Rotating Black Holes

Muhammad Firdaus Mohd Soberi
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Introduction

• Einstein’s field equation:

\[ R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi \frac{G}{c^4} T_{\mu\nu} \]

• Sets of 16 coupled non-linear PDEs
  • Reduces to 10 due to symmetry of tensors (10 independent components)
  • 4 coordinate system choice freedom \(\rightarrow\) 6 independent eqs.

• where \(R_{\mu\nu}\) is the Ricci curvature tensor, \(g_{\mu\nu}\) is the (symmetric 4x4) metric tensor, \(\Lambda\) is the cosmological const., \(G\) is Newton’s gravitational cont., \(c\) is the speed of light in vacuum, \(R\) is the scalar curvature/tensor contraction of Ricci tensor and \(T_{\mu\nu}\) is the stress-energy tensor.

• Hard to find exact solutions
No hair theorem

• Every black holes decay rapidly into stable black holes
• Can be described by 11 numbers:
  • Mass M
  • Linear Momentum P (3 components)
  • Angular Momentum J (3 components)
  • Position X (3 components)
  • Electric charge Q
• Other information ‘swallowed’/lost past event horizon
No hair theorem

• Change reference frame
• Can be described by 11 numbers:
  • Mass M
  • Linear Momentum P (3 components) - set to 0
  • Angular Momentum J (3 components) - orient spin of J along z
  • Position X (3 components) - set to 0
  • Electric charge Q

• So most general black hole can be described only by mass M, angular momentum J and electric charge Q!
Exact solutions of black hole

• 4 known “exact” black hole solutions to Einstein’s field equations
  • Schwarzschild (Uncharged/Q=0, Nonrotating/J=0) – 1916
  • Reissner–Nordström (Charged/Q ≠ 0, Nonrotating/J=0)
  • Kerr (Uncharged/Q=0, Rotating/J ≠0)-1963
  • Kerr–Newman (Charged/Q ≠ 0, Rotating/J ≠0)

• Astrophysical objects electrically neutral (Net charge=0)
  • Schwarzschild and Kerr represent physical Universe
Schwarzchild review

- Schwarzchild metric solution(-+++ convention)

\[
\begin{pmatrix}
-1 + \frac{r_s}{r} & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1 - \frac{r_s}{r} & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]

- Schwarzchild radius : \( r_s = \frac{2GM}{c^2} \)
- Spherically symmetric
- Schwarzchild metric line element (with c=1):

\[
ds^2 = -\left(1 - \frac{2GM}{r}\right)dt^2 + \left(1 - \frac{2GM}{r}\right)^{-1}dr^2 + r^2(d\theta^2 + \sin^2 \theta d\phi^2)
\]
Rotating black hole?

- Rotating black holes are of particular interest in astrophysics: they are thought to power quasars and other active galaxies, X-ray binaries, and gamma-ray bursts.

- Unlike for Schwarzschild black holes, it is possible to devise mechanisms that permit energy and angular momentum to be extracted from a (classical) rotating black hole
  - Penrose process
  - Blandford–Znajek process
Kerr solution

• Found by Roy Kerr in 1963
• Easier to represent in new coordinate system
• Boyer-Lindquist coordinate
  • $x = \sqrt{r^2 + a^2 \sin \theta \cos \phi}$
  • $y = \sqrt{r^2 + a^2 \sin \theta \sin \phi}$
  • $z = r \cos \theta$
• Kerr parameter: $a=J/M$ (J angular momentum); a limited value
• Set $J=0$; $a=0 \rightarrow$ Schwartzchild solution
• Breaks spherical symmetry if $J \neq 0$, only axially symmetric on rotation axis
Kerr solution (cont.)

- Kerr metric solution (++++ conversion)

\[
g_{\mu\nu} = \begin{bmatrix}
-1 + rr_s/\rho^2 & 0 & 0 & -rr_s a \sin^2 \theta/\rho^2 \\
0 & \rho^2/\Delta & 0 & 0 \\
0 & 0 & \rho^2 & 0 \\
-rr_s a \sin^2 \theta/\rho^2 & 0 & 0 & \sin^2 \theta [(r^2+a^2)^2 - a^2 \Delta \sin^2 \theta]/\rho^2 \\
\end{bmatrix}
\]

- \( r_s = 2GM \) (c=1)
- \( \rho^2 = r^2 + a^2 \cos^2 \theta \)
- \( \Delta = r^2 - r_s r + a^2 \)
- With assumption \( Q=0 \); if \( Q \neq 0 \), \( \Delta = r^2 - r_s r + a^2 + r_q^2 \); \( r_q^2 = G Q^2 / 4\pi \varepsilon_0 \)
- Off-diagonal terms: inertial frame dragging
• Kerr metric line element (++++)

\[ ds^2 = -\left(1 - \frac{2GMr}{\rho^2}\right)dt^2 - \frac{4GMr a \sin^2 \theta}{\rho^2} d\phi dt + \frac{\rho^2}{\Delta} dr^2 + \rho^2 d\theta^2 + \left(r^2 + a^2 + \frac{2GMr a^2 \sin^2 \theta}{\rho^2}\right)d\phi^2 \]
Singularities

• From metric singularities when $\rho^2 = 0$ and $\Delta = 0$

• Occur at:
  - $r=0$ and $\theta = \pi/2$; ring shape with radius “a” in Boyer-Lindquist coordinate
  - $r_\pm = GM \pm \sqrt{G^2M^2 - a^2}$, solve quadratic; event horizon sphere
  - Smaller than event horizon of non-rotating
  - $GM \geq a$;
  - $a = J/M \rightarrow J \leq GM^2$;
  - Accretion disk physics limit $J_{max} \leq 0.998GM^2$ (extreme Kerr solution)
  - “a” non-negative by choice of coordinate
Ergosphere

• When $g_{tt} = 0$, can find the “stationary limit” surface in which particle can remain stationary without being always in motion

• Kerr solution:
  • $-1 + rr_s/\rho^2 = 0 \rightarrow$ Quadratic equation: $r_{es \pm} = GM \pm \sqrt{G^2 M^2 - a^2 \cos^2 \theta}$
  • Takes shape of ellipsoid
  • Surface called “ergosphere”, lies outside event horizon
  • At $r_{es}$ cannot become stationary without falling into horizon. However, possible to corotate with BH within the region and maintain radial distance from horizon
• Close inspection of ergosphere \(\rightarrow\) Frame dragging/Lense-Thirring effect

• Inside ergosphere if want to remain close to horizon and stationary:
  • \(u_{\text{obs}}^{\mu} = \{u_{\text{obs}}^0,0,0,0\}; \quad (u_{\text{obs}}^0)^2 > 0\)
  • \(u_{\text{obs}}^\mu u_{\mu,\text{obs}} = -1 = g_{tt} (u_{\text{obs}}^0)^2\)
  • \(r_{es} = GM \pm \sqrt{G^2 M^2 - a^2 \cos^2 \theta}\) ….. \(g_{tt}=0\) on surface, \(g_{tt}<0\) outside surface, \(g_{tt}>0\) inside surface
  • \(u_{\text{obs}}^\mu u_{\mu,\text{obs}} = g_{tt} (u_{\text{obs}}^0)^2 = -1\) can’t be satisfied inside ergosphere

• Inside ergosphere spacetime moves faster than light
  • Not possible to be stationary inside ergosphere from outside observer, require \(v>c\)
  • Not the case with Schwartzchild, can be stationary from outside observer given sufficient propulsion since no ergosphere/no frame dragging
A striking feature of the Kerr solution is frame dragging: loosely, the black hole drags spacetime with it as it rotates. This arises ultimately because the Kerr metric contains off-diagonal components $g_{03} = g_{30}$. Consequence: particle dropped radially onto a Kerr black hole will acquire non-radial components of motion as it falls freely in the gravitational field.
Outer event horizon
\[ r_+ = m + \sqrt{m^2 - a^2} \]

Inner event horizon
\[ r_- = m - \sqrt{m^2 - a^2} \]

Outer ergosurface
\[ r_E^+ = m + \sqrt{m^2 - a^2 \cos^2 \theta} \]

Inner ergosurface
\[ r_E^- = m - \sqrt{m^2 - a^2 \cos^2 \theta} \]

Ring singularity
\[ x^2 + y^2 = a^2 \text{ and } z = 0 \]

Symmetry axis \( \theta = 0, \pi \)
Schwarzschild vs Kerr

[Diagram showing the differences between Schwarzschild and Kerr black holes, highlighting key features such as the horizon, singularity, photon sphere, and ergosphere.]
Extreme Kerr
Trajectories

- \( V_{\text{eff}} = -\frac{\kappa GM}{r} + \frac{L^2}{2r^2} + \frac{1}{2} (\kappa - E)^2 \left( 1 + \frac{a^2}{r^2} \right) - \frac{GM}{r^3} (L - aE)^2 \)
  - \( \kappa = 0 \) for light ray geodesics, \( \kappa = 1 \) for massive particle, \( L = \) conserved ang. mom./unit mass of test particle; \( E = \) conserved energy/unit mass of test particle

- Stable (photon) orbits

- \( r_1 \equiv 2M \left[ 1 + \cos \left( \frac{2}{3} \arccos \left( -\frac{|a|}{M} \right) \right) \right] \)
- \( r_2 \equiv 2M \left[ 1 + \cos \left( \frac{2}{3} \arccos \left( \frac{|a|}{M} \right) \right) \right] \)
- For \( a=1 \) (unit \( M=1 \)) photon stable orbit for Schwartzchild \( \rightarrow r = 1.5r_s = 3GM \)
  - Massive \( r = 3r_s = 6GM \)
- For \( a=1 \) (unit \( M=1 \)) photon stable orbit for Kerr \( \rightarrow r = 0.5r_s = GM \) (prograde)
  - same as its horizon radius and \( r = 2r_s = 4GM \) (retrograde)
  - Massive \( r = 0.5r_s = GM \) and \( r = 4.5r_s = 9GM \)

- Scatter orbits
Radius of innermost stable circular orbit as function of $\alpha = a/\mu$: the upper curve for $L<0$, the lower one for $L>0$. The horizon is shown by the dashed line.
Even a particle with a contrary angular momentum is swept along by the rotation of the black hole.

Effect of Frame Dragging
Extracting work

• In Schwarzchild, no further energy extraction possible
• Kerr, there are some process that can extract energy
• Made possible because rotational energy of BH not located inside horizon but outside (in ergosphere), ergo=work

• Two famous energy extraction schemes:
  • Penrose Process: particle splitting inside the ergosphere
  • Blandford-Znajek Process: BH spin twists magnetic field (manipulating B field inside ergosphere)
  • Penrose process theoretically possible but not feasible
  • Blandford-Znajek: best description how quasar is powered
Penrose process

• Inside ergosphere particle can have positive/negative energy (time n one of angular coord swap meaning)
• Shoot particle A into ergosphere $\rightarrow$ split A into particle B, C
• Particle B moves opposite direction out of ergo (to $\infty$) with greater than initial energy
• Particle C goes into event horizon with negative energy
• Total energy of BH decreases
• Can keep extracting energy until rotating BH reduces into Schwartzchild
• The maximal energy that can be extracted corresponds to 20.7% of the initial black hole mass. Larger efficiencies are possible for charged rotating black holes.
Measure frame dragging

• NASA Gravity Probe B experiment (launched Apr 2004, ended Aug 2005)

• Sent four gyroscopes to orbit earth, measure precession
  • Geodetic effect
  • Frame dragging effect

• Measured geodetic drift rate of \(-6,601.8\pm18.3\) mas/yr and a frame-dragging drift rate of \(-37:2\pm7.2\) mas/yr

• GR predictions: \(-6,606.1\) mas/yr and \(-39.2\) mas/yr, respectively
Guide Star
IM Pegasi
(HR 8703)

Frame-dragging Precession
39 milliarcseconds/year
(0.000011 degrees/year)

Geodetic Precession
6,606 milliarcseconds/year
(0.0018 degrees/year)

\[
\Omega = \frac{3GM}{2c^2R} (R \times v) + \frac{G\ell}{c^2R^2} \left[ \frac{3R}{R^2} (\omega \cdot R) - \omega \right]
\]

642 kilometers
(≈400 miles)
Measurement of BH spin

• Spin rate of black hole tells us about its ‘past’ information
  • How they are formed over time

• If black holes grow mainly from collisions and mergers between galaxies
  • accumulate material in a stable disk, and the steady supply of new material from the disk should lead to rapidly spinning black holes.

• If black holes grow through many small accretion episodes
  • accumulate material from random directions
  • Make the black hole spin more slowly (like a merry go round that is pushed both backwards and forwards)

• Spinning black hole drags space around with it
  • allows matter to orbit closer to the black hole than is possible for a non-spinning black hole
Measurement of BH spin (cont.)
Measurement of BH spin (cont.)

• Look for light distortions in X-rays streaming off material near black holes → researchers can gain information about their spin rates.

• Different types of spin: retrograde rotation (accretion disk/disk of matter falling onto the hole moves in the opposite direction of the black hole); no spin; and prograde rotation (disk spins in the same direction as the black hole).

• The faster a black hole spins, the closer its accretion disk can lie to it (frame dragging)

• Scientists assess how close the inner edge of an accretion disk comes to a black hole by breaking the X-ray light up into a spectrum of different colors, or energies.
  • Sharp peak is X-ray radiation from iron atoms circulating in the accretion disk.
  • If the accretion disk is close to the black hole, the X-ray colors from the iron will be spread out by the immense gravity of the black hole.
  • The degree to which the iron feature is spread out ("red wing“) reveals how close the accretion disk is to the black hole.
  • Because this distance depends on the black hole's spin, the spin rate can then be determined.
• Several black hole spin measured
• NASA : Chandra and XMM-Newton satellite Provide Direct Measurement of Distant Black Hole's Spin (March 5 2014)
    • Multiple images of a distant quasar known as RX J1131-1231 are visible in this combined view from Chandra (pink) and Hubble (red, green, and blue).
    • Gravitational lensing (the 4 pink dots are same image of quasar)
References

General Relativity and Quantum Cosmology (gr-qc)
• Black Holes, P.K Townsend (4 Jul 1997)

Areas of the Event Horizon and Stationary Limit Surface for a Kerr Black Hole
• arXiv:gr-qc/0001053v1 19 Jan 2000

The Kerr Metric
• http://arxiv.org/abs/1410.2130v2

The Kerr Space-time: A brief introduction
Thank You
Questions?