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**Airtanker initial attack: a spreadsheet-  
based modeling procedure**

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# Airtanker initial attack: a spreadsheet-based modeling procedure

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**Abstract:** Airtankers are widely employed in wildland fire control operations. This paper presents a spreadsheet procedure for modeling airtanker initial-attack performance within a protection region. Easily written and modified, this spreadsheet-based model provides the user with substantial flexibility when specifying the spatial distribution of fire occurrence within the initial-attack zone of an airtanker base. Once this spatial distribution has been specified, the expected flight distance from the airtanker base to a random fire ignition point as well as higher moments of the flight distance distribution are readily computed and applied using the spreadsheet. The chief advantages of this spreadsheet approach are described. Calculation procedures and potential applications of flight distance moments including the optimization of airbase location are illustrated. Detailed numerical examples and opportunities for future development are indicated.

**Résumé :** Les avions-citernes sont largement utilisés dans les opérations de lutte contre les feux de forêt. Cet article présente une méthode basée sur un chiffrier pour modéliser la performance des avions-citernes lors de l'attaque initiale dans une zone de protection. Cette méthode est facile à élaborer et à modifier et procure à l'utilisateur une grande flexibilité pour délimiter la distribution spatiale du feu dans la zone d'attaque initiale d'une base d'avions-citernes. Une fois que cette distribution spatiale a été définie, la distance estimée de vol entre la base des avions-citernes et un point d'allumage du feu au hasard de même que les moments plus élevés de la distribution des distances de vol sont rapidement calculés et appliqués grâce au chiffrier. Les principaux avantages de cette méthode sont décrits. Les procédures de calcul et les applications potentielles des moments des distances de vol, incluant l'optimisation de la localisation de la base aérienne, sont illustrées. Des exemples numériques détaillés et les possibilités de développement futur sont présentés.

[Traduit par la Rédaction]

## Introduction

The purpose of this paper is to describe and apply a spreadsheet-based procedure for modeling airtanker initial-attack performance. A brief review of the pertinent literature underscores the importance of airtankers and the need for their careful evaluation prior to deployment for initial attack. A common contemporary method of modeling travel distance using area centroids is presented. A well-known source of error is associated with this particular methodology and some attempts to eliminate it are noted. An analytical procedure of particular efficacy and eminently suitable for spreadsheet calculation of the statistical moments of travel distance is then presented. Detailed spreadsheet examples illustrate the use of these flight distance moments. The paper concludes with some thoughts regarding opportunities for future research in this area.

## Initial-attack flight distance for airtankers

The operational objective of controlling a wildfire during the first work period together with its governing policy of strong initial attack on all fires has long been employed by successful fire-control agencies (Davis 1959). Given this

emphasis on early control of fires, it is not surprising that the element of initial separation distance between fire and fire-fighting resources is of singular importance. Davis (1959) noted the development of "origin-to-arrival elapsed time standards" as providing "a basis for transportation planning". Planning the spatial distribution of agency resources, which will strongly influence travel time to fires and hence initial-attack effectiveness, requires the effective assessment of potential fire-start locations. Various methods have been applied to estimate initial-attack travel distances. The more relevant of these estimation methods as applied to airtanker initial-attack allocation will be examined following a brief discussion of airtanker initial attack.

## Airtanker initial attack

The first operational use of airtankers by the U.S. Forest Service occurred in 1956, and within 15 years the role of the airtanker in fire control had been firmly established within that organization. Monte Pierce, National Air Officer for the U.S. Forest Service, reported that "the greatest value of air tankers is in initial attack to slow the fire down and buy time until ground forces can arrive and control it" (Pierce 1970<sup>1</sup>). He also stressed the need for careful planning given the high costs associated with airtanker use.

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<sup>1</sup>M.K. Pierce, 1970. Organization and management of air tanker operation. Presented at the National Forest Fire Seminar on Aircraft Management, Petawawa Forest Experiment Station, 26 October 1970, Chalk River, Ont.

Simard and Forster (1972) stated that the operational use of airtankers in Canada seems to have started in 1957 with the successful application of skimmer-type (water-based) airtankers in Ontario. Synthesizing the comments of many knowledgeable fire-control professionals across Canada in 1969, Simard noted that "it is generally agreed that aircraft are most effective during the early stages of a fire". This observation is forcefully restated in their 1972 report where Simard and Forster wrote: "The most effective use of airtankers is in an initial attack role. Within this role it is not uncommon that success or failure of a suppression action depends on the timely arrival of an appropriate number and type of aircraft at the scene of a fire." and "Aircraft are by far the most expensive single forest fire suppression tool. It is essential therefore, in developing a balanced forest fire protection program, which includes airtankers, that all of the characteristics, cost and benefits of available airtankers be known to make a rational decision on their use in a forest fire protection program."

Among other factors that determine airtanker performance during initial attack is the drop cycle time. Airtanker cycle time is composed of several elements that differ between land-based and water-based operations (Simard and Forster 1972). In either case, however, flight times are critical and must be estimated. A typical approach is to develop an explanatory model of cycle times based, in part, on travel distance and aircraft speed.

### Flight distance estimation

A recent paper by Islam and Martell (1998) illustrates a basic approach to the estimation of airtanker flight distance. They explicitly incorporated the spatial heterogeneity of fire occurrence into their model by partitioning the protection area into rectangular cells. They assumed that fire starts occurring within any given cell will be uniformly distributed in probability over that cell and that the centroid of the cell may be taken as the representative fire-start location. Since they were evaluating water-based airtankers, they employed both the distance from the airbase to the fire-start location and the distance from the fire-start location to the nearest airtanker-accessible water. Individual fires and their initial attack by airtankers are then simulated within the computer model. Their use of protection area partitioning allowed them to incorporate fire occurrence patterns as determined by vegetation, weather, and land use. Quite similar uniform distribution assumptions were used by Maloney (1973) and Greulich and O'Regan (1975, 1983) in their studies of the California airtanker system.

The specification of fire-start locations as occurring with uniform probability across an enclosed area is found to be a computationally convenient assumption. The usual observation of nonuniform fire occurrence within a protection region may be approximated by partitioning of the protection region into subregions in the general manner of Islam and Martell (1998).

Since fire-start location is a random variable, the corresponding flight distance from the airbase to the fire location is also a random variable. The expected travel distance for models of this type are quite typically approximated using the straight-line distance from the facility to the centroid of the area. Such a procedure leads to what is referred to as ag-

gregation error (Ghosh and Rushton 1987). Models developed by Suddarth and Herrick (1964), Love (1972), Drezner and Weslowsky (1980), and Cavalier and Sherali (1986) have recognized and sought to eliminate or reduce this source of error. The issue of how to address aggregation error when working with continuous demand models continues to be an active research topic (Drezner and Drezner 1997). The actual magnitude of the aggregation error in any specific application is difficult to predict; however, for timber harvesting applications, Suddarth and Herrick (1964) very cautiously suggested that such approximations might be deemed acceptable if a margin of error of about 5% is permissible. After a thorough review of pertinent work in this, field Okabe and Miller (1996) concluded that accuracy benchmarking of approximation methods under a variety of conditions is still wanting. Okabe and Miller (1996) found that the computation times for their exact procedures in point-to-line and point-to-area calculations are quite short, increasing at most linearly with the number of line segments used to define the region. For theirs and related analytical procedures, such as those given in this paper, there would appear to be little if any time advantage to using other than these exact methods. Accordingly, under what are exceptionally general assumptions, the exact value of expected travel distance can be readily calculated.

### Moment calculations for flight distance

The assumptions underlying the methodology to be presented here are not very restrictive. In this specific application to airtanker flight distance, there are only four major assumptions that need be considered. These assumptions are quite flexible, permitting their easy accommodation by existing conditions in most applications.

Formulas derived from these assumptions as well as procedures for their application may appear to be somewhat lengthy. However, they are readily entered and reproduced within a computer spreadsheet. In fact, their computational structure will be found to be most convenient for spreadsheet application.

### Assumptions

There are four principal assumptions upon which the moment computation methodology of this paper is based. Any potential application of the procedure should meet these assumptions to a degree judged adequate for the specific purpose of the analysis.

The first assumption is that the initial-attack zone with its relevant initial-attack fire occurrence attributes may be adequately defined in a horizontal plane by three classes of geometric figures: polygonal regions, straight-line segments, and points. There may be as many of these individual figures as needed by the analyst to acceptably define the protection zone. An area of uniform fire-start probability, even if it has noncontiguous parts and (or) holes, can be modeled as a single polygonal region. Polygonal regions can be overlain with lines and points, all of which can be given uniquely defined fire occurrence probabilities. This assumption implies that real-world boundaries can be adequately approximated for modeling purposes by connected straight-line segments in the plane and that topographic relief can be safely ignored. It

is worth noting that if topographic relief must be recognized, as Maloney (1973) did in his evaluation of airtanker performance, then areas of similar topographic character can be collected into separate polygonal regions. It is anticipated that features such as roads may be quite adequately described by connected straight-line segments. Likewise, edges of rivers and lakes can also be modeled using connected straight-line segments where these locations have distinctly different fire occurrence probabilities when compared with the adjoining area. Campgrounds can be modeled as polygonal regions, connected straight-line segments, or points, whichever geometric figure or combination of figures meets the analyst's requirements in any given situation.

It is assumed that the fire-start location, given that a fire will start somewhere within any given well-defined area, can be described as a random variable distributed with uniform probability across the horizontal projection of that area. Likewise, the fire-start location on a line is assumed to be adequately described by a uniform probability distribution along the length of the horizontally projected line. The probability of the fire-start location at a point is concentrated at the point itself.

It is assumed that fire-start locations can be treated as independently distributed random variables within the planning period of the model. Precaution with this assumption would be most relevant if the procedure were to be applied in a day-to-day operational decision context. In such applications where relevant information regarding current fire-start locations is available, the treatment of future fire-start locations as being statistically independent may be questionable (e.g., in the case of either localized lightning storms or arsonist activity along a road). Dynamic updating of probabilities and recalculation of travel distance moments might be considered as a possible accommodating modification under such circumstances. Protection area partitioning and assignment of probabilities to the individual partitions should reflect the best available information at the beginning of the planning period for which the model is to be applied. The joint spatial-temporal distribution of fire-start locations becomes most important when a model is designed to handle the transfer of resources from one ongoing fire location to a new fire-start location. The assumption of statistical independence becomes of lesser concern and is more readily justified where the spatial-temporal occurrence of sequential fires is irrelevant to the decision process being modeled.

Flight distances are calculated as the straight-line horizontal distance from the airbase to the fire-start location. No allowance is made for indirect routing due to air traffic control, weather, or topographic features. If the analyst has information upon which to base an estimate, then a "wander factor" might be incorporated to adjust for indirect lines of flight. One-way map distance is used and a map projection of accuracy suitable to the application should be employed.

### Formulas

Airtanker flight distance from the airtanker base to a randomly located fire-start location is itself a random variable. The distribution function for the flight distance depends on the spatial definition of fire occurrence probability across the protection zone, which is of course uniquely defined for different protection zones and which may also vary for even a

single protection zone under temporally changing environmental conditions. The abstracted model of a protection zone is described by three geometric elements: triangular areas, line segments, and points. Moments of these individual elements may be calculated, weighted by their corresponding probabilities, and progressively aggregated to ultimately yield the exact moment(s) for the entire protection zone.

### Areas

Each line segment on the boundary of a polygonal region is used as the base of a triangle formed by placing its opposite vertex at the airbase (Fig. 1). These triangular elements are sequentially numbered in a counterclockwise direction around the perimeter of the area (Fig. 2). The end point of the last ( $n$ )th segment that closes the loop is also the beginning point of the first segment so that in terms of their respective coordinates,  $(X_{n+1}, Y_{n+1}) = (X_1, Y_1)$ . In Fig. 1, an arbitrary triangular element  $i$  is shown. The coordinates of the airbase location are  $(X_0, Y_0)$ , the lengths of the sides are denoted  $L_{i,1}$  and  $L_{i,2}$ , and the length of the base is  $L_{i,3}$ . Note that for the next triangular element  $(i + 1)$  of the polygon,  $L_{i+1,1} = L_{i,2}$ , and from the previous element  $(i - 1)$  of the polygon,  $L_{i,1} = L_{i-1,2}$ . For (fire-start) locations uniformly distributed in probability over the area of this triangular element, the respective first and second moments of (flight) distance from the (airbase) vertex at  $(X_0, Y_0)$  have been shown (Peters 1978; Greulich 1987) to be given exactly by

$$[1] \quad E\{D_{A_i}\} = \left(\frac{L_{i,1} + L_{i,2}}{6}\right) \left[1 + \left(\frac{L_{i,1} - L_{i,2}}{L_{i,3}}\right)^2\right] + \left(\frac{4A_i^2}{3L_{i,3}^3}\right) \left[\log_e \left(\frac{1+r_i}{1-r_i}\right)\right]$$

and

$$[2] \quad E\{D_{A_i}^2\} = \left(\frac{1}{12}\right) (3L_{i,1}^2 + 3L_{i,2}^2 - L_{i,3}^2)$$

where

$$[3] \quad A_i = \left(\frac{1}{2}\right) [(X_i - X_0)(Y_{i+1} - Y_0) - (X_{i+1} - X_0)(Y_i - Y_0)]$$

and

$$[4] \quad r_i = \left(\frac{L_{i,3}}{L_{i,1} + L_{i,2}}\right)$$

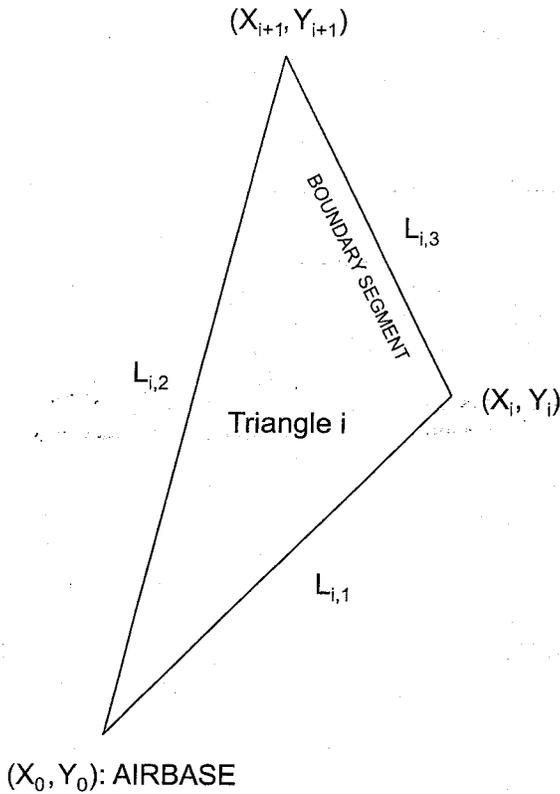
Line lengths are to be found by the usual formula for straight-line distance in the plane, e.g., the length of the line segment from  $(X_j, Y_j)$  to  $(X_k, Y_k)$  is

$$[5] \quad L = [(X_j - X_k)^2 + (Y_j - Y_k)^2]^{0.5}$$

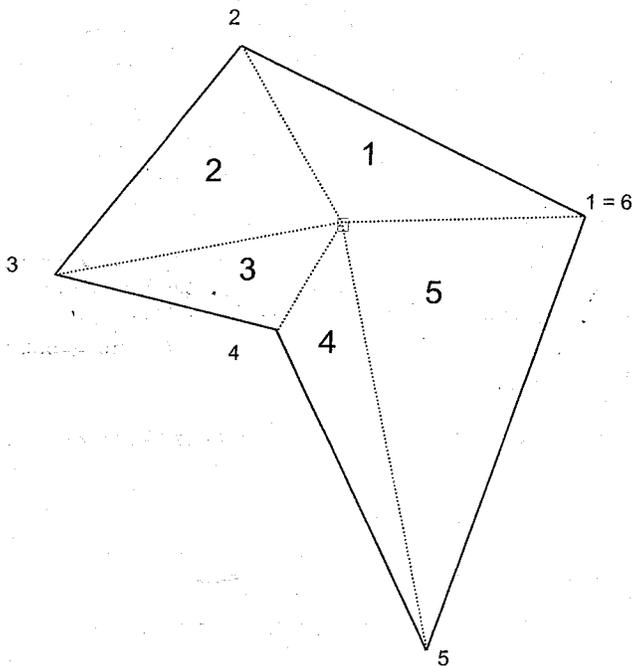
### Lines

The first and second moments about the origin for the random distance from a given point to a point randomly distributed along a straight-line segment are readily developed. Once again referring to Fig. 1, it is now assumed that fire starts are uniformly distributed in probability along the

**Fig. 1.** Triangular element, the basic geometric shape used to describe polygonal regions.



**Fig. 2.** Polygonal region comprising five triangular elements sharing a common vertex at the airbase location. Indexing proceeds in a counterclockwise direction.



length  $L_{i,3}$  of the triangle's base. The airbase is still located at the vertex  $(X_0, Y_0)$ . The above formulas for the triangular area are simply multiplied by the appropriate constant in each case:

$$[6] \quad E\{D_{L_i}\} = \left(\frac{3}{2}\right)E\{D_{A_i}\}$$

and

$$[7] \quad E\{D_{L_i}^2\} = (2)E\{D_{A_i}^2\}$$

A connected series of line segments may be used to approximate a curvilinear feature, for example, a contour-following road in mountainous terrain. The number of line segments ( $i = 1, n$ ) required for any given feature depends on the length and curvature of the feature in conjunction with the approximation accuracy sought by the analyst.

**Points**

The first and second moments about the origin for the distance to a point are trivial, since the fire occurrence location is constant. Denoting the distance from the airbase to the point as  $L_i$ , the first and second moments are

$$[8] \quad E\{D_P\} = L_i$$

and

$$[9] \quad E\{D_P^2\} = L_i^2$$

These two trivial and immediately obvious results provide a quick partial check of the preceding formulas. By letting  $L_{i,3}$  go to zero in the formulas for a line segment, the above two results for a point should be obtained, and in fact, that is what is observed. For details on taking the limit of  $E\{D_{A_i}\}$ , see Greulich (1987); the limit of  $E\{D_{A_i}^2\}$  is more straightforward and the anticipated result can be verified by inspection.

**Weighting procedure**

For a polygonal region, the moments are calculated using the following formulas:<sup>2</sup>

$$[10] \quad E\{D_A\} = \left(\frac{1}{A}\right)\sum_{i=1}^n A_i E\{D_{A_i}\}$$

and

$$[11] \quad E\{D_A^2\} = \left(\frac{1}{A}\right)\sum_{i=1}^n A_i E\{D_{A_i}^2\}$$

where

$$[12] \quad A = \sum_{i=1}^n A_i$$

As previously noted, the polygonal regions are described by a counterclockwise sequence of connected line segments. In applying a counterclockwise orientation, the area being encompassed is always kept to the left of the line of progress in proceeding around the area. The coordinate area formula

<sup>2</sup>The first use of the coordinate area formula for area weighting in a forestry application is found in Donnelly's (1978) publication. Somewhat earlier applications of this very general and extraordinarily efficient and easily used weighting technique are to be found in geography (Blair and Bliss 1967) and geology (Hall 1976).

will affix the appropriate sign, either positive or negative as required, to each triangular element making up the area, and the final sum (total area encompassed) will be positive if this counterclockwise orientation is observed.

The weighted moments for a feature described by a sequence of  $n$  line segments are calculated in a straightforward manner as (Okabe and Miller 1996)

$$[13] \quad E\{D_L\} = \left(\frac{1}{L}\right) \sum_{i=1}^n L_i E\{D_{L_i}\}$$

and

$$[14] \quad E\{D_L^2\} = \left(\frac{1}{L}\right) \sum_{i=1}^n L_i E\{D_{L_i}^2\}$$

where

$$[15] \quad L = \sum_{i=1}^n L_i$$

Individual line segments need not be oriented. However, for purposes of spreadsheet modeling, it has been found to be more efficient to use oriented chains whenever the termination of one segment can be taken as the beginning of another.

For  $n$  points, each of which has equal probability of being the location of a fire start, the weighted moments are given by

$$[16] \quad E\{D_P\} = \left(\frac{1}{n}\right) \sum_{i=1}^n E\{D_{P_i}\}$$

and

$$[17] \quad E\{D_P^2\} = \left(\frac{1}{n}\right) \sum_{i=1}^n E\{D_{P_i}^2\}$$

The first two moments of the model are calculated as described above for each feature within the protection zone. The concluding step is the calculation of a single weighted value for each of the two moments for the entire protection zone. To do this last calculation, every feature within the protection zone is assigned its conditional probability that a fire has started within it given that a fire has occurred somewhere within the protection zone. These final two weighted values are the first two moments of the flight distance distribution for the protection zone. They are the exact values of the first two moments for the abstracted model as defined by the analyst.

### Advantages

For those airtanker models where the previously described assumptions are appropriate, the proposed moment calculation procedure has much to offer the analyst.

Exact values for the first and higher moments of the travel distance distribution may be calculated for the analyst-defined model of the protection region. As calculated by distance-to-centroid procedures, the expected travel distance (first moment of the flight distance distribution) is only an approximation of its true value for the modeled region. Unless subject to sensitivity analysis, such approximations are

of unknown precision. In general, their precision can be improved by increasingly small subdivision of the modeled region, but this augmented precision is achieved through increased computational effort.

The proposed procedure offers relatively short computation times. Computationally equivalent to the procedure of Okabe and Miller (1996), the time required grows no more than linearly with the number of line segments used to describe the protection area. For the distance-to-centroid methodology, a computationally efficient procedure for calculating the centroid of a polygonal region is given in Hall (1976). The computation time for Hall's (1976) procedure also increases in a linear fashion with the number of line segments. Accordingly, there is no clearly significant time advantage for either computational process.

The geographic shape of the protection zone can be closely modeled. Curvilinear boundaries can be tightly followed with connected line segments of user-selected lengths. This process provides polygonal regions and connected line segments that can spatially depict protection zone conditions and features to the appropriate levels of resolution and detail.

The geographical distribution of fire occurrence can be accurately described. Not only areas, but line and point sources of fire start within the protection zone can be singled out and included in the model. Features with atypical fire occurrence probabilities compared with surrounding areas, e.g., roads and campgrounds, can be easily incorporated.

The proposed procedure not only offers improvements in model description of the protection zone but also provides the exact calculation of model moments with no significant increase in computational time. Taken in combination, these two improvements provide a more accurate model of the real-world protection area and its observed travel distance statistics. A final, very significant characteristic of the foregoing procedure is its straightforward development within a spreadsheet context. All of these analytical possibilities and potential benefits are best illustrated by numerical example.

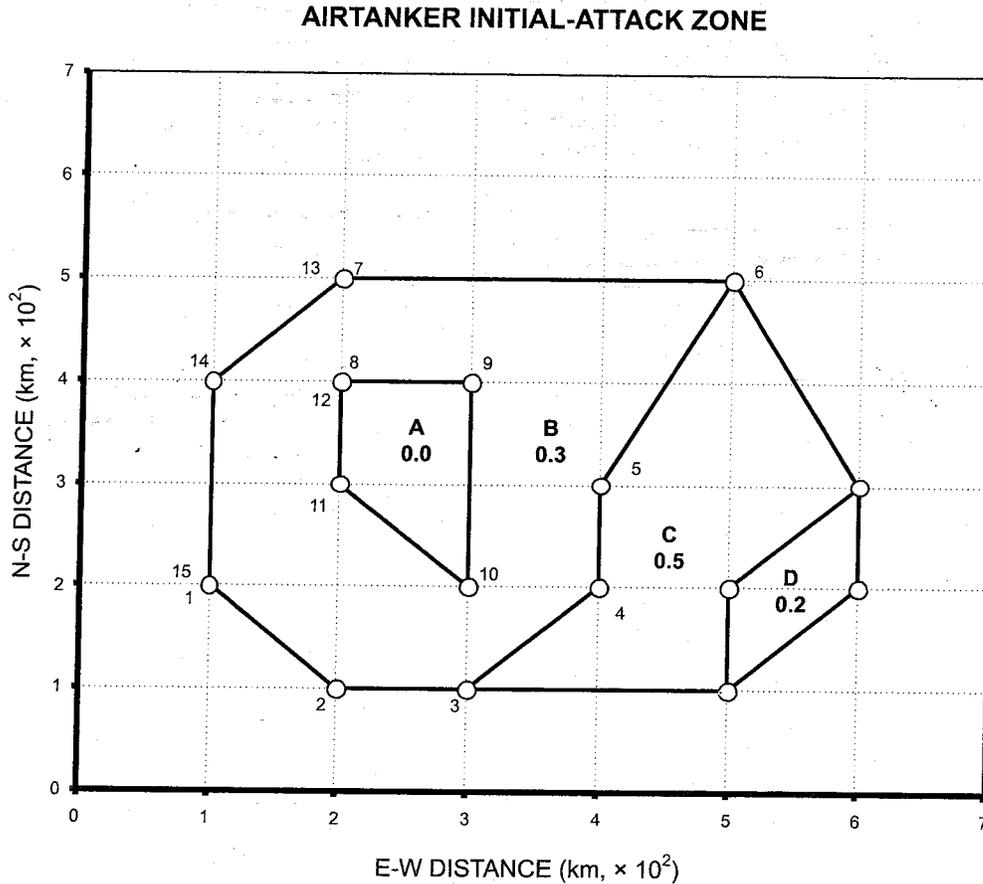
### Numerical examples

Several numerical examples are given in this section to illustrate application of the methodology. These examples, while relatively simple, are of sufficient complexity to capture essential elements of the procedure. While most of the examples are of summary form, sufficient detail is provided at various points to facilitate model development by the interested reader.

#### Area moments, calculation, and application

A single airtanker base with its associated initial-attack zone is examined in this first example. The entire initial-attack zone is partitioned into mutually exclusive areas of uniform fire-start conditions. Partition boundaries are drawn such that within any given partition, the probability of fire-start location is uniformly distributed across the partition area, i.e., in mathematical terms, each infinitesimal area,  $dA$ , of the partition has the same probability of being the fire-start location. Given that a fire starts somewhere within the initial-attack zone, the conditional probability of it having

Fig. 3. Map of an initial-attack protection zone consisting of three polygonal regions, one of which has a protection excluded enclave.



started within each partition is specified. These conditional probabilities when summed across all partitions must equal 1.

The initial-attack zone, associated partitions, and their conditional initial-attack fire occurrence probabilities are shown in Fig. 3. Partition A has zero probability assigned to it and may represent a lake, a separate fire-control jurisdiction, or any other reason for which initial-attack probability is considered nil. As illustrated here, there is no requirement that the initial-attack zone or its individual partitions be convex; the methodology works equally well regardless of partition or initial-attack zone shape. The coordinates of the line segments defining each partition are entered counterclockwise in accordance with the usual mathematical convention for oriented polygons. As an example, the entry sequence for polygonal region B is given in the first three columns of Table 1. An arbitrary starting point on the boundary is selected; this point is indexed as 1 and subsequent points sequentially numbered. As indexing and entry of coordinates proceeds, the area of interest is always kept to the left of the line of traverse. Note that at point 7, a jump is made to a point on the boundary of the interior region A. This jump can be made between any two arbitrarily selected points, one on each boundary. Once on the boundary of interior region A, points are now entered in a clockwise direction around this region, since it is an enclave within region B and must be excluded (note that the area of interest, region B, is still to the left of the line of traverse). Having encircled this interior region, a jump back to point 7 (now also indexed as point 13) is made. Area circumscription now continues counter-

clockwise terminating at the point where it began (ending point 15 is also starting point 1). The balance of Table 1 shows numerical results from the application of formulas 1–5 and 10–12 to region B if the airbase were to be located at (4, 4).

Table 1 was developed using the Microsoft Excel® spreadsheet program. The spreadsheet is easily constructed and only one precautionary remark is warranted. When  $A_i$  is zero for any triangle  $i$ , then the weighted value  $(A_i/A)E\{D_{A_i}\}$  for triangle  $i$  should be zero. However, before calculating  $E\{D_{A_i}\}$ , the value of  $A_i$  should be tested. If  $A_i$  equals zero, it may be because  $L_{i,1} + L_{i,2} = L_{i,3}$ , in which case,  $r_i = 1$  and the operation  $(1 + r_i)/(1 - r_i)$  in eq. 1 would result in a division by zero error. It is therefore recommended that a logical test for  $A_i$  equal to zero be entered in the spreadsheet column where  $E\{D_{A_i}\}$  is to be calculated; if the test result is true, then immediately set  $E\{D_{A_i}\}$  equal to zero; otherwise, use eq. 1 for its calculation. It should be noted that this same caveat applies to the calculation of  $E\{D_L\}$ , except that at the limit, its value, rather than zero, is given by

$$[18] \quad E\{D_L\} = \left(\frac{3}{2}\right) \left(\frac{L_{i,1} + L_{i,2}}{6}\right) \left[1 + \left(\frac{L_{i,1} - L_{i,2}}{L_{i,3}}\right)^2\right]$$

The results of the calculations shown in Table 1 have been transferred over to Table 2 where they are to be found as entries on the first line. The continued and final step in the weighting process is illustrated in this table where the mo-

**Table 1.** Microsoft Excel spreadsheet for polygonal region B of Fig. 3 illustrating steps in first and second moment calculation.

$i$	$X_i$	$Y_i$	$A_i$	$L_{i,1}$	$L_{i,2}$	$L_{i,3}$	$r_i$	$E\{D_{A_i}\}$	$(A_i/A)E\{D_{A_i}\}$	$E\{D_{A_i}^2\}$	$(A_i/A)E\{D_{A_i}^2\}$
1	1	2	2.5	3.6056	3.6056	1.4142	0.1961	2.3726	0.5932	6.3333	1.5833
2	2	1	1.5	3.6056	3.1623	1.0000	0.1478	2.2427	0.3364	5.6667	0.8500
3	3	1	1.0	3.1623	2.0000	1.4142	0.2740	1.7066	0.1707	3.3333	0.3333
4	4	2	0.0	2.0000	1.0000	1.0000	0.3333	0.0000	0.0000	1.1667	0.0000
5	4	3	0.5	1.0000	1.4142	2.2361	0.9262	0.5134	0.0257	0.3333	0.0167
6	5	5	1.5	1.4142	2.2361	3.0000	0.8219	0.9124	0.1369	1.0000	0.1500
7	2	5	1.0	2.2361	2.0000	1.0000	0.2361	1.3870	0.1387	2.1667	0.2167
8	2	4	0.0	2.0000	1.0000	1.0000	0.3333	0.0000	0.0000	1.1667	0.0000
9	3	4	1.0	1.0000	2.2361	2.0000	0.6180	0.9860	0.0986	1.1667	0.1167
10	3	2	-1.5	2.2361	2.2361	1.4142	0.3162	1.4400	-0.2160	2.3333	-0.3500
11	2	3	-1.0	2.2361	2.0000	1.0000	0.2361	1.3870	-0.1387	2.1667	-0.2167
12	2	4	-1.0	2.0000	2.2361	1.0000	0.2361	1.3870	-0.1387	2.1667	-0.2167
13	2	5	1.5	2.2361	3.0000	1.4142	0.2701	1.7148	0.2572	3.3333	0.5000
14	1	4	3.0	3.0000	3.6056	2.0000	0.3028	2.1396	0.6419	5.1667	1.5500
15	1	2									

$A = 10.0$   $E\{D_A\} = 1.9057$   $E\{D_A^2\} = 4.5333$

**Table 2.** Summarizing table of final weighting by conditional probabilities for the initial-attack protection zone of Fig. 3 (five different basing conditions are shown).

Airbase location	Coordinates	Partition	Weight	$E\{D\}$	$E\{D^2\}$	Expected cost
Arbitrary point	(4.00, 4.00)	B	0.3	1.9057	4.5333	1.5233
		C	0.5	1.7628	3.5667	1.4672
		D	0.2	2.5385	6.5000	1.6680
		Agg.*	1.9608	4.4433	1.5242	
Arbitrary point	(5.00, 3.00)	B	0.3	2.5905	7.4333	1.7146
		C	0.5	1.0746	1.5000	1.3131
		D	0.2	1.1942	1.5000	1.3238
		Agg.	1.5533	3.2800	1.4357	
Min{ $E\{D\}$ }	(4.55, 2.59)	B	0.3	2.3011	5.9143	1.6206
		C	0.5	1.0416	1.3273	1.3024
		D	0.2	1.2090	1.5006	1.3251
		Agg.	1.4529	2.7381	1.4024	
Min{ $E\{D^2\}$ }	(4.27, 2.69)	B	0.3	2.0687	4.8667	1.5528
		C	0.5	1.1175	1.4843	1.3162
		D	0.2	1.4839	2.2390	1.3827
		Agg.	1.4761	2.6500	1.4005	
Min{cost function}	(4.37, 2.66)	B	0.3	2.1482	5.2145	1.5755
		C	0.5	1.0817	1.4078	1.3096
		D	0.2	1.3874	1.9625	1.3617
		Agg.	1.4628	2.6608	1.3998	

\*Weighted mean.

ments for each of the three partitions are combined using their conditional probabilities. The first and second moments for the entire protection zone are thereby calculated as 1.9608 and 4.4433, respectively. The variance of the flight distance distribution may be calculated to be 0.5986 by the usual formula:

$$[19] \quad \text{Var}\{D\} = E\{D^2\} - [E\{D\}]^2$$

As just illustrated, it is possible to quickly develop the numerical values for the first and second moments of the flight distance distribution. In many cases, these results would be carried over as one small component within a more sophisti-

cated model. In a few cases, however, it may be possible to continue an analysis in the spreadsheet itself. Two ancillary examples of such continued model development follow.

Once programmed into the Excel spreadsheet, it is easy to evaluate basing alternatives for the protection area. In an extension of this first numerical example, a second arbitrary point, (5, 3), is specified and the first two moments immediately displayed. In this case, the expected flight distance has been reduced to 1.5533 but the variance increases to 0.8673. Simple comparative information of this type may be sufficient for some basing decisions without further formal development of the model. However, it is also worth noting

that some very simple optimization models can be applied within the context of the current spreadsheet without much additional effort.

The location that minimizes the expected straight-line flight distance is readily found using the existing spreadsheet in conjunction with the Excel add-in program called Solver (Fylstra et al. 1998). This add-in software comes bundled with Excel and is exceedingly efficient and easy to use. The minimum expected flight distance solution for the given protection area is shown in Table 2. If the airtanker(s) were to be positioned at (4.55, 2.59), the expected flight distance would be at a minimum with an expected value of 1.4529.

If airtanker system costs increase as the square of the distance from the fire, then the airbase location that minimizes the expected square of the distance provides the most economic location. This location is the centroid of the protection area and in this case is found to be (4.27, 2.69), once again having been quickly located using the Solver software.

While these last two solutions provide answers to the question of optimal location given that cost increases as either a linear or a squared function of flight distance, a more general approach is possible. The next example illustrates this more general approach to identifying the airbase location that minimizes operating costs.

#### Extended cost functions

Cost functions can be analyzed using distribution moments. This example demonstrates the use of an empirical cost function that was developed in the context of a real-world evaluation.

In this modification of McDonald's "one-strike initial-attack" model, as described by Hodgson and Newstead (1978), it is assumed that the airtanker(s) will be dispatched from the airbase, fly a straight line to the fire location, and make airdrops until forced to return because of low fuel. Water for drops is obtained by skimming nearby sources. These are the approximate assumptions under which data for the following cost equation were obtained.

Dobson (1965) developed cost information for the Canadair CL-215. Dobson's (1965) units of nautical miles and imperial gallons have been changed in the formula given here to kilometres and litres. An extremely close fit to his original information is provided by the following equation:

$$[20] \quad C = 1.15 + (0.0891)D + (0.0449)D^2$$

where  $C$  (cents per litre) is the delivered cost of the air-dropped water and  $D$  (km,  $\times 10^2$ ) is the flight distance from the airbase to the fire-start location.

The expected cost as defined by this equation is given by

$$[21] \quad E\{C\} = 1.15 + (0.0891)E\{D\} + (0.0449)E\{D^2\}$$

The airbase locations identified in the previous examples can be evaluated using this equation and the results are shown in the last column of Table 2.

The location that minimizes this expected cost function can be found using the Solver program. The spreadsheet results of this minimization are shown as the last series of row entries in Table 2. Because of the strict convexity of this problem, there is only one optimal location and it is the global optimum (Greulich 1991).

#### Moments for a general area

The final example is a protection area that represents a recreational area with its own fire suppression organization. In this example, all three geometric elements are employed to describe the area. Figure 4 shows the area being protected by aircraft stationed at coordinate location (1.0, 3.5).

All significant fire occurrence related features within the protection area have been described by the analyst using appropriate geometric figures. Thus, moments for the general area have been calculated using triangular area elements; for the highway and access road, line segments have been employed, and for the campsites and rest area, the formulas and procedures for points were used.

Each of the geometric elements has been evaluated using eqs. 1–18 and the calculated values are listed in Table 3. The first and second moments for the five features of the protection area are weighted by their conditional probabilities to arrive at the final moment estimates. The expected flight distance (km,  $\times 10^2$ ) is 2.3197, and the second moment is 5.9412.

In a simple extension of these results, assume that the initial attack will be flown on fires by an aircraft that has a cruise speed of  $V$  (km/h,  $\times 10^2$ ). Initial-attack flight time  $T$  (h) to a fire located a distance  $D$  (km,  $\times 10^2$ ) from the base is given by

$$[22] \quad T = \frac{D}{V}$$

with an expected value and variance of

$$[23] \quad E\{T\} = \frac{E\{D\}}{V} = \frac{2.3197}{V}$$

and

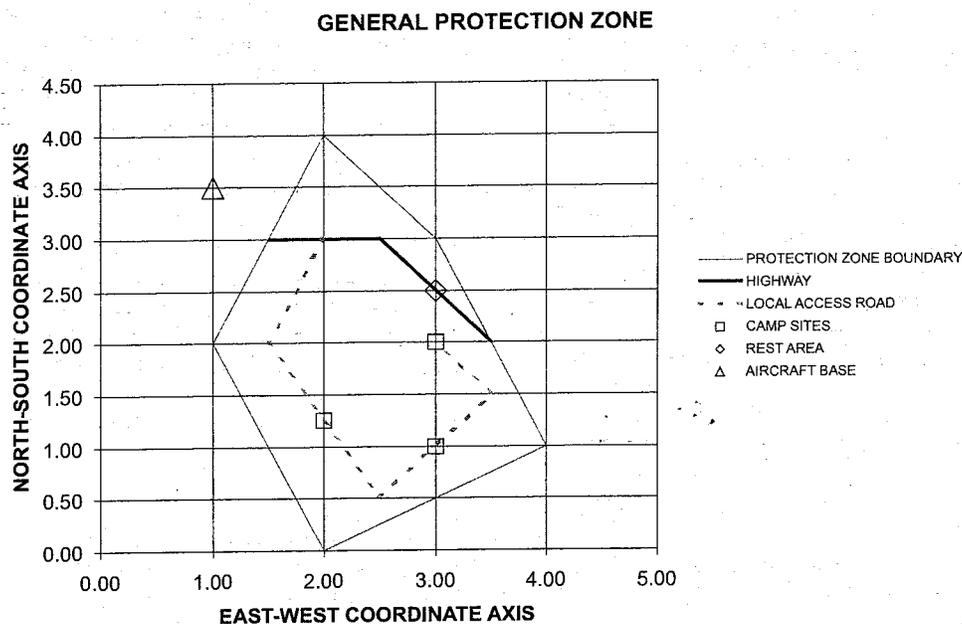
$$[24] \quad \text{Var}\{T\} = \frac{\text{Var}\{D\}}{V^2} = \frac{0.5602}{V^2}$$

These simple, very quickly obtained relationships may be of considerable value to fire-control officers who are weighing the effectiveness of alternative aircraft types.

#### Travel distance models: a historical perspective

The use of the expected travel distance parameter, with particular emphasis on forestry applications, has been discussed by Greulich (1987, 1991), but continuing investigation has yielded additional background information. Wilhelm Launhardt was perhaps the first to give a strong analytical basis to transportation analysis. An English-language publication that collects some of his work done during the latter half of the 19th century presents the derivation and use of expected travel distance to the center of a circular region (Launhardt 1900–1902). This extremely important result first appears in a forestry context in the 1930s when optimization of transportation systems using expected travel distances was an important research topic (Greulich 1987). After some misleading work based on the use of distance-to-centroid calculations by Matthews (1942), the correct formula for the expected travel distance to one of the corners of a rectangular region was given by Sundberg

Fig. 4. Map of an initial-attack protection zone described by a polygonal region, two connected line segment sequences, and two sets of points.



**Table 3.** Final weighting by conditional probabilities for the initial-attack protection zone shown in Fig. 4.

Feature	Weight	$E\{D\}$	$E\{D^2\}$
General area	0.40	2.2722	5.8141
Highway	0.25	1.7787	3.5788
Access road	0.10	3.1016	9.6667
Campsites	0.20	2.7213	7.5208
Rest area	0.05	2.2361	5.0000
Total zone	1.00	2.3197	5.9412

(1952, 1953). This formula is attributed to G. Almqvist by Larsson (1959). In his publication, Larsson (1959) did an in-depth analysis of optimal forest road spacing using expected travel distances during harvesting operations. In 1964, Suddarth and Herrick published what remains a key paper on the calculation of expected travel distance. Their paper includes, among others, a formula for the expected travel distance to an acute vertex of a right triangle. In 1978, Peters derived and applied the expected travel distance formula for a general triangle in a traditional forestry optimization model. Peter's (1978) formula was subsequently employed by Donnelly (1978) in conjunction with the coordinate area formula to calculate the expected travel distance from any polygonal region to a point. In 1981, Okabe and Miki derived the density function for travel distance from a fixed point to random points in the plane where both the fixed point and the random points are bounded by a convex polygonal region. In 1984, they presented formulas for the first two moments of this distribution. In 1987, Okabe extended their procedure to any polygonal region and gave an iterative procedure for developing the formula for any desired moment of the distribution. Also in 1987, the general polygonal model of Donnelly (1978) was extended by Greulich to include higher moments of the travel distance distribution. This extended model of Donnelly (1978) subsequently

formed the basis for a continuous-location optimization program (Greulich 1991). In this paper, the new optimization program was compared with alternatives found in the management science and operations research literature. This new model and optimization program, while significantly more general and easier to use than the alternative models, showed no notable sacrifice of computational speed.

### Future developments

There are several areas of application and potential research. One of the more exciting practical areas for future development is the use of a geographic information system (GIS) as the "front end" to the spreadsheet model.

In the examples of this paper, data for the geometric elements were entered from the keyboard. In most practical applications, it would be most advantageous to automate this data entry process using existing GIS databases. Fournier et al. (2000) noted that not only is this strategy the simplest formulation of the model-to-GIS linkage, but it is also the most flexible. Creating a GIS linkage to the model described in this paper would seem to be an important next step in developing a practical fire management tool from the procedures given in this paper. Subsequent integration of the model into the GIS environment itself may follow as both the model and GIS environment mature. At the present time, the GIS environment does not typically afford the tools necessary for a higher level of model integration. Work is progressing in this area, however, and Prof. A. Okabe of the University of Tokyo is actively developing GIS tools that may facilitate a higher level of integration (personal communication dated 15 February 2001).

The development of equations for higher moments would be of particular interest where variance estimates of strongly curvilinear cost equations are needed. First-order second-moment (FOSM) methods and their extension to higher moments are well established in the literature. Ang and Tang

(1975) discussed how other, more complex functions can be evaluated yielding not only estimates of the expected value but also estimates of variance. The development of equations for higher moments of the flight distance distribution can take advantage of some simplifying relationships given in the literature (Okabe 1987; Greulich 1987).

Martell (1982) has argued that researchers should examine the possibility of near-simultaneous fires and the congestion that such multiple fire starts might imply. Another implication of multiple initial-attack fires is the redirection of the in-use aircraft resource. Islam and Martell (1998) explicitly identified this issue in their recently published initial-attack model. At the moment, their model as well as those of other researchers "assume that airtankers are never dispatched directly from one fire to another". Relaxation of this model assumption requires the development of the travel distance moments from one randomly located point to another. First moments, or expected values, for the travel distance distribution from one randomly located point to another for all combinations of points, lines, and polygonal areas are given in Okabe and Miller (1996). The development of higher moments for these distributions remains to be done. The possible lack of statistical independence between the location of the two random points, i.e., the assumption that fire-start locations are uniformly and independently distributed random variables, should also be critically examined.

Another topic that has not been touched upon in this paper is that of constrained optimization. Many constraints are quite easily incorporated using the Solver add-in program that comes with Excel. If a new airbase is to be optimally sited and its potential spatial location is confined to a convex region, then constraint conditions for achievement of global convex cost function minimization are met. For nonconvex regions, the optimization process is typically more involved and it may in fact converge to a suboptimal location. The general topic of constrained optimization opens an area of interesting research possibilities in facility siting.

In some applications, an expected travel time expression might be called for in the constraint set. For example, a fire protection agency may wish to locate its base so as to satisfy a maximum mean travel time constraint for a region or sub-region within its jurisdiction while minimizing a more general cost function. Such model formulations are quite easily accomplished using the procedures given here and could lead to interesting results of practical utility.

Much of the theory and many of the computational techniques given here were originally developed for the evaluation of timber harvesting operations. In this paper, it has been shown that these general results are equally applicable to a variety of airtanker evaluation problems. It will be most interesting to see where and how future researchers apply these and similar continuous-location techniques to other location siting problems in forestry.

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