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ARTICLE

An economic model for integrated roading, yarding, and hauling operations on two alternative harvest unit landings

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Abstract: This paper presents an economic model for the optimization of a vertically integrated timber harvesting operation. The operations of road construction, timber yarding, and log truck hauling are collectively optimized. The harvest unit has two centralized landings that are to be accessed by truck road from a single existing road takeoff point. The harvest unit is located on level, unvarying terrain with uniformly distributed log turns. Formulas for the optimal yarding boundary between the landings, areas yarded to each landing, and the average yarding distances are derived. These formulas are then used in the bilevel optimization of a vertically integrated timber harvesting operation. Numerical examples are presented and discussed.

Key words: forest engineering, operations research, timber harvest, transportation, bilevel planning.

Résumé : Cet article présente un modèle économique permettant d'optimiser une opération de récolte de bois verticalement intégrée. Les opérations de construction de chemin, de débardage et de transport du bois par camion sont optimisées ensemble. L'unité de récolte compte deux jetées centralisées qui doivent être accessibles par une route de camionnage à partir d'un seul point de départ sur une route existante. L'unité de récolte est située en terrain plat et uniforme où les endroits pour faire demi-tour sont uniformément distribués. Les formules pour déterminer la limite optimale de débardage entre les jetées, la superficie des zones de débardage vers chaque jetée et les distances moyennes de débardage sont dérivées. Ces formules sont ensuite utilisées dans l'optimisation à deux niveaux d'une opération de récolte de bois verticalement intégrée. Des exemples numériques sont présentés et font l'objet d'une discussion. [Traduit par la Rédaction]

Mots-clés : génie forestier, recherche opérationnelle, récolte de bois, transport, planification à deux niveaux.

Introduction

The purpose of this paper is to present an optimization model that recognizes the economic trade-off that may exist between access-road construction, skidder yarding of turns, and log truck hauling. The scope of this paper is limited to the examination of two alternative centralized landings on a harvest unit. Both of these landings are fixed in place and are to be accessed by a truck road from a single existing road takeoff point. The managerial structure and behavior being modeled is likewise limited in scope. Among other restrictions, it is also confined to the integrated economic optimization of these three harvesting related activities.

The paper presents the model in two main sections, the first section is the analytical development of the individual components of the model, followed by the section on model placement and optimization within an organizational structure. In this latter section, several applications are presented and discussed. The paper concludes with a few comments on model limitations and future research directions.

Relation to previous work

A previous paper in this Journal presented a continuous location model for two landings (Greulich 2012). A serious deficiency of that model was the absence of an economic trade-off between yarding and the construction and use of truck roads. Road construction and truck hauling activities were optimized only after the two landing locations had already been determined by optimizing the yarding operation. This paper presents a first step in remedying that model deficiency.

Analytical development of model components

The development of an economic model to explore the trade-off between road construction, yarding of turns, and truck hauling from two adjacent landing locations on a harvest unit requires the sequential development of three new analytical elements. Of first importance is the analytical definition of the boundary curve separating the yarding operations between the two landings. Once this boundary has been mathematically defined, the associated areas to be yarded to each landing can be analytically determined. Using equations for the boundary curve and its enclosed yarding area, a computational formula for the average yarding distance to each landing can then be developed. These then are the three major analytical components of the model.

A final step, addressed in the second section of this paper, is the development of an optimization procedure, in which the costs of access-road construction, yarding of logs to the landings, and truck hauling operations are brought into economic balance within a specific organizational context. Some initial remarks about this targeted organizational structure and behavior are pertinent to the component development that follows.

The development of access-road infrastructure is an investment decision, and once the roads are built, they will be used operationally in the most economic fashion. A plan to maximize the benefits of the road investment will be an integral part of the capital investment decision in this integrated timber harvesting process. In this regard, it is assumed in this model that during the capital investment decision process, access-road construction costs will be treated as being uniformly amortized over the entire volume of timber to be harvested. It is also understood that once the road

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investment has been made, subsequent operational decisions will treat this access-road investment as an irretrievable economic cost. A more detailed consideration of other investment decision factors e.g., investment timing, interest payments, tax considerations, road usage fees, etc., are beyond the scope of this paper.

The boundary equation

The general cost structure for the integrated planning of roading, yarding, and hauling operations and its application to the derivation of the optimal boundary equation between two alternative landings is described in this portion of the paper.

It is assumed in this presentation that the yarding cost in dollars (US dollars are used throughout) per unit volume for a turn brought into landing A may be based on an equation of the following form:

$$\text{(1)} \qquad YC_A = (\beta_0 + \beta_1 \omega \rho_A)(1/v)$$

where ρ_A is the straight-line distance from any turn location in the harvest unit to landing A. The parameters in this equation are as follows: the wander factor, ω; the timber volume per turn, v; and the coefficients β_0 and β_1. A yarding cost equation, YC_B, of similar form to eq. 1, applies for landing B. This formula for the yarding cost is like that used in previous publications (Greulich 1991, 2012). It differs in that the factor $(1/v)$ has been inserted here and that only a linear relationship with yarding distance is retained.

In the analysis of this paper, it is assumed that a nonredundant truck road system will be built. It will depart from a single road takeoff point and will only provide access to these two landings. It is to be designed, built, and used in the most economic fashion (Greulich 1995, 1999). As in previous publications (Greulich 1997, 2012), the road use cost in dollars per unit volume hauled from landing A to the access-road takeoff point may be estimated as the following:

$$\text{(2)} \qquad RUC_A = h_j(S_{AJ} + S_{JT})$$

where S_{AJ} is the length of road used exclusively for hauling from landing A to the road junction point J, S_{JT} is the length of road from the junction point to the takeoff point T, and h_j is the truck haul cost in dollars per unit volume–distance using road standard j. Only one road standard will be used for the access-road system. A truck road use cost equation, RUC_B, of similar form to eq. 2, applies for landing B. Trucks hauling from both landings pass through the junction point J and travel on to the road takeoff point T.

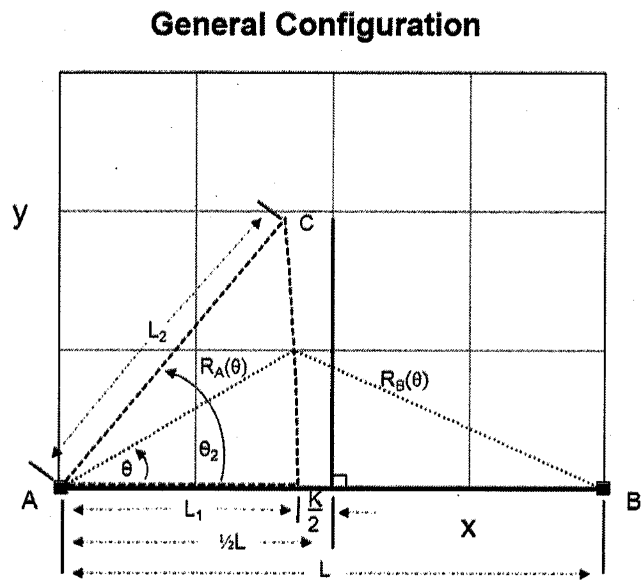
The road construction cost for the entire access-road system in dollars per unit volume is given by the following:

$$\text{(3)} \qquad RCC = \frac{(S_{AJ} + S_{BJ} + S_{JT})r_j}{V_J}$$

where $V_J = V_A + V_B$. The total volume on the harvest unit, V_J, consists of the volumes yarded to each of the two landings, and r_j is the cost of road construction per unit length of road built to standard j. The construction cost of the road system will be amortized over the total timber volume removed. Once built, these are sunk costs, and road-use decisions will only be based on the immediate operationally incurred costs.

At any point on the yarding boundary between the two landings, it is a matter of economic indifference as to which landing a turn should be taken. That is to say that the cost of yarding the volume of a boundary-location turn to either landing and then hauling it by truck to the road takeoff point should be the same for both landings. The following equation expresses that condition for a boundary-location point.

Fig. 1. The general configuration of the two-landing harvest unit showing labeled distances between key points.



$$\text{(4)} \qquad YC_A + RUC_A = YC_B + RUC_B$$

Substitution into and reduction of this equation leads to the following equation, which defines the boundary line in polar coordinates.[1]

$$\text{(5)} \qquad R_A(\theta) = \frac{L^2 - K^2}{2[K + L\cos(\theta)]}$$

In deriving this equation, ρ_A and ρ_B were replaced by R_A and R_B, respectively, in accordance with notation used in previous publications. $R_A(\theta)$ is the straight-line distance from landing A to the yarding boundary at the angle θ as measured in a positive counterclockwise rotation about landing A using the line between A and B as the axis of reference. This equation is sketched in Fig. 1, where L is the straight-line distance between landings A and B. The distance K in this equation and in Fig. 1 is given by the following relationship:

$$\text{(6)} \qquad K = \left(\frac{vh_j}{\omega\beta_1}\right)(S_{AJ} - S_{BJ})$$

In developing Fig. 1, it was assumed that the truck hauling distance from the landing to the junction point is greater for landing A than for landing B, which gives K a positive value as shown in this figure. If K has a negative sign, the boundary curve will start on the opposite side of the midpoint and bend away from landing A. It is also noted that $K^2 < L^2$ is a necessary condition for the economic feasibility of using both landings. It is convenient from this point on to restrict attention to landing A and drop the subscript on $R_A(\theta)$.

[1]Formula derivations are available upon request of the author.

In rectangular coordinates, the boundary line equation is found to be given by the following:

$$(7)\qquad y = \left[\frac{(L^2 - K^2)^{\frac{1}{2}}}{2K}\right][(L^2 - K^2) - 4Lx + 4x^2]^{\frac{1}{2}}$$

where landing A is the origin of the coordinate system and the line between landings A and B is taken as the positive x axis. The range of x is from $(L - K)/2$ to $-\infty$ for the illustrated example. This rectangular coordinate form of the boundary curve is convenient for finding where the yarding boundary curve intersects line segments defining the border of the harvest unit or any partition line segment within the unit. In regard to these intersection points, it is to be noted that when $K = 0$, eq. 7 becomes a vertical line, with $x = L/2$. In this case, it is observed that $y = m(L/2) + d$, where the equation of an intersected line is assumed to be given by $y = mx + d$. Equations 5 and 7 are also valid for negative values of K.

These are two alternative formulas for a hyperbola with its center at landing A. The development of similar results, albeit in a different application, were initiated by Rau (1841), as described by Shieh (1985). Early papers that developed this topic in more detail are those of Launhardt (1885) and Cheysson (1887). A more contemporary presentation is provided in the text by Paelinck and Nijkamp (1975).

Having developed an analytical description of the boundary curve, it is now possible to derive a formula for the yarded area lying between that curve and the landing.

The area equation

A formula for the area enclosed by the hyperbolic boundary curve and radial line segments L_1 and L_2 as shown on Fig. 1 can now be derived. First, it is noted that the line segment lengths L_1, L_2, and L_3 may be calculated using the following equations:

$$(8)\qquad L_1 = R(0) = (L - K)/2$$

$$(9)\qquad L_2 = \left[(x_A - x_C)^2 + (y_A - y_C)^2\right]^{\frac{1}{2}}$$

where (x_A, y_A) and (x_C, y_C) are the coordinates of landing A and the other end of line segment L_2 as shown in Fig. 1, and

$$(10)\qquad L_3 = \left[\left(L_1^2 + L_2^2 - LL_1\right) + \left(\frac{K}{L}\right)(KL_1 + 2L_1L_2)\right]^{\frac{1}{2}}$$

where L_3 is the length of the straight-line segment connecting the end points of segments L_1 and L_2.

The area formula is determined by the following integration:

$$(11)\qquad A_h = \frac{1}{2}\int_0^{\theta_2} R(\theta)^2 d\theta$$

Carrying out the integration and doing some algebraic manipulation leads to the following closed-form equation for the yarded area, A_h, as defined by the hyperbolic boundary curve:

$$(12)\qquad A_h = A_s\left(\frac{L}{L - K}\right) - \left(\frac{K}{2}\right)\left(\frac{(L^2 - K^2)^{\frac{1}{2}}}{4}\right)\ln\left|\frac{1 + r'}{1 - r'}\right|$$

For notational and developmental convenience, the following variable has been defined and used in this equation:

$$(13)\qquad r' \equiv \left\{\frac{(K - L)\left[L_3^2 - (L_1 - L_2)^2\right]}{(K + L)\left[L_3^2 - (L_1 + L_2)^2\right]}\right\}^{\frac{1}{2}}$$

and from previous usage (Greulich 1987), the area of the triangle formed by straight-line segments L_1, L_2, and L_3 is denoted A_s and is calculated as follows:

$$(14)\qquad A_s \equiv \left\{\frac{-\left[L_3^2 - (L_1 - L_2)^2\right]\left[L_3^2 - (L_1 + L_2)^2\right]}{16}\right\}^{\frac{1}{2}}$$

Using eq. 12 for the yarded area together with the boundary curve eq. 5, the average yarding distance for the area being examined can now be developed.

The average yarding distance equation

The average yarding distance (AYD_h) formula for the area enclosed by the hyperbolic boundary curve and radial lines L_1 and L_2 is derived by carrying out the following integration (Greulich 1991):

$$(15)\qquad AYD_h = \frac{1}{3A_h}\int_0^{\theta_2} R(\theta)^3 d\theta$$

Integration and some algebraic manipulation yield the following closed-form equation for the average yarding distance:

$$(16)\qquad AYD_h = \left(\frac{1}{48A_h}\right)\left\{\left[\frac{4LA_s(2L_2 - 3K)}{L_1}\right] + \left[(L^2 - K^2)^{\frac{1}{2}}(2K^2 + L^2)\right]\ln\left|\frac{1 + r'}{1 - r'}\right|\right\}$$

Before proceeding to an application of these formulas, an effort to verify their correctness was made.

Verification of formulas

One check on the formulas was done by comparing them with the well-known formulas for a right triangle (Sundberg 1952–1953; Suddarth and Herrick 1964). As K goes to zero in the formulas of this paper, the boundary curve should become a straight line; concomitantly, the yarded area and average yarding distance formulas should become those for the right triangle. These anticipated results were confirmed.

A second check of the formulas was performed by numerical integration of eqs. 11 and 15 for a general harvest unit design, where K is a nonzero value. The formulas used were, respectively,

$$(17)\qquad A_h \approx \frac{1}{2}\sum_{i=0}^{n-1}\left[\frac{R(\theta_i) + R(\theta_{i+1})}{2}\right]^2 \Delta\theta$$

$$(18)\qquad AYD_h \approx \frac{1}{3A_h}\sum_{i=0}^{n-1}\left[\frac{R(\theta_i) + R(\theta_{i+1})}{2}\right]^3 \Delta\theta$$

For one of the comparisons, the results of the last optimization example in Greulich (2012) was used, with $K = 0.391036$ and $L = 4.253667$. Setting $n = 500$, $\theta_0 = 0°$, $\theta_n = 60°$, and $\Delta\theta = 0.12°$, the results were $A_h \approx 3.03869$ and $AYD_h \approx 1.69664$. A comparison of these numerical results for the yarded area and average yarding distance with those provided by the formulas confirmed their accuracy to five significant figures.

With these confirmatory results of the listed formulas, they were employed in the development of an optimal economic trade-off model.

Organizational placement and optimization of the model

The model of this paper was developed for a specific organizational structure and behavior. It is a vertically integrated harvesting operation in which access-road development, yarding activity, and truck hauling operations are all managed within a single organization that bases its decisions on the minimization of economic cost. A discussion of the economic behavior of the organization and a model that mathematically describes its optimized decision-making process follows.

Organizational structure and behavior

At the tactical level, the organization seeks to design and construct an access-road system that will minimize the total cost of building the access road into a single harvest unit, yarding all logs on this unit to two alternative landings, and then hauling the logs by truck to the existing access-road takeoff point. The total cost to be minimized at the tactical level is given by the following function:

$$\text{(19)}\quad \mathrm{TC_t} = r_j(S_{\mathrm{AJ}} + S_{\mathrm{BJ}} + S_{\mathrm{JT}}) + (\beta_0 + \beta_1\omega\mathrm{AYD_A})\left(\frac{V_\mathrm{A}}{v}\right) + (\beta_0 + \beta_1\omega\mathrm{AYD_B})\left(\frac{V_\mathrm{B}}{v}\right) + h_j(S_{\mathrm{AJ}} + S_{\mathrm{JT}})V_\mathrm{A} + h_j(S_{\mathrm{BJ}} + S_{\mathrm{JT}})V_\mathrm{B}$$

or in equivalent abbreviated notation:

$$\text{(20)}\quad \mathrm{TC_t} = V_j\mathrm{RCC} + V_\mathrm{A}\mathrm{YC_A} + V_\mathrm{B}\mathrm{YC_B} + V_\mathrm{A}\mathrm{RUC_A} + V_\mathrm{B}\mathrm{RUC_B}$$

At the operational level of yarding and hauling, the expense of the now constructed access-road network will be treated as a sunk cost, and the objective at this level will be to minimize the cost of yarding and hauling the total harvested volume from the site given the constructed road system. That cost function is given by the following:

$$\text{(21)}\quad \mathrm{TC_o} = (\beta_0 + \beta_1\omega\mathrm{AYD_A})\left(\frac{V_\mathrm{A}}{v}\right) + (\beta_0 + \beta_1\omega\mathrm{AYD_B})\left(\frac{V_\mathrm{B}}{v}\right) + h_j(S_{\mathrm{AJ}} + S_{\mathrm{JT}})V_\mathrm{A} + h_j(S_{\mathrm{BJ}} + S_{\mathrm{JT}})V_\mathrm{B}$$

Cost minimization at the tactical level requires knowledge of the volumes yarded to each of the two landings. However, this volume-allocation decision will only be made at the operational level once it is provided with a road-design decision from the tactical level. Overall optimization at the tactical level requires knowledge of the optimal operational level response to their road construction decision. This organizational structure with its two differing objectives represents a bilevel decision process.

Integrated optimal decision making

This particular bilevel programming problem can be treated as an ordinary, single level optimization problem due to its fundamental characteristics described as follows (Dempe 2002).

The decision maker at the tactical level can know the unique, optimal response of the cooperating operational level to any specified road-design decision. Given a proposed road network connecting the takeoff point and the two landings, the operational response will be to optimize the location of the yarding boundary

Table 1. Values given and calculated for the optimized numerical example.

Variables	Units	Value
Given		
β_0	\$·turn^{-1}	5.6
β_1	\$·turn^{-1}·hectometre^{-1}	3.046
ω	Pure number	1.0
v	Metre^3·turn^{-1}	1.5
V	Metre^3·hectare^{-1}	300
h_j	\$·metre^{-3}·hectometre^{-1}	0.06
r_j	\$·hectometre^{-1}	1000
Calculated		
L	Hectometre	4.1231
A	Hectare	36.00
*K	Hectometre	0.0624
*V_A	Metre^3	5873.05
*V_B	Metre^3	4926.95
*$\mathrm{AYD_A}$	Hectometre	1.7930
*$\mathrm{AYD_B}$	Hectometre	1.7989
*S_{AJ}	Hectometre	3.5514
*S_{BJ}	Hectometre	1.4408
*S_{JT}	Hectometre	4.3949
*$\mathrm{YC_A}$	\$·metre^{-3}	7.3743
*$\mathrm{YC_B}$	\$·metre^{-3}	7.3862
*$\mathrm{RUC_A}$	\$·metre^{-3}	0.4768
*$\mathrm{RUC_B}$	\$·metre^{-3}	0.3501
*RCC	\$·metre^{-3}	0.8692
*$\mathrm{TC_o}$	\$	84 226.65
*$\mathrm{TC_t}$	\$	93 613.75

Note: Optimized values are indicated with an asterisk.

and subsequent volumes sent to each landing using the procedures given in the first part of this paper. This functional relationship is indicated as follows:

$$\text{(22)}\quad Y(S_{\mathrm{AJ}}, S_{\mathrm{BJ}}, S_{\mathrm{JT}}) \xrightarrow{\text{opt}} V_\mathrm{A}, V_\mathrm{B}, \mathrm{YC_A}, \mathrm{YC_B}, \mathrm{RUC_A}, \mathrm{RUC_B}$$

where the model, $Y(\ldots)$, determines the values associated with optimized yarding and hauling activities given the road network values S_{AJ}, S_{BJ}, and S_{JT}.

At the tactical level, the organization can elicit this optimized response of the lower level by selecting any arbitrary junction point $(x_\mathrm{J}, y_\mathrm{J})$ for the truck road network. Road segment lengths and their associated construction cost are calculated straight away. This modeled functional relationship, $R(\ldots)$, is indicated as follows:

$$\text{(23)}\quad R(x_\mathrm{J}, y_\mathrm{J}) \rightarrow \mathrm{RCC}, S_{\mathrm{AJ}}, S_{\mathrm{BJ}}, S_{\mathrm{JT}}$$

Given the explicit and unique optimal response at the operational level to any road location decision at the tactical level, it is possible to directly minimize the total cost by optimizing over the location of the road junction point $(x_\mathrm{J}, y_\mathrm{J})$.

$$\text{(24)}\quad \underset{x_\mathrm{J}, y_\mathrm{J}}{\mathrm{MIN}}\{\mathrm{TC_t}[R(x_\mathrm{J}, y_\mathrm{J}), Y(S_{\mathrm{AJ}}, S_{\mathrm{BJ}}, S_{\mathrm{JT}})]\}$$

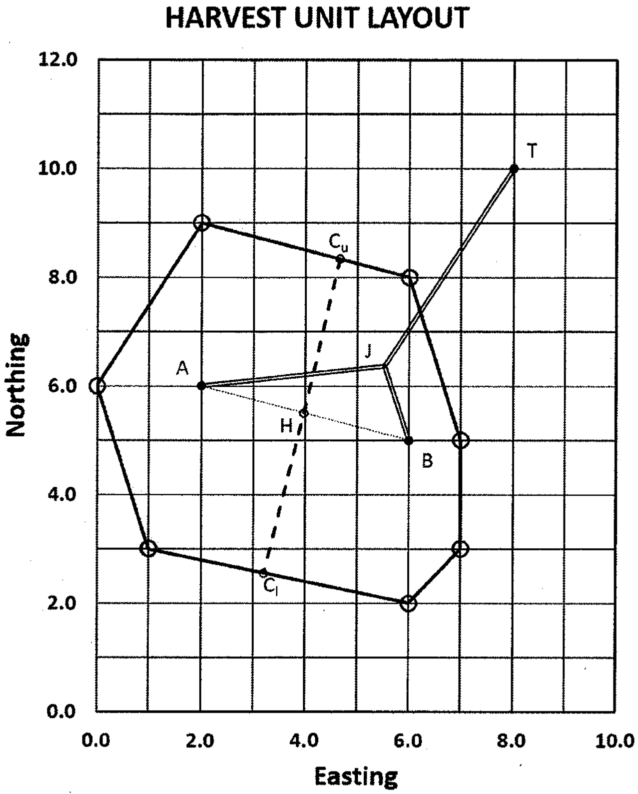
As an example of this optimization process, an Excel spreadsheet model has been developed.[2] This model and some of its results are now examined.

Model application

Data from previously published papers present an opportunity to explore the potential impact of the results of this paper on

[2]The MS Excel models discussed in this paper are available at http://faculty.washington.edu/greulich/Research.htm.

Fig. 2. Plan view of the example harvest unit layout. Access road and yarding boundary are shown after optimization has been done. See Table 2 for list of variables and coordinates for map locations.



harvesting activities under what may be fairly realistic harvesting conditions. Harvest unit parameter values taken as given, based on data used by Contreras and Chung (2007), are listed in the upper portion of Table 1. An effort was also made to use arguably realistic values for the harvest unit physical layout. This layout is illustrated in Fig. 2, with coordinates of key points listed in the upper portion of Table 2. Coordinates of the traverse turning points circumscribing the harvest unit may be scaled from Fig. 2. By using what might be considered reasonable harvesting conditions in this example, it may be possible to draw some very tentative conclusions regarding the planning of harvest operations under the organizational structure and behavior assumptions of this paper.[3]

The optimization problem described in the previous section was programmed and then optimized using Solver.[4] Unconstrained nonlinear programming was used, and the results are shown in Tables 1, 2, 3, and 4. Table 3 presents output from the "show iteration results" option of Solver. Given a user-specified starting point for (x_J, y_J), Solver first calculates the road-segment lengths and their construction cost. Then, it finds the volume to be moved to each landing so that their associated yarding and road-hauling costs are minimized. Summing these construction, yarding, and hauling costs gives Solver the current value of the objective function, TC_t. Solver then automatically revises the value of (x_J, y_J) in accordance with its nonlinear optimization routine and continues the converging optimization cycle. The lower portion of Table 1 and the middle portion of Table 2 give the final

Table 2. Coordinates for map locations of Fig. 2.

	Map symbol	Easting coordinate (x)	Northing coordinate (y)
Given values for the example			
Road takeoff	T	8.0000	10.0000
Landing A	A	2.0000	6.0000
Landing B	B	6.0000	5.0000
Values calculated by the optimization model and shown on Fig. 2			
Road junction	*J	5.5328	6.3629
Upper intersection	*C_u	4.6535	8.3366
Line AB intersection	*H	3.9698	5.5076
Lower intersection	*C_l	3.2073	2.5585
Road junction locations calculated under the case assumptions			
Road junction (case I)	J	5.5271	6.3697
Road junction (cases II and III)	J	5.5028	5.9129
Boundary curve locations under the case assumptions			
Upper intersection (case I)	C_u	4.6539	8.3365
Line AB intersection (case I)	H	3.9699	5.5075
Lower intersection (case I)	C_l	3.2076	2.5585
Upper intersection (case II)	C_u	4.6448	8.3388
Line AB intersection (case II)	H	3.9647	5.5088
Lower intersection (case II)	C_l	3.1981	2.5604
Upper intersection (case III)	C_u	4.7059	8.3235
Line AB intersection (case III)	H	4.0000	5.5000
Lower intersection (case III)	C_l	3.2619	2.5476

results of this optimization process. The coordinates optimized by this process are plotted in Fig. 2.

It is interesting, and perhaps informative, to examine the results of this optimization model with results obtained under other decision making procedures. Side by side results in Table 4 are shown the comparative results of the model of this paper and three alternative decision procedures.

Cases I, II, and III shown in Table 4 were analyzed using separate optimization models, one for road network optimization and a second for optimizing the yarding and hauling operations. Road network design is optimized under two different scenarios. In all three cases, it is assumed at the tactical planning level that turns eventually will be yarded to the nearest landing (the landing volumes are shown in Table 4). In case I at the tactical level, the road network is determined by minimizing the road construction and use cost. In cases II and III, the minimization of road construction cost alone determines road layout.

The independently determined operational response to each of these tactical level decisions is examined under the following assumptions. In cases I and II, yarding and hauling is optimized based on the road network that has been constructed, but in case III, turns are simply yarded to the nearest landing without any consideration of the road network that has been built.

An inspection of actual, realized expenditures at the tactical level shows that, although the cost under the optimization process of this paper is the lowest of the four scenarios, the difference is certainly not of practical significance. In fact, however, this comparative result is quite encouraging. Even when yarding to the nearest landing at the operational level, case III, the cost difference is not significant at either the tactical or the operational level. These modeling results would suggest that, in practice, making access-road location decisions based on the assumption that turns will be taken to the nearest landing is very close to optimal. In fact, considering construction costs alone for access-road location decisions would appear quite adequate.

[3]Readers are invited to use the available MS Excel models under their own alternative design–cost structure assumptions.

[4]Solver is an optimization software tool developed by Frontline Systems, Inc. that comes bundled with MS Excel.

Table 3. Iteration results for the example problem generated by the MS Excel 2013 Solver add-in using the default nonlinear programming options.

	Solver imput to $R(x_J, y_J)$		$R(x_J, y_J)$ output	Input to $Y(S_{AJ}, S_{BJ}, S_{JT})$			$Y(S_{AJ}, S_{BJ}, S_{JT})$ output						Objective function
Solver values at iteration	x_J	y_J	V_JRCC	S_{AJ}	S_{BJ}	S_{JT}	V_A	V_B	YC_A	YC_B	RUC_A	RUC_B	TC_t
1	0.00	0.00	$26 941	6.3246	7.8102	12.8062	5994	4805	$7.41	$7.33	$1.14	$1.23	$119 465.26
2	4.61	6.40	$9559	2.6386	1.9742	4.9462	5922	4878	$7.39	$7.37	$0.46	$0.42	$93 976.75
3	5.49	6.59	$9443	3.5412	1.6670	4.2348	5881	4919	$7.38	$7.38	$0.47	$0.35	$93 629.05
4	5.54	6.36	$9387	3.5546	1.4387	4.3940	5873	4927	$7.37	$7.39	$0.48	$0.35	$93 613.76
5	5.53	6.36	$9387	3.5546	1.4387	4.3940	5873	4927	$7.37	$7.39	$0.48	$0.35	$93 613.75

Table 4. Comparative results of alternative model assumptions.

Variable	Optimized example	Case I	Case II	Case III
Tactical level: access road network layout decision				
K	0.0624	0.0000	0.0000	0.0000
V_A	5873.05	5944.43	5944.43	5944.43
V_B	4926.95	4855.57	4855.57	4855.57
S_{AJ}	3.5514	3.5464	3.5039	3.5039
S_{BJ}	1.4408	1.4490	1.0395	1.0395
S_{JT}	4.3949	4.3926	4.7896	4.7896
Total roading cost	$9387.10	$9387.97	$9333.01	$9333.01
Total hauling cost	$4525.28	$4533.42*	$0.00*	$0.00*
Roading and hauling	$13 912.38	$13 921.39*	$9333.01*	$9333.01
Operational level: yarding and hauling response				
K	Same as above	0.0620	0.0728	0.0000
V_A	Same as above	5873.49	5861.08	5944.43
V_B	Same as above	4926.51	4938.92	4855.57
AYD_A	1.7930	1.7931	1.7912	1.8041
AYD_B	1.7989	1.7988	1.8012	1.7850
Total yarding cost	$79 701.37	$79 701.32	$79 703.02	$79 696.85
Total hauling cost	$4525.28	$4524.50	$4643.90	$0.00*
Yarding and hauling	$84 226.65	$84 225.82	$84 346.92	$79 696.85*
Total realized expenditures at operational and tactical levels				
TC_o	$84 226.65	$84 225.82	$84 346.92	$84 353.09
TC_t	$93 613.75	$93 613.79	$93 679.93	$93 686.10

*The model calculated level of expenditure when planning under the assumptions of the specific case but not actually observed in the total tactical level or total operational level cost when the actualized costs are summed at the completion of harvest.

At the operational level, the lowest cost is experienced under the assumptions of case I. It certainly cannot be considered a significant cost difference; however, it is worth examining. It is observed in this comparison that when the total harvesting system is optimized, the reduction in road construction cost, $0.87, more than offsets the comparative increase in yarding and hauling cost, $0.83. It is this quantification of the economic trade-off illustrated by this comparison that stimulated the development of this economic model.

Although it might seem from these examples that the use of a hyperbolic boundary curve yields no advantage to the harvest unit planning process, it does in fact depend on the particular application. In this regard, the interested reader who explores the Excel models will discover that changing the general parameters that appear in eq. 6 for K and moving the location of the road takeoff point can significantly shift the hyperbolic boundary resulting in a notable impact on yarding cost if not the total realized tactical level cost.

In addition, it has been assumed in this paper that there is one shared road takeoff point. If the two landings were reached from different takeoff points with greatly differing truck haul costs, there almost certainly would be a quite noticeable trade-off between yarding and hauling. Under such conditions, an economically significant shifting of the yarding boundary away from the perpendicular bisector would be a likely result.

Commentary

The Excel optimization model used to develop the numerical results of the previous section is only a proof of concept. It is severely limited in its scope and flexibility, especially in the specification of the harvest unit boundary. Its most appropriate use is for illustration of the optimization techniques employed in this paper and as a limited exploratory research tool.

Many simplifying assumptions were made during model development, e.g., a common, linear yarding cost function for both landings, a single road construction standard, etc. More realistic assumptions, or at least assumptions more consistent with particular applications, may be possible. For example, two landings may differ in their economic suitability as destinations for yarded logs. This economic disparity may be due to ground conditions, different skidder types assigned to each landing, or other causes. Incorporating and exploring the potential economic impact of modified assumptions such as these would seem quite feasible using modifications of this type of model.

The model is also based on a specific organizational structure and behavior. There are certainly other organizational alternatives for which this particular model might not be appropriate, for example, if the roading, yarding, and hauling operations were each done by independently contracted firms. The cooperative operational behavior assumed in this integrated model is unlikely when there are conflicting economic objectives and no integrated management planning, oversight, and control. Here it should be noted that the comparative numerical results generated by the model, however tentative, do raise questions about the economic significance of these organizational differences. These organizational differences and their economic impact certainly merit further research because of their potential management policy implications.

Finally, a more definitive evaluation of the economic trade-off between roading, yarding, and hauling must wait until this model is integrated into the landing location optimization model (Greulich 2012). Shifting the landing locations, which are fixed in this current analysis, will also enter into the balance to be struck between roading, yarding, and hauling expenditures. The merger of the two models is a necessary next step in this particular line of research.

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