# Average Yarding Slope and Distance on Settings of Simple Geometric Shape 

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#### Abstract

Previous studies have found that the slope and distance of a yarding operation can significantly influence harvesting system productivity. Consistently accurate prediction of production levels depends on a good understanding of these two variables and how they are parameterized. On a specific setting, average yarding slope and average yarding distance are two possible production parameters. Different definitions of yarding slope and yarding distance are possible, however. In this study one definition is selected for each of these variables and the average values for a setting with uniform turn distribution are examined. The general equations for average yarding slope and average yarding distance are then applied to four simple geometric shapes. Through relative weighting procedures application is extended to some variation of these four shapes. The results of this study should find use in verification of computer programs, development of theoretical models, and field estimation of operating conditions on particular settings. Forest Sci. 26:195-202.


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The important influence of slope and yarding distance on production of timber harvesting systems has undoubtedly been recognized since man first started logging. Efforts to apply scientific methodology to the collection, analysis, and use of data in this area, however, is of more recent origin.

Brandstrom (1933), in one of the best examples of the early work in this area, reported the results of time study observations and their analysis. Both slope and distance were examined. Referring to yarding distance he noted that it had a significant impact on the productivity of both cable and tractor yarding operations, and he presented quantitative estimates of this effect for several different harvesting systems. The effect of slope on tractor yarding production was also quantified. Some difficulty was encountered in attempting to identify the effect of slope on cable operations. This difficulty was probably due to the limited analytical techniques available to him at that time. He did state however that some differences due to slope were apparent in the lower horsepower high-lead systems. This early evidence has been substantiated by subsequent work. Peters (1974) found, in a review of more recent articles dealing with cable time studies, that among the most frequently occurring factors of identified significance are slope distance and skyline slope. ${ }^{1}$

Scientific observation and quantification of the effects of topographic factors

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on production prefaced the scientific use of that information in decisionmaking. With data being collected and summarized using scientific procedures it was then possible to proceed in a logically rigorous fashion toward its use in decisionmaking. The seminal work in this field is that of Matthews (1942).

In his treatment of slope and distance as factors influencing production, Matthews placed primary emphasis on yarding distance. That yarding distance receives this attention in preference to slope is understandable. Yarding distance was, and continues to be, the variable of most practical importance to the logger in that it offers more opportunities to be controlled by his decisions than does slope. Additionally, quantitative estimates of the relationship between distance and production were either available or readily obtained and the results of the theory could be applied immediately.

There were weak points in these first developments of the theory. Suddarth and Herrick (1964) reported and corrected an error in the logic used in the calculation of average yarding distance. In practical terms the magnitude of the error is small to nonexistent depending on the shape of the setting. Another weakness is encountered in his treatment of average yarding slope. Matthews proceeded in a nonrigorous ad hoe fashion. For example, referring to a landing serving a semicircular sidehill setting below a contour road, Matthews argues intuitively that the average grade would be approximately half the slope grade. It will be shown here that in fact the average grade is closer to two-thirds that of the slope. Once again it might be argued convincingly that in practical terms the error is small. In consideration of the estimation techniques which were commonly applied to yarding time studies data during this period and the moderate demands for cost estimation accuracy, Matthews' work certainly met the needs of the time.

Today, available analytical techniques and economic conditions are persuasive forces in seeking a higher degree of production estimation accuracy. Activity directed toward more accurate quantification of topographic factors has centered on minicomputers and digitizers (Perkins and Lynn 1979, Gibson 1978, Dykstra and Riggs 1977, Young and Lemkow 1976, Peters and Burke 1972).

It is interesting to note that while these application-oriented procedures have moved forward rapidly, the theory has lagged. The last major contribution to theory was that made by Suddarth and Herrick in 1964. Their principal contribution, as mentioned earlier, was the correct mathematical specification of average yarding distance (AYD) and its application to several simple geometric figures.

The failure of theory to keep pace with application has meant on occasion that erroneous procedures are used which might otherwise be detected and avoided. For example, in at least one applications model a naive slope correction for the AYD is made after numerical integration of horizontal yarding distances. It will become evident from what follows that the likelihood is very small that such procedures will give correct results.

The lack of a sufficiently abstract model which incorporates slope has also delayed the development of answers to several related questions. All of the applications models use techniques of numerical integration. Such procedures are time consuming and not very revealing in their results when applied to such problems as optimal spacing of roads, yarding system selection, and economic external yarding distance. Decision rules, similar to those developed by Matthews for level terrain, are needed for areas where slope is a significant factor.

It is the purpose then of this paper to extend the theory in two areas. First a definition of yarding slope will be offered and some particular results presented when it is applied to several simple geometric shapes. Secondly, the slope element will be introduced in the calculation of AYD. Once again results of application to some specific setting shapes will be presented.


Figure 1. Plan view of a setting with radius R uphill yarding through $90^{\circ}$ with the landing at " o ."

## Derivation and Example

This development shares the following assumptions with the Suddarth and Herrick study:

1. There is a uniform distribution of turns over the area of the setting. ${ }^{2}$
2. There is one central landing with turns being brought straight into it.
3. The setting has a simple geometric shape (for those cases where explicit results are obtained).

In addition to these assumptions the following will also be needed for the specific examples given herein:
4. The setting is located on a uniform sidehill slope, and has a specific orientation with respect to that slope.
Average yarding slope, AYS, for a setting of area " $A$ " is given by

$$
\mathrm{AYS}=\frac{1}{\mathrm{~A}} \int_{\mathrm{A}} \mathrm{~s} \mathrm{dA}
$$

where " $s$ " is defined to be the slope of the straight line from the landing to a point in the setting, both points being at the same height above the ground.

As a specific example consider the setting of Figure 1. Given the uniform contours, all points along the line o-h have the same slope, $s$, calculated as

$$
\mathrm{s}=\mathrm{Scos} \theta
$$

where " $S$ " is the slope perpendicular to the contour and " $\theta$ ' is the angle between the perpendicular and the line o-h. The infinitesimal area for which this slope applies is calculated as

$$
\mathrm{dA}=1 / 2 \mathrm{R}^{2} \mathrm{~d} \theta
$$

[^1]with the total area of the setting equal to
$$
\mathrm{A}=1 / 4 \pi \mathrm{R}^{2} .
$$

Substitution into the definition of average slope and simplification yields

$$
\mathrm{AYS}=\frac{2 \mathrm{~S}}{\pi} \int_{0}^{\pi / 2} \cos \theta \mathrm{~d} \theta
$$

which then gives

$$
A Y S=\frac{2 S}{\pi}
$$

Consider now the calculation of AYD in a situation where there is significant ground slope. The formula given by Suddarth and Herrick for a setting of area " $A$ " is

$$
\mathrm{AYD}=\frac{1}{\mathrm{~A}} \int_{\mathrm{A}} \mathrm{xdA}
$$

Where " $x$ " is the length of the direct yard from a point in the setting to the landing. The same definition is used here, recognizing that " $x$ " is a slope distance. Turning to the example of Figure 1, the slope distance from the landing to any horizontal distance ' p " on the line $\mathrm{o}-\mathrm{h}$ is calculated as

$$
\mathrm{x}=\frac{\mathrm{p}\left(100^{2}+\mathrm{S}^{2} \cos ^{2} \theta\right)^{1 / 2}}{100}
$$

The infinitesimal area for which this yarding distance applies is given as

$$
\mathrm{dA}=\mathrm{pd} \theta \mathrm{~d} \mathrm{p}
$$

with a total area once again of

$$
\mathrm{A}=1 / 4 \pi \mathrm{R}^{2}
$$

Substitution gives

$$
\mathrm{AYD}=\frac{4}{\pi \mathrm{R}^{2}} \int_{0}^{\pi / 2} \int_{0}^{\mathrm{R}} \frac{\mathrm{p}^{2}\left(100^{2}+\mathrm{S}^{2} \cos ^{2} \theta\right)^{1 / 2}}{100} \mathrm{dp} \mathrm{~d} \theta
$$

It is observed in this equation that the slope correction is not independent of all the variables of integration. Slope corrections must therefore be made during integration of yarding distance in this case if numerical integration were to be performed.

Carrying out the first integration leaves

$$
\mathrm{AYD}=\frac{\mathrm{R}}{75 \pi} \int_{0}^{\pi / 2}\left(100^{2}+\mathrm{S}^{2} \cos ^{2} \theta\right)^{1 / 2} \mathrm{~d} \theta
$$

The second integration is that of an elliptic integral, for which a second order approximation yields

$$
\mathrm{AYD}=\left[\frac{2 \mathrm{R}}{3}\right]\left[\frac{0.0075 \mathrm{~S}^{2}+100}{\left(\mathrm{~S}^{2}+100^{2}\right)^{1 / 2}}\right]
$$

The first term is the AYD of a spherical sector on level ground. The second term is the slope correction.


Figure 2. Plan diagrams of geometric settings for which average yarding slope and distance have been calculated.

## Results

The average yarding slope and distance have been calculated in four specific cases. The four cases are diagrammed in Figure 2(A-D). The results derived are based on the figure drawn in the first quadrant. It is noted however that, because of symmetry, application is immediately and easily extended to combinations of all four quadrants. In the Appendix are given the formulae for the calculation of average yarding slope and distance for the diagrams in Figure 2.

As an example consider a rectangular setting with a downhill section as displayed in Figure 3 together with a mirror image of that section in the second quadrant. The total area is now 2 LW . The average yarding distance remains the same. The average slope however is now zero since the uphill and downhill sections cancel out.

Further extention is obviously possible through relative weighting procedures. ${ }^{3}$ Continuing with the previous example, if the rectangular section above the road has the same width as that below the road but only half the length the following formulae apply:

$$
\mathrm{AYS}=\frac{\mathrm{LW} \mathrm{AYS} S_{1}+1 / 2 \mathrm{LW} \mathrm{AYS}}{2} \text { }
$$

and

[^2]

Figure 3. Diagram of a rectangular setting with an equivalent uphill section.

$$
\mathrm{AYD}=\frac{\mathrm{LW} \mathrm{AYD} D_{1}+1 / 2 \mathrm{LW} \mathrm{AYD}}{2} \text { },
$$

where the subscripts 1 and 2 apply to the sections below and above the road respectively.

A second example illustrates another useful application of the weighting procedure. Consider a rectangular area of width W and length L located below the road as in Figure 4. A third of this area consists of a view-protecting leave strip along the road. The width of this leave strip is $W$ and its length is L/3. The following formulae apply to the balance of the area which is to be yarded:

$$
\begin{aligned}
\mathrm{AYS} & =(\mathrm{LW} \mathrm{AYS} \\
1 & -1 / 3 \mathrm{LW} \mathrm{AYS} \\
\mathrm{AYD} & =(\mathrm{LW} \mathrm{AYD} \\
1 & \left.-1 / 3 \mathrm{LW} \mathrm{AYD}_{2}\right) /(2 / 3 \mathrm{LW})
\end{aligned}
$$

where the subscripts 1 and 2 apply to the total area and the leave strip area, respectively.

## Discussion

It is the purpose of this paper to make a modest extension to the theory of yarding parameter estimation. Attention has been directed at the influence of slope. One


Figure 4. Diagram of a rectangular setting with a leave strip.
possible definition for yarding slope has been offered. Using this definition, average yarding slope on a setting has been derived for several cases. The effect of slope on average yarding distance is also examined. Continuing the work of Suddarth and Herrick, slope distance is entered in their average yarding distance formula. Once again results were derived for several different setting designs.

Specific results are presented for four simple geometric shapes, and their inclusion should serve the following purposes:

1. Existing applications programs can be tested against these results to verify their correctness.
2. The results may be used in the further development of theoretical models (Peters 1978).
3. The results may be used in the field where quick estimates are needed for settings with conditions approximately equal to those assumed here.

## Literature Cited

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## Appendix: Equations for Yarding Slope and Distance Parameters on Geometric Settings

Average yarding slope and distance for the settings diagramed in Figure 2. The variables are defined as follows: $S$, maximum slope (percent); $R$, radius of circular setting; $a, b$, elliptic intercepts of major and minor axes, respectively; W, width of the rectangular setting; L, horizontal length of the rectangular setting perpendicular to contours; H , horizontal length of the triangular setting perpendicular to contours; $D$, length of the base on the triangular setting; $A \equiv\left(100^{2}+S^{2}\right)^{1 / 2} /$ $100 ; \mathrm{C} \equiv \mathrm{W} / \mathrm{A} ; \mathrm{B} \equiv \mathrm{C}^{2}+\mathrm{L}^{2}$.

Circle $\left[\frac{2 S}{\pi}\right]$
Ellipse $\left[\frac{S}{\pi\left(a^{2}-b^{2}\right)^{1 / 2}}\right]\left[a \pi-2 a \arcsin \left(\frac{b}{a}\right)\right]$
Rectangle $\left[\frac{S}{2 W L}\right]\left[L^{2} \log _{e}\left(\frac{\mathrm{~W}+\left(\mathrm{W}^{2}+\mathrm{L}^{2}\right)^{1 / 2}}{\mathrm{~L}}\right)+\mathrm{W}\left(\mathrm{W}^{2}+\mathrm{L}^{2}\right)^{1 / 2}-\mathrm{W}^{2}\right]$
$\underset{\text { Triangle }}{\text { Right }}\left[\frac{S H}{2 \mathrm{D}}\right]\left[\log _{e}\left(\frac{\left(\mathrm{H}^{2}+\mathrm{D}^{2}\right)^{1 / 2}+\mathrm{D}}{\left(\mathrm{H}^{2}+\mathrm{D}^{2}\right)^{1 / 2}-\mathrm{D}}\right)\right]$
Average Yarding Distance
Circle $[\mathrm{R}]\left[\frac{0.005 \mathrm{~S}^{2}+66.67}{\left(\mathrm{~S}^{2}+100^{2}\right)^{1 / 2}}\right]$
Ellipse $\left[\frac{3 a^{2}\left(S^{2}+100^{2}\right)+b^{2} 100^{2}}{600 a\left(S^{2}+100^{2}\right)^{1 / 2}}\right]$
Rectangle $\left[\frac{\mathrm{AB}^{1 / 2}}{3}\right]+\left[\frac{\mathrm{AC}^{2}-3 \mathrm{CW}}{12 \mathrm{~L}}\right]\left[\log _{\mathrm{e}}\left(\frac{\mathrm{C}}{\mathrm{L}+\mathrm{B}^{1 / 2}}\right)\right]$
$+\left[\frac{\mathrm{AL}^{2}}{6 \mathrm{C}} \log _{\mathrm{e}}\left(\frac{\mathrm{C}+\mathrm{B}^{1 / 2}}{\mathrm{~L}}\right)\right]$
$\underset{\text { Triangle }}{\text { Right }}\left[\frac{\left(\mathrm{D}^{2}+\mathrm{A}^{2} \mathrm{H}^{2}\right)^{1 / 2}}{3}\right]+\left[\frac{\mathrm{A}^{2} \mathrm{H}^{2}}{3 \mathrm{D}}\right]\left[\log _{e}\left(\frac{\mathrm{D}+\left(\mathrm{D}^{2}+\mathrm{A}^{2} \mathrm{H}^{2}\right)^{1 / 2}}{\mathrm{AH}}\right)\right]$


[^0]:    ${ }^{1}$ The articles referenced by Peters use either a yarding slope definition similar to that which will be given here or a skyline chord slope definition. What is relevant is that slope, in some form, is almost always found to be a significant factor in estimating production.

[^1]:    ${ }^{2}$ For a discussion on the relaxation of this assumption the interested reader is referred to Donnelly (1978).

[^2]:    ${ }^{3}$ A detailed description of the procedure applied here is given by Suddarth and Herrick (1964).

