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The Quantitative Description of Cable Yarder Settings—Parameters for the Triangular Setting with Apical Landing

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ABSTRACT. The design and operation of an economically efficient timber harvesting system requires the effective characterization of the individual setting. Two contributions are made toward this objective. First, closed-form solutions for several well-established parameters of turn location, including average yarding distance and average yarding slope, are derived for triangular settings on uniform slope with an apical landing. These derivations are greatly facilitated by a new expected value formulation. Second, the new concept of an effective external boundary is presented with one possible definition of its measure for any setting. Closed-form solutions are then obtained for the effective external boundary and the average external yarding distance on any triangular setting with an apical landing. FOR. SCI. 33(3):603–616.

ADDITIONAL KEY WORDS. Logging, timber harvesting, production analysis, forest engineering.

FROM THE TIME OF ITS INCEPTION during the first decade of this century, the profession of logging engineering has addressed the challenge of forest harvesting and transportation on difficult terrain. The development of an economically sound forest transportation system was then, as it is now, contingent on accurate prediction of the costs of primary and secondary log transportation (Peed 1910, Greulich 1985). Central to the prediction of primary transportation cost is the objective description of the individual setting. Terms such as *yarding distance* and *slope* must be given unambiguous definitions amenable to engineering measurement.

By the early 1930s, numerical procedures for the estimation and use of average yarding distance (*AYD*) were well established (Hughes 1930, Bradner et al. 1933). These procedures and the assumptions on which they were developed are in general accordance with those found in current theory and practice. It was commonly known, for example, that for a yarding circle in stands of uniformly distributed density and size, the *AYD* is two-thirds the external yarding distance (Munger and Brandstrom 1931, Brandstrom 1933, Worthington 1932).

Unfortunately, it would appear that no analytical model of significant depth was developed and published as the basis for these numerical procedures. Perhaps as a consequence, the later introduction of erroneous estimation procedures based on "center-of-gravity" and "equal area" arguments (Matthews 1942) went unchallenged for more than 20 years.

Logging engineers were finally provided with an appropriate analytical model in 1964 (Suddarth and Herrick.)[1] This model has a firm theoretical basis in geometrical probability where the properties of figures formed by

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[1] The apparently independent development of a similar analytical model occurred in transportation engineering (Smeed 1963).

the random distribution of points along lines, over plane surfaces, and in three-dimensional space has been studied by statisticians for more than 100 years (Kendall and Moran 1963). The analytical model of Suddarth and Herrick was confined to the derivation of *AYD* for level settings of simple geometric shape. Peters (1978) later derived the *AYD* for any triangle under the same assumptions. The general applicability of Peters' result was quickly appreciated. Donnelly (1978) described a general analytical procedure for the *AYD* of any polygonal setting on level ground, and Garner (1979), following Donnelly's procedure, applied Peters' *AYD* formula to polygonal settings on ground with significant slope. Further extension of the analytical model to settings located on uniform side hill slopes resulted in the derivation of formulas for the average yarding slope (*AYS*) as well as the *AYD* for several simple geometric shapes (Greulich 1980).

Based on their analytical model, Suddarth and Herrick suggested a numerical procedure for practical problems. This procedure is comparable to that employed by Hughes (1930) and Bradner and other early practitioners (1933). Suddarth and Herrick also suggest that the effect of slope on yarding distance can be incorporated into the numerical procedure. Computerized numerical methods developed since the seminal paper by Suddarth and Herrick have generally allowed for some treatment of slope when it is present (Peters and Burke 1972, Twito and Mann 1979, Young and Lemkow 1976, Gibson 1978, Dykstra and Riggs 1977, Perkins and Lynn 1979). Most of these papers do not discuss the manner in which they handle slope, and none of them offer any theoretical justification. Simulation procedures have also been employed (Dykstra 1976, Greulich 1981). While simulation procedures may have great generality and theoretical justification, their application is somewhat hampered by long computational times, the calculation of model statistics rather than the parameters themselves, and computer hardware requirements.

This paper has a two-fold purpose. First, an analytical model applicable to the evaluation of turn location parameters (e.g., *AYD* and *AYS*) for settings located on uniform slopes will be provided. These new formulas for settings of plane triangular shape when used in conjunction with the well-known rules for composite areas will now permit the analytical evaluation of polygonal settings over uniform slopes. The second purpose of the paper is to introduce the concept of the effective external boundary. This new concept will then provide the basis for the derivation of the average external yarding distance for a triangular setting. The application of rules for composite lines will permit the calculation of average external yarding distance for any polygonal setting.[2]

A GENERAL FORMULA

A useful and quite general formula to be used in subsequent calculations will now be derived.[3] The following assumptions are required:

1. The setting has the shape of a plane triangle and is located on a uniform slope.
2. There is a uniform distribution of turns over the horizontal area of the setting.

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[2] Numerical procedures for the field evaluation of settings based on these results have been developed and will be presented elsewhere.

[3] In the interest of economy, derivations are either much abbreviated or omitted altogether. The interested reader may obtain the complete derivations by request from the author.

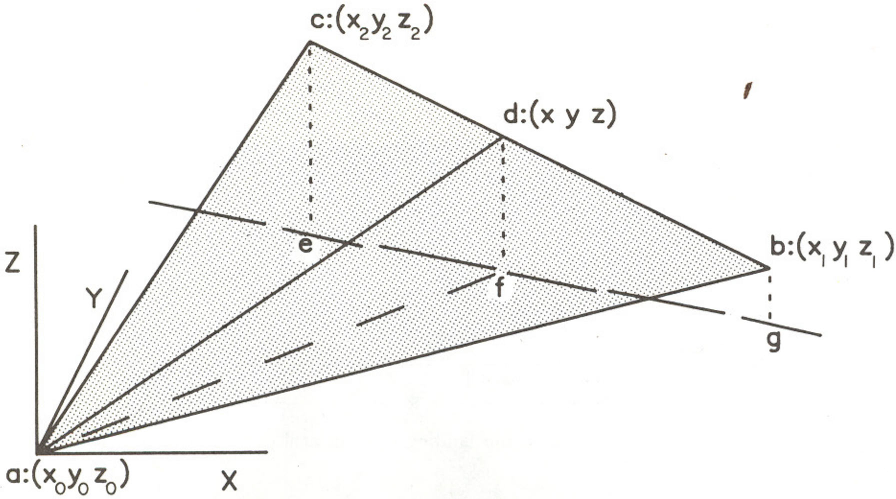

FIGURE 1. Perspective view of the setting ground surface (plane triangle *abc*) with the landing at point *a* and the external boundary along *bc*. The line segment *efg* is the projection of the external boundary onto the horizontal *XY* plane.

3. The central landing is located at one vertex of the plane triangle, and turns are yarded straight into it.[4]

In cylindrical coordinates, where a point in space is given by its polar coordinates *R* and θ in the *XY* plane and its cartesian coordinate *Z*, a curve *C* may be represented by a pair of equations

C:

$$F(R,\theta,Z) = 0 \tag{1a}$$

$$G(R,\theta,Z) = 0 \tag{1b}$$

where the curve is the intersection of these two surfaces. A line may be written in the explicit form

C:

$$R = R(\theta) \tag{2a}$$

$$Z = Z(\theta) \tag{2b}$$

Hence the straight line, segment *bc* of which forms the external boundary of the setting, may be written in this fashion. Without loss of generality, the origin of the coordinate system is placed at the landing, and the *XY* plane is made horizontal (Figure 3).

Consider now a line from the landing *a* to an arbitrary point *d* located on the external boundary (Figure 2) with horizontal and vertical components, *R* and *Z*. Under the previously stated assumptions and by the usual definition of the expected value of a function (Mood and Graybill 1963):

$$E\{k\delta^a\zeta^b\rho^c\} = \frac{\int_{\theta_1}^{\theta_2}\int_0^{R(\theta)} k\delta^a\zeta^b\rho^c\ \rho\ d\rho\ d\theta}{\int_{\theta_1}^{\theta_2}\int_0^{R(\theta)} \rho d\rho d\theta} \tag{3}$$

[4] A perspective view of the setting is given in Figure 1. Frequent reference to this figure and related Figures 2 and 3 will assist the reader in the development that follows.

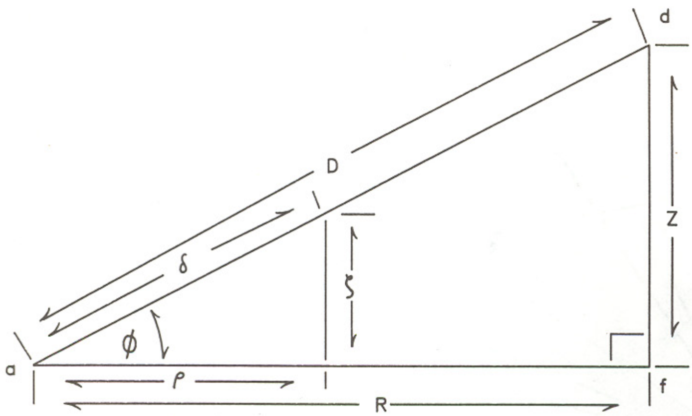

FIGURE 2. Terrain cross-section from the landing *a* to an arbitrary point *d* on the external boundary *bc*.

Where k is an arbitrary constant and a, b and c are arbitrary rational numbers except that $a + b + c \neq -2$. Using the following relationships from Figure 2,

$$\delta = (\rho^2 + \zeta^2)^{1/2} \tag{4a}$$

$$\zeta = (\rho/R)Z \tag{4b}$$

$$D = (R^2 + Z^2)^{1/2} \tag{4c}$$

it is found that

$$\delta^a \zeta^b \rho^c = D^a R^{-a-b} Z^b \rho^{a+b+c} \tag{5}$$

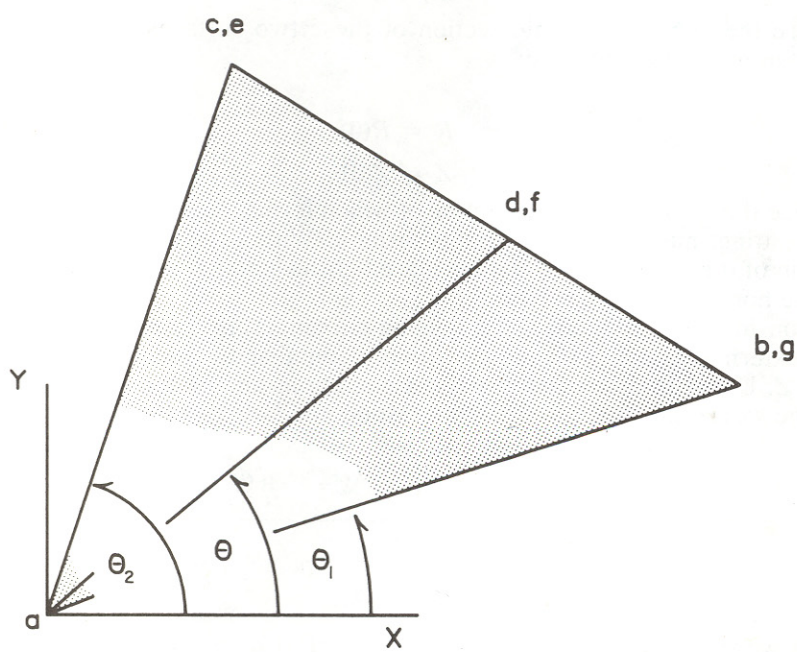

FIGURE 3. Plan view of the setting *abc*.

Substitute (5) into (3) and integrate out ρ to obtain:

$$E\{k\,\delta^a\zeta^b\rho^c\} = \left[\frac{2k}{a+b+c+2}\right]\frac{\int_{\theta_1}^{\theta_2} D^aZ^bR^cR^2d\theta}{\int_{\theta_1}^{\theta_2} R^2d\theta} \tag{6}$$

A variable transformation is now carried out based on the following parametric equations for the projected line segment *eg:*

$$x = x_1 + \lambda(x_2 - x_1) \tag{7a}$$

$$y = y_1 + \lambda(y_2 - y_1) \tag{7b}$$

for $0 \leq \lambda \leq 1$,
then

$$\theta = \tan^{-1}\left\{\frac{y_1 + \lambda(y_2 - y_1)}{x_1 + \lambda(x_2 - x_1)}\right\} \tag{8}$$

and

$$\lambda = \frac{x_1 \tan\theta - y_1}{(x_1 - x_2)\tan\theta + (y_2 - y_1)} \tag{9}$$

with

$$d\lambda = \left[\frac{(x_1y_2 - x_2y_1)\,(1 + \tan^2\theta)}{[(x_1 - x_2)\tan\theta + (y_2 - y_1)]^2}\right] d\theta \tag{10}$$

It is easily shown that

$$R^2 = \left[\frac{x_1y_2 - x_2y_1}{(x_1 - x_2)\tan\theta + (y_2 - y_1)}\right]^2 [1 + \tan^2\theta] \tag{11}$$

so that

$$d\lambda = \left[\frac{R^2}{x_1y_2 - x_2y_1}\right] d\theta \tag{12}$$

Using Equation (12) and limits of integration obtained with Equation (9), the expected value is given in much simplified form as:

$$E\{k\delta^a\zeta^b\rho^c\} = \left[\frac{2k}{a+b+c+2}\right]\int_0^1 D^aZ^bR^c d\lambda \tag{13}$$

Where D, Z, and R, previously functions of θ, are now yet to be specified functions of λ. The formulation of the expected value given by Equation (13) can now be used to calculate some of the setting parameters associated with turn location.

AYD and AYS

Before proceeding to the calculation of setting parameters, it is convenient to define the following directed vectors with reference to Figure 1:

$$P_i^T: \quad (x_i - x_0, y_i - y_o, z_i - z_0) \qquad i = 1,2 \tag{14a}$$

$$H_i^T: \quad (x_i - x_0, y_i - y_0, 0) \qquad i = 1,2 \tag{14b}$$

$$V_i^T: \quad (0, 0, z_i - z_0) \qquad i = 1,2 \tag{14c}$$

Then for an arbitrary point d on the segment bc:

$$P = P_1 + \lambda(P_2 - P_1) \tag{15a}$$

$$H = H_1 + \lambda(H_2 - H_1) \tag{15b}$$

$$V = V_1 + \lambda(V_2 - V_1) \tag{15c}$$

for some $0 \leqslant \lambda \leqslant 1$.
Noting that

$$D = \|P\| = (P^T P)^{1/2} \tag{16}$$

the slope distance ad can be written:

$$D(\lambda) = [P_1^T P_1 + 2\lambda P_1^T(P_2 - P_1) + \lambda^2(P_2 - P_1)^T(P_2 - P_1)]^{1/2} \tag{17a}$$

likewise:

$$R(\lambda) = [H_1^T H_1 + 2\lambda H_1^T(H_2 - H_1) + \lambda^2(H_2 - \mathrm{H}_1)^T(H_2 - H_1)]^{1/2} \tag{17b}$$

and

$$Z(\lambda) = (z_1 - z_0) + \lambda(z_2 - z_1) \tag{17c}$$

It is also convenient to define

$$L_1^2 \equiv D^2(0) = P_1^T P_1 \tag{18a}$$

$$L_2^2 \equiv D^2(1) = P_1^T P_1 + 2P_1^T(P_2 - P_1) + (P_2 - P_1)^T(P_2 - P_1) \tag{18b}$$

$$L_3^2 \equiv (P_2 - P_1)^T(P_2 - P_1) \tag{18c}$$

noting also then that

$$2P_1^T(P_2 - P_1) = L_2^2 - L_1^2 - L_3^2 \tag{19}$$

Equation (13) can now be employed to find the AYD. Since the AYD is the expected value of δ, it can be written as:

$$AYD = E\{1\delta^1\zeta^0\rho^0\} = E\{\delta\} = \frac{2}{3}\int_0^1 D(\lambda)d\lambda \tag{20}$$

Straightforward integration with utilization of Equations (18a-c), (19) and simplification lead to the final result:

$$AYD = \left\{\frac{1}{6L_3^2}\right\}\left\{[L_1 + L_2]\,[L_3^2 + (L_1 - L_2)^2] - \left[\frac{1}{2L_3}\right][L_3^2 - (L_1 - L_2)^2]\,[L_3^2 - (L_1 + L_2)^2] \left[\log_e\left(\frac{L_1 + L_2 + L_3}{L_1 + L_2 - L_3}\right)\right]\right\} \tag{21}$$

This same formula for AYD may be more easily derived by restricting attention to horizontal triangles. In fact, Equation (21) is the same as that given in a publication by Peters (1978) where it was initially applied to horizontal triangles. It was later applied by Garner (1979) to triangles in three dimensions. That an AYD formula derivable in two dimensions is immediately applicable to similar plane figures in three dimensions may be somewhat unexpected. This basic equivalence, anticipated by Garner, is based on the fact that any arbitrarily selected infinitesimal area on the surface of a three-dimensional plane figure differs only by a constant factor, invariant across the figure, from its infinitesimal projected area on the horizontal plane. Hence the integration for the *AYD* of three-dimensional plane figures can be carried out (in two dimensions) across the plane of the figure yielding the same basic formulas for *AYD* in three dimensions as were obtained for two dimensions. All lengths appearing in formulas so derived represent slope measurements.

Using the fact that the surface area A_s of the triangle *abc* may be written as

$$A_s = \left\{ \frac{-[L_3^2 - (L_1 - L_2)^2][L_3^2 - (L_1 + L_2)^2]}{16} \right\}^{1/2} \tag{22}$$

and defining

$$r \equiv L_3/(L_1 + L_2) \tag{23}$$

Equation (21) may be written in more compact form:

$$AYD = \left[\frac{L_1 + L_2}{6}\right]\left[1 + \left(\frac{L_1 - L_2}{L_3}\right)^2\right] + \left[\frac{4A_s^2}{3L_3^3}\right]\left[\log_e\left(\frac{1 + r}{1 - r}\right)\right] \tag{24}$$

Either Equation (21) or Equation (24) will give the exact value for the *AYD;* however, for some purposes, an approximation may be more convenient. By employing the first term of the following expansion:

$$\log_e\left(\frac{1 + r}{1 - r}\right) = 2\left[r + \frac{r^3}{3} + \frac{r^5}{5} + \ldots\right] \qquad r^2 < 1 \tag{25}$$

the following approximation may be obtained:

$$AYD \cong \left[\frac{L_1 + L_2}{3}\right] - \left[\frac{L_3^2 - (L_1 - L_2)^2}{(6)(L_1 + L_2)}\right] \tag{26}$$

Equation (26) gives an underestimation of the *AYD*. The approximation is quite good, even for values of r relatively close to 1.

A second approximation of very simple form may be obtained for those applications where L_3 is, or can be made, relatively small when compared to both L_1 and L_2. The angle *bac* of Figure 1 is denoted as τ, and the law of cosines is used to examine the relationship between τ, L_1, L_2, and L_3. From this general relationship it may be seen that as $L_3 \rightarrow 0$ both L_1 and L_2, if only constrained to be positive real numbers, must converge upon a common limit. Denote this common limit, an arbitrary positive real number, as L. At the limit then, $L_3 = 0$ and $L_1 = L_2 = L$. At points other than the limit it is assumed that there are no restrictions on the values of τ, L_1, and L_2 except that they be positive real numbers and satisfy the law of cosines. Since τ, also is unspecified (except at the limit), there is no unique functional relationship between the L_i; specifically it is possible to set $L_1 = L_2 = L$ for all values of L_3. Now taking the limit of Equation (21) as $L_3 \rightarrow 0$ and using $L_1 = L_2 = L$ it is found after several applications of L'Hôpital's rule that $AYD \rightarrow (2/3)L$.

When for a specific triangular setting the adjacent sides L_1 and L_2 do not greatly differ in length and in comparison with them L_3 is relatively small, a good approximation to the *AYD* can be obtained by setting $L = (L_1 + L_2)/2$ yielding the formula

$$AYD \cong (L_1 + L_2)/3 \tag{27}$$

This formula is the same as that suggested by Perkins and Lynn (1979) as an approximation to the average skidding distance on rough terrain. The simplicity of Equation (27) as compared to Equation (26) must be weighed against its significantly lower accuracy when L_3 is not sufficiently small relative to both L_1 and L_2. It should also be noted that having a small angle τ for a specific setting does not necessarily imply that this relative side length condition is adequately achieved.

What appears to be a somewhat better approximation than Equation (27) is one suggested by Donnelly (1978). In this approach L is set equal to the slope length of the line from the landing to the midpoint of the external boundary. The same caveat with regard to the relative lengths of the triangle sides applies to this approximation as well.

The *AYS* is found by evaluating

$$AYS = E\{-100\delta^0\zeta^1\rho^{-1}\} = -100 \int_0^1 \frac{Z(\lambda)}{R(\lambda)}\, d\lambda \tag{28}$$

Where k has been set equal to a negative 100 in order to convert to percent and to conform with the usual sign convention on yarding slope. Substitutions of Equations (17b–c) followed by integration lead to

$$AYS = \left[\frac{(100)(z_2 - z_1)(L_{1H} - L_{2H})}{L_{3H}^2}\right] + \left[\frac{50}{L_{3H}^3}\right]\left[(z_1 - z_2)(L_{1H}^2 - L_{2H}^2) + (2z_0 - z_1 - z_2)(L_{3H}^2)\right]\left[\log_e\left(\frac{L_{1H} + L_{2H} + L_{3H}}{L_{1H} + L_{2H} - L_{3H}}\right)\right] \tag{29}$$

Where the subscript H denotes the horizontal component of the indicated setting side length.

Using the first term of Equation (25), the following approximation is obtained:

$$AYS \cong [-100]\left[\frac{(z_1 - z_0) + (z_2 - z_0)}{L_{1H} + L_{2H}}\right] \tag{30}$$

Other parameters may also be calculated in a similar fashion. As a final example, some researchers have found that the square of the yarding distance is a significant variable in predicting yarding cycle time (Tennas et al. 1955, Hensel and Johnson 1979). In order to avoid bias in the estimation of cycle time, the expected value of yarding distance squared should be calculated (Greulich 1981). The expected value of this variable is easily found

$$E\{\delta^2\} = \frac{1}{12}[3L_1^2 + 3L_2^2 - L_3^2] \tag{31}$$

It should be noted that since

$$\text{Var}\{\delta\} = E\{\delta^2\} - E^2\{\delta\} \tag{32}$$

the variance of the yarding distance may now be calculated. This variance estimate can be used in the establishment of confidence intervals when predicting cycle times or production levels as a linear function of yarding distance. The variance of yarding slope can be calculated and employed in a like manner.

EFFECTIVE EXTERNAL BOUNDARY

In the development that immediately follows, all of the ground in the setting covered with timber to be cut and yarded to a single landing (the "setting cut area") is described mathematically as three-dimensional surface(s) bounded by a finite set of smooth twisted curves $\{C_i | i = 1, n\}$. No assumption with regard to the nature of the ground relief across the setting is required. The external yarding boundary of a cable setting is then defined to be the set of points in the setting cut area, each of which is more distant horizontally from the landing than any other point within the setting cut area on its particular bearing. Figure 4 illustrates this concept where it may be seen that the external yarding boundary of the setting can be described by a set of curves $\{C_j | j = 1, m\}$ selected from curves or portions of the curves in the set $\{C_1 \ldots\ldots C_n\}$. It is assumed that these external yarding boundary curves may be represented by continuously differentiable vector functions:[5]

$$C: \qquad P = P(\lambda) \qquad a \leqslant \lambda \leqslant b \tag{33}$$

The tangent vector of C at an arbitrary point is given as:

$$\dot{P} = dP(\lambda)/d\lambda \tag{34}$$

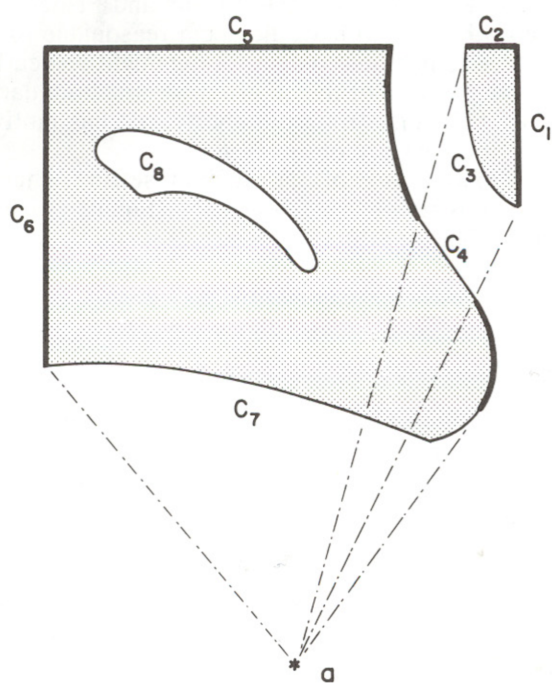

FIGURE 4. Plan view of a cable setting. The bolder solid lines indicate the external yarding boundary when the landing is located at a.

[5] The subscript j is understood to apply although omitted for simplicity of notation.

The unit vector perpendicular to P and in the plane of P and $\dot{P}$ is denoted U. The restrictions on U can be written as

$$\|U\| = 1 \tag{35a}$$

$$U^T P = 0 \tag{35b}$$

$$U = \alpha P + \beta \dot{P} \qquad \alpha, \beta\text{:scalars} \tag{35c}$$

from which it follows that

$$U = \frac{\dot{P} - \left(\dfrac{\dot{P}^T P}{P^T P}\right) P}{\left\|\dot{P} - \left(\dfrac{\dot{P}^T P}{P^T P}\right) P\right\|} \tag{36}$$

The effective external boundary (EEB) will now be defined by

$$EEB \equiv \sum_j \int U^T \dot{P} d\lambda \tag{37}$$

where the integral is evaluated for each curve C_j on the external yarding boundary and summed to a total for the setting. Before continuing with an application of this definition of the effective external boundary, a parenthetical comment is in order.

While it is unmistakably clear that the number of cable roads per setting is an important variable in production estimation, the calculation of its value currently follows ad hoc procedures (Robinson and Fisher 1983). For some cable systems (e.g., high lead) it would seem reasonable to anticipate that the number of cable roads required on a setting might be nearly proportional to the ratio of some measure of the external yarding boundary to the length of the choker line. It is this strong possibility that has motivated the foregoing definition of the measure of the EEB.

Returning now to the case of a triangular setting where the external yarding boundary consists of line segment bc (Figure 1) it may be shown, after some algebra, that

$$U = \frac{P_2 - KP_1}{\|P_2 - KP_1\|} \tag{38}$$

where

$$K = \frac{P_2^T P_1 + \lambda P_2^T (P_2 - P_1)}{P_1^T P_1 + \lambda P_1^T (P_2 - P_1)} \tag{39}$$

and from (15a) that

$$\dot{P} = P_2 - P_1 \tag{40}$$

With reference to Figure 5, it is seen that an infinitesimal segment $(P_2 - P_1)d\lambda$ of the external yarding boundary is projected onto the unit length vector U. Integration of this projected length over the full course of the external yarding boundary yields the EEB. It should be noted here that the EEB so defined is a slope distance. No restrictions have been placed on the nature of the terrain across the setting; specifically, the assumption of uniform slope is not required.

Evaluation of the integral

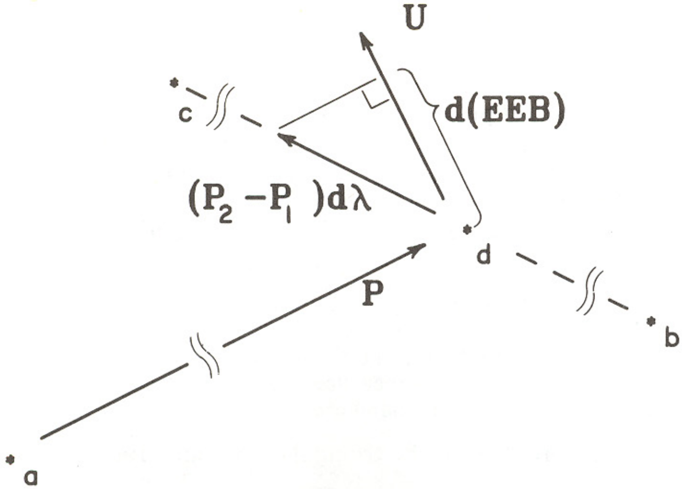

FIGURE 5. The infinitesimal $d(EEB)$ of the effective external boundary at arbitrary point d shown as the component of the external yarding boundary infinitesimal $(P_2 - P_1)d\lambda$ projected onto the unit length vector U, which is perpendicular to P in the plane of the triangle abc.

$$EEB = \int_0^1 U^T \dot{P} d\lambda \quad (41)$$

involves tedious algebra but eventually yields the gratifyingly compact result that

$$EEB = \left[\frac{2A_s}{L_3}\right]\left(\log_e\left(\frac{1 + r}{1 - r}\right)\right) \quad (42)$$

Once again, using the first term of Equaton (25) yields an approximation

$$EEB \cong \frac{4A_s}{L_1 + L_2} \quad (43)$$

AVERAGE EXTERNAL YARDING DISTANCE

In defining the average external yarding distance ($AEYD$), as in the other definitions presented, it is essential to know the ultimate use of the parameter. For example, $AEYD$ could be based on at least two different definitions. First, the time required to rig a cable road is sometimes related to the external yarding distance (Peters 1972). In this case an arguably appropriate definition might be given by:

$$AEYD_1 \equiv \left[\frac{1}{EEB}\right] \int_{EEB} \|P\| d(EEB) \quad (44)$$

However, it has also been found that yarding cycle time delays may be related to the external yarding distance (Tennas et al. 1955). In this second case the following definition would appear to be more appropriate than the first

$$AEYD_2 \equiv \left[\frac{1}{A}\right] \int_{\theta_A} \int_{\rho_A} D_e(\theta)\ \rho d\rho d\theta \quad (45)$$

where A is the horizontal projection of the setting cut area surface and $D_e(\theta)$ is the straight line distance from the landing to the external yarding boundary. Whether one or both definitions should be called on clearly depends on the specific application.

Restricting attention to the first application and definition the following assumptions apply:

1. The tailholds will be located along the external yarding boundary of the setting.
2. Every small interval on the *EEB* has the same probability of being selected as the location for a cable tailhold (uniform distribution).
3. The external yarding distance is measured as the straight line distance from the landing to the external yarding boundary.

In applying this definition to the triangular setting, a fourth assumption is that:

4. The external yarding boundary consists of a single line segment.

The derivation is quite easy and yields

$$AEYD = \frac{L_3}{\log_e\left(\frac{1 + r}{1 - r}\right)} \quad (46)$$

It is observed that

$$A_s = \tfrac{1}{2}(AEYD)(EEB) \quad (47)$$

which might have been anticipated from a careful consideration of the definitions for *AEYD* and *EEB*.

An approximation to Equation (46) may be obtained by letting $L_3 \rightarrow 0$ with $L_1 = L_2 = L$ at the limit. Under these conditions it is found that $AEYD \rightarrow L$. The use of $L = (L_1 + L_2)/2$ provides the estimate

$$AEYD \cong (L_1 + L_2)/2 \quad (48)$$

Since this same approximation formula can also be obtained through the use of the first term of the series in Equation (25) it should provide good, though slightly high, estimates of the *AEYD* for those settings where L_3 is relatively small in comparison to $L_1 + L_2$.

Finally, then, it should be noted that for the apical landing–triangular setting of uniform slope, uniform turn distribution, and no dead ground

$$\lim_{L_3 \rightarrow 0} \{AYD\} = (2/3) \lim_{L_3 \rightarrow 0} \{AEYD\} \quad (49)$$

For a circular setting on level terrain, all of the infinitesimally small circular sectors into which the area can be divided have the same weight when calculating the *AYD* for the entire circle. Likewise each of the infinitesimal arcs of the circle, collectively forming the external boundary, have equal weight in the calculation of the *AEYD*. From these two observations and Equation (49) it readily follows that yarding circles in level, continuous stands of uniformly distributed density and size have an *AYD* that is two-thirds their external yarding distance. All of those concepts first linked by very similar statements attributable to logging engineers of the 1930s have now been firmly grounded in the theory of timber harvesting.

CONCLUDING STATEMENT

The logging engineer faces the interesting, albeit difficult task of designing and operating a production facility spread across thousands of acres of forestland. The single decision that most significantly impacts the profitability of this enterprise is the tradeoff to be struck between roading and yarding expenditures. Of these two costs it is that associated with yarding that presents the greater challenge to accurate estimation. One prerequisite to accurate cost estimation is the adequate, as well as accurate, characterization of the individual setting. The analytical models and specific results presented in this paper are a step in that directon. Many of the ideas presented in this paper should stimulate both theoretical and applied research activity since the results, though quite general, are clearly not the last word on the topic.

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