Turn-location parameters for a continuous landing model. I. Road tangents

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Timber harvesting operations often employ continuous landings on or along truck road right-of-ways. During the harvest-unit design process forest engineers describe the spatial distribution of turns with respect to a proposed landing by distribution parameters such as average yarding distance and average yarding slope. In this two-part paper these parameters and others are derived for a continuous landing model. In this first paper, parameters are derived and applied to a continuous landing located on or along a road center-line tangent. In the second paper, a similar development is applied to a continuous landing located on or along the circular curve of a road.

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Les opérations de récolte de bois d'oeuvre utilisent souvent des jetées continues sur les emprises de chemins de camionnage ou le long de celles-ci. Durant le processus de conception de l'unité de récolte, les ingénieurs forestiers décrivent la distribution spatiale des virées relativement à une jetée proposée par des paramètres de distribution tels que la distance moyenne et la pente moyenne de débardage. Dans cette communication en deux parties, ces paramètres ainsi que d'autres sont dérivés pour un modèle de jetée continue. Dans cette première partie, les paramètres furent dérivés puis appliqués à une jetée continue située sur une tangente à la ligne centrale d'une route ou le long de cette tangente. Dans la seconde partie, un développement similaire est appliqué à une jetée continue située sur la courbe circulaire d'une route ou le long de cette courbe.

[Traduit par la rédaction]

Introduction

A landing is a collection point to which logs are yarded pending secondary transportation (McCulloch 1958). Most landings can be classified as either centralized or continuous depending upon their spatial configuration (McGonagill 1978). As the names imply, centralized landings are relatively small, compact areas when compared with continuous landings. Centralized landings are spaced at intervals along the length of a truck road passing through a cutting unit. Continuous landings, however, may extend along the entire length of the road as it passes through the cutting unit.

The accurate appraisal of primary transportation costs for a cutting unit depends in part upon the identification and accurate evaluation of relevant physical factors. Innovative research begun during the 1920s in the western United States led to landmark publications in American forest engineering (Krueger 1929; Bradner et al. 1933; Brandstrom 1933; Brundage et al. 1933). These studies confirmed distance and slope from stump to landing as key determinants of yarding productivity. Methods by which these factors could be estimated and employed in cost estimation and subsequent optimization of logging operations were developed and published by these early researchers. The development and publication of a rigorous theoretical basis for these procedures was to come later. Matthews (1942) served as a focal point for development of the theory. This readily available textbook contained formally developed models and served to stimulate research. Suddarth and Herrick (1964) published an alternative average yarding distance model for centralized landings. Subsequent research then led to the development and application of some very general and practical models. The current status of this work on centralized landings has recently been described elsewhere (Greulich 1992).

The analysis of continuous landings has not received a comparable level of critical examination nor has its development proceeded apace of that for centralized landings. Unlike Matthews' treatment of centralized landings, there appears to be general agreement that the analytical procedures collected and presented in his textbook for continuous landings are generally correct. Over the past 50 years, however, logging and its operational environment have changed; so also has the planning technology available to the forest engineer. The assumptions and procedures of Matthews' textbook are now too restrictive.

The purpose here is to extend the scope of timber harvesting theory as it pertains to the evaluation of continuous landings. Previous assumptions will be relaxed, new parameters and their calculating formulas will be given, and new procedures will be presented for faster as well as more accurate evaluation of settings with continuous landings. In this paper the case of a continuous landing located along a road tangent will be examined. In a companion paper the case of a continuous landing located on a curve is considered.

Road tangents and continuous landings

After presentation of definitions, assumptions, and some minor preliminary analysis, a single geometric element (the trapezium) will be examined. Results obtained from this elemental analysis and the engineer's coordinate area formula are then combined. The result is a very general and efficient procedure for the calculation of average yarding distance and other turn-location parameters. Average yarding slope and the impact of adjustments to straight-line yarding distance are subsequently examined. This first paper concludes with two examples.

Preliminaries

Model definition is best provided within the context of on-the-ground operational conditions that the model attempts to describe. The appropriateness of the model's application

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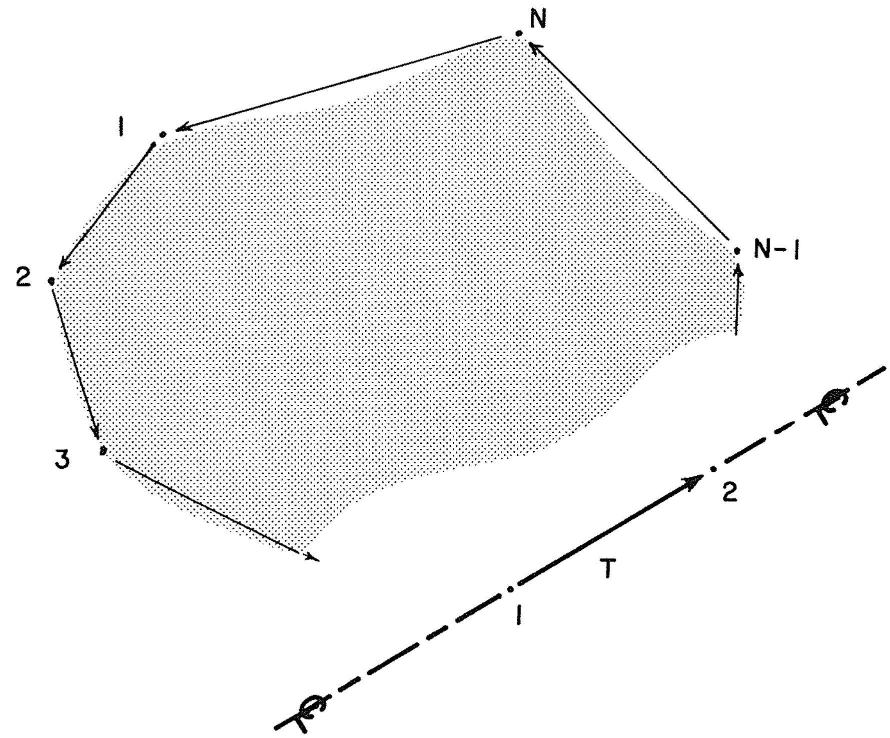

FIG. 1. Plan view of a portion of the cutting unit as delineated by a sequence of directed line segments. The correct orientation of the road center-line tangent vector is also illustrated.

to any specific operation should be judged by the degree to which the following assumptions are approximated:

(1) The ground surface of the cutting unit is described by a well defined region on a plane. The road tangent also lies within this same plane. (The plane need not be horizontal.)
(2) Turn locations are described by points on the planar region. These points are uniformly and independently distributed across the projected horizontal area of the region.
(3) For any selected turn a straight line describes the route followed during the yarding cycle. The yarding cycle starts and ends at the point on the road nearest the turn location.

The region representing the harvest cut area is "traversed" by a sequence of straight line segments (Fig. 1). The polygonal region thus defined may be both complex and nonconnected. The coordinates of the ith turning point of this boundary traverse are given by the position vectors $\mathbf{B}_i$

$$[1] \qquad \mathbf{B}_i = \begin{bmatrix} x_i \\ y_i \\ z_i \end{bmatrix} \qquad i = 1, 2, \ldots, N$$

If coordinates of two arbitrary points on the road center line are given by position vectors $\mathbf{T}_1$ and $\mathbf{T}_2$, then a road tangent vector, $\mathbf{T}$, may be calculated

$$[2] \qquad \mathbf{T} = \mathbf{T}_2 - \mathbf{T}_1$$

The polygonal region must lie on or to the left of the road center line. Subscripting of the two road center line position vectors is to be determined by this criterion. Left and right sides of center line are here defined by occupying centerline position one while facing position 2.[1] If a road tangent passes through a cutting unit it effectively divides that area into two parts. The two parts are analyzed separately exchanging subscripts on the road center line position vectors as required.

For each of the N courses of the boundary point traverse calculate the corresponding course vector $\mathbf{C}_i$:

$$[3] \qquad \mathbf{C}_i = \mathbf{B}_{i+1} - \mathbf{B}_i \qquad \text{for } i = 1, 2, \ldots, N \text{ and } \mathbf{B}_{N+1} \equiv \mathbf{B}_1$$

Calculate the unit normal vector of the plane, $\mathbf{U}_\mathrm{P}$, by selecting from the set $\{\mathbf{C}_i | i = 1, 2, \ldots, N\}$ any two noncollinear course vectors $\mathbf{C}_r$ and $\mathbf{C}_s$; then

$$[4] \qquad \mathbf{U}_\mathrm{P} = \frac{\mathbf{C}_r \times \mathbf{C}_s}{\| \mathbf{C}_r \times \mathbf{C}_s \|}$$

where the cross product indicates the vector product operation. Orient this normal vector so that it points upward; i.e., if the z component is negative change the sign on the vector.

The cosine of the angle between the plane of the ground surface and the horizontal plane is given by

$$[5] \qquad \cos(\psi) = \mathbf{U}_\mathrm{P} \cdot \mathbf{e}_3$$

where the dot product indicates the scalar product operation, and by definition

$$[6] \qquad \mathbf{e}_3 \equiv \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

The percent slope, S, of the plane is calculated as

$$[7] \qquad S = \left| \, 100 \tan[\cos^{-1}(\mathbf{U}_P \cdot \mathbf{e}_3)] \, \right|$$

Having obtained these preliminary results the development will now focus on a single geometric element. One line segment of the traverse is isolated for examination. The imme-

[1]Note the unavoidable departure from the usual road engineering convention with regard to left and right of center line.

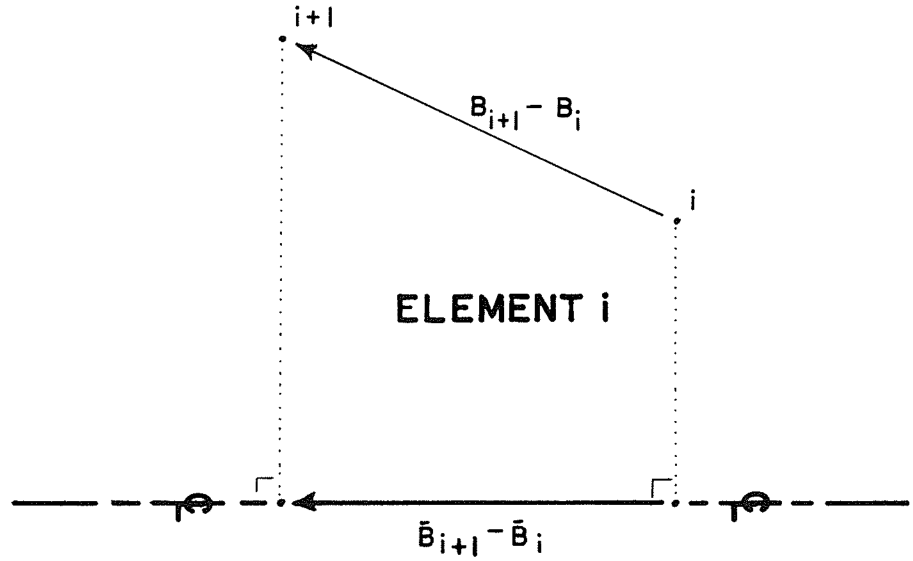

FIG. 2. The geometric element (trapezium) used for turn location parameter estimation. The view is perpendicular to the ground surface.

diate goal is to obtain a general formula for the expected value of the yarding distance raised to any positive integer.

The trapezium

Consider the single, arbitrarily selected traverse course extending from boundary point i to point $i + 1$ (Fig. 2). The boundary points are projected on the road tangent. The location of projected boundary point i is given by a position vector $\bar{\mathbf{B}}_i$ calculated as

$$[8] \qquad \bar{\mathbf{B}}_i = \mathbf{B}_i + \delta_i \mathbf{U}_L = \begin{bmatrix} \bar{x}_i \\ \bar{y}_i \\ \bar{z}_i \end{bmatrix}$$

where $\mathbf{U}_L$, the unit vector perpendicular to the line, is given by

$$[9] \qquad \mathbf{U}_L = \frac{\mathbf{U}_P \times \mathbf{T}}{\|\mathbf{U}_P \times \mathbf{T}\|}$$

and

$$[10] \qquad \delta_i = (\mathbf{T}_1 - \mathbf{B}_i)\cdot\mathbf{U}_L$$

Similar calculations yield $\bar{\mathbf{B}}_{i+1}$ as the position vector of projected boundary point $i + 1$. The position vector for any point along the course from i to $i + 1$ may be written as

$$[11] \qquad \mathbf{B} = \mathbf{B}_i + \lambda(\mathbf{B}_{i+1} - \mathbf{B}_i)$$

for some $0 \le \lambda \le 1$.
with the position vector for the corresponding projected point on the road tangent written as

$$[12] \qquad \bar{\mathbf{B}} = \bar{\mathbf{B}}_i + \lambda(\bar{\mathbf{B}}_{i+1} - \bar{\mathbf{B}}_i)$$

Define the vector $\mathbf{P}(\lambda)$ as

$$[13] \qquad \mathbf{P}(\lambda) \equiv \mathbf{B} - \bar{\mathbf{B}}$$

After substitution, and some minor algebra,

$$[14] \qquad \mathbf{P}(\lambda) = D(\lambda)\mathbf{U}_L$$

where the scalar function $D(\lambda)$ is given by

$$[15] \qquad D(\lambda) = [\lambda\mathbf{C}_i + \mathbf{B}_i - \mathbf{T}_1]\cdot\mathbf{U}_L$$

then

$$[16] \qquad \|\mathbf{P}(\lambda)\| = D(\lambda)$$

It may be shown without difficulty that the projected horizontal area, A, of the trapezium (element i in Fig. 2) is given by

$$[17] \qquad A = \|\bar{\mathbf{B}}_{i+1} - \bar{\mathbf{B}}_i\|\cos(\psi)\int_0^1 D(\lambda)\, d\lambda$$

Evaluation of eq. 17 yields

$$[18] \qquad A = [\|\bar{\mathbf{B}}_{i+1} - \bar{\mathbf{B}}_i\|\cos(\psi)]\left[\frac{D(1)^2 - D(0)^2}{2\mathbf{C}_i\cdot\mathbf{U}_L}\right]$$

Consider now a random uniform distribution of points over the horizontal region A. Each random point will be projected vertically onto the plane surface and the minimum straight-line slope distance, δ, to the road tangent calculated. The expected value of this random distance raised to the ath power is given by

$$[19] \qquad E\{\delta^a\} = \frac{1}{A}\int_A \delta^a\, dA$$

The differential horizontal area by which δ^a is weighted

$$[20] \qquad dA = \|\bar{\mathbf{B}}_{i+1} - \bar{\mathbf{B}}_i\|\cos(\psi)\, d\delta\, d\lambda$$

is substituted into eq. 19 and the following expected value formula is obtained

$$[21] \qquad E\{\delta^a\} = \frac{\int_0^1\int_0^{D(\lambda)} \|\bar{\mathbf{B}}_{i+1} - \bar{\mathbf{B}}_i\|\cos(\psi)\delta^a\, d\delta\, d\lambda}{A}$$

Evaluation of this formula yields

$$[22a] \qquad E\{\delta^a\} = \left[\frac{2}{(a+1)(a+2)}\right]\left[\frac{D(1)^{a+2} - D(0)^{a+2}}{D(1)^2 - D(0)^2}\right]$$

In the case where $D(0) = D(1)$ an examination of the limit yields

$$[22b] \qquad E\{\delta^a\} = \left[\frac{D(1)^a}{a+1}\right]$$

Defining the counterclockwise direction to be positive the signed area of the horizontal region A of the projected trapezium is calculated by the coordinate area formula as

$$[23] \quad A = \frac{[(x_i - \bar{x}_{i+1})(y_{i+1} - \bar{y}_i) + (x_{i+1} - \bar{x}_i)(\bar{y}_{i+1} - y_i)]}{2}$$

This formula may be derived from Green's theorem for the calculation of the area of a plane region as a line integral over the boundary. In this case the boundary consists of straight line segments. The use of this formula facilitates the evaluation of complex nonconnected polygonal regions (Greulich 1992).

Using eqs. 22 and 23 a procedure for the analysis of composite regions can now be developed.

Composite regions

For each traverse line segment a value may be calculated from eqs. 22 for $E(\delta^a)$, hereafter also denoted EDa_i. Likewise the signed horizontal area, A_i, may be calculated from eq. 23. Applying the composite area rule (Suddarth and Herrick 1964) the expected value based on the entire region enclosed by the traverse is then computed as

$$[24] \quad EDa = \frac{\sum A_i \, EDa_i}{\sum A_i}$$

The average yarding distance may be found from this formula by setting $a = 1$ and the expected square of the yarding distance by setting $a = 2$. The variance of the yarding distance is then calculated as

$$[25] \quad \mathrm{Var}(\delta) = \mathrm{ED2} - \mathrm{ED1}^2$$

This last result is of considerable practical interest in the quantification of the uncertainty surrounding predicted values. The use of pertinent engineering techniques such as the first-order second-moment method assumes that such variances or their estimates are available.[2]

Higher powers of yarding distance, such as its cube, ED3, may also be needed upon occasion. When approximating strongly nonlinear production or cost relationships using a Taylor series, higher order terms may be statistically justified during regression analysis (Neter et al. 1990). Under such circumstances, a forced linear approximation during development of the equation, followed by use of only the average yarding distance parameter, ED1, in the estimation of production on a setting, may lead to serious prediction error.

Average yarding slope and distance adjustments

Because all turns face the same slope when yarded to the road the average yarding slope, AYS, may be easily obtained from the components of the vector $\mathbf{U}_\mathrm{L}$

$$[26] \quad \mathbf{U}_\mathrm{L} = \begin{bmatrix} x_\mathrm{u} \\ y_\mathrm{u} \\ z_\mathrm{u} \end{bmatrix}$$

that is

$$[27] \quad \mathrm{AYS} = \frac{-100 \; z_u}{[\mathrm{x}_\mathrm{u}^2 + \mathrm{y}_\mathrm{u}^2]^{1/2}}$$

It has been assumed in making this estimate that yarding follows a straight line to the nearest point on the road tangent. Experience has shown that in the case of tractive yarding systems some adjustment of this straight line distance is

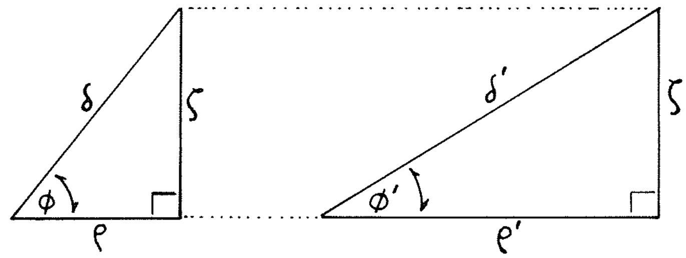

FIG. 3. The ground profiles of the constant slope yarding path before and after the incorporation of a wander factor. Variables associated with profile components are given.

required (Hughes 1930; von Segebaden 1964). The usual approach is to assume that the actual distance traveled is proportional to the minimum straight line distance to the road. The constant of proportionality, w, is sometimes referred to as the wander factor and it is here assumed constant for the given region

$$[28] \quad E\{(w\delta)^a\} = w^a E\{\delta^a\}$$

On steep ground a vehicle may follow a distinctly different path when loaded versus unloaded. Under these circumstances outhaul and inhaul distances are treated separately using different wander factors.

The impact of the wander factor on other turn-location parameters must also be considered. Another parameter of particular interest when evaluating tractive yarding systems is the average yarding slope. The analytical treatment of yarding slope is complicated by the introduction of a wander factor. One promising approach involves modification of the third assumption as given in the preliminary section. The modified assumption is:

(3′) For any selected turn paths of constant grade describe the routes followed during outhaul and inhaul elements of the yarding cycle. The yarding cycle starts and ends at the point on the road nearest the turn location.

The slope distance traveled by a single randomly located turn from its pickup location in the setting to the road is given by

$$[29] \quad \delta' = w\delta$$

where δ is, as previously defined, the straight line slope distance to the nearest point on the road. If the difference in elevation between the turn pick-up point in the cutting unit and its drop-off point on the road is denoted ζ, then the percent slope faced in yarding this turn is

$$[30] \quad 100 \tan(\phi') = \frac{100 \; \zeta}{[(w\delta)^2 - \zeta^2]^{1/2}}$$

Some algebraic manipulation using the variables defined in Fig. 3 leads to the result

$$[31] \quad 100 \tan(\phi') = [100 \tan(\phi)]\left[\frac{1}{w}\right] \left[1 + \left(\frac{\zeta}{\rho}\right)^2 \left(\frac{w^2 - 1}{w^2}\right)\right]^{-1/2}$$

It is noted that for small values of ζ/ρ (i.e., $\tan(\phi)$) and (or) values of w close to 1, the following approximation may be used

$$[32] \quad 100 \tan(\phi') \approx \left[\frac{1}{w}\right][100 \tan(\phi)]$$

[2]A good description of the use of first and higher order moments of random variables in conjunction with truncated Taylor series expansions of functions is given by Ang and Tang (1975).

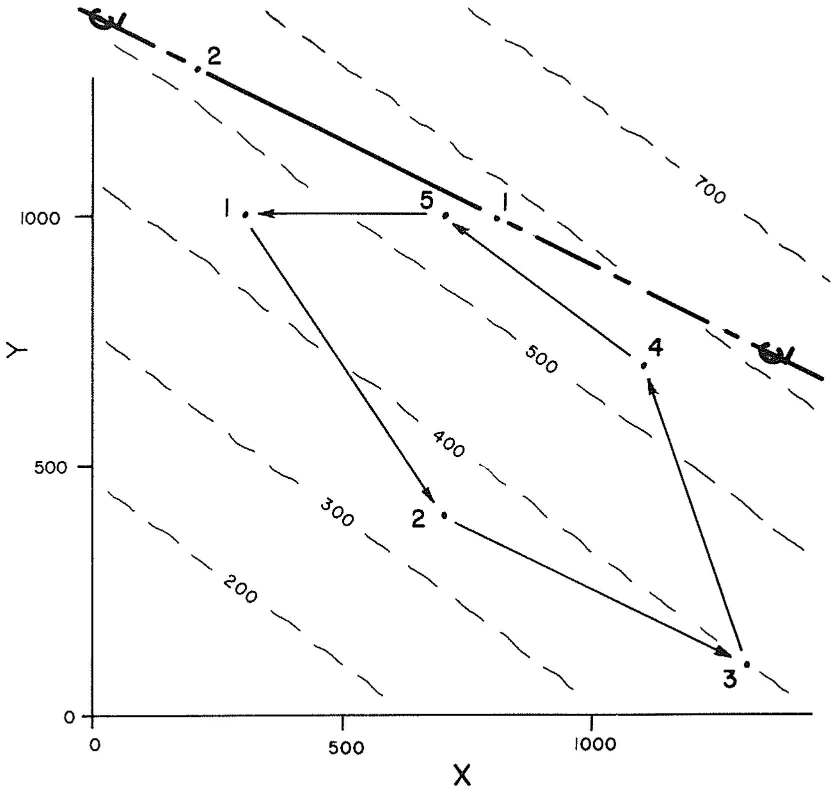

FIG. 4. Plan view of the cutting unit for example 1.

This approximation is extremely good; for example, with 100 tan(ϕ) = 30% and w = 1.5, the approximation yields an estimate of 20.0 versus an exact value of 19.5 from eq. 31. There seems little practical reason for not using this convenient, easily remembered approximation in most applications. Finally then, by extension, the wander-factor-adjusted AYS for the setting may be estimated by simply dividing the straight line AYS of eq. 27 by the wander factor. As previously mentioned, the adjusted AYD and AYS may differ for outhaul and inhaul because of different wander factors.

Examples

Two hypothetical examples are presented to illustrate the general procedure and to provide test cases for use during computer program development. The second example is somewhat more complex than the first and touches on some important ancillary issues relating to model application.

In the first example, a climbing road crosses a uniform sidehill slope (Fig. 4). A cutting unit has been located below the road. A length of road tangent will serve as a continuous landing for a cable yarding operation. Input coordinates for the continuous landing model are listed in Table 1. Based on these coordinates, the road is estimated to have a grade of 7.45%, the ground slope is estimated as 41.7%, and the cutting unit is estimated to enclose an area of 390 000 square units.

The analytical procedures of this paper are used to provide turn-location parameters for the model. Some of these parameters are:

ED1 (AYD)	3.58×10^2
ED2	1.50×10^5
ED3	6.96×10^7
ES1 (AYS)	4.08×10^1

These model parameters represent estimates of the actual on-the-ground parameter values. No wander factor adjustment of yarding distance and slope is indicated in the case of most cable systems.

TABLE 1. Turning point coordinates of the polygonal region and the coordinates for two points on the road tangent for example 1

		Coordinates		
	i	x_i	y_i	z_i
Traverse turning point number	1	300	1000	450
	2	700	400	350
	3	1300	100	400
	4	1100	700	550
	5	700	1000	550
Road point number	1	800	1000	575
	2	200	1300	525

The second example consists of a contour road that has been located along a break in the ground slope (Fig. 5). Here the road passes through the cutting unit. The steeper portion of the cutting unit lies above the road. A tractive yarding operation uses the road right-of-way as a continuous landing, decking logs above and below the road. Consideration of factors such as stocking and silvicultural prescriptions, log bucking rules, and tractive yarder capability leads to a partitioning of the cutting unit into three homogeneous areas. Partition boundaries are established such that other factors known to significantly affect yarding productivity are kept constant within each partition. As previously noted, partitioning must also separate otherwise homogeneous areas lying on opposite sides of the road center line.

Upon fitting the model, the spatial coordinates defining cutting unit partitions and road location are obtained (Table 2).

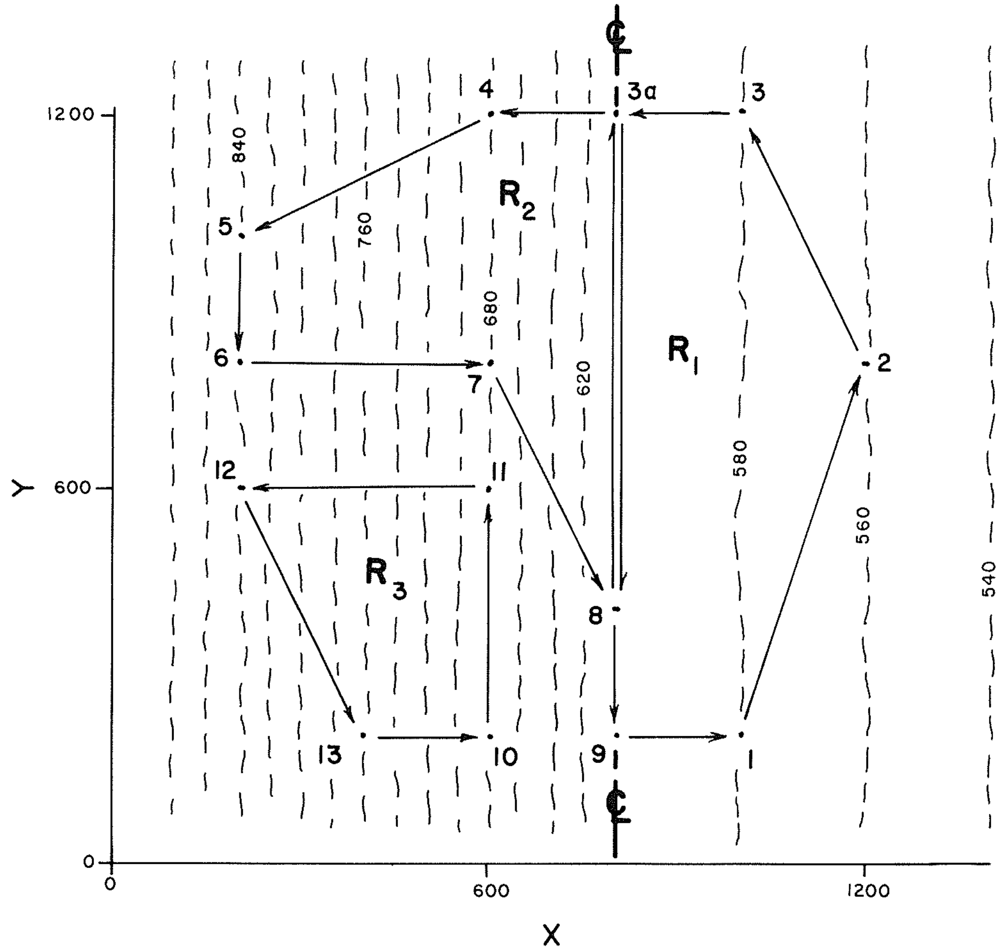

FIG. 5. Plan view of the cutting unit for example 2.

TABLE 2. Turning point coordinates of the partitioned region and the coordinates for two points on the road tangent for example 2

		Coordinates		
	i	x_i	y_i	z_i
Partition 1: traverse turning point number	1	1000	200	580
	2	1200	800	560
	3	1000	1200	580
	3a	800	1200	600
	8	800	400	600
	9	800	200	600
Partition 2: traverse turning point number	8	800	400	600
	3a	800	1200	600
	4	600	1200	680
	5	200	1000	840
	6	200	800	840
	7	600	800	680
Partition 3: traverse turning point number	11	600	600	680
	12	200	600	840
	13	400	200	760
	10	600	200	680
Road point number	1	800	400	600
	2	800	1400	600

For each partition, unadjusted turn location parameters are calculated (Table 3).

Illustrative ancillary data for this example are given in Table 4. When appropriate, the parameters of Table 3 can be combined to form a weighted average using these ancillary data. For example, the wander factor adjusted outhaul AYD for the cutting unit is 469 with an associated AYS of −15.5%.

TABLE 3. Selected turn location parameters for the model of example 2

	Partition		
	R_1	R_2	R_3
ED1 (AYD)	1.56×10^2	2.51×10^2	3.83×10^2
ED2	3.37×10^4	9.67×10^4	1.57×10^4
ED3	8.39×10^6	4.40×10^7	6.86×10^7
ES1 (AYS)	1.00×10^1	-4.00×10^1	-4.00×10^1

TABLE 4. Ancillary data for example 2

	Partition		
	R_1	R_2	R_3
Area	300 000	240 000	120 000
Turns per unit area	0.001	0.002	0.003
Wander factor			
Inhaul	1.25	1.50	1.50
Outhaul	1.25	1.75	1.75

Concluding statement

Yarding cost evaluation requires accurate information about turn location. Distance and slope to the landing during yarding are two random variables of particular interest. Formulas yielding summary distribution parameters of these two random variables and others have been developed for an elemental geometric shape, the trapezium. These latter formulas, when joined with the engineer's coordinate area formula, yield a very general procedure for the analysis of continuous landings. Computer implementation and subsequent user application are extremely easy. In this regard it is anticipated that forest engineers, already familar with the use of these contemporary evaluation techniques for centralized landings, will find the similar procedures of this paper readily accessible and of immediate utility. Yarding systems where a wander-factor adjustment is applied to the straight line yarding distance have also been considered. Under a uniform grade assumption, simply dividing average yarding slope by the wander factor provides a very accurate adjustment. In the second of these two papers, continuous landings along circular curves are examined and similar estimation procedures developed.

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Turn-location parameters for a continuous landing model. II. Circular curves

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Timber harvesting operations often employ continuous landings on or along truck road right-of-ways. During the harvest-unit design process forest engineers describe the spatial distribution of turns with respect to a proposed landing by distribution parameters such as average yarding distance and average yarding slope. In this two-part paper these parameters and others are derived for a continuous landing model. In the first paper, parameters were derived and applied to a continuous landing located on or along a road center-line tangent. In this second paper, a similar development is applied to a continuous landing located on or along the circular curve of a road.

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Introduction

In the first installment of this two-part paper a turn-location model was developed for a continuous landing along a road tangent. Turn-location parameters such as average yarding distance (AYD) were derived for continuous landings long a road tangent. Modern estimation procedures for composite areas were applied. These contemporary procedures, which offer great flexibility and general application, have the additional advantage of being readily translatable into computer programs. These programs are exceptionally easy for others to understand and use. Theoretical development and practical extension of the theory now turn to a continuous landing along the circular curve of a road.

Circular curves and continuous landings

Within geometric road design practice, one of the simplest descriptions of the horizontal projection of the road center line is provided by using a sequence of tangents and circular curves. Even roads of nongeometric design can be approximated for evaluation purposes using these two design elements. In point of fact, any road center line can be approximated to an acceptable level of accuracy using only straight line segments if they are appropriately spaced and of sufficient number. However, during the development that follows it will become clear that two considerations motivate the introduction of circular curves. First, there is the conceptual advantage that comes from enhanced model similarity to the actual road description. Second, and of more practical importance, is the greater accuracy that is achieved with far less effort.

The paper starts with a very simple and direct analytical development for circular curves. Calculating formulas for two specific geometric elements including computational procedures for their use in composite area evaluation follow. Examples illustrating the use of these procedures and some final thoughts on future research directions conclude the paper.

General development

A more restrictive first assumption than previously stated will be employed in the development that follows. Analytical development and application of the model will now be restricted to harvest areas and road design elements lying in the horizontal plane. All other assumptions including the third, as modified to allow circuitous yarding paths to the nearest point on the road, will remain the same.

By assumption the harvest-area boundary is definable by an oriented piecewise smooth closed curve. The orientation is such that the harvest area is kept to the left of the line of traverse along the curve. This region is partitioned as necessary with subregions classified as either inside, R_{inside}, or outside, R_{outside}, the circular curve (Figs. 1*a*, 1*b*, and 2). Region boundaries lying either on or concentric with road center lines are defined by circular arcs.

At this point a very different approach is taken to the development of relevant equations and analytical procedures. Much tedious developmental detail is avoided by utilizing a geometric relationship that permits the use of previously derived and published results.

In Figs. 1*a* and 1*b* a point selected at random within the region of interest is shown. For points distributed with uniform probability over the region the expected shortest straight line distance to the road center-line arc is one parameter of particular interest. This key parameter is denoted $E\{\delta\}$; or, in more specific notation, ED1_a. Another parameter, the expected straight line distance to the center of the

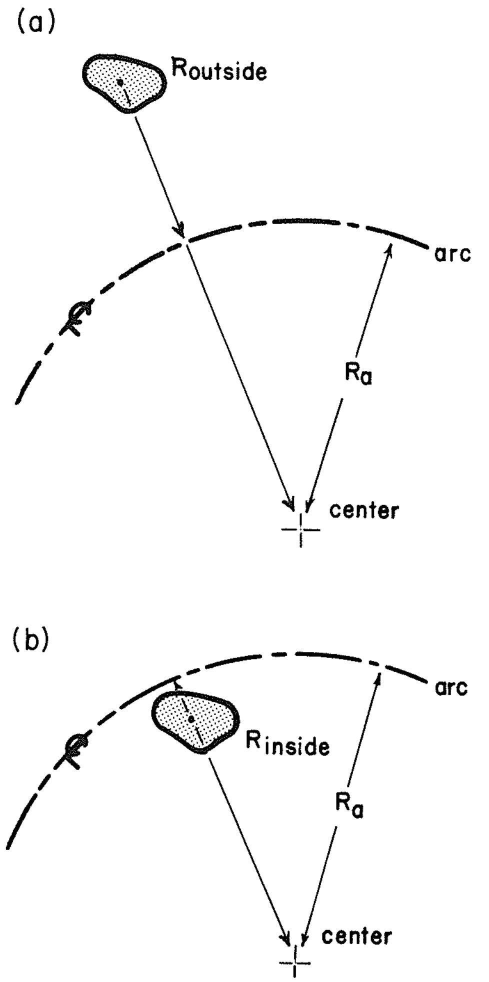

FIG. 1. Cutting units for which turns will be yarded to a circular curve along a road. Regions wholly outside (*a*) and inside (*b*) of the curve are shown.

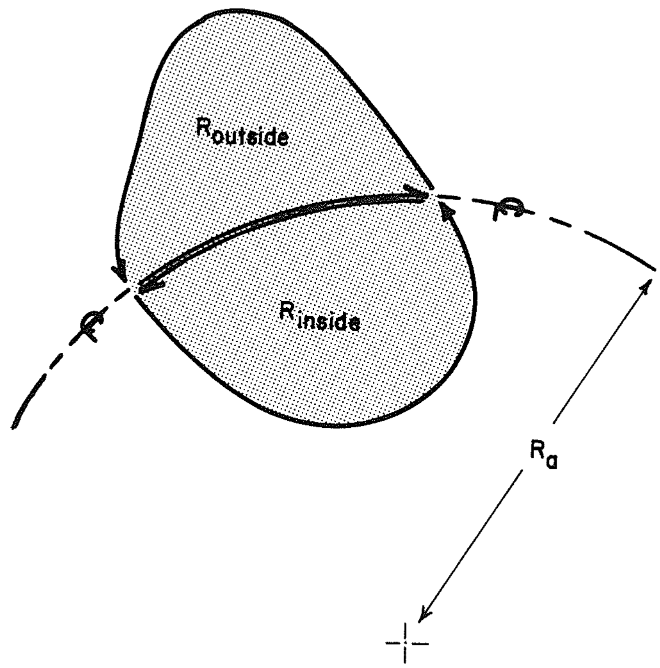

FIG. 2. A cutting unit partitioned into inside and outside regions of the circular curve to which the turns will be yarded.

circular curve, is denoted $\mathrm{ED1_c}$. The radius of the arc is denoted R_a.

Turning special consideration to Fig. 1*a* it is noted that for any randomly selected point in the region R_outside the distance to the center of the circular curve is the sum of two parts, the shortest straight line distance to the arc and the radius of the arc. Since this relationship holds for any point within the region it follows that

$$[1] \quad \mathrm{ED1_c} = \mathrm{ED1_a} + R_\mathrm{a}$$

which, solving for $\mathrm{ED1_a}$, yields

$$[2a] \quad \mathrm{ED1_a} = \mathrm{ED1_c} - R_\mathrm{a}$$

For a region (R_inside) inside the curve (Fig. 1*b*) similar reasoning leads to the equation

$$[2b] \quad \mathrm{ED1_a} = R_\mathrm{a} - \mathrm{ED1_c}$$

The two terms on the right-hand side of the last two equations are examined. The radius of the arc is assumed to be known. If the expected distance from the region to the center of the circular curve can be found, then, it follows that the parameter of interest, average yarding distance to the circular arc, may be immediately calculated.

Proceeding along similar lines, and referring once again to Fig. 1*a*, a formula for the expected square of the distance from random points in the region to the arc can be determined. The expected square of the shortest straight line distance to the arc is denoted $E\{\delta^2\}$ or, more specifically, $\mathrm{ED2_a}$. The expected square of the straight line distance to the center of the circular curve is denoted $\mathrm{ED2_c}$. By geometry and definition the following can be written

$$[3] \quad \mathrm{ED2_c} = E\{(\delta + R_\mathrm{a})^2\}$$

Upon squaring the parentheses, taking the expectation term by term, and then rearranging parameters the following equation is obtained

$$[4a] \quad \mathrm{ED2_a} = \mathrm{ED2_c} - 2R_\mathrm{a}\mathrm{ED1_a} - R_\mathrm{a}^2$$

Similarly, for regions inside the curve

$$[4b] \quad \mathrm{ED2_a} = \mathrm{ED2_c} + 2R_\mathrm{a}\mathrm{ED1_a} - R_\mathrm{a}^2$$

Consider now the parameters appearing on the right-hand side of eqs. 4*a* and 4*b*. The radius of the curve, R_a, is known. The expected distance to the arc, $\mathrm{ED1_a}$, may be found on solution of eqs. 2*a* and 2*b*. Only $\mathrm{ED2_c}$, the expected square of the distance from a randomly selected point in the region to the center of the arc, remains to be found. If $\mathrm{ED2_c}$ can be calculated then eqs. 4*a* and 4*b* will yield the desired parameter values.

In fact, the two missing parameters, $\mathrm{ED1_c}$ and $\mathrm{ED2_c}$, are readily calculated for any region and their computation is now examined.

Computational procedure

The calculation of $\mathrm{ED1_c}$ and $\mathrm{ED2_c}$ can be based on formulas and procedures developed for the analysis of centralized landings (Greulich 1982, 1987). In general, the boundary of the region may be completely and adequately described by a sequence of connected line segments. The computational formulas and procedures for $\mathrm{ED1_c}$ and $\mathrm{ED2_c}$ are well known in this case (Greulich 1992). However, when the harvest area boundary is defined with respect to a circular curve, it may on occasion be computationally or otherwise expedient to use a circular sector. Calculating formulas for

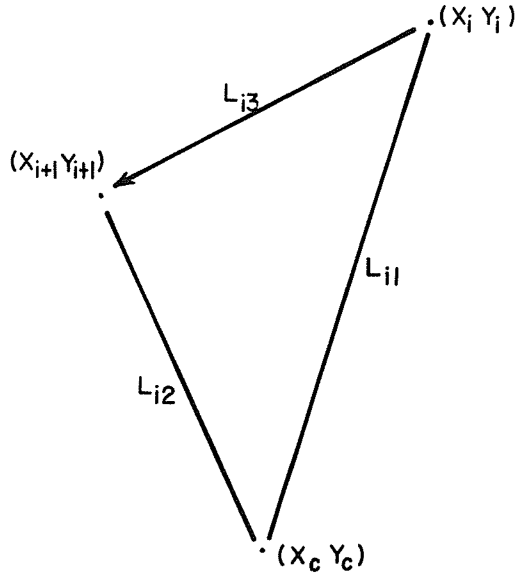

FIG. 3. A triangular element and associated variables.

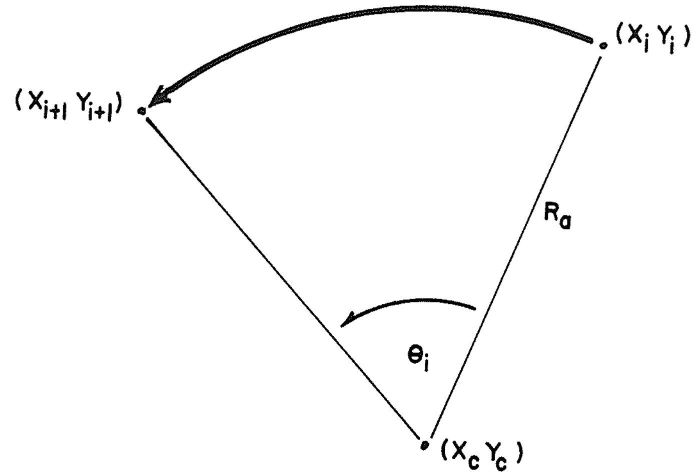

FIG. 4. A circular sector element and associated variables.

both triangular and circular sector elements should therefore be available for use as required.

In general, the harvest area boundary is defined by a piecewise smooth closed curve. The analyst then approximates this curve with a sequence of straight-line segments (and possibly with circular arcs). The center of the circular curve and each line segment (and circular arc) define a geometric element. These geometric elements are combined and form a composite region for which $ED1_c$ and $ED2_c$ can be estimated.

Geometric elements

For geometric elements that are triangular in shape (Fig. 3), the following formulas are employed for the average yarding distance and the expected square of the distance to an apical location

$$[5] \quad ED1_i = \frac{2}{3}\left\{\left[\frac{L_{i,1}+L_{i,2}}{4}\right]\left[1+\left(\frac{L_{i,1}-L_{i,2}}{L_{i,3}}\right)^2\right] + \left[\frac{2A_{i,s}^2}{L_{i,3}^3}\right]\left[\ln\left(\frac{1+r_i}{1-r_i}\right)\right]\right\}$$

and

$$[6] \quad ED2_i = \frac{[3L_{i,1}^2+3L_{i,2}^2-L_{i,3}^2]}{12}$$

where

$$[7] \quad A_{i,s} \equiv \frac{\{-[L_{i,3}^2-(L_{i,1}-L_{i,2})^2][L_{i,3}^2-(L_{i,1}+L_{i,2})^2]\}^{\frac{1}{2}}}{4}$$

and

$$[8] \quad r_i \equiv \frac{L_{i,3}}{(L_{i,1}+L_{i,2})}$$

The terms $L_{i,1}$, $L_{i,2}$, and $L_{i,3}$ in the above equations are the three side lengths of triangular element i as shown in Fig. 3. The signed area is calculated as

$$[9] \quad A_i = \frac{[(x_i-x_c)(y_{i+1}-y_c)-(x_{i+1}-x_c)(y_i-y_c)]}{2}$$

where the xs and ys are the coordinates of the vertices of the triangular elements.

Circular sectors (Fig. 4) provide a more realistic description of that portion of the harvest area boundary defined by the circular curve of the road. Consequently, the region might be defined by a traverse consisting of both line segments and circular arcs.

The average yarding distance and the expected square of the distance for any element, i, that is a circular sector are given by

$$[10] \quad ED1_i = \frac{2R_a}{3}$$

and

$$[11] \quad ED2_i = \frac{R_a^2}{2}$$

The area is calculated as

$$[12] \quad A_i = \frac{\Theta_i R_a^2}{2}$$

where

$$[13] \quad \Theta_i = 2\sin^{-1}\left\{\frac{[(x_{i+1}-x_i)^2+(y_{i+1}-y_i)^2]^{1/2}}{2R_a}\right\}$$

The value for Θ_i, the central angle of the curve, is calculated in radians. Application is restricted to curves with central angles falling in the interval $(0,\pi)$.[1] Before inserting Θ_i into eq. 12 it must be given the correct sign. The sign is either positive or negative, respectively indicating either counterclockwise or clockwise movement around the sector.[2]

Composite region

The expected distance and the expected square of the distance to the center of the circular curve can now be calculated as area weighted means:

[1]This restriction rules out curves with central angles of 180° or greater, e.g., a hairpin curve on a switchback.

[2]For computer programming purposes the appropriate sign (but not the magnitude) may be most easily determined using eq. 9.

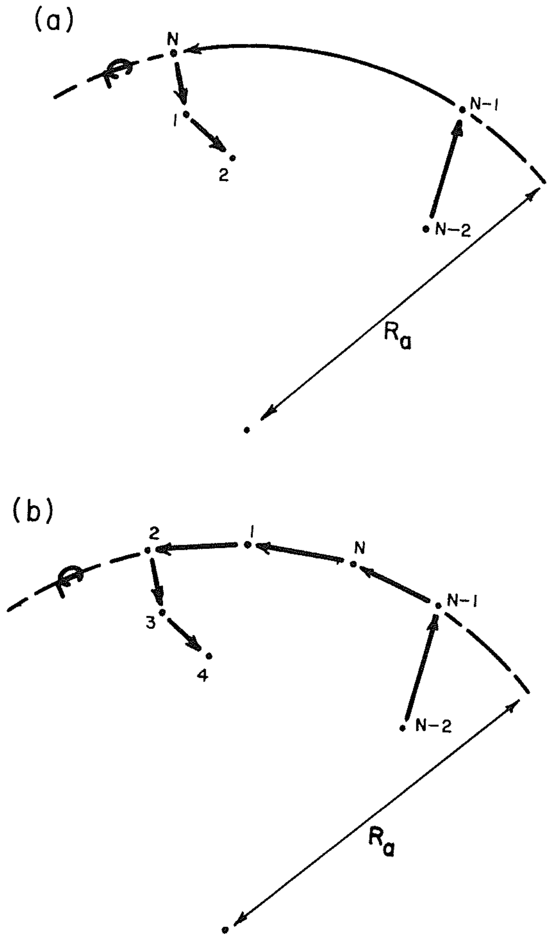

FIG. 5. A region inside a circular curve which is bounded by that curve. That portion of the region defined by the circular curve may be described by either a circular sector (*a*) or by one or more triangular elements (*b*).

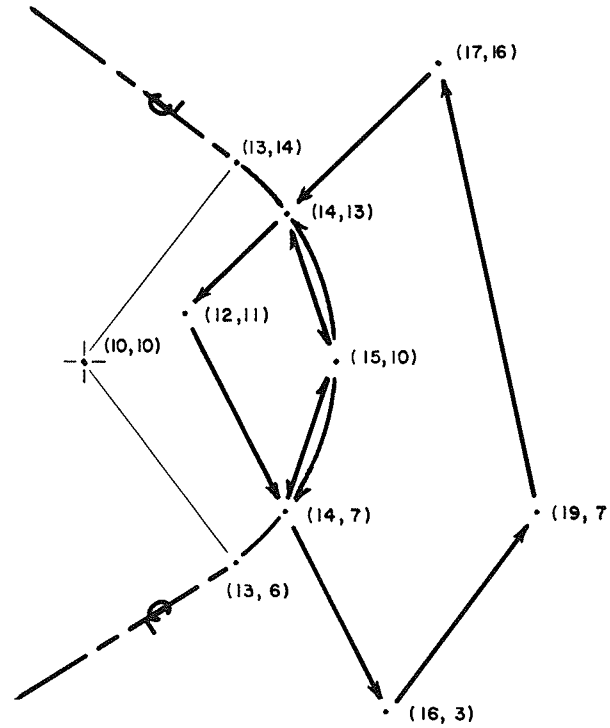

FIG. 6. Plan view of the cutting unit for example 1.

[14] $$\mathrm{ED1}_c = \frac{\sum_{i=1}^{N} A_i \mathrm{ED1}_i}{A}$$

and

[15] $$\mathrm{ED2}_c = \frac{\sum_{i=1}^{N} A_i \mathrm{ED2}_i}{A}$$

respectively, with

[16] $$A \equiv \sum_{i=1}^{N} A_i$$

Any portion of the boundary lying on the circular curve may be analyzed using either a circular sector or triangle(s) as illustrated in Figs. 5*a* and 5*b*. Which geometric elements are most appropriate in any application may be decided by the analyst.

Examples and commentary

Figure 6 shows a setting lying on both sides of a circular curve. All turns will be yarded and continuously decked along the curve of the road. Yarding distances are to be calculated to the road center line. Tables 1A and 1B show the analysis when circular elements are used to describe the road center line. In Table 1A, each element, identified by its coordinates, is evaluated. These results are then combined to yield the composite area parameters shown in Table 1B.

Here it has been assumed that turns will be decked on the road. If turns are decked off the road center line a significant distance then the analysis can be done using a more appropriate curve radius. If turns are decked on both sides of the road then two different radii may be used for turns outside and inside the curve.

Tables 2A and 2B show the results of an analysis of the same problem. However, here the circular sector has been approximated using two triangles. A comparison of Tables 1B and 2B shows that little accuracy was sacrificed by using this approximation in calculating the two parameters ED1 and ED2 for the total unit. A more significant loss of accuracy shows up in the parameter calculations for the individual regions inside and outside the curve. For the region outside the curve the approximation gives estimates that are too low. For the region inside the curve the estimates are too high. When combined, these two errors compensate for one another to produce a deceptively good total unit estimate.

Figure 7 shows a setting for which the continuous landing extends along both the circular curve and two adjacent tangents. Results of an analysis using circular sectors are given in Table 3. The setting has been partitioned to carry out the analysis. In this example, partitioning is based on three considerations; the division of the cutting unit into two parts by the road, the yarding of turns to the nearest point on the road, and whether turns are brought to a road tangent or curve. Procedures given in part I of this paper have been used to evaluate those regions to be yarded to a tangent. Means and variances associated with the random distribution of yarding distances have been calculated and are given in Table 3. Some generally applicable statistical procedures can be based on these parameters.

TABLE 1. (A) Geometric element analysis for example 1 employing circular sectors

Region element	i	Coordinates	$ED1_i$	$ED2_i$	A_i
$R_{outside}$					
Triangle	1	(10,10) (19,7) (17,16)	5.70	36.7	37.5
Triangle	2	(10,10) (17,16) (14,13)	4.74	26.0	−1.50
C-Sector	3	(10,10) (14,13) (14,7)	3.33	12.5	−16.1
Triangle	4	(10,10) (14,7) (16,3)	4.72	25.8	−5.00
Triangle	5	(10,10) (16,3) (19,7)	6.09	41.7	22.5
R_{inside}					
Triangle	1	(10,10) (12,11) (14,7)	2.20	5.83	−5.00
C-Sector	2	(10,10) (14,7) (14,13)	3.33	12.5	16.1
Triangle	3	(10,10) (14,13) (12,11)	2.41	6.83	−1.00

(B) Results of composite area analysis when employing circular sector elements in example 1

Region	$ED1_c$	$ED2_c$	A	$ED1_a$	$ED2_a$	ED1	ED2
$R_{outside}$	7.12	52.0	37.4	2.12	5.75		
R_{inside}	3.98	16.4	10.1	1.02	1.57		
Total unit						1.89	4.86

TABLE 2. (A) Geometric element analysis for example 1 employing triangles as approximations to the circular sectors

Region element	i	Coordinates	$ED1_i$	$ED2_i$	A_i
$R_{outside}$					
Triangle	1	(10,10) (19,7) (17,16)	5.70	36.7	37.5
Triangle	2	(10,10) (17,16) (14,13)	4.74	26.0	−1.50
Triangle	3	(10,10) (14,13) (15,10)	3.22	11.7	−7.50
Triangle	4	(10,10) (15,10) (14,7)	3.22	11.7	−7.50
Triangle	5	(10,10) (14,7) (16,3)	4.72	25.8	−5.00
Triangle	6	(10,10) (16,3) (19,7)	6.09	41.7	22.5
R_{inside}					
Triangle	1	(10,10) (12,11) (14,7)	2.20	5.83	−5.00
Triangle	2	(10,10) (14,7) (15,10)	3.22	11.7	7.50
Triangle	3	(10,10) (15,10) (14,13)	3.22	11.7	7.50
Triangle	4	(10,10) (14,13) (12,11)	2.41	6.83	−1.00

(B) Results of composite area analysis when employing triangular elements to approximate circular sectors in example 1

Region	$ED1_c$	$ED2_c$	A	$ED1_a$	$ED2_a$	ED1	ED2
$R_{outside}$	7.06	51.2	38.5	2.06	5.56		
R_{inside}	3.87	15.4	9.00	1.13	1.70		
Total unit						1.88	4.83

The density function of δ varies depending on the specific configuration of the setting. However, for any single randomly selected turn on a setting, Chebychev's inequality can be used to set a confidence interval for its to be observed yarding distance. Likewise, the central-limit theorem can be used to establish a confidence interval for the to be observed average yarding distance for a setting. For example, if there are m turns uniformly and independently distributed over the setting then the to be observed mean yarding distance is approximately normally distributed with a mean of $E\{\delta\}$ and a variance of $\mathrm{Var}\{\delta\}/m$.[3] In this context, it is appropriate to return to a consideration of the wander factor.

[3]On most settings, the number of turns will be large enough to give a very adequate approximation.

It should be recognized that the wander factor, w, is in fact a random variable. Denote this random variable by small ω and assume that for any setting it has a mean and variance. If yarding distance is statistically independent of the wander factor, then

$$[17] \quad E\{\omega\delta\} = E\{\omega\}E\{\delta\} = w\mathrm{AYD}$$

and

$$[18] \quad \mathrm{Var}\{\omega\delta\} = \mathrm{Var}\{\delta\}\mathrm{Var}\{\omega\} + \mathrm{AYD}^2\mathrm{Var}\{\omega\} + w^2\mathrm{var}\{\delta\}$$

These relationships and their assumption of statistical independence must be considered when applying Chebychev's inequality or the central-limit theorem to tractive yarding settings. However, it is anticipated that under most circumstances the variance of the dimensionless wander factor will

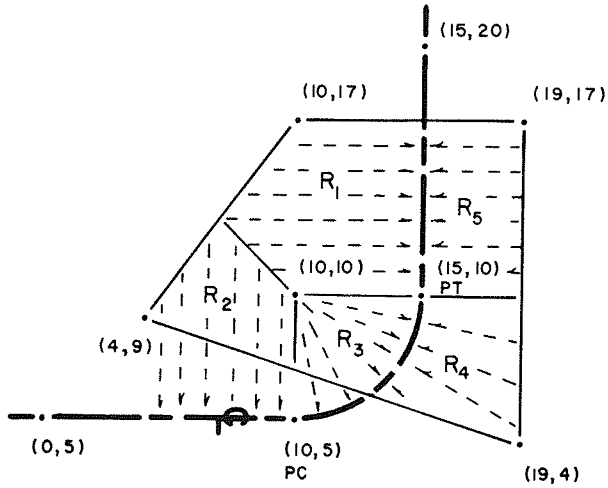

FIG. 7. Plan view of the cutting unit for example 2.

be quite small, perhaps on the order of 10^{-2}. When at the same time the variance of the straight line yarding distance is large it may be appropriate to use

[19] $\mathrm{Var}\{\omega\delta\} \approx w^2\mathrm{Var}\{\delta\}$

Concluding observations

It has been the purpose of this two-part paper to propose procedures appropriate to the analysis of continuous landing models. The procedures given should greatly improve the speed and accuracy with which turn location parameters are calculated. However, much work remains to be done.

On the theoretical side future research topics might include: (*i*) extension of these procedures to curves on sloping ground; (*ii*) relaxation of the assumption of statistical independence between yarding distance and the wander factor; (*iii*) development of optimal location models for continuous landings; and, (*iv*) derivation of formulas for shortest distance yarding parameters from a triangular planar region to a noncoplanar line segment.

On the application side, future work is needed in: (*i*) implementation of these procedures in computerized planning models; (*ii*) statistical testing of model predicted parameter values against observed values for actual settings; and (*iii*) development of equations for predicting expected values and variances of the wander factor under different yarding conditions.

TABLE 3. Intermediate and final composite area results for example 2

Region	$E\{\delta\}$	$E\{\delta^2\}$	$\mathrm{Var}\{\delta\}$	A
R_1	3.31	14.8	3.89	45.5
R_2	4.74	24.1	1.56	19.5
R_3	1.88	4.92	1.40	16.1
R_4	2.44	7.91	1.94	24.4
R_5	2.00	5.33	1.33	28.0
Total unit	2.91	11.7	3.24	133.5

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