The calculation of average yarding distance to a centralized landing, given a polygonal mesh approximation of the setting surface

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A new formula for average yarding distance is presented. This formula permits the development of an algorithm suitable for calculating average yarding distance under very general conditions when cable yarding to centralized landings. The minimal assumptions are that the surface of the setting is adequately approximated by a polygonal mesh and that a straight line segment provides an adequate description of the path followed by each turn from its setting location to the landing. Examination of the results obtained from a specific example offers tentative validation of the formula and the algorithm. The algorithm-calculated parameters are shown to closely match the corresponding statistics obtained by way of an independent simulation model. Both the theoretical basis and the computational efficiency of turn location parameter estimation in general logging engineering practice should be enhanced by this new formula and computational algorithm.

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Cet article présente une nouvelle formule pour calculer la distance moyenne de débusquage. La formule permet l'élaboration d'un algorithme pour calculer la distance moyenne de débusquage dans des conditions très générales de débusquage par câble vers une jetée centrale. Les hypothèses minimales sont à l'effet que la surface de l'emplacement soit évaluée de façon adéquate au moyen d'un filet polygonal et qu'un segment de ligne droite procure une description correcte du chemin suivi par chaque tour depuis son emplacement jusqu'à la jetée. L'examen des résultats d'un exemple concret présente une validation expérimentale de la formule et de l'algorithme. Les paramètres calculés de l'algorithme ressemblent de près aux valeurs statistiques correspondantes obtenues d'après un modèle de simulation indépendant. Le fondement théorique et l'efficacité de calcul de l'estimation des paramètres de localisation des tours dans la pratique générale du génie forestier devraient tous deux être haussés par cette nouvelle formule de l'algorithme qui l'accompagne.

[Traduit par la revue]

Introduction

A review of the conceptual development of average yarding distance (AYD) for central landings has been given in a recent publication (Greulich 1987). As noted in that review, a key paper in the theoretical development is that of Suddarth and Herrick (1964). Their paper also made a substantial contribution to the practical application of the theory. Among these contributions is a clear description of the practical yet analytically rigorous evaluation of composite areas. Donnelly (1978) extended their results and presented a computationally convenient and theoretically sound evaluation procedure for polygonal settings on level terrain. Garner (1979[1]), incorporating elevational differences, then applied these procedures to settings on steep terrain. In this procedure Garner employed a formula for the AYD of any triangular setting with an apical landing, a formula first given by Peters (1978). Because of the apical landing assumption, however, the procedure employed by Garner is, in a strict sense, only applicable to settings that exhibit uniform ground slope in all yarding directions from the landing (Greulich 1987).

The paper by Suddarth and Herrick (1964) suggests an alternative approach to the estimation of AYD for settings on steep, broken terrain. In this approach the horizontal area of the setting is subdivided into a finite number of mutually exclusive rectangles. The distance from the landing to the (terrain-projected) geometric center of each rectangle is weighted by its corresponding horizontal area. Summation of these weighted distances and division by the total horizontal area of the setting yields an estimate of the AYD. At the limit, as the number of subdividing rectangles goes to infinity and the maximum diagonal of the rectangles approaches zero, this summation yields the exact value for the AYD of the setting. It is simply a refined variation of this fundamental concept that is presented in the present paper.

The imprecision associated with, and the impracticality of, directly measuring yarding distance on proposed settings has favored the development of model-based estimation procedures. Models typically employed are contour maps or polygonal meshes of the setting surface. One source of estimation error in this approach is that arising from discrepancies in fit between an abstracted model surface and the actual physical surface of the setting. A second, independent source of error is the procedure by which the AYD of the fitted model is then calculated. It is the purpose of this paper to present a calculation procedure that can entirely eliminate this second source of error for polygonal mesh type models. It should be possible to amend this particular procedure in most AYD estimation programs that are based on the finite division of a setting surface into polygonal elements. For a given level of computational effort, very significant gains in the total precision of the AYD estimate should be realized from such modification.

The AYD formula

The computational algorithm to be presented employs a new formula for AYD. With reference to Fig. 1, consider a right-angled triangle *opi* that could be drawn in the $X'Y'$

[1]G. J. Garner. 1979. Cut-block area and average primary transport distance. Internal report of the Forest Engineering Research Institute of Canada, Pointe Claire, Quebec.

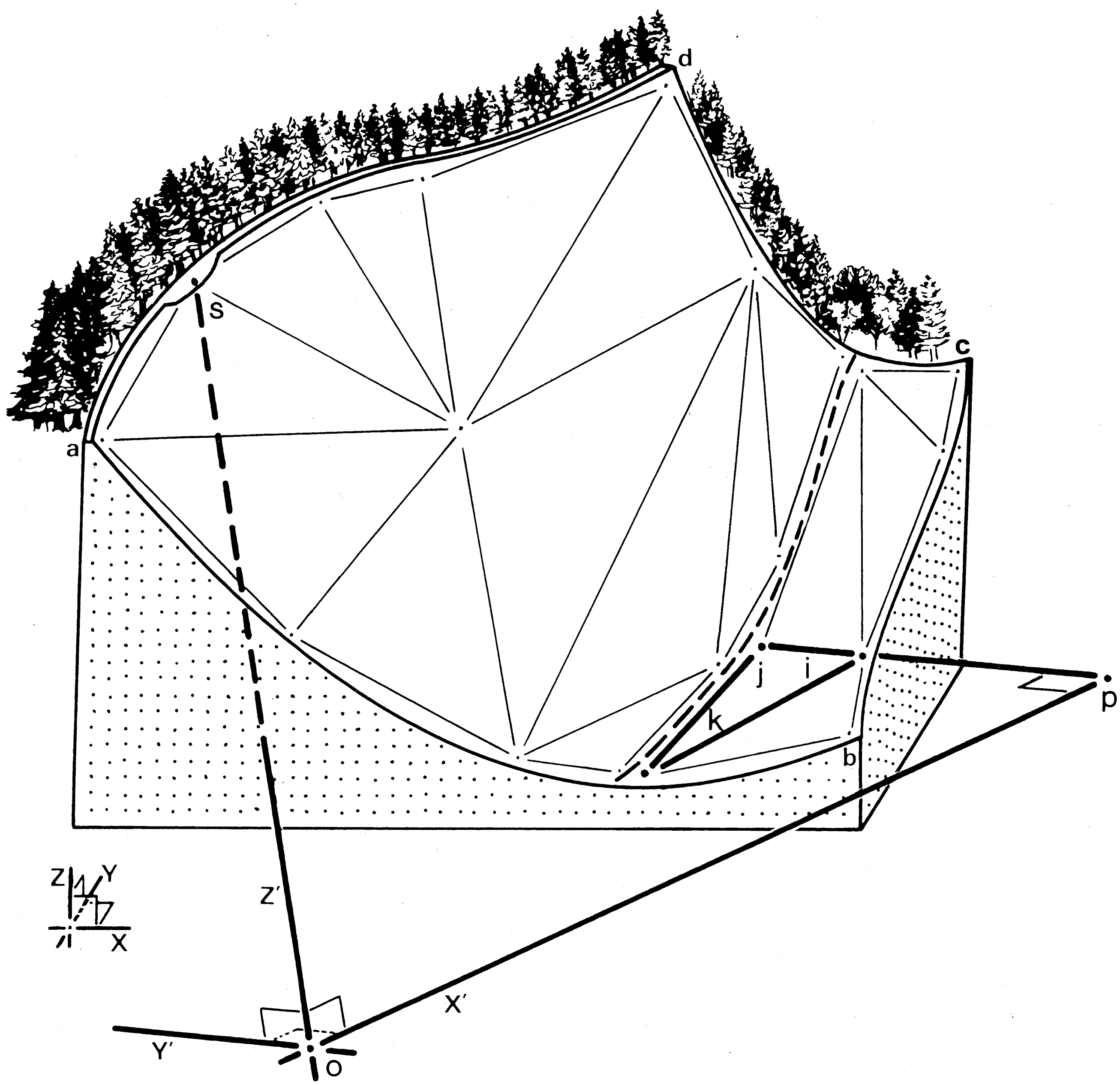

FIG. 1. Setting surface *abcd* has been fitted with two triangular meshes separated by a streamside leave strip. The heavier lines indicate one of the triangular elements, *ijk*, defining a plane $X'Y'$. The right-hand coordinate system $X'Y'Z'$ used in the analysis of side *ij* of this triangular element is placed as follows. First, the Z' axis is established. It is normal to the $X'Y'$ plane and passes through the landing location *s*. The direction of increasing elevation, with reference to the original *XYZ* coordinate system, is treated as positive. The X'-axis is then rotated within the $X'Y'$ plane to the position where it is perpendicular to the extended side *ij* of the triangular element.

plane, defined as being the plane of the triangle *ijk*. The lengths of its two legs will be denoted L_{op} and L_{pi}. This triangle is constructed so that the normal to the $X'Y'$ plane passing through the triangle's vertex *o* also passes through the landing location *s*. The distance from the landing to the three vertices of the triangle *opi* will be denoted L_{so}, L_{sp}, and L_{si}.

With reference to triangle *opi*, and beginning with the usual assumptions, which are (*i*) a uniform distribution of turns over the horizontal area of the triangle, and (*ii*) the straight line yarding of these turns to the landing from their lay on the triangle, the following formula for the AYD of triangle *opi* can be derived:[2]

$$[1] \quad \text{AYD} = \left[\frac{L_{si}}{3}\right] + \left[\frac{L_{op}^2 + 3L_{so}^2}{3L_{pi}}\right]\left[\ln\left(\frac{L_{pi} + L_{si}}{L_{sp}}\right)\right] - \left[\frac{2L_{so}^3}{3L_{op}L_{pi}}\right]\left[\tan^{-1}\left(\frac{L_{op}L_{pi}}{L_{sp}^2 + L_{so}L_{si}}\right)\right]$$

[2]The derivation is quite long. Copies may be obtained from the author upon written request.

TABLE 1. Example of a triangular mesh evaluated for average yarding distance by both analytical and simulation methods

Triangular element	Vertex	Coordinates (X,Y,Z)	Analytical model Area	Analytical model AYD	Simulation model Sample size	Simulation model $\overline{\text{AYD}}$	Simulation model SE
A	1	1000, 1000, 500	125 000	559.01	250 191	559.02	0.29
	2	500, 500, 500					
	3	500, 0, 500					
B	1	500, 0, 500	625 000	901.33	1 249 459	901.25	0.22
	2	2000, 500, 1000					
	3	1000, 1000, 500					
C	1	1000, 1000, 500	250 000	926.29	499 561	926.69	0.34
	2	2000, 500, 1000					
	3	1000, 1500, 1000					
Landing		500, 1000, 500					
Setting*			1 000 000	864.78	1 999 211	864.78	0.19

*Setting values have been calculated by aggregation of the results listed in the table for the individual triangular elements.

Computational algorithm

Given this new AYD formula it is now possible to formulate a general algorithmic solution for a polygonal mesh. This sequential procedure will yield the AYD for any setting surface that is everywhere visible from the landing location and that has turns uniformly distributed over its projected horizontal area.

1. Fit polygonal mesh(es) to the setting surface (*abcd* of Fig. 1).
2. Select a polygonal element (e.g., *ijk* of Fig. 1) for evaluation.
3. Find the plane $(X'Y')$ of the polygonal element.
4. Calculate the coordinates of the point *o* where the plane has a normal that passes through the location of the landing (*s*).
5. Select and extend a side (*ij*) of the polygonal element.
6. Calculate the coordinates of the point (*p*) where the extended side has a perpendicular that passes through the point (*o*) found in step 4.
7. Calculate the AYD and the horizontal area for each of the two right-angled triangles (*opi*, *opj*) formed by the two points found in steps 4 and 6 and the vertices associated with the current polygonal element side, each vertex being taken in turn.
8. Using the composite area rule and the results of step 7, calculate the AYD for the triangle (*oij*) formed by the point found in step 4 and the two vertices associated with the current side. Find the horizontal area of this triangle.
9. If all sides of the current polygonal element have been analyzed, continue to step 10, otherwise return to step 5.
10. Using the AYDs and horizontal areas associated with each side of the polygonal element, calculate the composite AYD of the polygonal element. Calculate its horizontal area.
11. If all polygonal elements of the mesh(es) have been evaluated, continue to step 12, otherwise return to step 2.
12. Employing AYDs and horizontal areas which have now been determined for all polygonal elements in the mesh(es), calculate the composite AYD for the mesh(es).

The first step in this algorithm is discussed elsewhere and the interested reader should consult the paper by Eli et al. (1984) for an example of its application within the context of forest engineering. Analytical procedures based on composite areas and vector algebra can be used to facilitate programming implementation of subsequent steps. The interested reader should refer to Donnelly (1978) and Perkins and Suddarth (1970) for forest engineering related examples of these respective procedures.

Application

The algorithm given in the previous section will now be applied to a specific example. The objectives are threefold: to establish an initial level of confidence in the validity of the algorithm, to further illustrate the nature of its application, and to provide a numerical check for subsequent program development by others.

The previously listed algorithm (excluding the first step) was programmed for the Hewlett-Packard HP-41CX. The triangulation mesh listed in Table 1, in the first three columns ("triangular element," "vertex," and "coordinates"), was evaluated for the specified landing location, and the results are given in Table 1 in the columns headed "analytical model".

A simulation model was then written and used as a standard of comparison. Each triangular element was examined in turn. Points were randomly distributed over the horizontal projected area of each triangular element at an intensity of approximately two points per unit of horizontal area. The actual number of such points in each case is given in the column showing the sample size of the simulation model in Table 1. Each point was projected onto its corresponding triangular surface in three dimensions and its distance from the landing location was then computed. The mean distance was calculated for each of the three samples and is recorded in the column headed $\overline{\text{AYD}}$. The standard error for each of these estimates of AYD is given in the last column. The close agreement between the corresponding values of AYD and $\overline{\text{AYD}}$ is offered as supporting evidence of procedural validity. In one case (triangular element A) the AYD can also be calculated by means of a previously derived and generally accepted formula (Peters 1978). This alternative calculation was performed and the same result was obtained for the AYD.

Final observations

The algorithm presented in this paper can provide the basis for a computationally efficient means of estimating AYD. The procedure is quite general and should be applicable to most cable settings encountered in practice. Even so, a further generalization of this basic algorithm is possible. Nonuniform distribution of turns can be handled by additional surface partitioning and the application of appropriate weights in the composite formulas (Donnelly 1978). It is worth noting that in general, the assumption of uniform turn distribution over an area is not as restrictive as it might initially seem. Within any given partition it simply reflects the lack of more specific knowledge with regard to the actual spatial distribution of the turns (see Appendix). Thus, the only major restriction on the generality of the algorithm would be the straight line yarding assumption. For those settings with topography that places a significant proportion of the turns well out of sight of the landing, the algorithm may not describe with sufficient accuracy the path followed by a typical turn.[3]

Other turn location parameters, such as average yarding slope and the expected square of the yarding distance, might be developed in a similar fashion. This approach to their estimation would provide both a firm theoretical formulation and a very general range of applicability, in addition to increased computational efficiency.

The analytical underpinning of this procedure may also provide the basis for some interesting advances in the theory, the development and comparison of approximating formulas being one especially challenging area of investigation.

[3]A necessary and sufficient condition for the existence of landing-concealed ground on a setting is that $Z_o > Z_s$ for at least one element of the mesh.

Appendix

If no information other than boundary location is available with regard to the horizontal distribution of turns across an area, what can be said about the nature of an "appropriate" distribution? One approach to this question might involve distribution characterization through maximum entropy (ME). Applying this methodology, as suggested by Shannon (1948), the problem stated previously may be written as follows:

$$\text{[A1]} \quad \text{maximize } I = \iint_D -W(x,y)\ln W(x,y)\, dx\, dy$$

$$\text{[A2]} \quad \text{subject to } \iint_D W(x,y)\, dx\, dy = 1$$

A (density) function $W(x,y)$ is sought which will maximize the entropy I over the area D in the XY plane while concurrently satisfying a normalization constraint across the same area. This is a two independent variable isoperimetric problem of the calculus of variations. Following Weinstock (1952), the unknown function $W(x,y)$ must satisfy

$$\text{[A3]} \quad \frac{\partial}{\partial \mathrm{W}} [-\mathrm{W}(\mathrm{x,y})\ln W(x,y) + \lambda W(x,y)] = 0$$

from which

$$\text{[A4]} \quad W(x,y) = e^{\lambda - 1}$$

As λ (the Lagrange multiplier) is a constant, $W(x,y)$ must also be constant. From the constraint [A2] it is then concluded that

$$\text{[A5]} \quad W(x,y) = 1 \Big/ \iint_D dx\, dy = 1/\text{area}$$

The uniform density is thus seen to represent the "appropriate" choice by the ME criterion. By the principle of ME, it is then the broadest possible distribution consistent with the given information (boundary location); or, in the words of Jaynes (1957), "It is the least biased estimate possible on the given information; i.e., it is maximally noncommittal with regard to missing information."

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