Compact Objects

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What are they?

- Blanket term for high density astrophysical objects
  - Usually white dwarf stars, neutron stars, and black holes
- High mass, low volume objects held in equilibrium with gravitational infall
  - Makes use of quantum degeneracy pressure
- Labs of high energy and exotic phenomena

Pauli Exclusion Principle

Suppose two particles are in the states $\psi_a(\boldsymbol{r})$ and $\psi_b(\boldsymbol{r})$ respectively so that we can write a combined wavefunction:

$$\psi(r_1, r_2) = \psi_a(r_1)\psi_b(r_2)$$

If the two particles are identical, we must build a wavefunction that is noncommittal:

$$\psi_{\pm}(r_1, r_2) = A(\psi_a(r_1)\psi_b(r_2) \pm \psi_a(r_2)\psi_b(r_1))$$

Particles which admit the minus sign above are fermions. If the particles are identical,

$$\psi_a = \psi_b \Rightarrow \psi_{-}(r_1, r_2) = 0$$

Hence fermions cannot occupy the same state.

Quantum Degeneracy Pressure

Suppose we have a rectangular solid with dimensions l_x , l_y , and l_z . If we treat fermions interior to this as a free fermionic gas with potential

$$V(x,y,z) = \begin{cases} 0 \; if \; (0 < x < lx, 0 < y < ly, 0 < z < lz \\ \infty \; otherwise \end{cases}$$

we find solutions, $\psi = \sqrt{\frac{8}{l_x l_y l_z}} \sin(\frac{nx\pi}{l_x}) \sin(\frac{ny\pi}{l_y}) \sin(\frac{nz\pi}{l_z})$

and allowed energies, $E = \frac{\hbar^2\pi^2}{2m}\left(\frac{{n_x}^2}{{l_x}^2} + \frac{{n_y}^2}{{l_y}^2} + \frac{{n_z}^2}{{l_z}^2}\right) = \frac{\hbar^2 k^2}{2m}$

Quantum Degeneracy Pressure

From this we can create a k-space in which the wavefunction of any two fermions occupy the first octant of a sphere with volume $\frac{\pi^3}{l_x l_y l_z}$ and radius k_f given by $\frac{1}{8}\left(\frac{4}{3}\pi {k_f}^2\right) = \frac{Nq}{2}(\frac{\pi^3}{l_x l_y l_z}) = \frac{Nq}{2}(\frac{\pi^3}{V})$

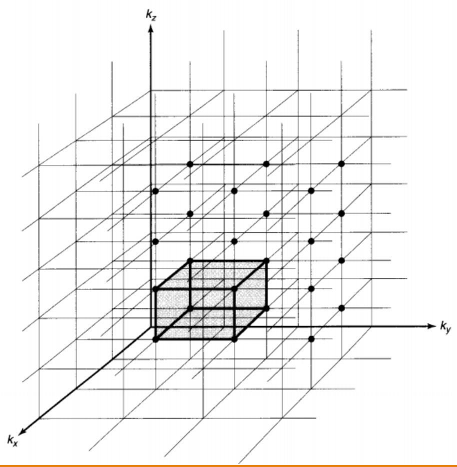
We find that the total energy of the system is given by

$$E_{tot} = \frac{\hbar^2 {k_f}^5 V}{10\pi^2 m}$$

so since $dW = PdV$, this energy shows up as work done on the outside by a quantum pressure

$$P = \frac{2\,Etot}{3\,V} = \frac{\hbar^2 {k_f}^5}{10\pi^2 m}$$

dependent on the material density and mass of the constituent particles



Stellar Evolution

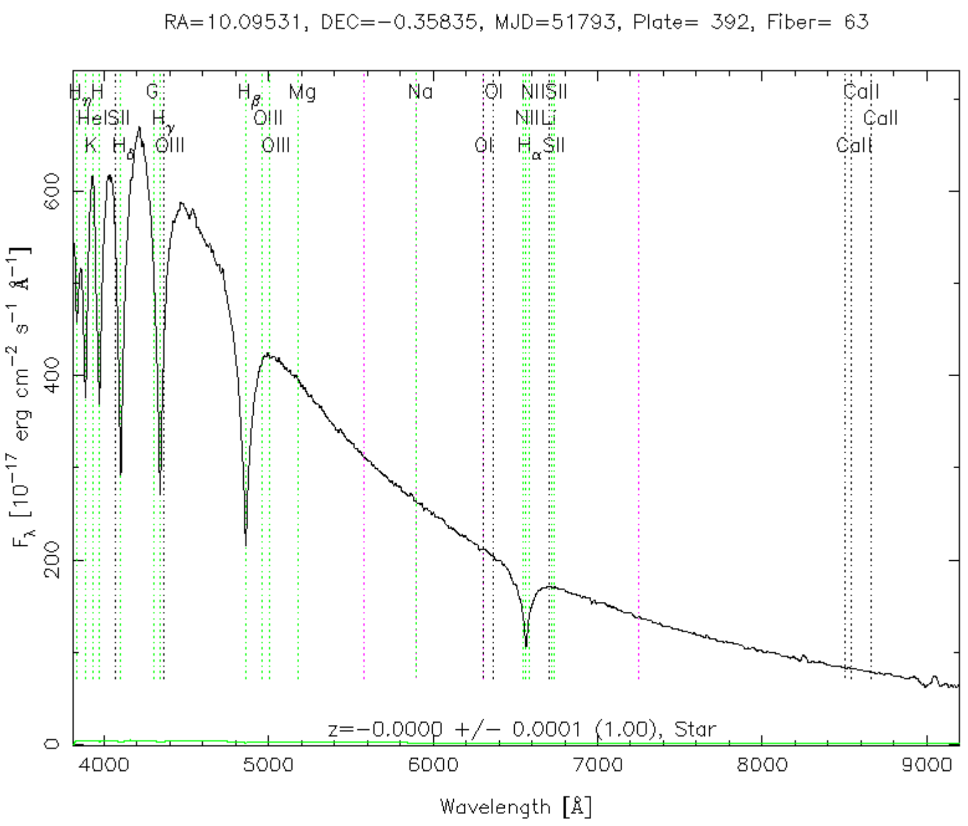
- The outcome of a star's death is decided by its initial mass
  - After fusing all available fuel:
    - Stars of very low mass (~0.5 – 10 $M_{\odot}$) eject material, become white dwarf
    - Stars of intermediate mass (~10 – 20 $M_{\odot}$) go supernova, become neutron star
    - Stars of high mass (~>20 $M_{\odot}$) go supernova, become black hole

White Dwarf Stars

- Roughly solar mass star with Earth-sized volume
- Composition decided by initial mass of progenitor star
  - Very low mass star cannot bulk-fuse helium to carbon/oxygen → Helium dominated
  - Higher mass progenitors hot enough to create C/O and potentially oxygen/neon/magnesium
- Held against gravitational collapse to 1st order by electron degeneracy pressure
- Observed magnetic fields strengths up to 30kT

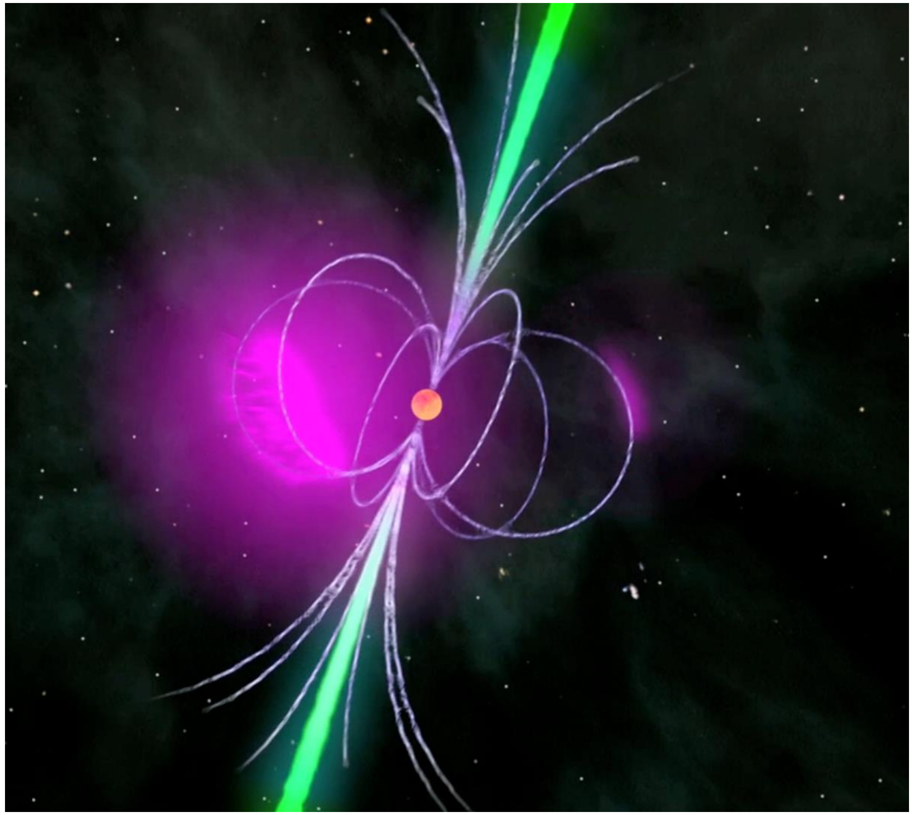
Observing White Dwarf Stars

- Heavy materials separated gravitationally from lighter elements
  - Creates H/He envelope
- Similar spectrum as H/He star, but surrounding nebula show traces of heavy elements
- Evidence of high-order Zeeman-splitting
- Binary systems may create novas



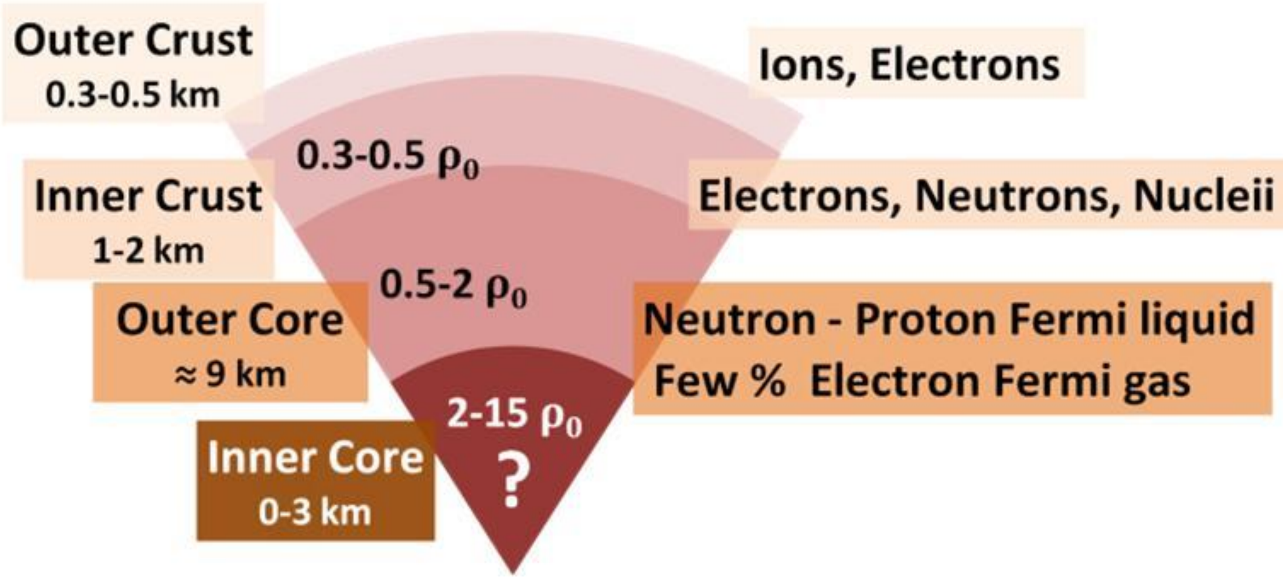
Neutron Stars

- Roughly solar mass star with radius ~10km
- Theoretical internal structure complex mixture of electron-captured neutrons and electrons
- Held against gravitational collapse by neutron degeneracy pressure
- Magnetic field strengths 1MT-1GT
  - Emits beamed synchrotron radiation
- Intense thermal x-ray and UV radiation



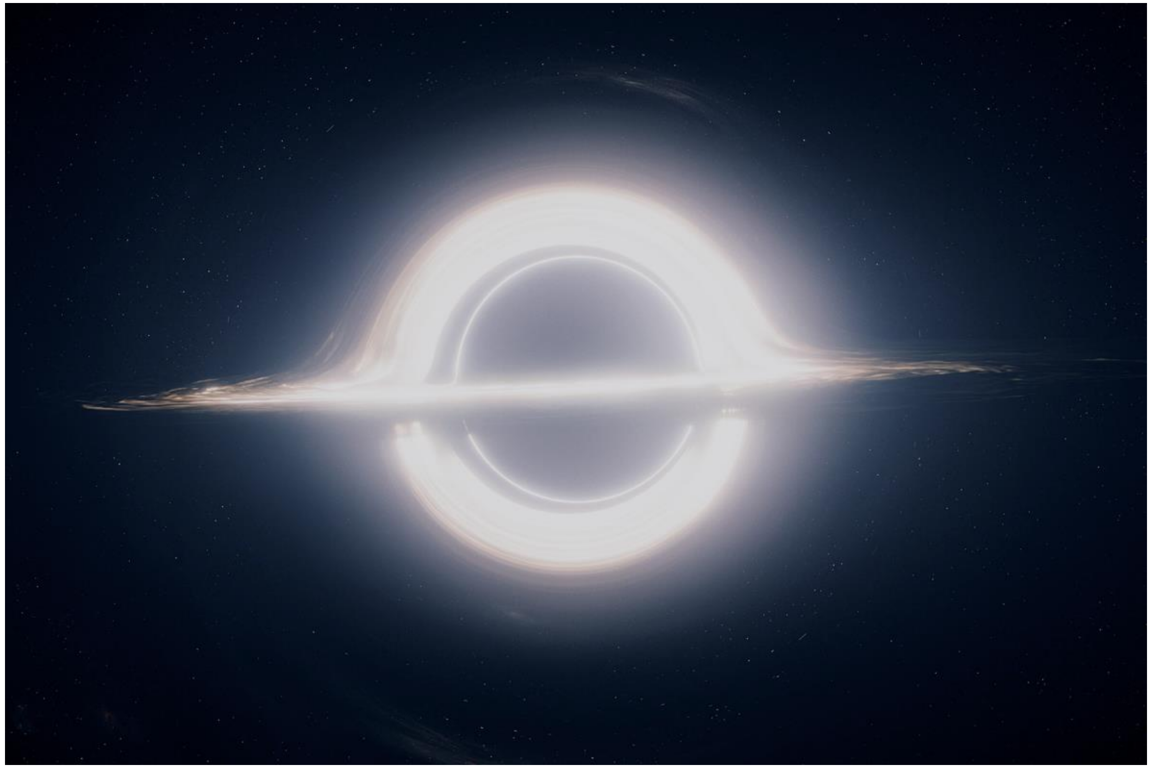
Internal Structure

- Astroseismology
  - Oscillatory modes resonate along and inside star
- Variability
- Magnetic field
  - Degree of Zeeman-splitting
  - Mechanisms for producing fields



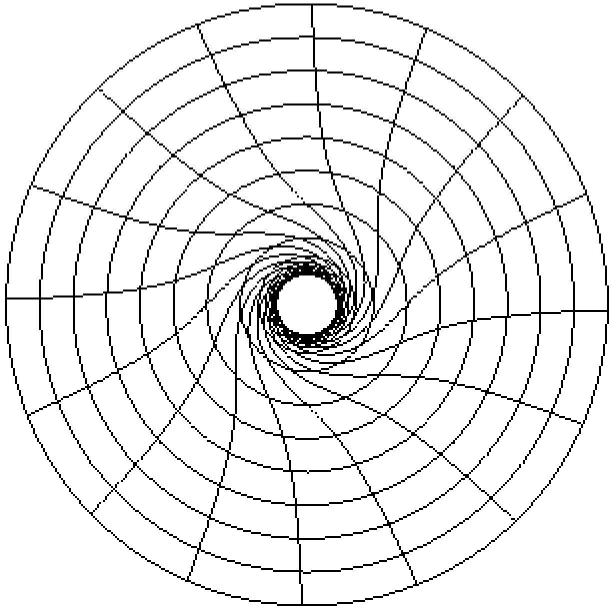
Black Holes

- Degeneracy pressure loses to gravitational collapse
  - Finite mass object in infinitesimally small volume
- Matter falling in tidally rips and creates accretion disk
  - Emits intense x-ray and UV
- Different variability from neutron stars
  - No emission from material hitting solid surface
- Can reach masses of ~10^8 $M_\odot$ through accretion feeding black hole



Stable Orbits

- For non-rotating bodies, the escape velocity exceeds the speed of light at the Schwarzschild Radius, $r_s = \frac{2Gm}{c^2}$
  - Event horizon skewed for rotating black holes
- In rotating black holes, frame-dragging creates a region outside event horizon of local velocities faster than light speed
- Inside the marginally stable orbit, $r_{ms} = \frac{6Gm}{c^2}$, free-particles are not able to maintain stable circular motion



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