Rotating Black Holes

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PHYS 486

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Introduction

- Einstein's field equation:

$$R_{\mu\nu} - \frac{1}{2}Rg_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi\frac{G}{c^4}T_{\mu\nu}$$

- Sets of 16 coupled non-linear PDEs
  - Reduces to 10 due to symmetry of tensors (10 independent components)
  - 4 coordinate system choice freedom→ 6 independent eqs.
- where $R_{\mu v}$ is the Ricci curvature tensor, $g_{\mu v}$ is the (symmetric 4x4) metric tensor, Λ is the cosmological const., G is Newton's gravitational cont., c is the speed of light in vacuum, R is the scalar curvature/tensor contraction of Ricci tensor and $T_{\mu v}$ is the stress-energy tensor.
- Hard to find exact solutions

No hair theorem

- Every black holes decay rapidly into stable black holes
- Can be described by 11 numbers:
  - Mass M
  - Linear Momentum P (3 components)
  - Angular Momentum J (3 components)
  - Position X (3 components)
  - Electric charge Q
- Other information ‘swallowed’/lost past event horizon

No hair theorem

- Change reference frame
- Can be described by 11 numbers:
  - Mass M
  - ~~Linear Momentum P (3 components)~~ -set to 0
  - ~~Angular Momentum J (3 components)~~ -orient spin of J along z
  - ~~Position X (3 components)~~ -set to 0
  - Electric charge Q
- So most general black hole can be described only by mass M, angular momentum J and electric charge Q!

Exact solutions of black hole

- 4 known "exact" black hole solutions to Einstein's field equations
  - Schwarzschild (Uncharged/Q=0, Nonrotating/J=0) – 1916
  - Reissner–Nordström (Charged/Q ≠ 0, Nonrotating/J=0)
  - Kerr (Uncharged/Q=0, Rotating/J ≠0)-1963
  - Kerr–Newman (Charged/Q ≠ 0, Rotating/J ≠0)

- Astrophysical objects electrically neutral (Net charge=0)
  - Schwarzschild and Kerr represent physical Universe

Schwarzchild review

- Schwarzchild metric solution(-+++ convention)
- $g_{\mu v} = \begin{bmatrix} -1 + r_s/r & 0 & 0 & 0 \\ 0 & \frac{1}{1 - r_s/r} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin^2\theta \end{bmatrix}$
- Schwarzchild radius : r_s=2GM/c^2
- Spherically symmetric
- Schwarzchild metric line element (with c=1):
  - $ds^2 = -\left(1 - \frac{2GM}{r}\right)\mathrm{dt}^2 + (1 - 2GM/r)^{-1}dr^2 + r^2(d\theta^2 + \sin^2\theta\, d\emptyset^2)$

Rotating black hole?

- Rotating black holes are of particular interest in astrophysics: they are thought to power quasars and other active galaxies, X-ray binaries, and gamma-ray bursts.
- Unlike for Schwarzschild black holes, it is possible to devise mechanisms that permit energy and angular momentum to be extracted from a (classical) rotating black hole
  - Penrose process
  - Blandford–Znajek process

Kerr solution

- Found by Roy Kerr in 1963
- Easier to represent in new coordinate system
- Boyer-Lindquist coordinate
  - $x = \sqrt{r^2 + a^2}\,\sin\theta cos\,\emptyset$
  - $y = \sqrt{r^2 + a^2}\,\sin\theta \sin\emptyset$
  - $z = rcos\theta$
  - Kerr parameter: a=J/M (J angular momentum) ;a limited value
  - Set J=0; a=0 →Schwartzchild solution
  - Breaks spherical symmetry if J ≠ 0, only axially symmetric on rotation axis

Kerr solution (cont.)

- Kerr metric solution (-+++ convertion)
- $g_{\mu v} = \begin{bmatrix} -1 + rr_s/\rho^2 & 0 & 0 & -\frac{rr_s a \sin^2\theta}{\rho^2} \\ 0 & \rho^2/\Delta & 0 & 0 \\ 0 & 0 & \rho^2 & 0 \\ -\frac{rr_s a \sin^2\theta}{\rho^2} & 0 & 0 & \frac{\sin^2\theta[(r^2+a^2)^2 - a^2\Delta\sin^2\theta]}{\rho^2} \end{bmatrix}$
- r_s=2GM (c=1)
- $\rho^2 = \mathrm{r}^2 + \mathrm{a}^2\cos^2\theta$
- $\Delta = \mathrm{r}^2 - r_s r + a^2$
- With assumption Q=0; if Q ≠ 0,$\Delta = \underline{\mathrm{r}^2 - r_s r + a^2} + r_q^2$; $r_q^2 = GQ^2/4\pi\varepsilon_o$
- Off-diagonal terms: inertial frame dragging

Kerr metric

- Kerr metric line element (-+++)
  - $ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right)\mathrm{dt}^2 - \frac{4GMra\sin^2\theta}{\rho^2} d\emptyset \mathrm{dt} + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + (r^2 + a^2 + 2GMra^2 \sin^2\theta\,/\rho^2) d\emptyset^2$

Singularities

- From metric singularities when $\rho^2 = 0$ and $\Delta = 0$
- Occur at:
  - r=0 and $\theta = \pi/2$;ring shape with radius "a" in Boyer-Lindquist coordinate
  - $r_{\pm} = GM \pm \sqrt{G^2M^2 - a^2}$, solve quadratic ; event horizon sphere
  - Smaller than event horizon of non-rotating
  - $GM \geq a$;
  - a=J/M → $J \leq GM^2$;
  - Accretion disk physics limit $J_{max} \leq 0.998GM^2$ (extreme Kerr solution)
  - "a" non-negative by choice of coordinate

Ergosphere

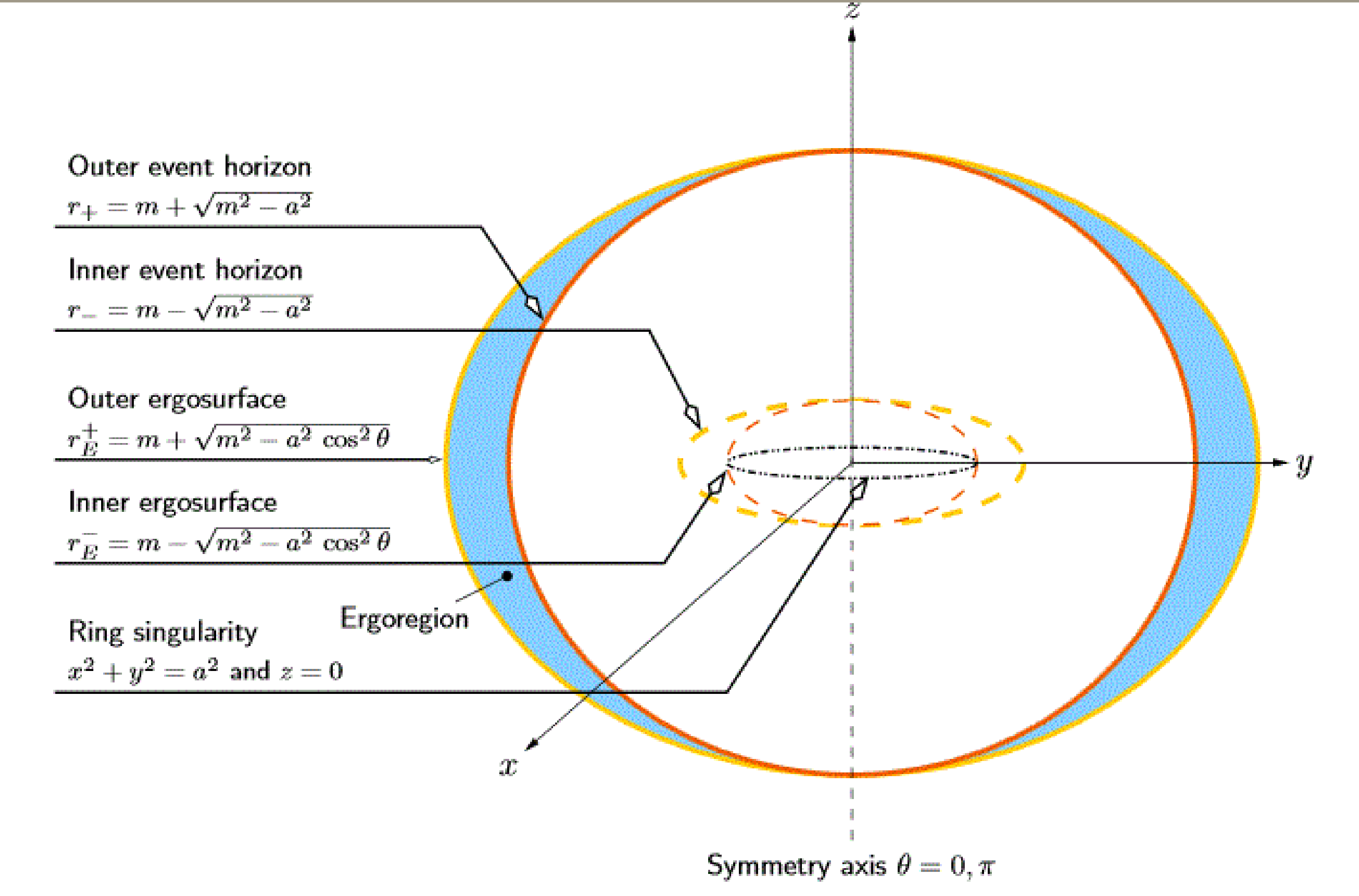
- When $g_{tt} = 0$, can find the "stationary limit" surface in which particle can remain stationary without being always in motion
- Kerr solution:
  - $-1 + rr_s/\rho^2$=0 → Quadratic equation: $r_{es\pm} = GM \pm \sqrt{G^2M^2 - a^2\cos^2\theta}$
  - Takes shape of ellipsoid
  - Surface called "ergosphere", lies outside event horizon
  - At r_{es} cannot become stationary without falling into horizon. However, possible to corotate with BH within the region and maintain radial distance from horizon

Ergosphere(cont.)

- Close inspection of ergosphere→ Frame dragging/Lense-Thirring effect
- Inside ergosphere if want to remain close to horizon and stationary:
  - $u^{\mu}_{obs} = \{u^{0}_{obs},0,0,0\};\ (u^{0}_{obs})^2 > 0$
  - $u^{\mu}_{obs} u_{\mu,obs} = -1 = g_{tt}\ (u^{0}_{obs})^2$
  - $r_{es\pm} = GM \pm \sqrt{G^2M^2 - a^2\cos^2\theta}$ ….. g_{tt}=0 on surface, g_{tt}<0 outside surface, g_{tt}>0 inside surface
  - $u^{\mu}_{obs} u_{\mu,obs} = g_{tt}\ (u^{0}_{obs})^2 = -1$ can't be satisfied inside ergosphere
- Inside ergosphere spacetime moves faster than light
  - Not possible to be stationary inside ergosphere from outside observer, require v>c
  - Not the case with Schwartzchild, can be stationary from outside observer given sufficient propulsion since no ergosphere/no frame dragging

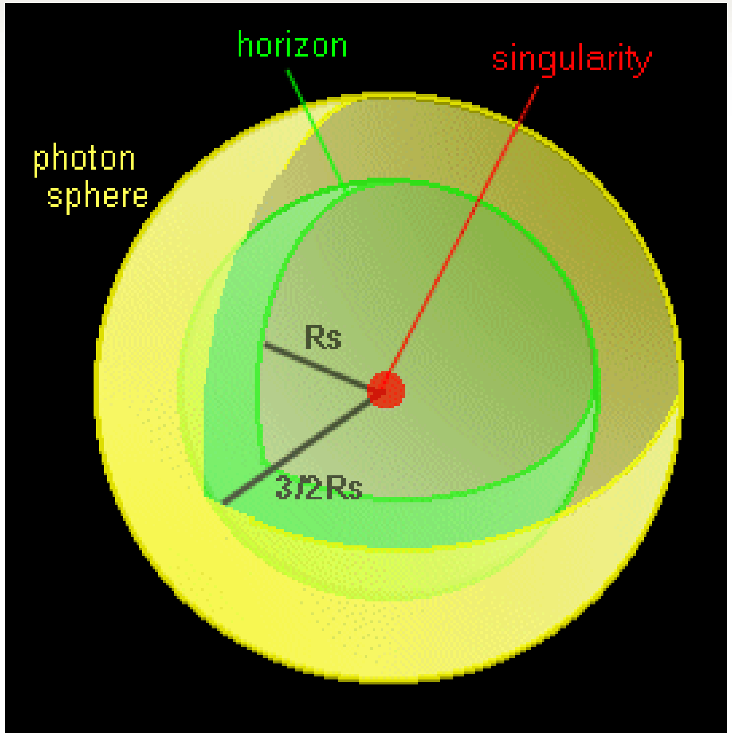
Frame Dragging

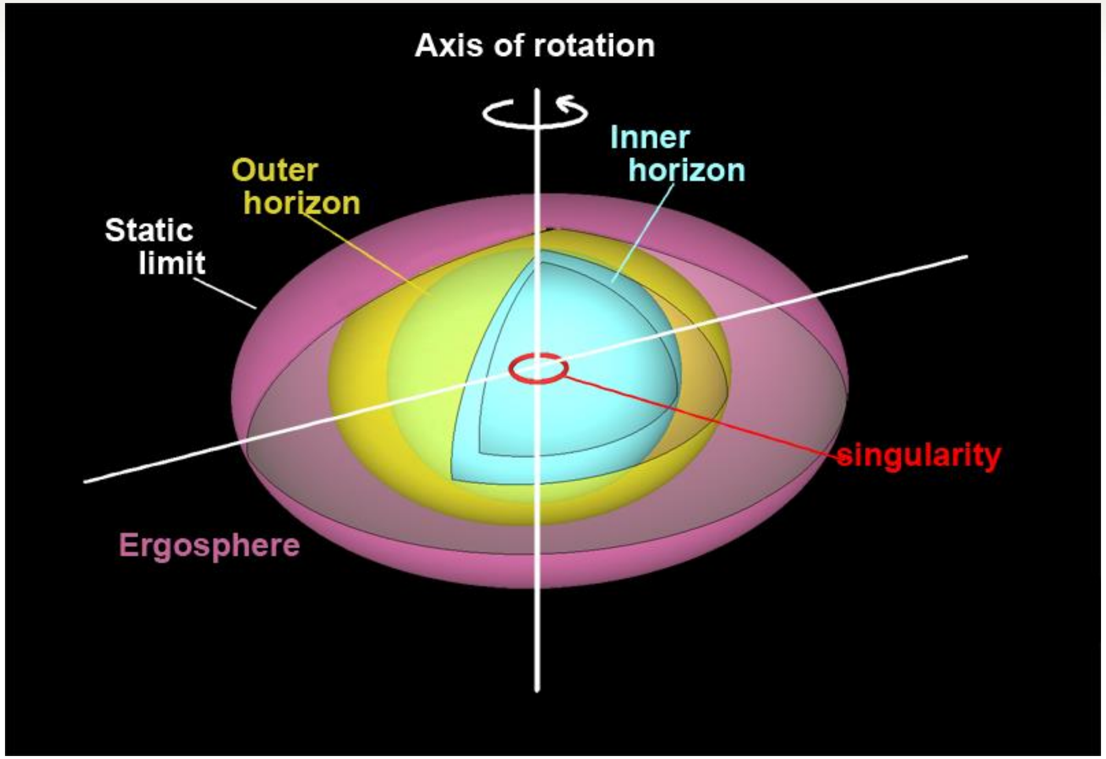
- A striking feature of the Kerr solution is frame dragging: loosely, the black hole drags spacetime with it as it rotates.
  - This arises ultimately because the Kerr metric contains off-diagonal components $g_{03} = g_{30}$
  - Consequence: particle dropped radially onto a Kerr black hole will acquire non-radial components of motion as it falls freely in the gravitational field

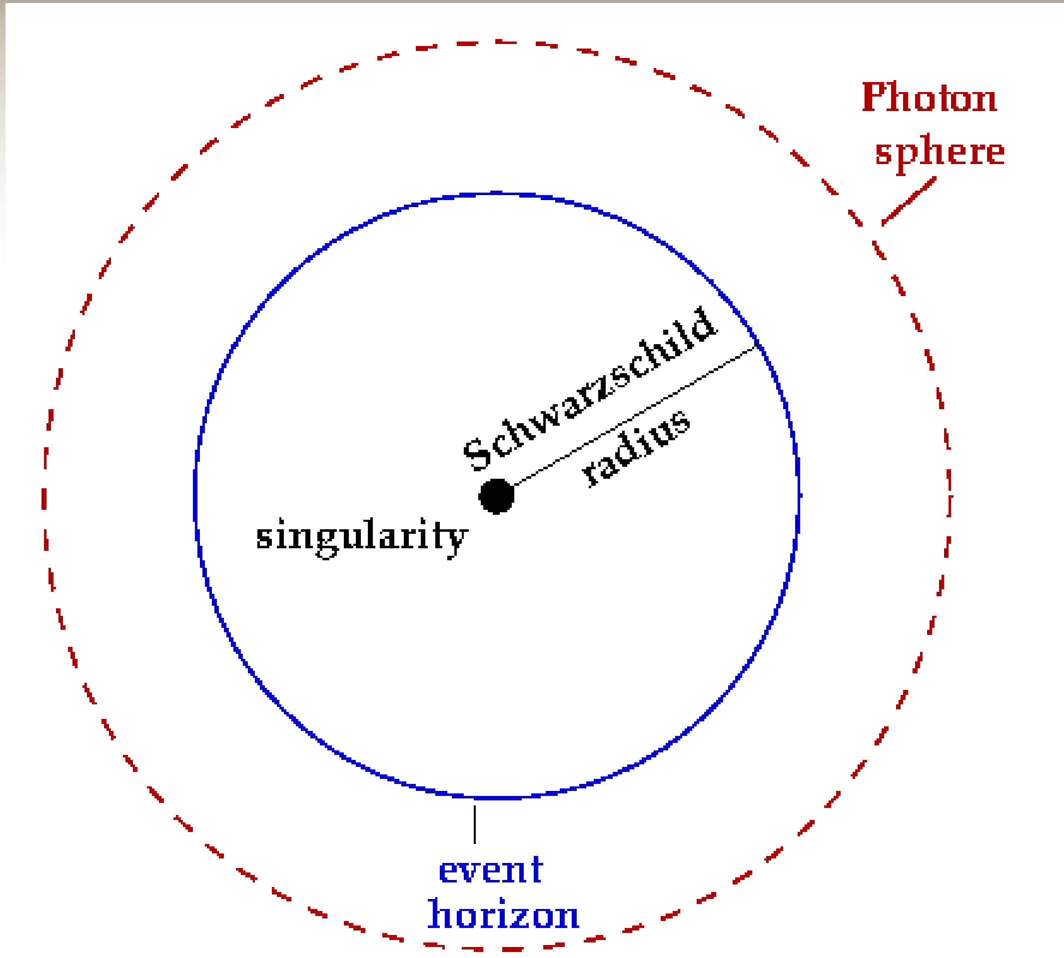


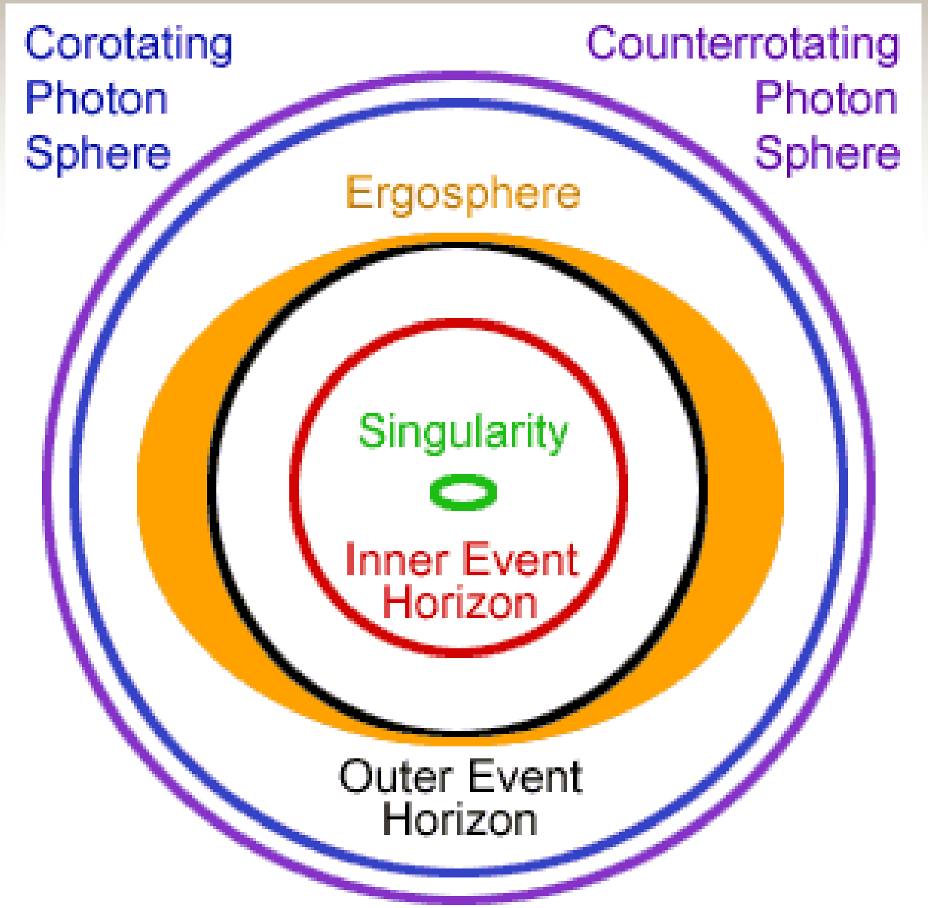
Outer event horizon
$r_+ = m + \sqrt{m^2 - a^2}$
Inner event horizon
$r_- = m - \sqrt{m^2 - a^2}$
Outer ergosurface
$r_E^+ = m + \sqrt{m^2 - a^2 \cos^2\theta}$
Inner ergosurface
$r_E^- = m - \sqrt{m^2 - a^2 \cos^2\theta}$
Ergoregion
Ring singularity
$x^2 + y^2 = a^2$ and $z = 0$
z
y
x
Symmetry axis $\theta = 0, \pi$

Schwarzchild vs Kerr

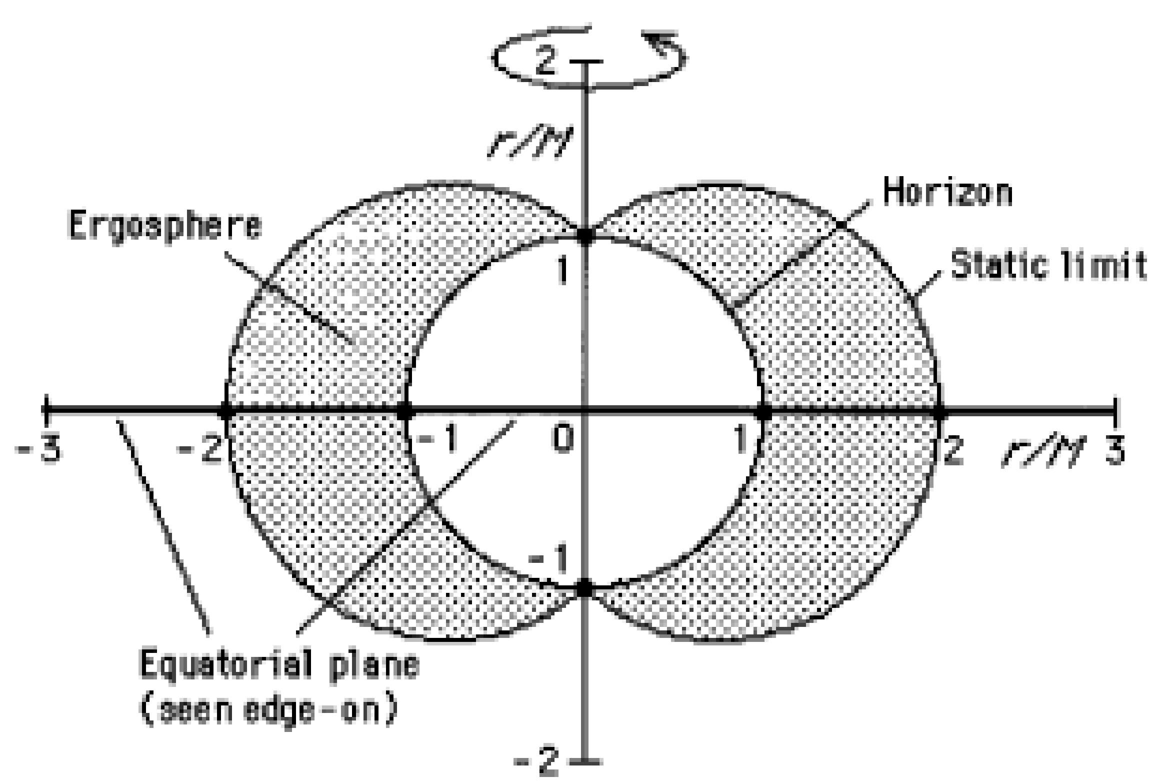





Photon
sphere
Schwarzschild
radius
singularity
event
horizon

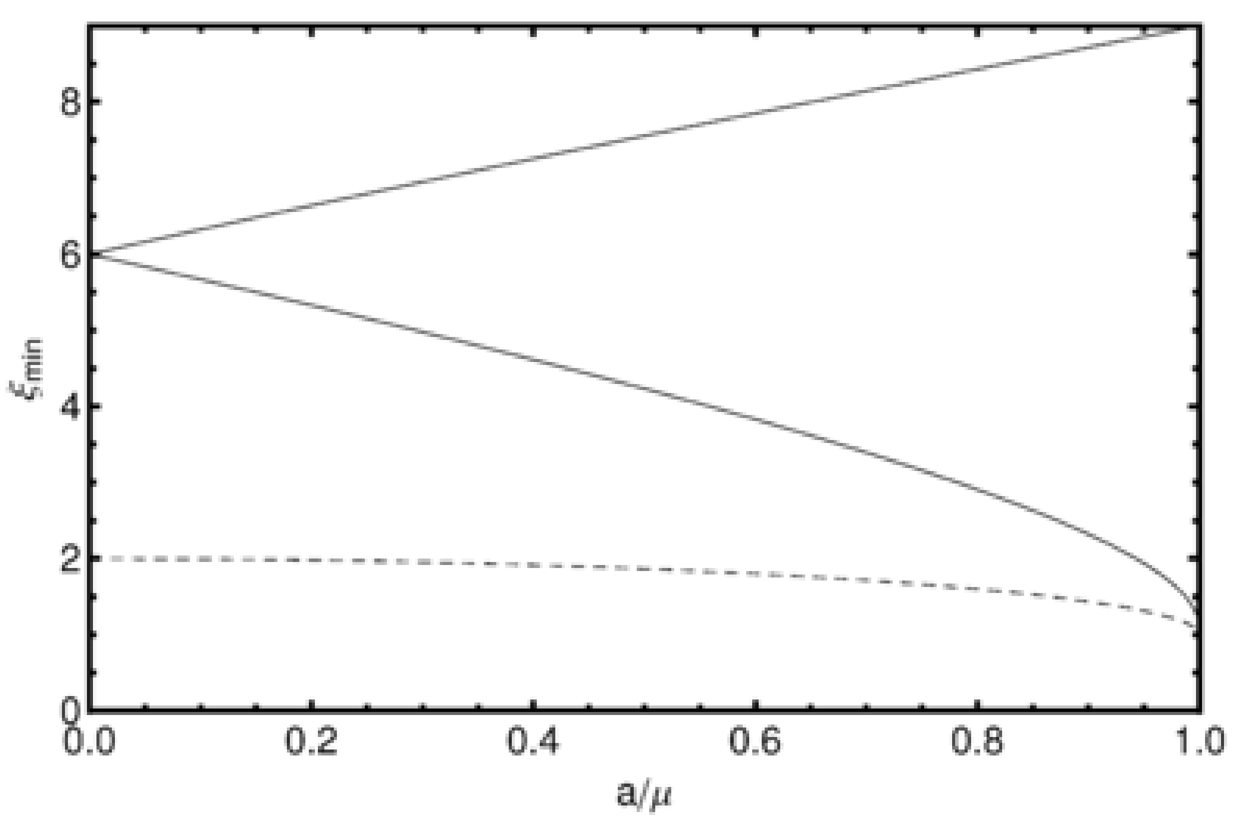

Corotating
Photon
Sphere
Counterrotating
Photon
Sphere
Ergosphere
Singularity
Inner Event
Horizon
Outer Event
Horizon

Extreme Kerr

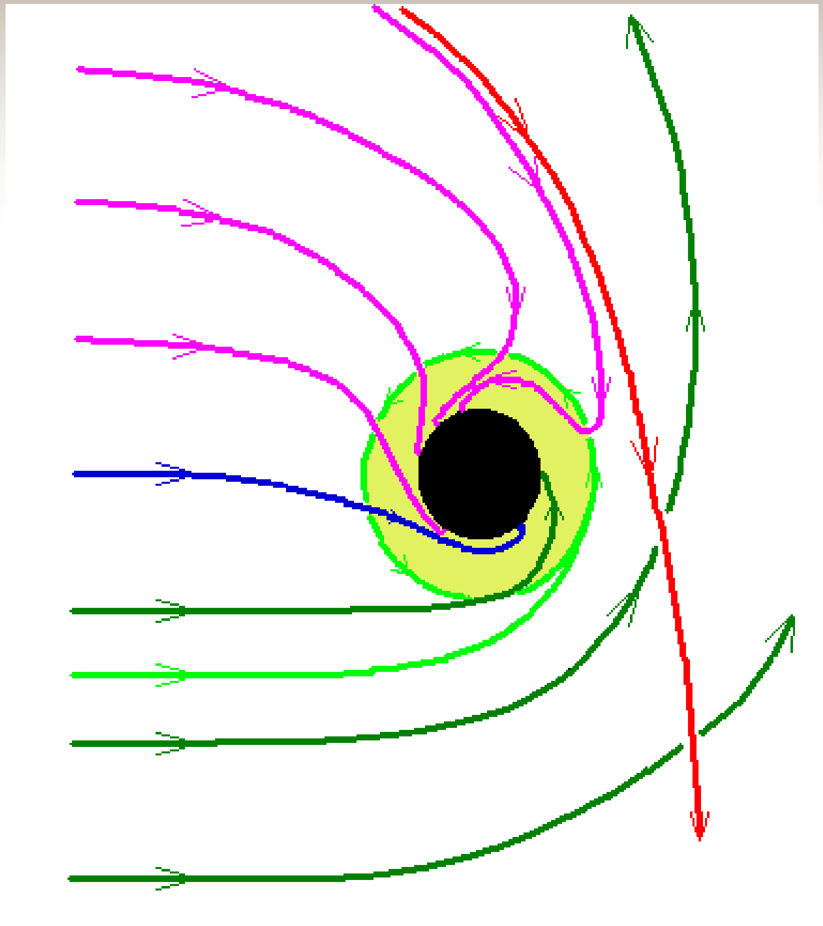


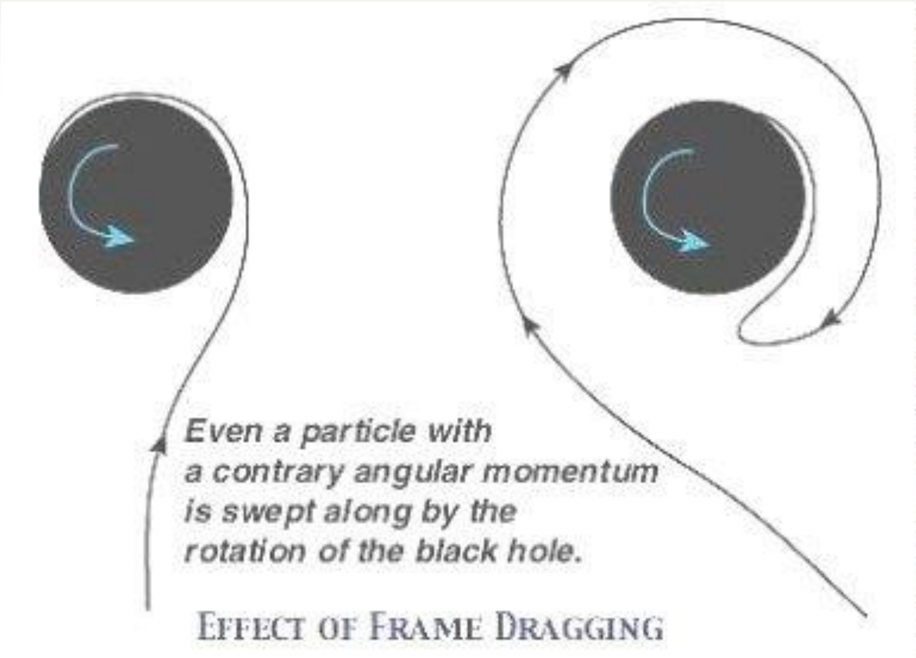
Trajectories

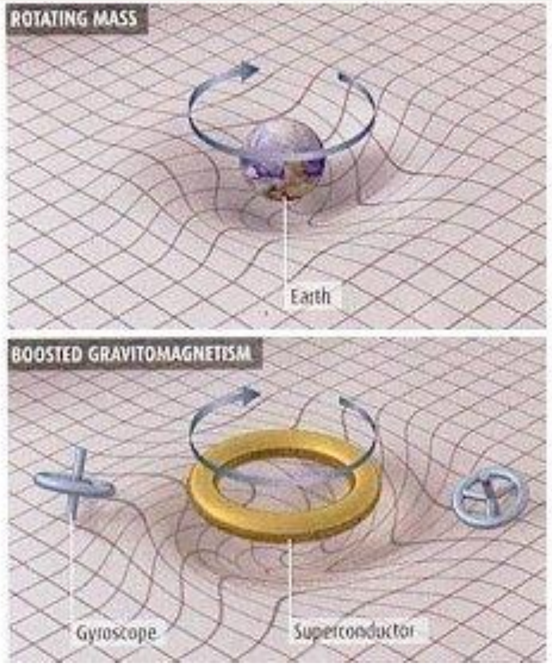
- $V_{eff} = -\frac{\kappa GM}{r} + \frac{L^2}{2r^2} + \frac{1}{2}(\kappa - E)^2\left(1 + \frac{a^2}{r^2}\right) - \frac{GM}{r^3}(L - aE)^2$
  - κ=0 for light ray geodesics, κ=1 for massive particle, L=conserved ang. mom./unit mass of test particle ; E=conserved energy/unit mass of test particle
- Stable (photon) orbits
- $r_1 \equiv 2M\left[1 + \cos\left(\frac{2}{3}\arccos\left(-\frac{|a|}{M}\right)\right)\right]$
- $r_2 \equiv 2M\left[1 + \cos\left(\frac{2}{3}\arccos\left(\frac{|a|}{M}\right)\right)\right]$
- For a=1(unit M=1) photon stable orbit for Schwartzchild → $r = 1.5r_s = 3GM$
  - Massive $r = 3r_s = 6GM$
- For a=1(unit M=1) photon stable orbit for Kerr → $r = 0.5r_s = GM$ (prograde) same as its horizon radius and $r = 2r_s = 4GM$ (retrograde)
  - Massive $r = 0.5r_s = GM$ and $r = 4.5r_s = 9GM$
- Scatter orbits



Radius of innermost stable circular orbit as function of $\alpha = a/\mu$:
the upper curve for $L<0$, the lower one for $L>0$. The horizon is shown by the dashed line.




Even a particle with
a contrary angular momentum
is swept along by the
rotation of the black hole.
EFFECT OF FRAME DRAGGING

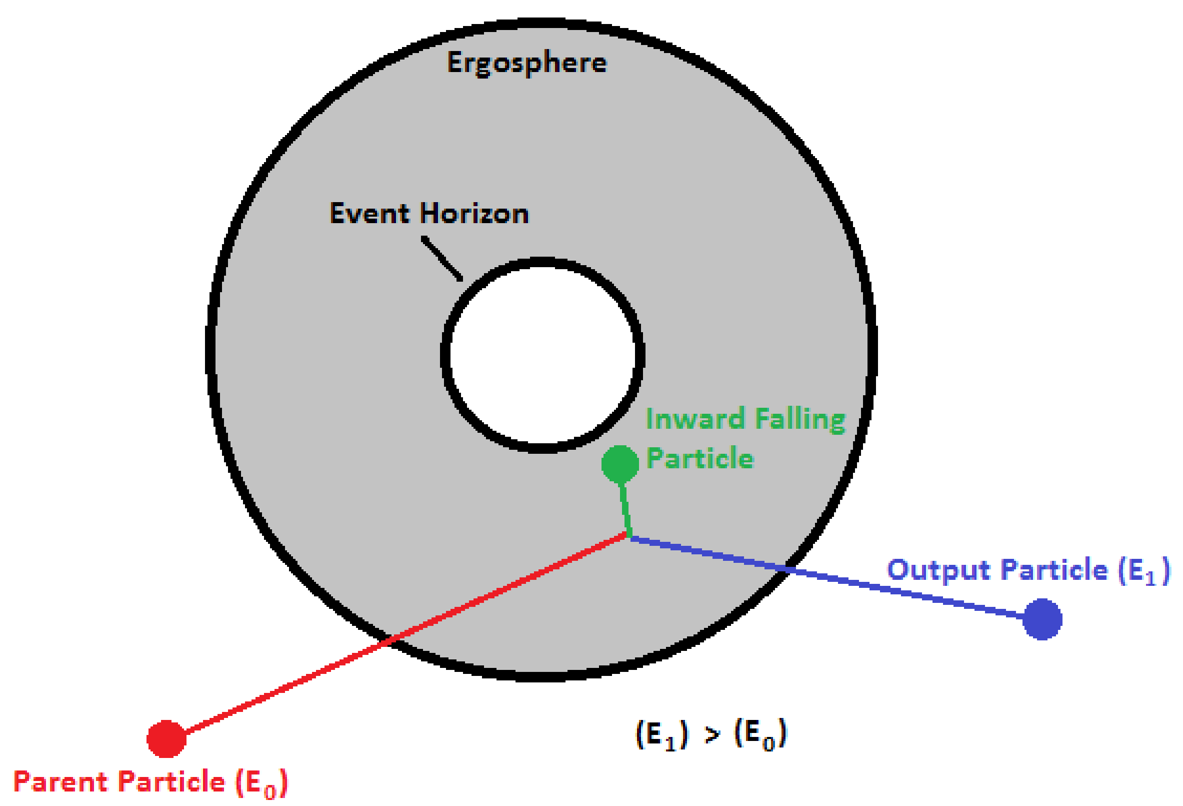

ROTATING MASS
Earth
BOOSTED GRAVITOMAGNETISM
Gyroscope
Superconductor

Extracting work

- In Schwarzchild, no further energy extraction possible
- Kerr, there are some process that can extract energy
- Made possible because rotational energy of BH not located inside horizon but outside (in ergosphere), ergo=work
- Two famous energy extraction schemes:
  - **Penrose Process:** particle splitting inside the ergosphere
  - **Blandford-Znajek Process:** BH spin twists magnetic field (manipulating B field inside ergosphere)
  - Penrose process theoretically possible but not feasible
  - Blandford-Znajek: best description how quasar is powered

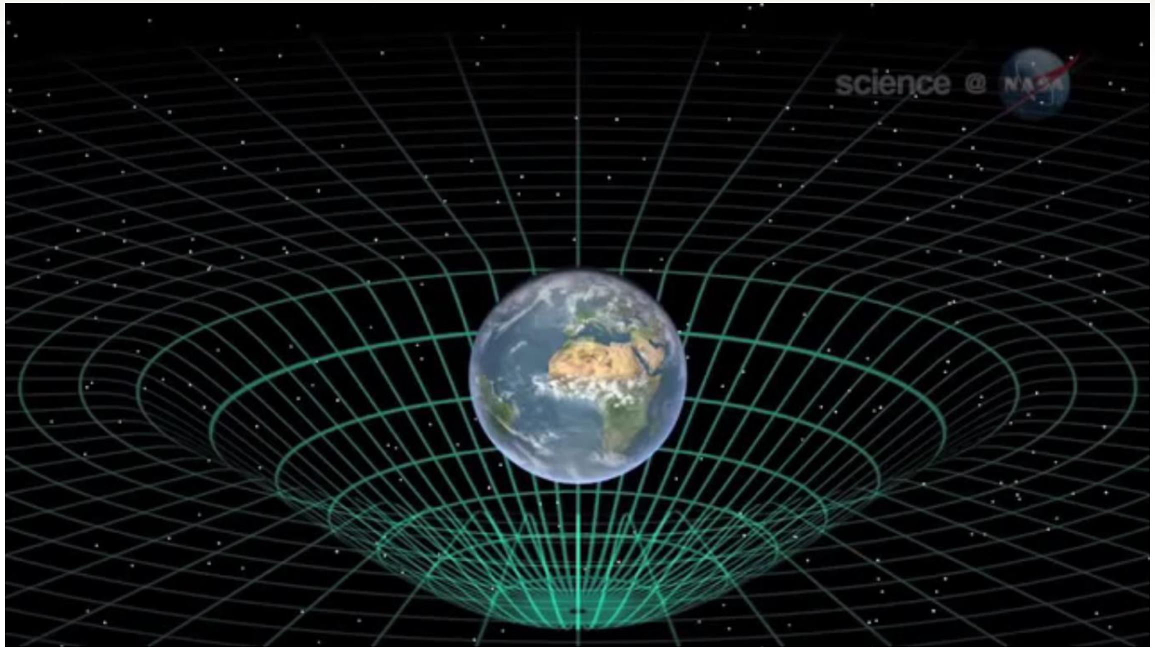
Penrose process

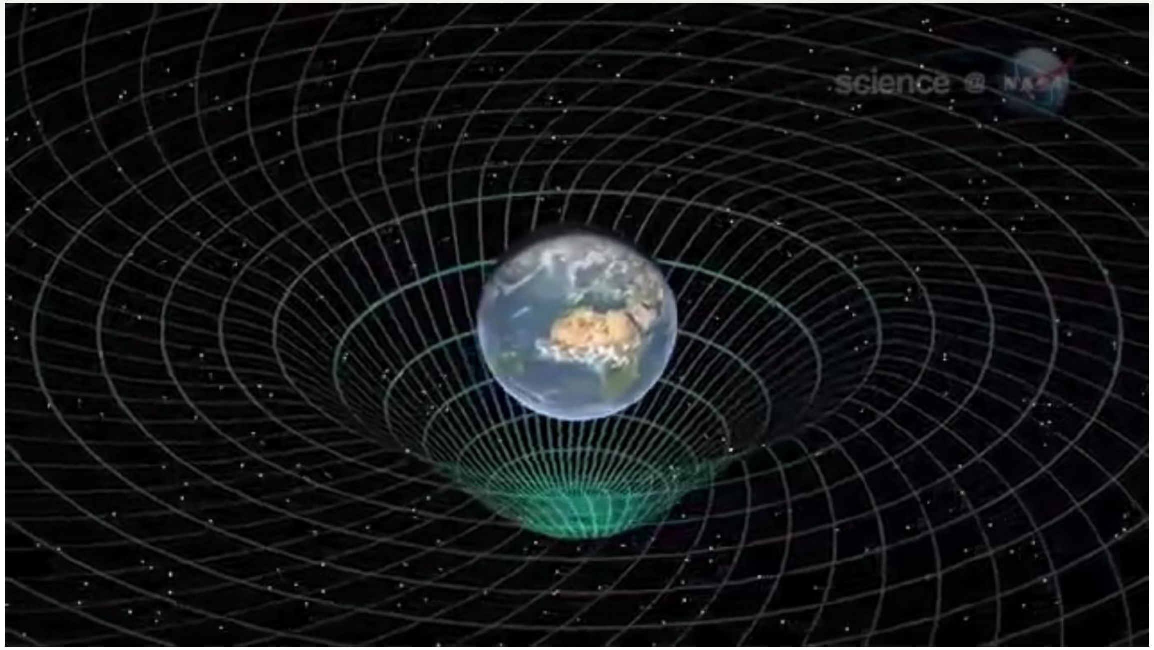
- Inside ergosphere particle can have positive/negative energy (time n one of angular coord swap meaning)
- Shoot particle A into ergosphere → split A into particle B, C
- Particle B moves opposite direction out of ergo (to ∞) with greater than initial energy
- Particle C goes into event horizon with negative energy
- Total energy of BH decreases
- Can keep extracting energy until rotating BH reduces into Schwartzchild
- The maximal energy that can be extracted corresponds to 20.7% of the initial black hole mass. Larger efficiencies are possible for charged rotating black holes.

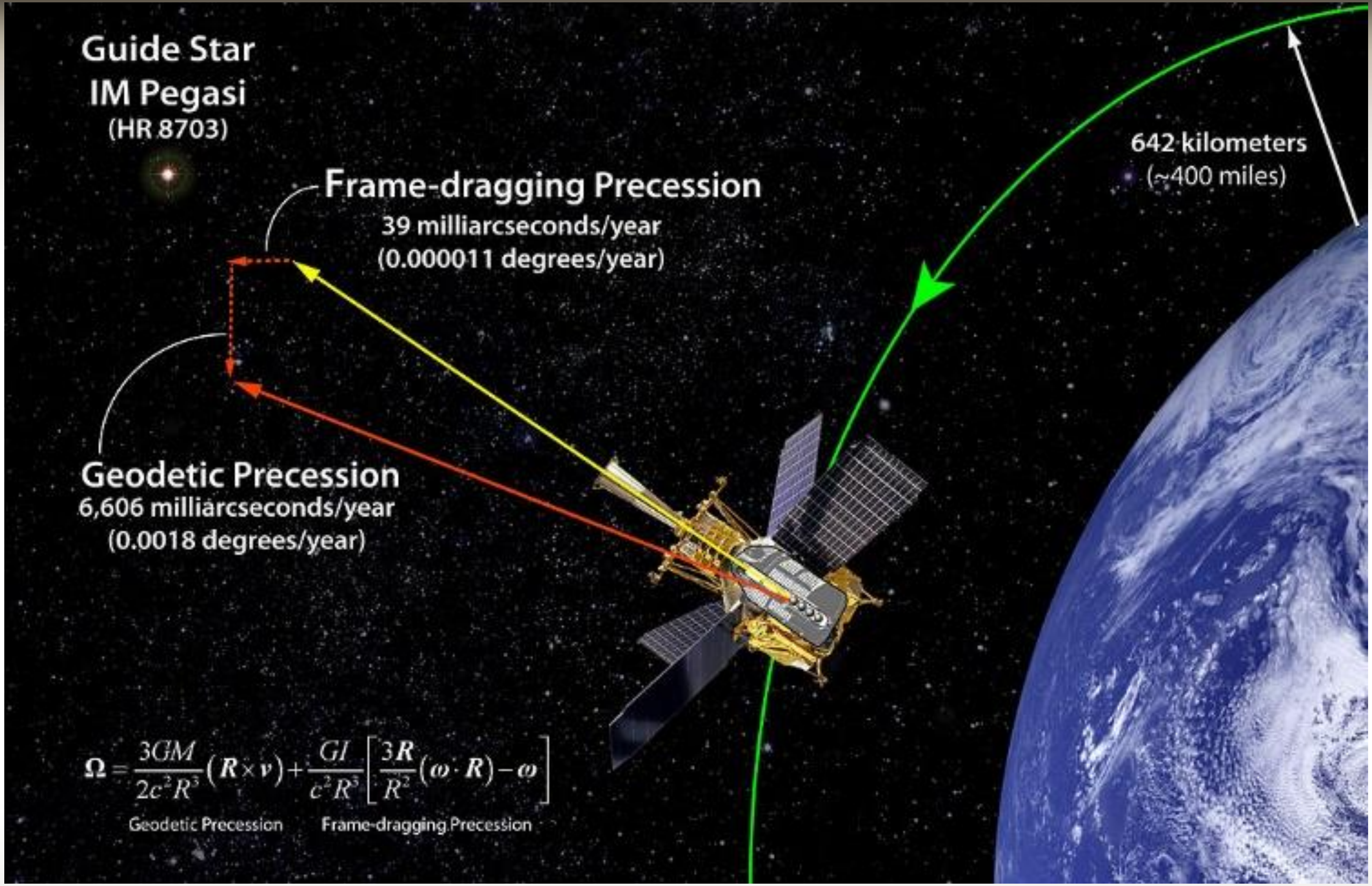

Ergosphere
Event Horizon
Inward Falling
Particle
Output Particle (E_1)
(E_1) > (E_0)
Parent Particle (E_0)

Measure frame dragging

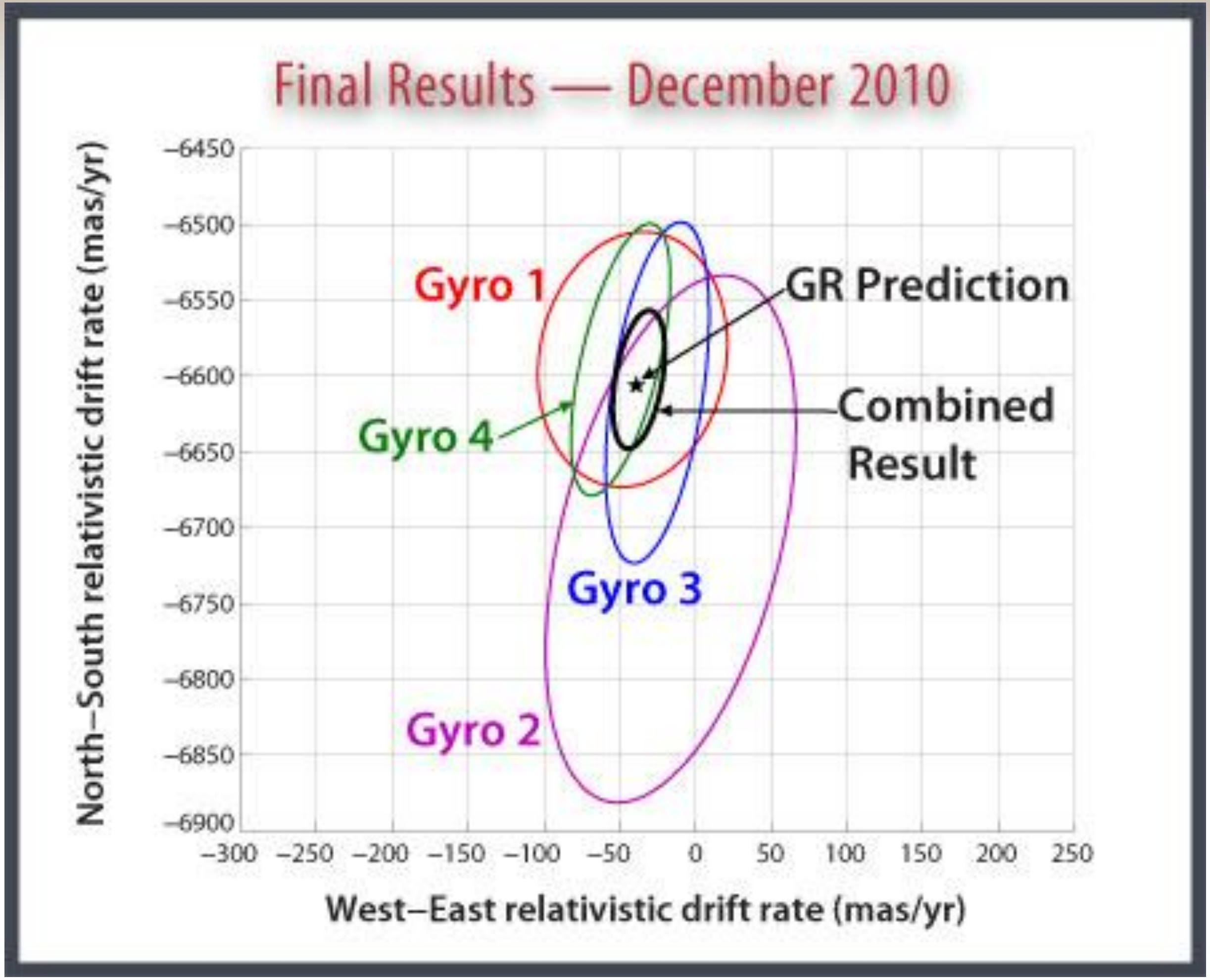
- NASA Gravity Probe B experiment (launched Apr 2004, ended Aug 2005)
- Sent four gyroscopes to orbit earth, measure precession
  - Geodetic effect
  - Frame dragging effect
- Measured geodetic drift rate of -6,601.8±18.3 mas/yr and a frame-dragging drift rate of -37:2±7.2 mas/yr
- GR predictions: -6,606.1 mas/yr and -39.2 mas/yr, respectively


science @ NASA


science @ NASA



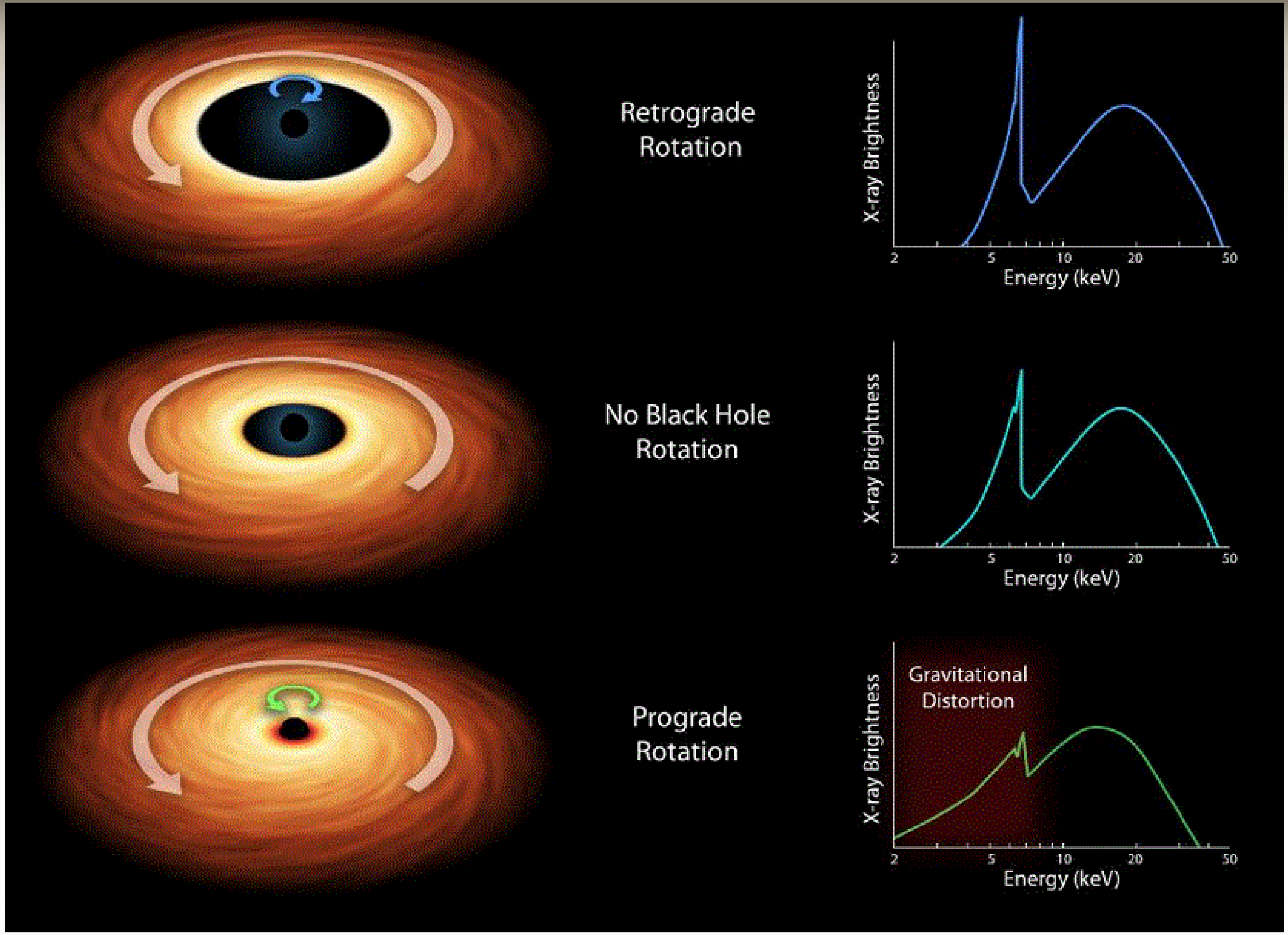
Guide Star
IM Pegasi
(HR 8703)
Frame-dragging Precession
39 milliarcseconds/year
(0.000011 degrees/year)
642 kilometers
(~400 miles)
Geodetic Precession
6,606 milliarcseconds/year
(0.0018 degrees/year)
$$\Omega = \underbrace{\frac{3GM}{2c^2R^3}(R \times v)}_{\text{Geodetic Precession}} + \underbrace{\frac{GI}{c^2R^3}\left[\frac{3R}{R^2}(\omega \cdot R) - \omega\right]}_{\text{Frame-dragging Precession}}$$


Final Results — December 2010
North–South relativistic drift rate (mas/yr)
−6450
−6500
−6550
−6600
−6650
−6700
−6750
−6800
−6850
−6900
−300 −250 −200 −150 −100 −50 0 50 100 150 200 250
West–East relativistic drift rate (mas/yr)
Gyro 1
Gyro 2
Gyro 3
Gyro 4
GR Prediction
Combined Result

Measurement of BH spin

- Spin rate of black hole tells us about its ‘past’ information
  - How they are formed over time
- If black holes grow mainly from collisions and mergers between galaxies
  - accumulate material in a stable disk, and the steady supply of new material from the disk should lead to rapidly spinning black holes.
- If black holes grow through many small accretion episodes
  - accumulate material from random directions
  - Make the black hole spin more slowly (like a merry go round that is pushed both backwards and forwards)
- Spinning black hole drags space around with it
  - allows matter to orbit closer to the black hole than is possible for a non-spinning black hole

Measurement of BH spin(cont.)

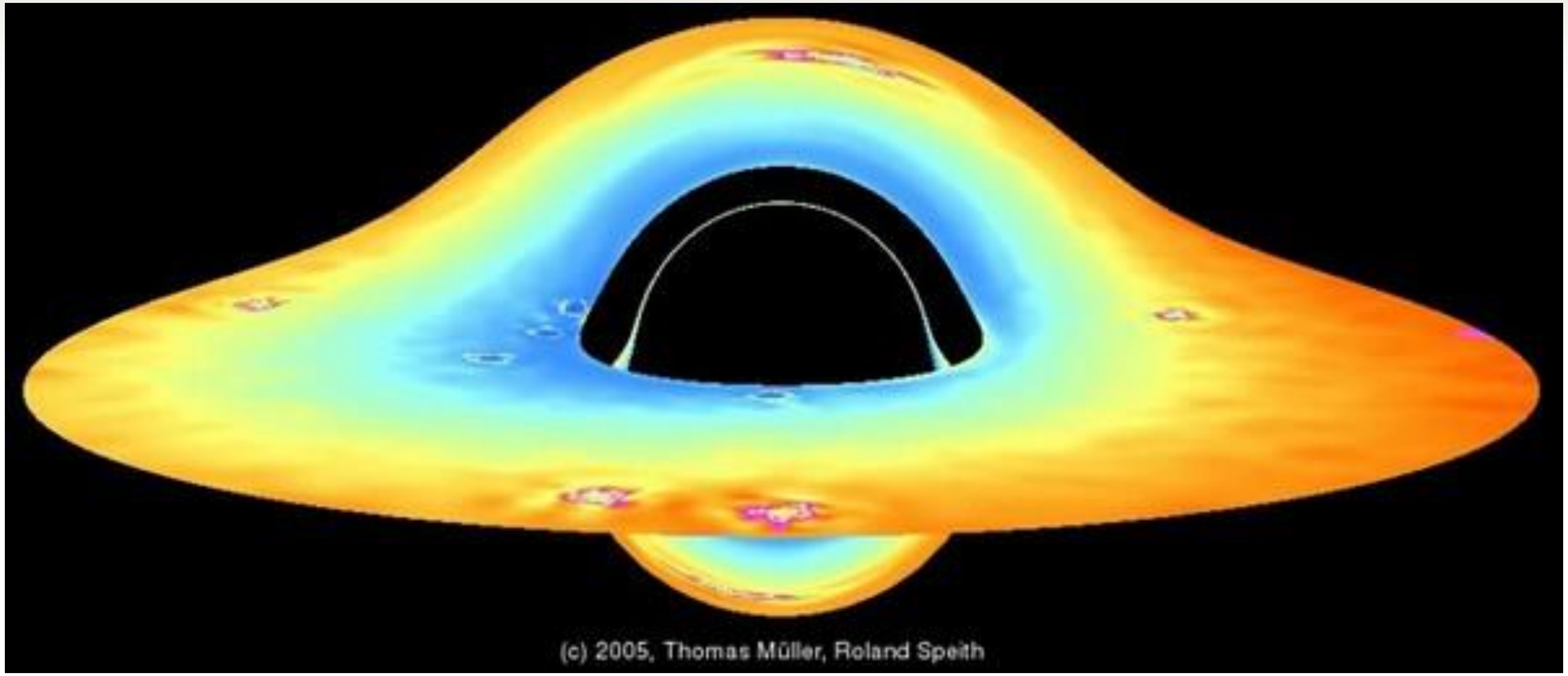


Measurement of BH spin (cont.)

- Look for light distortions in X-rays streaming off material near black holes→ researchers can gain information about their spin rates.
- Different types of spin: retrograde rotation (accretion disk/disk of matter falling onto the hole moves in the opposite direction of the black hole); no spin; and prograde rotation (disk spins in the same direction as the black hole).
- The faster a black hole spins, the closer its accretion disk can lie to it (frame dragging)
- Scientists assess how close the inner edge of an accretion disk comes to a black hole by breaking the X-ray light up into a spectrum of different colors, or energies.
  - Sharp peak is X-ray radiation from iron atoms circulating in the accretion disk.
  - If the accretion disk is close to the black hole, the X-ray colors from the iron will be spread out by the immense gravity of the black hole.
  - The degree to which the iron feature is spread out ("red wing“) reveals how close the accretion disk is to the black hole.
  - Because this distance depends on the black hole's spin, the spin rate can then be determined.

- Several black hole spin measured
- NASA :Chandra and XMM-Newton satellite Provide Direct Measurement of Distant Black Hole's Spin (March 5 2014)

- Multiple images of a distant quasar known as RX J1131-1231 are visible in this combined view from Chandra (pink) and Hubble (red, green, and blue).
- Gravitational lensing (the 4 pink dots are same image of quasar)


(c) 2005, Thomas Müller, Roland Speith

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- http://arxiv.org/pdf/gr-qc/9707012v1.pdf

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The Kerr Metric

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The Kerr Space-time: A brief introduction

- arXiv:0706.0622v3 [gr-qc] 15 Jan 2008

Thank You

Questions?