is given in Fig. 4 and its associated data listed in Table 2. Based on timber stand conditions, this unit has been partitioned into four mutually exclusive divisions. Each partition is defined by a traverse, the turning point coordinates of which are listed in the table. This unit has a small interior area that will not be harvested, but it is assumed that the skidders can pass through it unhindered. There is also another area to be concurrently yarded that is somewhat separated from the main harvest area. The total area to be yarded is 22.5 ha in extent but turns are not uniformly distributed over the entire area; it is for this reason that the harvest unit has been partitioned. Within each partition, those turns found therein are uniformly distributed in probability over the area of the partition. The total volume to be yarded in this unit is 6750 m^3. There is an existing road system terminating at (7,8). This takeoff point will be used to provide road access to the landings, and its optimal configuration, which is to be discussed later, is shown in Fig. 4.

Harvesting costs for the example

There are three major cost categories, roading, yarding, and truck hauling, that vary according to landing locations within the unit. For these cost categories, the recent work by Contreras and Chung (2007) was used as a reference.

For skidder cycle time, Contreras and Chung (2007) acknowledged the contribution of Han and Renzie (2005). The downhill skidding cycle time equation of Contreras and Chung, obtained by modifying a regression model developed by Han and Renzie, is used in this paper. (No cycle time equation was provided for skidding on level ground.) Their cycle time equation is a linear function of only yarding distance; no other factors, such as turn volume, are identified as being significant in either of the two papers. The cost coefficients used in eq. 3 are obtained upon conversion of measurement units and application of the equipment rental cost used by Contreras and Chung.

For each partition k, the expected yarding cost per turn will be calculated as

$$[3] \qquad E\{YC\} = 5.6 + 3.046E\{\rho\}$$

Table 2. Boundary coordinates, area, proportion of all turns, and wander factor for each partition of the more complex example.

Partition number k	Coordinates x_{ki}	y_{ki}	Area A_k (ha)	Proportion of turns p_k	Wander factor w_k
I	0	6	3.00	0.2	1.0
	4	8			
	1	8			
II	1	2	4.00	0.3	1.0
	2	4			
	2	5			
	0	6			
III	5	1	13.00	0.1	1.0
	4	8			
	0	6			
	2	5			
	3	5			
	4	3			
	2	5			
	2	4			
IV	6	2	2.50	0.4	1.0
	6	4			
	5	7			
	5	4			
Total			22.50	1.0	

where the expected value of ρ is evaluated by integrating over that portion of partition k yarded to a specific landing, i.e., either A or B. The notation follows that of Greulich (1991) and the indices for the partition and landing have been dropped to simplify notation. The one-way yarding distance ρ is measured in hectometres for this equation and the cost is in dollars per yarded turn. This cost equation could be unique for each partition, but here, it will be assumed to be the same for all four partitions. It should be noted, as a general modeling restriction, that the yarding cost equation cannot differ by landing destination. Comparing this specific expected cost equation with eq. 1, it is observed that there is no quadratic component, i.e., β_{2k} equals zero for all k for both landings.

In the yarding operation, it was assumed, also following Contreras and Chung (2007), that the skidder yards 1.5 m^3 per turn regardless of the partition in which it is working. The total number of turns on the unit is then 4500 and they are distributed as shown in Table 2.

The truck haul rate calculation is based on the truck cost, speed, and capacity used by Contreras and Chung (2007). The number of truck loads taken from a landing is the volume yarded to that landing divided by the truck capacity. The trucking cost is incurred while traveling (both ways) over a spur road segment, measured in hectometres, of length S_i. The total volume in cubic metres hauled over spur road segment i is denoted V_i.

The spur road construction cost, using the lower range given by Contreras and Chung (2007), is assumed to be \$1000/hm. For each road segment i, the total road construction and truck hauling cost is then calculated as

$$[4] \qquad \mathrm{TRC}_i = (0.06V_i + 1000)S_i$$

There is but one road standard to be used for this particular spur road system regardless of the volume hauled. It should be noted, however, that different road construction standards are typically determined by the haul volume over a segment; higher haul volumes can justify higher road standards. Procedures for the analytical determination of the economic road standard are available for those situations (Greulich 1997). Here, however, the total cost of constructing and using the haul-road system is the sum over all road segments of this single TRC_i equation. The development of two landings implies that there will be at least two, and possibly three, road segments built for nonredundant truck road access to the harvest unit landings.

No sensitivity analysis of the cost coefficients of eqs. 3 and 4 was done, but clearly such an evaluation is possible. Caution is warranted, however, in changing some parameters that go into the calculation of the cost coefficients. For example, if the yarded volume per turn is changed, it will change the number of turns to be collected at a landing. A change in turn volume will also change the yarding cycle time and that relationship, which must be known for a valid sensitivity analysis, is not specified in the cited references. Likewise, changes in truck haul volumes will have an unspecified impact on truck speeds, and hence the hauling cost. These interrelated parameter values are exogenously determined and any corresponding sensitivity analysis is outside the scope of this presentation.

Modeling results for the example

Starting from a variety of different locations, there appears to be only one point-pair $[(x_A,y_A),(x_B,y_B)]$ of coordinates that meets optimality conditions: [(1.2627,5.2406), (5.4038,4.2688)]. An example of results typical of the itera-

Table 3. Results from the iterative process for the more complex example starting from the arbitrary point-pair [(3,6),(4,4)].

Iteration cycle	Landing locations before iteration cycle				Objective function value after each step		
	Landing A		Landing B				
No.	*x*	*y*	*x*	*y*	Area allocation step	Location optimization step	Change
1	3.000 00	6.000 00	4.000 00	4.000 00	11.742 359 139 42	10.546 874 214 91	−1.195 484 924 51
2	1.503 96	6.055 33	5.219 23	4.082 18	9.981 563 273 17	9.778 292 121 26	−0.203 271 151 91
3	1.266 82	5.258 00	5.403 65	4.255 71	9.773 115 946 05	9.772 953 478 92	−0.000 162 467 13
4	1.263 30	5.241 40	5.404 12	4.267 88	9.772 944 377 62	9.772 943 705 36	−0.000 000 672 25
5	1.262 72	5.240 66	5.403 85	4.268 70	9.772 943 653 77	9.772 943 648 41	−0.000 000 005 37
6	1.262 66	5.240 62	5.403 81	4.268 77	9.772 943 645 14	9.772 943 645 09	−0.000 000 000 05
7	1.262 65	5.240 61	5.403 81	4.268 77			

tive optimization process is given in Table 3. The iterative process was halted when the objective function failed to decrease by more than 1.0×10^{-10}. A Microsoft Fortran compiler (version 5.0) was used and the double precision program ran in under 0.01 s on an Intel Pentium 3.40 GHz platform with a Microsoft XP version 2002 operating system. The value of the objective function at this solution is $9.7729/turn giving a total cost of $43 978 for yarding the entire unit. The partition areas yarded to landing A are, in numerical order, 3.0000, 4.0000, 7.3519, and 0.0000 ha. For landing B, the respective partition areas are 0.0000, 0.0000, 5.6481, and 2.5000 ha. These yarded areas with their turn counts, and associated volumes, provide an estimate of the total truck haul volume coming off each landing, 3757 and 2993 m^3 for landings A and B, respectively. Construction and use cost for road segments originating at landings A and B are, respectively, $1225.40 and $1179.60 per hectometre of road. If the two haul volume streams coming off the two landings are brought together and hauled over a common, third road segment, the cost for that segment is $1405.00 per hectometre of road. With this information, an optimal spur road system that connects the takeoff point with the two landings can be determined analytically (Greulich 1995, 1999). This independently run appended procedure determines that the construction of three road segments with a junction point located at (5.0328,5.3106) will yield an optimal estimated cost of constructing and using the spur road system of $10 607. The total estimated cost for the harvest unit of roading, yarding, and truck hauling under the given conditions is then $54 585.

Comparative analysis

It is instructive to compare the foregoing results with those obtained using an earlier program that finds the optimal location for a single landing (Greulich 1991). Using the same cost structure, the following results are obtained for this unit. The optimal location of a single landing, considering only the yarding cost, is at (3.1141,4.9782) and has a cost of $59 388. Connecting this single landing with the road takeoff point costs $6916, which gives a total roading, yarding, and truck hauling cost of $66 304 for the harvest unit.

This earlier model permits the simultaneous optimization of roading, yarding, and truck hauling costs. By recognizing and accommodating the trade-off between moving logs via skidder versus truck, it is possible to reduce the combined cost of the two operations. This simultaneous optimization process yields a total yarding and roading cost of $65 952 with the landing located at (3.6165,5.1417). It has been assumed here that both roading and hauling costs and those of yarding are incurred by a logger who is trying to minimize the total cost of these three activities. The total savings due to joint optimization is $352, less than one half of one percentage of the total cost. This gain is obtained by moving the landing about 53 m. Its location is now closer to the road takeoff point, a move that increases the average yarding distance but reduces the truck haul distance for a net savings.

The current two-landing model is unable to simultaneously optimize roading, yarding, and truck hauling operations. The relatively small gain noted in the one-landing model due to this trade-off would suggest that this issue may be of minor financial importance, but it should be kept in mind when examining results provided by the two-landing model. Another consideration would be an induced curvilinearity and shifting of the dividing line between the two landings. This curvilinearity and shifting of the boundary is a consequence of different truck hauling costs for the two landings. These cost differences arise from the different truck haul distances for the two landings. Different road standards, when they exist, would also impact the degree of departure from the bisecting straight line that has been assumed to divide the yarded area between the two landings. This topic will be explored in more detail later in the paper.

A rather substantial reduction in total roading, yarding, and truck hauling cost has been achieved through the use of two rather than one landing on this particular unit. The total savings is \$11 367, or a cost reduction of about 17%. This savings is gained through the reduced cost of yarding, which drops from \$59 729 to \$43 978. The total road construction and use cost increases from \$6223 to \$10 607 but this is clearly a favorable trade-off.

Model verification

Using formulas provided by Jeffery and Dai (2008), a second program[1] was written to numerically estimate first- and second-order optimality conditions at any point-pair. For the reentrant quadrilateral, the usual first-order necessary condition was satisfied, i.e., the partial derivatives were found to be equal to zero. Determinants of the principal minors of the Hessian matrix were calculated and, using standard diagnostic patterns (Taylor and Mann 1983), the definitive classification of the point-pair as a cost minimizing location was made. This test, as written for this application, only applies when both landing locations are being optimized.

The complex example of the previous section also provided additional verification of the two-landing model results. The positive sequence of determinant values for the four principal minors of the Hessian matrix confirmed that it is in fact a minimum point.

Discussion

The site-generating LAA model of this paper addresses the landing location problem using continuous spatial definition and numerical optimization techniques. There are deficiencies in the model that may, under certain conditions, restrict its value. A review of some of the more notable deficiencies leads naturally into some final thoughts on future work with models of this type.

Deficiencies of the current model

There are deficiencies in the current two-landing model when compared with the one-landing model. Among the more notable deficiencies are the failure to guarantee that a global optimum will be found, the lack of simultaneous optimization of the roading, yarding, and truck hauling processes, and, related to this second issue, the use of a perpendicular bisector to divide the two yarding areas.

Global optimization

The search for optimization procedures that can guarantee a global optimum for nonconvex problems is an area of active theoretical research by mathematicians. Unlike the one-landing optimization process, there is no current procedure that can generally provide absolute assurance that a global optimum has been found for the multiple-landing problem described in this paper. Accordingly, recourse has been made to a standard heuristic practice whereby the user is encouraged to use professional judgement in selecting a variety of starting points for the optimization process. If more than one local extreme point is found, the user can then select that one for which the lowest value is attained. In most forestry applications, the user-directed selection of starting points by a professional with knowledge of on-the-ground conditions is likely to be preferable to an automated selection heuristic such as that employed by Cavalier and Sherali (1986). While global optimization of nonconvex problems is a formidable theoretical problem, it seems doubtful that it will present a serious impediment to finding solutions that are of practical assistance in forestry applications.

Simultaneous yarding and trucking optimization

In the one-landing optimization procedure, the cost of building and using a truck road may be simultaneously optimized in conjunction with the cost of yarding. An explicit economic trade-off can be made between moving the logs by skidder and truck. The two-landing model does not currently permit this economic trade-off between the two modes of transportation; each phase of the transportation process must be optimized separately. Determining the optimal nonredundant road network for multiple landing locations is, itself, a challenging problem; integrating the two issues into a simultaneous optimization process is an even more ambitious task. In the case of the takeoff and two-landing truck road problem of this paper, well-established optimization procedures are available (Greulich 1995, 1997, 1999). This understanding of the road network optimization process makes its eventual successful merger with the yarding optimization process appear quite promising in the two-landing case. The relative importance of this integrated approach to the optimal landing and road location decision is unclear, however. The comparative analysis of this paper done with the more complex, and realistic, cost inputs of Contreras and Chung (2007) seems to suggest the likelihood of a relatively significant movement of landing placement due to this trade-off with, albeit, only a minor reduction in cost. In this one-landing case, a displacement of 53 m was observed with a cost reduction of less than one half of one percentage of the total. It is speculated, however, that when optimizing the location of multiple landings, the individual landing displacements may not be quite this large. Only future research will provide a definitive answer to this issue.

Curvilinear division of yarding areas

The perpendicular bisector unambiguously defines the optimal allocation of the total area between the two landings, i.e., minimization of the yarding cost for both landings requires that turns be taken to the nearest landing in all cases. In fact, the transportation cost function can be any analytic function and, as long as it is the same cost function for both

[1]Executable files of this program and the optimization program itself may be downloaded from the following Web site: http://faculty.washington.edu/greulich/Research.htm.

landings, the perpendicular bisector defines the two separate yarding areas. The relevant conditions, for the purposes of this paper, are that the cost of yarding, after perhaps taking into account local turn-building conditions, is only a function of the distance to a landing and that it is the same function for each landing. If, however, the cost equations for transporting any specific turn from its location in the harvest unit to the mill differs for the two landing destinations, then the area-dividing curve departs from this straight line and a curvilinear partition boundary may be indicated for total cost minimization. Returning to the more complex example of this paper, it is observed that the assumptions for the use of the perpendicular bisector have been violated. At the identified "optimal point", the truck haul distance from landing A to the road-bifurcating junction point is greater than from landing B to that point. One cubic metre of wood taken from landing A incurs a higher roading and hauling charge than wood from landing B. This roading and hauling cost differential means that the boundary line between the yarding areas should be curved and also offset from the bisecting point on the connecting line between landings A and B. Early work addressing a similar issue was done by Launhardt (1885) and Cheysson (1887).

Design of the transportation system should be guided by how roading, yarding, and hauling costs will be paid. It has been generally assumed in this presentation that all of these costs will be borne by a vertically integrated cost-minimizing harvesting operation. Very specifically, amortized haul-road design and construction costs, as well as road maintenance costs, are applied against the volume hauled over each road segment. If these haul-road costs are otherwise assigned, in whole or part, then operational behavior, and hence related transportation system design decisions, will depart from those generally envisioned in this paper. The reassignment of truck hauling costs would have a similar, if generally more moderate, impact on design of the transportation system.

Future directions in model development

A high research priority is the extension of the current model to handle the optimization of more than two landings. The large cost reduction seen with the use of two landings rather than one in the cost-realistic example of this paper suggests that determining the optimal number of landings for a harvest unit should be a major research priority. By contrast, the same example suggests that determining the precise optimal location of landings provides a relatively small total cost reduction. Spatial relocation of a single landing in response to a trade-off with roading and hauling costs was quite large but it did not give a significant reduction in total cost. It appears that the harvesting cost function may be relatively flat around the optimal point and that landing location (s) need not be precisely determined to realize near-optimal costs.

If the above conclusions based on the very limited results of this paper are essentially correct, it would suggest that a somewhat lower priority might be given to developments that impact optimal landing placement, especially if that spatial impact is limited. Included among these model developments of lower priority might be inclusion of minor barriers to skidder movement, the trade-off between skidding versus truck hauling, curvilinear boundaries between yarded areas, and minor constraints on landing and (or) road locations. Harvest unit conditions can certainly be envisioned where these particular elements will have a significant impact on total cost as well as optimal landing location, but how prevalent are these conditions within the collection of likely real-world harvest areas for which this model might be applied? That determination should help establish future research priorities.

References

Cavalier, T.M., and Sherali, H.D. 1986. Euclidean distance location–allocation problems with uniform demands over convex polygons. Transport. Sci. **20**(2): 107–116. doi:10.1287/trsc.20.2.107.

Cheysson, É. 1887. La statistique géometrique: ses applications industrielles et commerciales. Le Génie Civil, **10**(13): 206–210 and **10**(14): 224–228. [29 January and 5 February 1887.] *Translated by* J. Hartley and E. Evans. *Edited by* R.F. Hebert. [Translation is available in the library of Lousiana State University, Baton Rouge, La. pp. 159–216.]

Chung, W., Sessions, J., and Heinimann, H.R. 2004. An application of a heuristic network algorithm to cable logging layout design. Int. J. For. Eng. **15**(1): 11–24.

Contreras, M., and Chung, W. 2007. A computer approach to finding an optimal log landing location and analyzing influence factors for ground-based timber harvesting. Can. J. For. Res. **37**(2): 276–292. doi:10.1139/x06-219.

Cooper, L. 1963. Location–allocation problems. Oper. Res. **11**(3): 331–343. doi:10.1287/opre.11.3.331.

Cooper, L. 1964. Heuristic methods for location–allocation problems. SIAM Rev. **6**(1): 37–53. doi:10.1137/1006005.

Cooper, L. 1974. A random locational equilibrium problem. J. Reg. Sci. **14**(1): 47–54. doi:10.1111/j.1467-9787.1974.tb00428.x.

Cooper, L. 1978*a*. Bounds on the Weber problem solution under conditions of uncertainty. J. Reg. Sci. **18**(1): 87–92. doi:10.1111/j.1467-9787.1978.tb00530.x.

Cooper, L. 1978*b*. The stochastic transportation–location problem. Comput. Math. Appl. **4**(3): 265–275. doi:10.1016/0898-1221(78)90038-X.

Dykstra, D.P., and Riggs, J.L. 1977. An application of facilities location theory to the design of forest harvesting areas. AIIE Trans. **9**(3): 270–277. doi:10.1080/05695557708975155.

Gibson, D.F., and Egging, L.T. 1973. A location model for determining the optimal number and location of decks for rubber-tired skidders. Pap. No. 73-1534. Proceedings of the American Society of Agricultural Engineers, 11–14 December 1973, Chicago, Ill. American Society of Agricultural Engineers, St. Joseph, Mich.

Gibson, D.F., and Rodenberg, J. 1975. Location models for the forest product industry and other applications. AIIE Trans. **7**(2): 143–152. doi:10.1080/05695557508974996.

Gorski, J., Pfeuffer, F., and Klamroth, K. 2007. Biconvex sets and optimization with biconvex functions: a survey and extensions. Math. Methods Oper. Res. **66**(3): 373–407. doi:10.1007/s00186-007-0161-1.

Greulich, F.E. 1991. Optimal landing location on flat, uniform terrain. Can. J. For. Res. **21**(5): 573–584. doi:10.1139/x91-079.

Greulich, F.E. 1995. Road network design: optimal economic connection of three horizontal control points on flat, uniform terrain. Int. J. For. Eng. **7**(1): 73–82.

Greulich, F.E. 1997. Optimal economic selection of road design standards for timber harvesting operations — a corrected analytical model. For. Sci. **43**(4): 589–594.

Greulich, F.E. 1999. The barycentric coordinates solution to the optimal road junction problem. Int. J. For. Eng. **10**(1): 111–114. 10 (2):81

Greulich, F.E. 2003. Transportation networks in forest harvesting: early development of the theory. *In* Proceedings of S3.04/3.06/3.07 Subject Area, IUFRO International Seminar on New Roles of Plantation Forestry Requiring Appropriate Tending and Harvesting Operations, 2–5 October 2002, Tokyo. IUFRO, Vienna, Austria, pp. 57–65.

Han, H.-S., and Renzie, C. 2005. Productivity and cost of partial harvesting method to control mountain pine beetle infestations in British Columbia. West. J. Appl. For. **20**(2): 128–133.

Hodgson, M.J., and Newstead, R.G. 1978. Location–allocation models for one-strike initial attack of forest fires by air tankers. Can. J. For. Res. **8**(2): 145–154. doi:10.1139/x78-024.

Hodgson, M.J., and Newstead, R.G. 1983. Location–allocation models for control of forest fires by air tankers. Can. Geogr. **27** (2): 145–162. doi:10.1111/j.1541-0064.1983.tb01469.x.

Jeffery, A., and Dai, H.H. 2008. Handbook of mathematical formulas and integrals. 4th ed. Elsevier Academic Press, Boston, Mass.

Katz, I.N., and Cooper, L. 1974. An always-convergent numerical scheme for a random locational equilibrium problem. SIAM J. Numer. Anal. **11**(4): 683–692. doi:10.1137/0711055.

Katz, I.N., and Cooper, L. 1976. Optimal facility location for normally and exponentially distributed points. J. Res. Natl. Bur. Stand. **80B**(1): 53–73.

Launhardt, W. 1885. Mathematische Begründung der Volkswirtshaftslehre. B.G. Teubner, Leipzig. *Translated by* H. Schmidt. *Edited by* J. Creedy. Edward Elgar Publishing Co., Brookfield, Vt.

Leamer, E.E. 1968. Locational equilibria. J. Reg. Sci. **8**(2): 229–242. doi:10.1111/j.1467-9787.1968.tb01327.x.

Love, R.F., Morris, J.G., and Wesolowsky, G.O. 1988. Facilities location: models & methods. North-Holland, New York.

Matthews, D.M. 1942. Cost control in the logging industry. McGraw-Hill, New York.

Peters, P.A. 1978. Spacing of roads and landings to minimize timber harvest cost. For. Sci. **24**(2): 209–217.

Taylor, A.E., and Mann, W.R. 1983. Advanced calculus. 3rd ed. John Wiley & Sons, Inc., New York.

Valero Franco, C., Rodríquez-Chía, A.M., and Espejo Miranda, I. 2008. The single facility location problem with average-distances. TOP (Madr.), **16**(1): 164–194. doi:10.1007/s11750-008-0040-9.