UNIFICATION AND QUANTUM GRAVITY

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Overview

- Basic General Relativity
- Unification with Quantum Mechanics
- Quantum Gravity
  - Early Theories
  - Recent Approaches

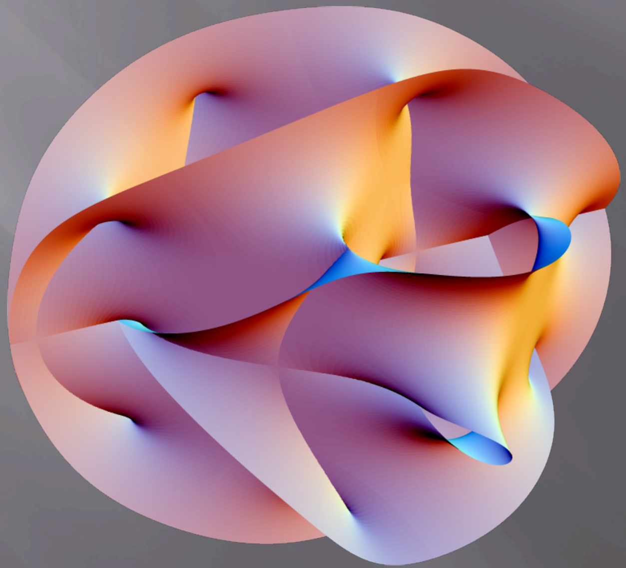
General Relativity

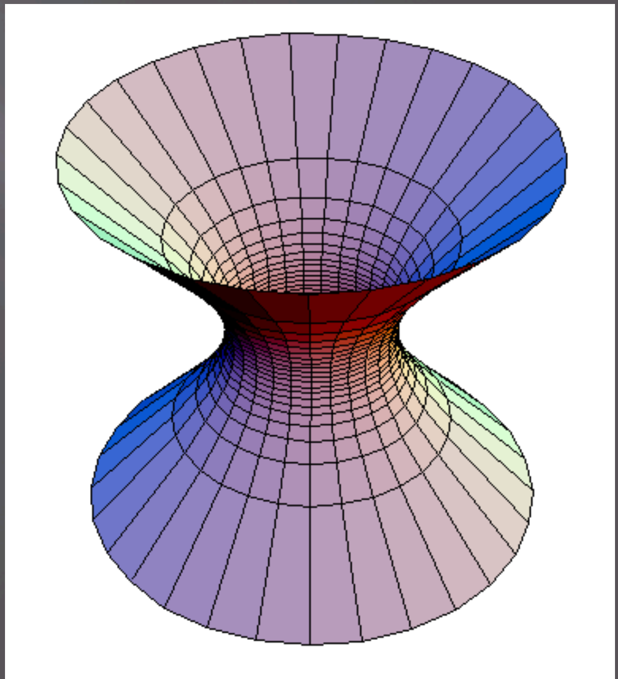
- Space-time manifold obeys Einstein's Field Equations:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi T_{\mu\nu}$$

General Relativity

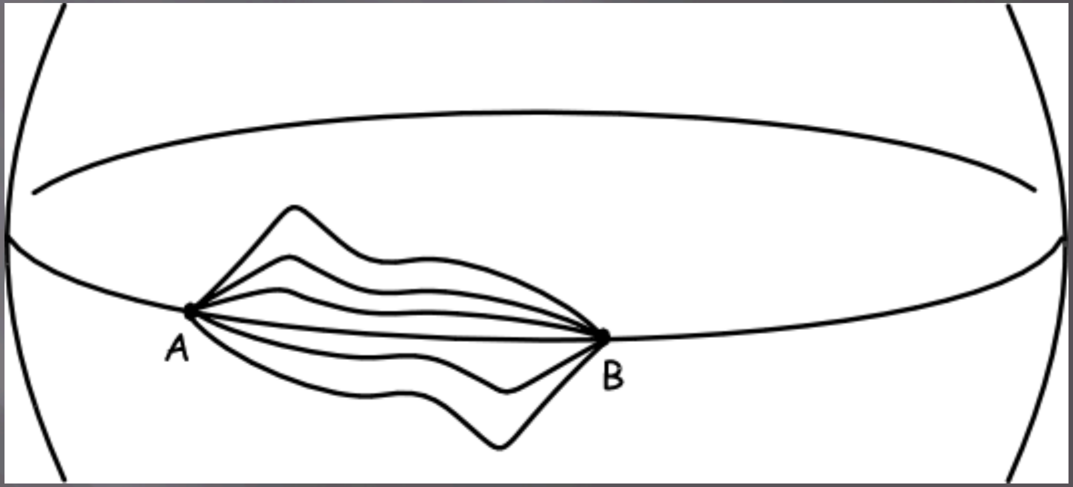
- Space-time is a (generally curved) 4-manifold
  - Objects in flat space-time traverse straight worldlines with no net force
  - What equivalent exists for objects in curved space-time?





Geodesics

- Geodesic – path of minimum distance joining two points on a surface (manifold)
  - Principle of least action implies objects follow geodesics

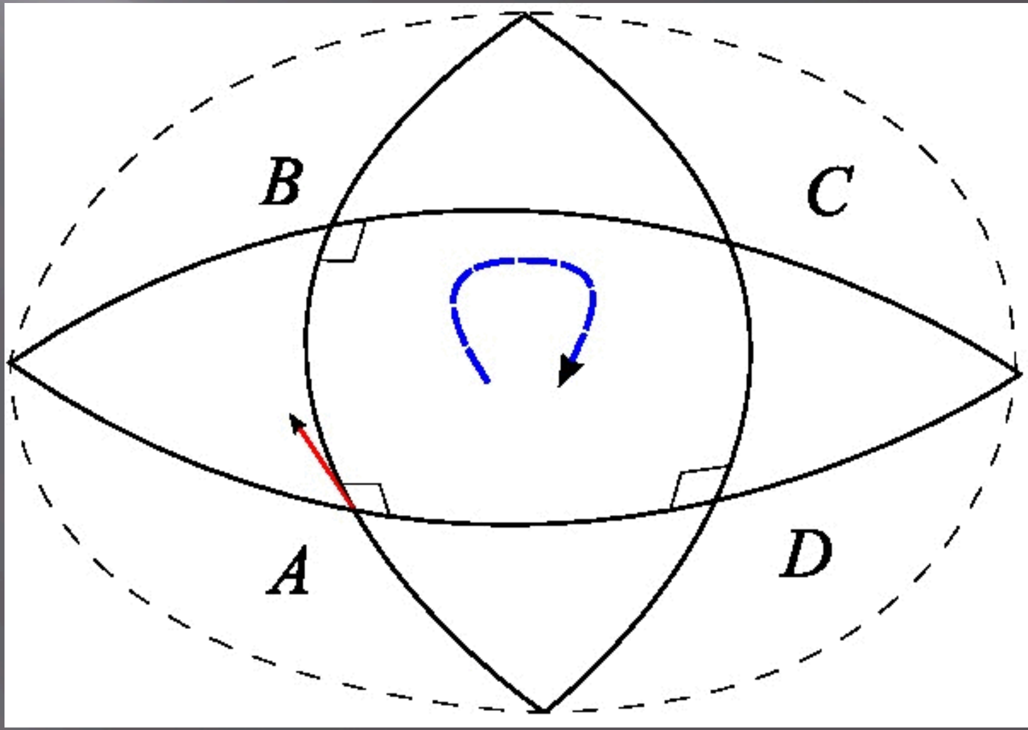


Geodesics in GR

- Objects moving between two points in curved space-time follow the geodesics between those points in absence of external forces
- Gravity doesn’t count:
  - Gravitational interactions contained in curvature of space-time

Riemann Curvature Tensor

- In flat space-time, continuous parallel transport is an identity transformation.
  - But *not* in curved space-time!
- How much parallel transport fails to be identical: Riemann curvature tensor



Riemann Curvature Tensor

- Riemann curvature tensor is given by

$$R^{\alpha}_{\beta\gamma\delta} = \partial_{\gamma}\Gamma^{\alpha}_{\beta\delta} - \partial_{\delta}\Gamma^{\alpha}_{\beta\gamma} + \Gamma^{\mu}_{\beta\delta}\Gamma^{\alpha}_{\mu\gamma} - \Gamma^{\mu}_{\beta\gamma}\Gamma^{\alpha}_{\mu\delta}$$

- $4^4 = 256$ components, but only 20 distinct
- In particular, contains Ricci tensor and scalar curvature (trace of Ricci tensor):

$$R_{\mu\nu} := R^{\lambda}_{\mu\lambda\nu}, \quad R := g^{\mu\nu} R_{\mu\nu}$$

Ricci Scalar and Ricci Tensor

- *Positive* scalar curvature at given point implies the volume of an epsilon ball at that point is *smaller* than in Euclidean space
- *Negative* scalar curvature implies epsilon ball at that point has *greater* volume than in Euclidean space

Recall from Special Relativity…

- Used to define products and magnitudes of 4-vectors

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \left(\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \right)$$

- For 4-vectors A, B, $AB = \eta_{\mu\nu} A^{\mu} B^{\nu}$ and $|A| = (AA)^{1/2}$.

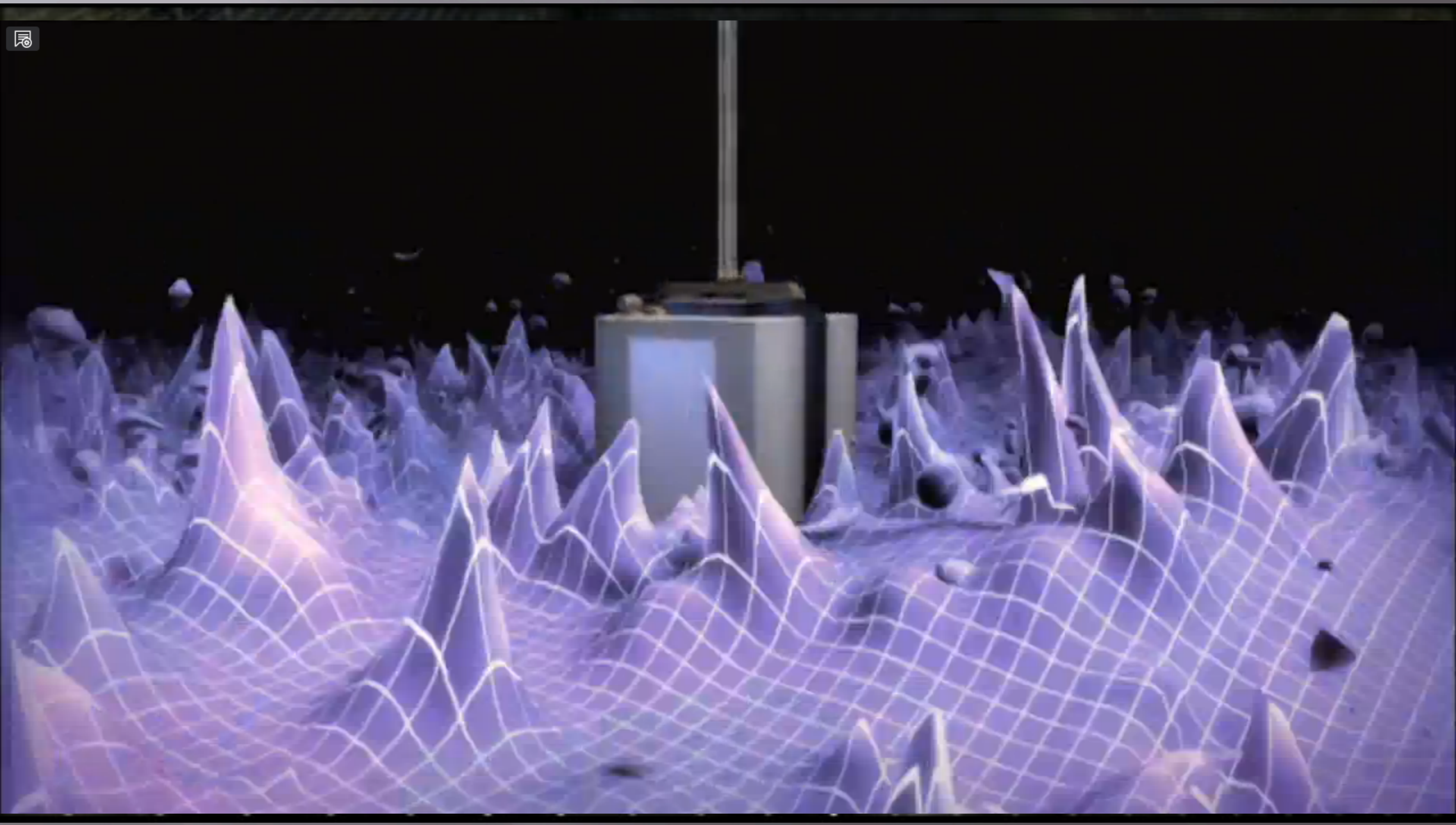
Recall from Special Relativity…

- For curved geometries, need different metrics

$$\eta_{\mu\nu} = \begin{bmatrix} -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad \left(\eta_{\mu\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{bmatrix} \right)$$

GR versus QM

- Difficult to reconcile theories:
  - GR space-time is a smooth (differentiable pseudo-Riemannian) manifold
  - QM space-time is "fluctuating" (nondeterministic)
  - GR and QM conflict at singularities
  - "Problem of time"
  - Gravitational fields and uncertain mass distribution
  - Nonrenormalizability of gravity



Black holes

- Solution to EFE: The Schwarzschild metric

$$d\tau^2 = \left(1 - \frac{r_s}{r}\right) dt^2 - \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 - r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2\right), \quad r_s = \frac{2GM}{c^2}$$

- At zero radius, this is singular

Nonrenormalizability

- Quantum theory of gravity can be constructed just as for other fundamental forces
  - Predictions accurate at low energies (good EFT)

$$S = \int \sqrt{g} \left\{ \Lambda + \frac{1}{16\pi G} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots + L_{\text{matter}} \right\} d^4 x$$

  - Yields the valid, (very) small quantum mechanical correction to Newtonian gravity

$$V(r) = -G \frac{m_1 m_2}{r} \left[1 - \frac{G(m_1 + m_2)}{2c^2 r} - \frac{122 G\hbar}{15\pi c^3 r^2} \right]$$

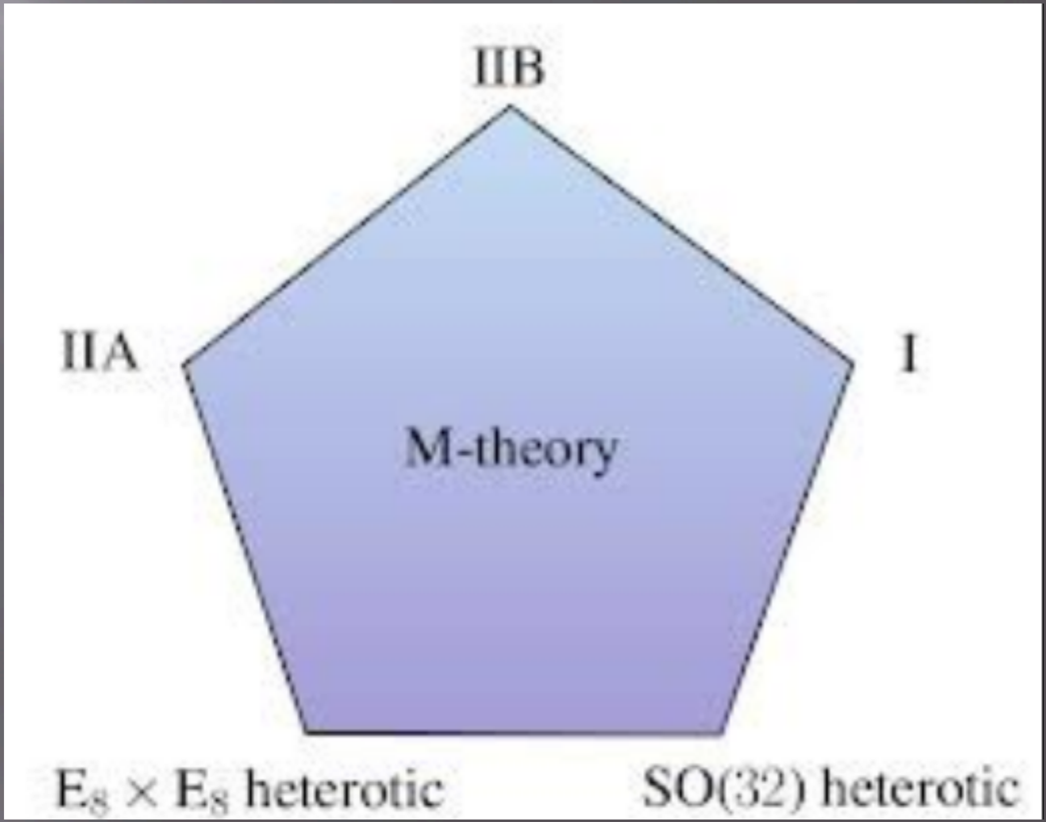
Nonrenormalizability

- Other fundamental forces take into account higher-energy corrections via *renormalization*
  - H.O.T. added and operators+H.O.T. re-normalized
- Gravity is nonrenormalizable
  - Two-loop contribution causes divergent action integral (at higher energies, spacetime is a point)
  - Dimensional regularization unable to yield a finite integral

$$S = \int \sqrt{g} \left\{ \Lambda + \frac{1}{16\pi G} R + c_1 R^2 + c_2 R_{\mu\nu} R^{\mu\nu} + \cdots + L_{\text{matter}} \right\} d^{4+i\varepsilon} x$$

Supergravity (SUGRA)

- "Five String Theories Problem"
- Strings 1995 conference: Ed Witten proposes M-Theory
- Candidate for unified theory of everything



Supergravity (SUGRA)

- Type I, IIA, IIB string theories require 10 dimensions, while Heterotic-O and Heterotic-E require 26
- M-theory captures 11 dimensional supergravity theory unifying supersymmetry with gravity

Eleven-dimensional SUGRA

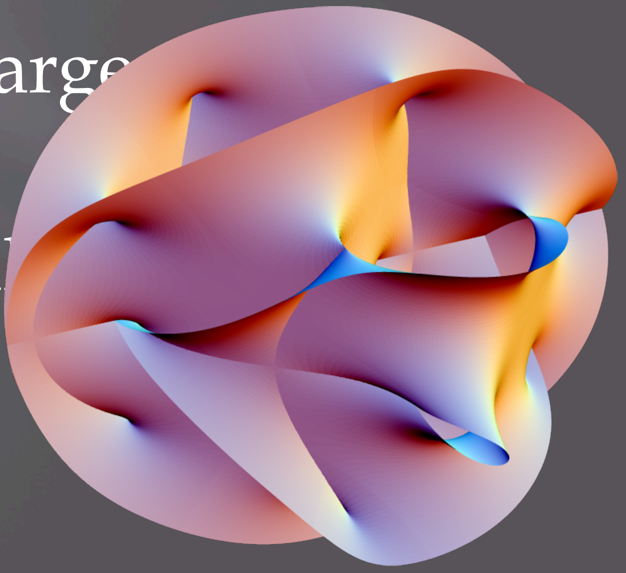
- Maximum dimensionality for a single graviton
  - More dimensions implies particles with spin > 2
  - Technically okay with d = 12 if two are light-like

Eleven-dimensional SUGRA

- Maximum dimensionality for a single graviton
  - More dimensions implies particles with spin > 2
- Minimum dimensionality to contain SU(3) (strong) and SU(2)×U(1) (EW)
- There exists a classical action for 11-dimensional supergravity such that all others are classically (even if not QM) equivalent
- Can compactify 7 (or 4) dimensions

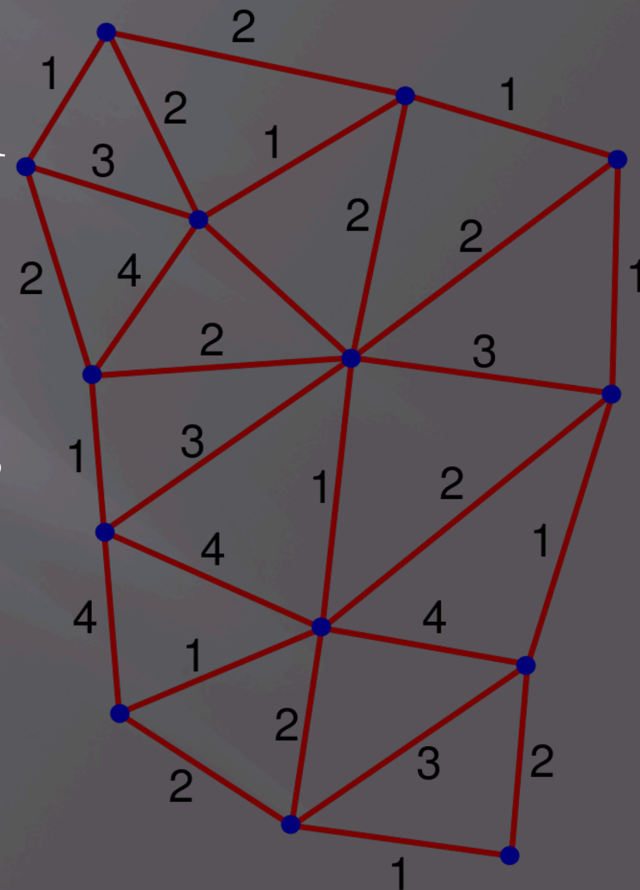
Shortcomings of Supergravity

- Can compactify 7 dimensions, but remaining 4 form anti de Sitter space (attractive Λ)
- Limited number of 7-manifolds; none compatible with SUSY or quarks/leptons
  - Suggested SO(8) or SO(5)×SU(2)
- Most SUGRA theories make Λ larg… fine-tuning to solve
- Quantization led to QFT anomal…



Loop Quantum Gravity

- Quantum states of the gravitational field are "spin networks"
- Distance and area are quantized
- Spin networks evolve in time, giving rise to a *spin foam*
- Feynman path integral becomes sum over all geometries
- Research is ongoing



Loop Quantum Gravity

- Constraints: Canonical position/momentum obey Poisson bracket relations
- Infinitely many Gauss constraints from regarding GR as SU(2) Yang-Mills theory
- More constraints from Hamiltonian and smeared diffeomorphisms
- Quantizing of constraints yields Quantum GR

$$\hat{G}_j \psi(A) = -iD_a \frac{\delta \Psi[A]}{\delta A_a^j} = 0.$$

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