A Nonlinear Programming Approach to Skidder Congestion

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Abstract

The reduction of yarding costs by alternative optimization methods is examined. A general nonlinear programming approach is presented for selecting an optimal yarding strategy from two different skidder types. A reason that an optimal yarding strategy is not apparent is due to the complex interactions of skidders queueing at the landing. The nonlinear programming approach captures the effect of skidders queueing at the landing with the use of nonlinear equations, whereas a linear programming model is unable to include this important effect. The solutions to both nonlinear and linear programming models are compared to provide some insight into the effect of skidder congestion on an optimal yarding strategy.

Keywords. Forest engineering, Logging, Operations research, Linear programming, Nonlinear programming.

Introduction

In this article we consider the problem of how to best allocate skidders of differing capacity within a timber harvest area where logs are being picked up and brought to a common landing. The best spatial allocation of the skidder resource may not be that which intuition, even intuition tempered by experience, initially favors. A major determinant of skidder productivity (and by implication a potential factor in its allocation) is congestion at the landing (McIntosh and Johnson, 1974). Failure to recognize and allow for the adverse productivity impact of this queueing effect has the potential of leading to serious overestimation of skidder system output. This overestimation may subsequently affect installed operational capacity of other harvesting subsystems when skidding is balanced with related activities such as felling, loading, and hauling. Under these conditions better estimates of skidding production will yield a more efficient overall harvesting system operation.

The primary objective is to develop a general optimization model that more realistically describes skidding operations. In this study, we achieved this goal by developing a nonlinear optimization model of a skidding yarding activity. A nonlinear programming approach was chosen because skidder congestion at the landing and its influence on productivity is fundamentally a nonlinear process. Estimation of this nonlinear production relationship under optimal allocation of the skidder resource is examined using some initial computer runs with realistic input data. A comparison of these numerical results with those obtained from a linear programming model of the same skidding operations ignoring congestion reveals the relative impact of the nonlinear queueing effect.

While it is clear that nonlinear effects exist in harvesting operations, very few applications of nonlinear programming have been made in this area. According to Bare (1971):

> "Much work needs to be done in exploring the potential of nonlinear programming methods in forest management. At present it is not clear if the use of more sophisticated programming methods will offer the forest manager any advantages over linear programming. However, it is an area that deserves much more attention than it has so far received."

The research presented in this article strongly suggests that a nonlinear programming approach could be very useful as an advanced analytical procedure applied to a harvesting problem.

Background

In a recent work, Koger and Webster (1986) presented an optimization procedure (LOST: logging optimization selection technique) to evaluate road and setting layout within a forest tract. They used linear programming to help identify the "best" transportation system design. The transportation system was considered "best" when the related costs of harvesting the tract were minimized within the confines of user-specified layout alternatives.

LOST (Koger and Webster, 1984) included several activity submodels which calculated the times and costs of skidding, trucking, and other required implementing activities for each user-specified layout. Multiple equipment was allowed within these activity submodels. The LOST model provided a general context within which to examine the relative value of applying linear and nonlinear optimization to typical harvesting activities. In this paper, we examine the skidding submodel for: (1) the production impact of optimal allocation of the skidder resource within a single setting; and (2) the impact on production of skidder congestion at the landing. In keeping with the LOST submodel, the assumption of concurrent

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use of all skidders is retained throughout. The evaluation of multiple-equipment performance within a LOST submodel is based on two additional important assumptions: 1) optimization within a submodel is not necessary because potential gains are insignificant; and 2) interaction effects with other submodels, if present, are small enough to ignore. Our research investigated the impact of optimization within the scope of a skidding submodel, considered potential impact of interaction effects, and demonstrated the differences of applying linear and nonlinear optimization.

MODEL DEVELOPMENT

A general optimization model of the skidding process has been developed. The problem can be stated as: given a setting that is to be yarded and several skidders, what is an optimal strategy for assigning skidders to regions so that the entire setting is yarded in the minimal amount of time. We assume a setting area with a central landing, and start by partitioning the setting area into subareas, specified by the subscript $j (j = 1, m)$. The model can easily incorporate irregular setting shapes and nonuniform turn distributions through appropriate partitioning of the setting area. The number of partitions reflects the desired accuracy of the solution as well as computational limitations.

Skidders were sorted into two classes (each class representing a different resource input) with the objective of aggregating skidders of similar capability and comparable yarding cycle times. We use the subscript i to denote a skidder class $(i = 1, 2)$. This article is limited to two classes, although an arbitrary number of skidder classes may be permitted in the future. A sample problem is presented later where class 1 contains small skidders, and class 2 contains medium-sized skidders.

Let p_{ij} be the decision variable, defined as the proportion of the volume (V_j) in a setting partition j that is to be yarded to the landing by those skidders comprising class i. The objective function is specified as minimizing task completion time, i.e., time to yard the entire setting. The constraint that all skidders of both classes start and finish the job together is included. The use of proportions as the decision variable in the optimization model leads to the inclusion of normalization and nonnegativity constraints. The general optimization problem is then written as:

Minimize: T_1

Subject to: $T_1 = T_2$

$$\sum_{i=1}^{2} p_{ij} = 1 \quad \text{for } j = 1, m$$

$$p_{ij} \geq 0 \qquad \text{for } i = 1, 2 \text{ and } j = 1, m.$$

It remains to derive the equations for T_1 and T_2, the expected time it takes for the skidders in each class to complete the job. Assuming that all skidders in class i work as a group until their assigned volume($\Sigma\, V_j\, p_{ij}$) has been yarded, their expected clock time from job start to job finish is estimated by:

$$T_i = \sum_{j=1}^{m} t_{ij}\, V_j\, p_{ij} \,/\, v_i\, K_i \tag{1}$$

where p_{ij} is the decision variable; V_j, v_i, and K_i are input parameters; and t_{ij} is the total expected cycle time for any skidder in class i when yarding from partition j. All of the input parameters for the optimization model are defined in figure 1. The total expected cycle time is approximated by:

$$t_{ij} = W_q + \mu_i^{-1} + b_{ij} \tag{2}$$

where μ_i^{-1} and b_{ij} are input parameters; and W_q is the expected time spent waiting in the queue at the landing. We now specify the calculations for the queueing time W_q, based on any randomly selected yarding cycle (selected from those of all skidders of both classes). It is also assumed that both unloading and backcycle times are adequately described by the exponential distribution. This is a common and convenient assumption, and an experimental justification may be researched in the future. The queueing time is calculated by the following formula (Gross and Harris, 1985:104-107):

$$W_q = \{K / [\bar{\mu}(1 - P_0)]\} - \bar{\mu}^{-1} - \bar{\lambda}^{-1} \tag{3}$$

where K is an input parameter; and $\bar{\mu}^{-1}$, $\bar{\lambda}^{-1}$, and P_0 are all calculated. The variable $\bar{\mu}^{-1}$ can be interpreted as an average unloading time for all skidders, and is estimated by (Gross and Ince, 1981):

$$\bar{\mu}^{-1} = (K_1\, \mu_1^{-1} + K_2\, \mu_2^{-1}) / (K_1 + K_2) \tag{4}$$

m	=	the number of partitions of the setting area, j = 1, m.
$\bar{V}$	=	average volume of logs per unit area, m^3/ha (or bd ft/acre).
V_j	=	total volume of logs in partition j, m^3 (or bd ft.).
X_j	=	average one-way straight-line yarding distance for partition j, meters (or ft).
K	=	total number of skidders in use concurrently.
K_i	=	number of skidders in class i, i = 1, 2. To be consistent, K_1 plus K_2 must equal K.
v_i	=	average turn capacity for skidders in class i, m^3/cycle (or bd ft/cycle).
μ_i^{-1}	=	expected unloading time at the landing for skidders in class i, min/cycle. This time does not include the time spent waiting for other skidders to unload.
b_{ij}	=	expected backcycle time for skidders in class i yarding partition j, min/cycle. This time is measured from the moment the skidder departs the landing to when it returns to the landing and enters the possible queue.

Figure 1–Definitions of the model's input parameters.

where K_1, K_2, μ_1^{-1}, and μ_2^{-1} are all input parameters. The variable $\bar{\lambda}^{-1}$ can be interpreted as an average backcycle time for all skidders, and is similarly estimated by (Gross and Ince, 1981):

$$\bar{\lambda}^{-1} = (K_1 \lambda_1^{-1} + K_2 \lambda_2^{-1}) / (K_1 + K_2) \quad (5)$$

where K_1 and K_2 are input parameters, and λ_i^{-1} is the average backcycle time for skidder class i, estimated by forming the weighted average:

$$\lambda_i^{-1} = \sum_{j=1}^{m} b_{ij} V_j p_{ij} / \sum_{j=1}^{m} V_j p_{ij} \quad \text{for } i = 1,2 \quad (6)$$

where p_{ij} is the decision variable as defined earlier; and b_{ij} and V_j are input parameters. Notice that the equations for λ_i^{-1} are nonlinear in terms of the decision variables. The variable P_0 is the probability that the landing is unoccupied, and is given by (Gross and Harris, 1985):

$$P_0 = \{\sum_{n=0}^{K} [K!/(K-n)!] [\bar{\lambda}/\bar{\mu}]^n\}^{-1} \quad (7)$$

where K is an input parameter; and $\bar{\mu}$ and $\bar{\lambda}$ can be obtained from equations 4 and 5.

As described above, this general model is a nonlinear programming model.

Numerical solution of this model requires the use of a nonlinear programming algorithm. The use of an algorithm not requiring derivatives (e.g, E04UAF of the Numerical Algorithms Group, 1982) greatly facilitates implementation. The nonlinearity of the above problem is introduced through the queueing effect at the landing, W_q. If the queueing term W_q is set equal to zero, implying that the total expected cycle time (t_{ij}) is specified without regard for possible congestion, then the above model becomes linear in the p_{ij} and can be evaluated via conventional linear programming packages. When two or more skidders are working on a setting having limited landing capacity, the recognition of queueing delays may lead to more realistic estimates of expected cycle time including delays due to congestion. The nonlinear programming model can be compared to the linear programming model to evaluate the impact of skidder interaction on the yarding strategies.

Results and Discussion

To gain some insight into the relative merits of the linear and nonlinear models, numerical results were examined. A sample problem is introduced for the purposes of this analysis. Values were assigned to input parameters that are based on those given in Koger and Webster (1984) and Koger (1976), and represent realistic values for skidding operations in southeastern United States. Other estimation procedures may be employed if available.

The sample problem is a circular setting on level terrain with a central landing that is to be skidder yarded. Values for all the input parameters are included in figure 2. The setting has an external yarding distance of 305 m (1,000 ft) and is partitioned into five concentric bands, each band is 61 m (200 ft) wide. Skidders are placed into either a small (52 kw, or 70 hp) class (i = 1) or a medium (89 kw, or 120 hp) class (i = 2). The actual number of skidders in each class was varied to examine the impact on task-completion time.

The numerical results of the optimization runs are listed in Tables 1 through 3. Results in Table 1 clearly illustrate the potential impact of queueing. Substantial increases in task-completion time due to queueing are indicated even for as few as two skidders assigned to the setting. Consider the case of one small-sized and one medium-sized skidder being assigned to the setting (Tables 1 and 3). The linear programming (LP) solution, which does not include the effect of queueing, estimates the task-completion time at 346 hours which is a difference of about 25% from the estimated solution of 458 hours using the nonlinear programming (NLP) model with the queueing effect. The magnitude of this difference increases dramatically as the number of skidders assigned to the setting increases and the queueing effect increases in importance. These results may be an indication of the difficulty possibly attending any attempt to increase daily production by assigning additional skidders to the setting. Prior estimation of

m = 5 partitions of concentric bands, each band 61 m (200 ft) wide.

$\bar{V}$ = 360 m³/ha (30,000 bd ft/ac).

Vj = $\bar{V}p\,\{(r_j)^2 - (r_{j-1})^2\}/10{,}000$ for j = 1, 5, where r_j is the outer radius of concentric band j. The constant 10,000 converts square meters into hectares (a constant of 43,560 converts square feet into acres).

	V_1	V_2	V_3	V_4	V_5
m³	421	1,263	2,104	2,946	3,788
(bd ft)	(86,545)	(259,636)	(432,726)	(605,817)	(778,907)

$X_j = (2/3)\,\{(r_j)^3 - (r_j - 1)^3\} / \{(r_j)^2 - (r_j - 1)^2\}$ for j = 1, 5, where r_j is defined above.

	X_1	X_2	X_3	X_4	X_5
m	40.7	94.9	154.5	215.0	275.6
(ft)	(133.3)	(311.1)	(506.7)	(704.8)	(907.3)

K, K_1, and K_2 are varied in the analyses.

v_1 = 2.01 m³/cycle (410 bd ft/cycle) v_2 = 6.54 m³/cycle (1,334 bd ft/cycle).

μ_1^{-1} = 6.38 minutes/cycle μ_2^{-1} = 11.56 minutes/cycle.

$b_{1j} = (A)X_j^{1.022} + (B)X_j^{1.098}$ $b_{2j} = (C)X_j^{1.022} + (D)X_j^{1.098}$ for j = 1, 5. where the values for A, B, C, and D are given below. The values of the exponents are independent of unit measurement. The calculations for the constants come from Koger and Webster (1984), and incorporate a skidder wander-correction factor of 1.86 applied to all straight-line average yarding distances; and an efficiency factor for each skidder class, L_1 = 0.80 and L_2 = 0.90.

	A	B	C	D
Xj in m	0.0097	0.0129	0.0188	0.0088
(Xi in ft)	(0.00287)	(0.00349)	(0.00557)	(0.00239)

Figure 2–Values for all input parameters used in the sample problem.

TABLE 1. Minimum total time to yard the setting (hours) by number of skidders in each class*

		Number of Small Skidders							
		0		1		2		3	
		LP†	NLP‡	LP	NLP	LP	NLP	LP	NLP
Number	0	—	—	1,146	1,146	573	710	382	602
of	1	511	511	346	458	265	441	215	446
medium	2	255	351	206	365	173	381	150	397
skidders	3	170	320	146	341	129	360	116	375

* Hours.
† Determined by linear programming (LP).
‡ Determined by nonlinear programming (NLP).

TABLE 3. Minimum time with best allocation vs. maximum time with worst allocation to yard the setting*

	Minimum Best Time		Maximum Worst Time	
	LP†	NLP‡	LP	NLP
1 Small skidder and 1 Medium skidder	346	458	358	460
2 Small skidders and 2 Medium skidders	173	381	179	395

* Hours.
† Determined by linear programming (LP).
‡ Determined by nonlinear programming (NLP).

loading and truck-hauling requirements based on the skidding LP solution will be biased upward, in contrast to the NLP solution where the queueing effect on skidder production is recognized.

An examination of the minimal task-completion times as calculated by the NLP model reveals an interesting effect. If the number of medium-class skidders is held constant (Table 1) at one skidder of this class, and the number of small skidders assigned to the setting is progressively increased from zero to three, a pattern emerges. The corresponding marginal reductions in task completion time calculated by successive subtractions are: from 0 to 1 skidder equals –53, from 1 to 2 skidders equals –17, and from 2 to 3 skidders equals +5. The sign on the last number is not a mistake, but rather a clear indication of how the more productive medium-sized skidders can be adversely affected through increased queueing time as more, less productive, small-sized skidders are employed. In the LP model, every additional skidder decreases the yarding time. Consider the case where there are zero medium-sized skidders. Then the yarding time for two small skidders is simply one-half the time for one small skidder, and the yarding time for three small skidders is one-third the time for one small skidder. Thus the LP solutions indicate linear relationships which do not incorporate queueing-related effects associated with multiple skidders.

Optimal allocation of the skidder resource is also influenced by the incorporation of queue waiting time. Typical results are given in Table 2, with an illustration given in figure 3. If the impact of queue formation at the landing is ignored (LP formulation) then the small-class skidders are assigned to work from the landing out, while the medium-class skidders start at the external yarding boundary and work towards the landing. The two classes of skidders will meet at a radial distance of 221.43 m (726 ft). Recognition of time lost in the queue (NLP formulation) leads to a reversal of this allocation scheme. Now the optimal allocation is for the medium-class skidders to work from the landing out, while the small-class skidders work the outer area. They now will meet at a radial distance of 235.46 m (772 ft). It would seem that the small (less productive) skidders should be forced to spend more time traveling (hence away from the landing) so that they do not tie up the more productive medium-class skidders in a longer landing queue.

Skidder allocation within the setting does not seem to greatly affect task completion time. Table 3 compares the time required to yard the setting under the worst possible allocation with the time required using the best possible allocation of skidders for two different resource levels. The best allocation was determined by minimizing task completion time, and are consistent with the times reported in Table 1. To contrast with extreme allocations, the worst allocation was determined by maximizing task completion

TABLE 2. Optimal yarding pattern showing the proportion (p_{ij}) of each setting partition (j) yarded by each skidder class (i) when two small and one medium-class skidder are available*

		LP		NLP	
		Small skidder class (i=1)	Medium skidder class (i–2)	Small skidder class (i=1)	Medium skidder class (i=2)
Setting	(j=1) band 1	1.00	0.00	0.00	1.00
	(j=2) band 2	1.00	0.00	0.00	1.00
partition	(j=3) band 3	1.00	0.00	0.00	1.00
	(j=4) band 4	0.63	0.37	0.14	0.86
(j)	(j=5) band 5	0.00	1.00	1.00	0.00

* Solutions are shown for linear programming (LP) and nonlinear programming (NLP) models. The five concentric bands are 61 m (200 ft) wide, with the first band being the innermost one in the center.

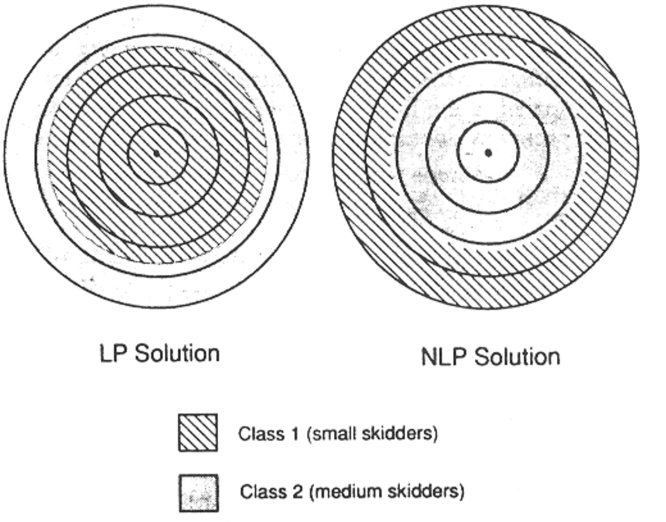

Figure 3–Diagram of the optimal skidder allocations corresponding to the LP and NLP solutions in Table 2.

time. This provides a range on task completion time. The variation in task completion time is small when comparing LP best with LP worst, or when comparing NLP best with NLP worst. However, the variation in task completion time is quite large when comparing LP best with NLP best, or when comparing LP worst with NLP worst. This indicates that time differences due to allocation are minimal, while time differences due to a queueing effect are substantial. The difference is greatest when queueing has the biggest effect; i.e., for the NLP formulation with four skidders.

Summary and Conclusions

In a recent trade journal article it was suggested that one solution to delays resulting from skidder congestion at the landing was to send the large capacity, slow skidders farther from the landing while the fast, small capacity skidders worked in close to the landing (Garland, 1989). The results obtained from our nonlinear programming model suggest that such intuitive allocation rules must be closely re-examined. For at least one numerical example when the effect of skidder queues was included, this allocation rule was reversed. Model results suggested a more efficient allocation where the smaller, less productive skidders should spend more time traveling, and hence away from the landing, so that they do not tie up the more productive medium-sized skidders in a longer landing queue.

Regardless of the allocation rule applied to the skidders, productivity was only slightly affected. Even under the worst possible allocation of the skidder resource, productivity was only slightly decreased from that achieved under optimal allocation. The suggestion here is that there are only minor productivity gains that result from intensive management of skidder spatial allocation on a setting. This observation does not rule out substantial productivity gains from more creative approaches to skidding subsystem design on a setting (Anonymous, 1988).

Failure to incorporate operational delays due to skidder queueing at the landing can lead to significantly different results in estimation of production. Through the use of a nonlinear program, our results indicate that the impact of congestion and subsequent queue formation during harvesting operations lead to substantial increases in overall harvest time, both on the immediately affected activity and on those activities further along in the production process. Total harvesting system efficiency may be adversely affected when other subsystems such as loading and hauling are designed for higher than necessary levels of production.

Overall system models, such as LOST, might significantly benefit from the inclusion of subsystem models that include nonlinear effects such as those associated with queueing. While linear programming has been widely used in forestry and the forest products industry, nonlinear applications have been relatively few (Bare, 1971; Bare et al., 1984). Nonlinear programming techniques have achieved a level of general applicability and ease of use that warrants their serious consideration as an alternative to more traditional approaches such as linear programming.

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PRESENTATION

LINEAR AND NONLINEAR OPTIMIZATION OF SKIDDER YARDING TIME

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SUMMARY:

The reduction of yarding costs by alternative optimization methods is examined. Linear and nonlinear programming approaches are presented and compared for selecting an optimal yarding strategy from two different skidder types. The nonlinear programming approach captures the effect of skidders queueing at the landing, whereas the linear programming model is unable to include this important effect.

KEYWORDS:

Forest engineering, linear programming, logging, operations research, nonlinear programming

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INTRODUCTION

This paper presents initial results from an exploratory application of nonlinear programming to timber harvest planning. A nonlinear model of skidder yarding activity was compared with a more conventional linear programming model. The increased flexibility of the nonlinear model permits skidding operations to be described more realistically than is possible with the linear model.

Within the exploratory scope of this work, a general optimization model was developed to capture the production impact of skidder interaction at the landing. Some initial computer runs using realistic input data have been made. The comparison of these preliminary numerical results from both linear and nonlinear models of the skidding process suggest the relative merit of a nonlinear formulation.

In the first section of this paper, the two alternative models are discussed and placed within the context of some previous research. The next section is devoted to a comparative examination of numerical results from these models. A summary discussion of current results and future opportunities for nonlinear modeling in the harvesting process concludes the paper. This study was sponsored by the USDA Forest Service, Pacific Northwest Research Station, in cooperation with the University of Washington (Agreement No. PNW-86-268, Amend. No.2.).

MODEL DEVELOPMENT

In a recent paper, Koger and Webster (1986) present an optimization procedure (LOST: logging optimization selection technique) to evaluate road and setting layout within a forest tract. They use linear programming to help identify the "best" transportation system design. The transportation system is considered "best" when the related costs of harvesting the tract are minimized within the confines of user-specified layout alternatives.

LOST has several submodels. The times and costs of skidding, trucking, and other required implementing activities are calculated for each user-specified layout. Within these activity submodels, multiple equipment may be used. The evaluation of multiple-equipment performance within a submodel is based on several important assumptions: (1) optimization within a submodel is not neccessary because potential gains are insignificant; (2) interaction effects, if present, are small enough to ignore; and (3) all equipment within a submodel is used concurrently.

It is not the immediate purpose of this paper to suggest changes to the optimization procedures of the LOST model. The LOST model provides, however, a general context within which to examine the relative value of applying linear and nonlinear optimization to typical harvesting activities. In particular, the skidding submodel will be examined for (1) the production impact of optimal allocation of the skidder resource within a single setting and (2) the impact on production of skidder congestion at the landing. In keeping with the LOST submodel, the assumption of concurrent use of all skidders will be retained throughout.

A general optimization model of the skidding process has been developed. (A detailed mathematical description of the model is given in Appendix A.) The model can easily incorporate irregular setting shapes and nonuniform turn distributions through appropriate partitioning of the setting area. We used the subscript j to denote a specific setting and partition. The number of partitions reflects the desired accuracy of the solution as well as computation time. Skidders are sorted into two classes with the objective of aggregating skidders of similar capability. We used the subscript i to denote a skidder class. This paper is limited to two classes, although future research may permit an arbitrary number of skidder classes. A decision variable, p_{ij}, indexed over all skidder class and setting-partition (s-c/s-p) combinations, was defined as the proportion of the volume in a setting partition j yarded to the landing by a skidder class i. The use of proportions as the decision variable (hence "p-model") in the optimization model leads to the inclusion of nonnegativity and normalization constraints. One additional constraint, that all skidders of both classes start and finish the job together, was also imposed. Because the total cost of skidding activity for the setting is proportional to the time required to complete that task, the objective function may be specified as minimizing task-completion time.

If the expected cycle time for each s-c/s-p combination is specified without regard for possible queueing delays at the landing, then the problem as just described is completely linear in the decision variables. Linear programming algorithms can be employed to find the optimal allocation of skidder classes to setting partitions, together with the associated task-completion time.

When two or more skidders are working on a setting having limited landing capacity, the recognition of queueing delays may lead to more realistic estimates of expected cycle time. Work by Gross and Ince (1981) suggests a method for easy approximation of the expected time spent in a finite source queue. When expected queueing time is incorporated into the expected cycle time, the problem becomes nonlinear in the decision variables. Nonlinear programming algorithms must now be applied to obtain the optimal allocation and associated minimal completion time. The nonlinear programming model can be compared to the linear programming model to evaluate the impact of skidder interaction.

NUMERICAL RESULTS

To gain some insight into the relative merits of the linear and nonlinear models, numerical results were examined. An effort was made during problem specification to assign realistic values to model parameters. (A complete numerical description of the problem is given in Appendix B.) A circular setting with an external yarding distance of 1,000 feet and a uniformly distributed volume of 30 thousand board foot (Mbf) per acre was partitioned into five 200-foot-wide concentric bands. Skidders were placed into either a small (70 horse power [HP]) class or a medium (120 HP) class. The actual number of skidders in each class was varied to examine the impact on task-completion time.

The numerical results of the optimization runs are listed in tables 1 through 3. Results in table 1 clearly illustrate the potential impact of queueing. Substantial increases in task-completion time due to queueing are indicated even for as few as two skidders assigned to the setting. Consider the case of one small and one medium-sized skidder being assigned to the setting. The linear programming (LP) solution, which does not include the effect of queueing, underestimates the task-completion time by about 25%. The magnitude of this difference increases dramatically as the number of skidders assigned to the setting increases and the queueing effect increases in importance. These results may be an indication of the difficulty possibly attending any attempt to increase daily production by assigning additional skidders to the setting. Prior estimation of loading and truck-hauling requirements based on the skidding LP solution will be biased upward, in contrast to the NLP solution where the queueing effect on skidder production is recognized.

		Number of Small Skidders							
		0		1		2		3	
		LP	NLP	LP	NLP	LP	NLP	LP	NLP
Number	0	------	------	68,743	68,743	34,372	42,604	22,914	36,124
of	1	30,636	30,636	20,785	27,465	15,905	26,460	12,880	26,730
medium	2	15,318	21,052	12,335	21,895	10,390	22,865	9,010	23,820
skidders	3	10,212	19,182	8,770	20,480	7,740	21,590	6,930	22,525

Table 1. Minimum total time to yard the setting (minutes) by number of skidders in each class as determined by linear programming (LP) and nonlinear programming (NLP) models.

An examination of the minimal task-completion times as calculated by the nonlinear programming (NLP) model reveals an interesting effect. If the number of medium-class skidders is held constant (refer to table 1), say one skidder of this class, and the number of small skidders assigned to the setting is progressively increased, say from zero to three, a pattern emerges. The corresponding marginal reductions in task completion time are -3,171, -1,005, and +270. The sign on the last number is not a mistake, but rather a clear indication of how the more productive medium-sized skidders can be adversely affected through increased queueing time as more, less productive, small-sized skidders are employed. A glance at the LP-calculated task-completion times shows that this queueing-related effect has not been captured by that model.

Optimal allocation of the skidder resource is also influenced by the incorporation of queue waiting time. Typical results are given in table 2. If the impact of queue formation at the landing is ignored (LP formulation) then the small-class skidders are assigned to work from the landing out, while the medium-class skidders start at the external yarding boundary and work towards the landing. The two classes of skidders will meet at a radial distance of 726 feet. Recognition of time lost in the queue (NLP formulation) leads to a reversal of this allocation scheme. Now the optimal allocation is for the medium-class skidders to work from the landing out, while the small-class skidders work the outer area. They now will meet at a radial distance of 772 feet. It would seem that the small (less productive) skidders should be forced to spend more time traveling (hence away from the landing) so that they do not tie up the more productive medium-class skidders in a longer landing queue. Another way to further mitigate this particular problem would be to give landing-use priority to the medium-class skidders.

The optimal allocations shown in table 2 indicate that the most efficient strategy is to divide the setting into two areas, one for each skidder class. The

		LP		NLP	
		Small skidder class (i=1)	Medium skidder class (i=2)	Small Skidder Class (i=1)	Medium skidder class (i=2)
	radial distance				
Setting	0'- 200' (j=1)	1.00	0.	0.	1.00
	200'- 400' (j=2)	1.00	0.	0.	1.00
partition	400'- 600' (j=3)	1.00	0.	0.	1.00
	600'- 800' (j=4)	0.63	0.37	0.14	0.86
(j)	800'- 1,000' (j=5)	0.	1.00	1.00	0.

Table 2. Optimal yarding pattern showing the proportion (p_{ij}) of each setting partition (j) yarded by each skidder class (i) when two small and one medium-class skidder are available. Solutions are shown for linear programming (LP) and nonlinear programming (NLP) models.

assumption that dividing the setting into two work areas is an optimal strategy led to the development of another optimization model using the radial distance as the decision variable (hence "r-model"). This model is discussed in Appendix C. During testing of this assumption, two concentric work areas were generally found to be an optimal strategy for typical values of skidder parameters; however, academic examples were constructed where this assumption led to suboptimal decisions. The p-model still found the optimal allocations under those circumstances. Sometimes the r-model was easier to run, partly because convexity of the objective function guaranteed convergence of the optimization method. Both models were used to cross verify internal consistency.

Skidder allocation within the setting does not seem to greatly affect task completion time. Table 3 compares the time required to yard the setting under the worst possible allocation with the best possible allocation of skidders for two different resource levels. The variation in task completion time between best and worst allocation decisions is small. The difference is greatest when queueing has the biggest effect; i.e., for the NLP formulation with four skidders.

	Minimum time		Maximum time	
	LP	NLP	LP	NLP
1 Small skidder and 1 Medium skidder	20,785	27,465	21,460	27,615
2 Small skidders and 2 Medium skidders	10,390	22,865	10,730	23,695

Table 3. Minimum time for best allocation vs maximum time for worst allocation to yard the setting (minutes) as determined by linear programming (LP) and nonlinear programming (NLP) models.

DISCUSSION

This study compared the application of linear and nonlinear programming to harvesting activities and revealed a considerable need and potential for nonlinear models in this area. Whereas linear programming has been widely used in forestry and the forest products industries, nonlinear applications have been few (Bare, 1971; Bare, et al., 1984). Bare (1971) states:

> "Much work needs to be done in exploring the potential of nonlinear programming methods in forest management. At present it is not clear if the use of more sophiscated programming methods will offer the forest manager any advantages over linear programming. However, it is an area that deserves much more attention than it has so far received."

Our results indicated that the impact of queue formation during harvesting operations may be substantial, both on the immediately affected activity and on those activities further along in the production process. The queueing effect is nonlinear and thus requires the use of nonlinear programming to effectively incorporate this impact into optimal decisions.

Future work is needed to extend this model to encompass other elements of the harvesting system including loading and hauling. Other activities such as road construction might also be more accurately portrayed by a nonlinear model.

On the practical side, efforts should be made to incorporate many of these ideas into the LOST model. Substantial gains in the descriptive power of the model might be achieved through the use of nonlinear submodels.

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Appendix A

Appendix A contains a complete description of the general optimization model used in the paper. It defines all the notation and includes all the equations used in the model.

The setting area with a central landing is partitioned into subareas (j=1,m) appropriate to requisite modeling precision. We define the average board foot volume per acre as, $\overline{V}$ (bd.ft./acre), and then calculate the total volume of logs in each partition, V_j (bd.ft.). The average one-way straight-line yarding distance, X_j (ft.), is also calculated. These calculations depend on the actual shape and partitioning of the setting area. (Numerical values are included in Appendix B to provide an example.)

A variety of skidders, K in total number, are to be used concurrently to yard the setting. These skidders are separated into two classes (i=1,2). This sorting is done to collect skidders with comparable characteristics, and hence comparable yarding cycle times, into the same class. The number of skidders in class i is denoted K_i ; K_1 plus K_2 then equals to K.

A decision variable, p_{ij}, is defined as the proportion of the volume V_j in partition j that is to be yarded by those skidders comprising class i. We refer to this model as the p-model because proportions are used in the solution.

For each of the two skidder classes, several variables are defined. Let v_i (bd.ft./cycle) denote the average turn capacity for skidders in class i. The expected unloading time at the landing, u_i^{-1} (minutes/cycle), is estimated. This time does not included that spent waiting for other skidders to unload. The expected backcycle time, b_{ij} (minutes/cycle), is also estimated for each skidder-class and setting-partition combination. This time is measured from the moment the skidder departs the landing to when it returns to the landing and enters the possible queue. A major factor in estimating this average backcycle time is the average yarding distance, X_j, associated with any given partition. In this analysis, the backcycle estimations are based on those given in Koger and Webster (1984) and Koger (1976) as explained in Appendix B. Other estimation procedures may be employed if available. Both unloading and backcycle times are assumed to be adequately described by the exponential distribution.

For any randomly selected yarding cycle (selected from those of all skidders of both classes), the expected time spent waiting in the queue at the landing is denoted W_q (minutes). This queueing time is calculated for a finite source queue (Gross and Harris, 1985) by the following formula:

$$W_q = [K/(\acute{u}(1 - P_0)] - (1/\acute{u}) - (1/\acute{\lambda}) \quad .$$

Values for $\acute{u}^{-1}$ and $\acute{\lambda}^{-1}$ are estimated, per Gross and Ince (1981), by the formulae:

$$\acute{u}^{-1} = (K_1 u_1^{-1} + K_2\ u_2^{-1})/(K_1 + K_2) \text{ and}$$

$$\acute{\lambda}^{-1} = (K_1 \lambda_1^{-1} + K_1 \lambda_2^{-1})/(K_1 + K_2) \quad .$$

where λ_i^{-1} has been estimated by forming the weighted average:

$$\lambda_{.}^{-1} = \sum_{j=1}^{m} b_{.j}^{-1} V_j p_{.j} / \sum_{j=1}^{m} V_j p_{.j} \quad .$$

The probability of an unoccupied landing, P_0, is given by (Gross and Harris 1985):

$$P_0 = \sum_{n=0}^{k} [(k\,!)/(k - n)!][\lambda/u]^n]^{-1} \quad .$$

The expected cycle time, t_{ij} for any skidder in class i when yarding from setting-partition area j is approximated by:

$$t_{ij} = W_q + u_i^{-1} + b_{ij} \quad .$$

Then, assuming that all skidders in class i work as a group until their assigned volume ($\Sigma V_j\ p_{ij}$) has been yarded, their expected clock time from job start to job finish is estimated by:

$$T_i = \sum_{j=1}^{m} t_{ij} V_j p_{ij}/v_i K_i \quad .$$

The general p-model optimization problem is then written as:

Minimize: T_1

Subject to:

$$T_1 = T_2$$

$$\sum_{i=1}^{2} p_{ij} = 1 \qquad \text{for } j = 1, m$$

$$p_{ij} \geq 0 \qquad \text{for } i = 1,2 \text{ and } j = 1, m.$$

As described above, this is a nonlinear programming model. Numerical solution of this model requires the use of a nonlinear programming algorithm. The use of an algorithm not requiring derivatives (e.g, E04UAF of the Numerical Algorithms Group (1982) greatly facilitates implementation. The nonlinearity of the above problem is introduced through the queueing effect at the landing, W_q. If this latter term is set equal to zero, then the above model becomes linear in the p_{ij} and can be evaluated via conventional linear programming packages.

Appendix B

Appendix B contains values for all model parameters used in the numerical analyses presented in this paper. The values of the parameters are given in the same order as they were defined in Appendix A.

A circular setting on level terrain with a central landing and a constant external yarding distance of 1,000 feet is to be skidder yarded. There is an average volume of 30,000 board feet per acre ($\overline{V}$ = 30,000) in turns assumed to be uniformly distributed over the setting. The setting is partitioned into five concentric bands, each 200 feet wide. The volume of logs in each band can be calculated by:

$$V_j = \overline{V}\,\pi\ \{(200\ j)^2 - [200(j-1)]^2\}/43{,}560\quad ,$$

$$j = 1,5$$

where the constant 43,560 converts square feet into acres. The average straight-line yarding distance for each partition is then calculated by:

$$X_j = (2/3)(\{[200\ j]^3 - [200(j-1)]^3\}/\{[200j]^2 - [200(j-1)]^2\})\quad .$$

$$j = 1,5$$

The skidders are separated into either a small, 70-horsepower, skidder class (i=1) or a medium, 120-horsepower, skidder class (i=2). The following values are given:

v_1 = 410 bd. ft./cycle $\qquad$ v_2 = 1,334 bd.ft./cycle

u_1^{-1} = 6.38 minutes/cycle $\qquad$ u_2^{-1} = 11.56 minutes/cycle

The expected backcycle times are calculated by using the following equations (Koger and Webster, 1984):

$$b_{1j} = (0.00287)X_j^{1.022} + (0.00349)X_j^{1.098}\quad \text{and}$$

$$j = 1,5$$

$$b_{2j} = (0.00557)X_j^{1.022} + (0.00239)X_j^{1.098}.$$

$$j = 1,5.$$

The constants in the above two equations incorporate a skidder wander-correction factor of 1.86 applied to all straight-line average yarding distances; and an efficiency factor for each skidder class, L_1=0.80 and L_2=0.90. Unless otherwise specified, mean values as given in Koger and Webster (1984) and Koger (1976) are employed in equation development.

Having developed these 10 backcycle values, the analysis proceeds as described in Appendix A for selected values of K_1 and K_2.

Appendix C

In the case of a level circular setting with the landing at the center, an alternative model (the r-model) may be developed. It is assumed that under optimal allocation, skidders of one class will be employed at shorter distances than skidders of the other. The radial distance, r (feet), must then be found that, under optimal allocation, divides the circular setting into two concentric bands (j=1,2), one work area for each skidder class.

In the r-model, it is assumed that skidder-class 1 is used closer to the landing (partition j=1) while skidder-class 2 is employed in the other work area (j=2). The situation is as described in Appendix B, with R=1,000 feet or the radius of the circular setting. Then the following equations apply:

$$\lambda_1^{-1} = (0.00187)r^{1.022} + (0.00224)r^{1.098}$$

$$\lambda_2^{-1} = (0.00374)[(R^3 - r^3)/(R^2 - r^2)]^{1.022}$$

$$+ (0.00152)[(R^3 - r^3)/(R^2 - r^2)]^{1.098}$$

and

$$t_{ii} = W_q + u_i^{-1} + \lambda_i^{-1} \qquad i = 1,2$$

$$t_{ij} = 0 \qquad i \neq j$$

with W_q and other variables as previously defined in Appendix A. Then the expected clock time from job start to finish is given for each respective skidder class by:

$$T_1 = (\pi\ r^2/43,560)(\overline{V}/v_1)(t_{11}/K_1) \text{ and}$$

$$T_2 = (\pi\ (R^2 - r^2)/43,560)(\overline{V}/v_2)(t_{22}/K_2)\ .$$

The r-model optimization problem is then written as:

Minimize: T_1

Subject to:

$$T_1 = T_2$$

$$0 \leq r \leq R$$

The r-model formulation, although nonlinear, is in a single variable, r, and thereby offers some computational advantages. It is convex in r, and thus convergence of the optimization algorithm is guaranteed. Also, it is easy to graph the objective function, T_1, and T_2 as functions of r and thereby gain some insight into the model. The main disadvantage of the r-model is that it is not easily generalized to a noncircular setting.

Another disadvantage to the r-model is that two separate runs are generally required because it is not usually known which skidder-class should be employed closer to the landing; hence, both options must be examined. This can be seen as an advantage of the model, however, because it makes comparison of suboptimal allocations very easy. Also, as discussed in the text, the assumption that two concentric work areas is an optimal strategy seems to be generally valid with the exception of contrived counter examples. To perform the analysis presented in the text, the combination of the r-model and the p-model was ideal because it included the generality of the p-model with the ease and graphic capabilities of the r-model. Both models were used to verify internal consistencies among solutions.