movies, most people would opt for the movies if the weatherman said that the probability of rain tomorrow were 95%. Person X's decision about whether or not to ask person Y for a date can depend on the probability that person Y will answer yes. A skier's decision about whether to ski down a very steep chute will depend on what the skier thinks the probability is that she will crash and get hurt, and so on.

As noted in the previous chapter, social scientists tend to be professionally interested in probability because conclusions based on experiments can typically be stated only probabilistically. This is the principal reason that probability is discussed in books such as this one. However, since so many everyday events are based on probabilistic happenings, it is to everyone's advantage (whether interested in social science or not) to have a good idea of what probability is all about.

PROBLEMS

1. Suppose that we have a universal set W consisting of all people. Verbally describe the complements of the following sets.
   a. All people who are left-handed.
   b. All people who are left-handed and color-blind.
   c. All people who are left-handed or color-blind.
   d. All left-handed, color-blind females.
   e. All people who are left-handed or male but not color-blind.
   (*Hint:* Use "normal" to indicate "not color-blind".)

2. Classify the following sets as finite, countably infinite, or uncountably infinite.
   a. The set of all four-letter strings.
   b. The set of all-letter strings.
   c. The set of all amounts that a human being could weigh.
   d. The set of exact weights of 100 human beings.
   e. The set of all grains of sand in the Sahara Desert.
   f. The number of times a pair of dice could be thrown before double 6s turn up.

3. Consider W = all people. Given this universal set, make up examples of pairs of sets that are:
   a. Mutually exclusive and exhaustive.
   b. Mutually exclusive but not mutually exhaustive.
   c. Not mutually exclusive but mutually exhaustive.
   d. Neither mutually exclusive nor mutually exhaustive.

4. Let S be the set of all restaurants in Walla Walla, Washington. Assume S contains 96 elementary events. Define F as the set of all French restaurants and assume that F contains 32 elementary events. Define T as the set of

restaurants that require men to wear ties and jackets, and assume that T contains 48 elementary events. Thirty-two of the French restaurants require ties and jackets.

a. Represent the situation as a Venn diagram—fill in the appropriate numbers of elementary events.
b. Represent the situation as a contingency table. Now compute the following:

(1) $p(F)$	(6) $p(F \cap \overline{T})$	(11) $p(\overline{F}\vert\overline{T})$
(2) $p(T)$	(7) $p(\overline{F} \cup \overline{T})$	(12) $p(T\vert F)$
(3) $p(F \cap T)$	(8) $p(F\vert T)$	(13) $p(T\vert\overline{F})$
(4) $p(F \cup T)$	(9) $p(F\vert\overline{T})$	(14) $p(\overline{T}\vert F)$
(5) $p(\overline{F} \cap T)$	(10) $p(\overline{F}\vert T)$	(15) $p(\overline{T}\vert\overline{F})$

5. There are 84 passengers on an airliner. Define the following two events:

    F: The person is traveling first class.

    M: The person is male.

    Suppose that there are 32 first-class passengers and 30 males. Suppose further that $p(F \cup M) = \frac{52}{84}$.

    a. Construct a contingency table depicting the preceding information. Compute the following, based on your contingency table:
    b. $p(F|M)$
    c. $p(M|F)$
    d. $p(\overline{F}|\overline{M})$
    e. $p(\overline{M}|\overline{F})$
    f. $p(\overline{M} \cap \overline{F})$
    g. $p(\overline{F}|M)$
    h. $p(M|\overline{F})$
    i. $p(\overline{M}|F)$

6. Consider the set S of all gas stations in the country. The set G is the set of stations open and selling gas, and the set E is the set of all Exxon stations.

    Suppose that:

    $$p(G) = 0.4$$
    $$p(G \cup E) = 0.5$$
    $$p(E) = 0.2$$

    Represent this situation as a contingency table and then calculate
    a. $p(G \cap E)$
    b. $p(G|E)$
    c. $p(G|\overline{E})$
    d. $p(\overline{E}|G)$
    e. $p(E|\overline{G})$

7. Assume that there are 100 cars in East Bend, Idaho. Of these cars, 25 are Fords, 25 are Jaguars, and the remaining 50 are Volkswagens. Twenty of the

100 cars are blue. The two events "type of car" and "being blue" are independent. Suppose now that a random car is picked from East Bend.

a. Make a contingency table that represents this situation.
b. What is the probability that the car will be a blue Ford?
c. What is the probability that the car will be a Jaguar that is not blue?
d. What is the probability that the car will be blue and either a Jaguar or a Volkswagen?
e. What is the probability that the car will be neither blue nor a Volkswagen?

8. Assume that there is a set S of 96 people traveling on an airplane. Of these 96 people, 24 are in the first-class section (set F) and 32 are airline employees (set E). Four airline employees are in the first-class section.

Find the following probabilities:

a. $P(E \cup F)$
b. $P(F \mid \overline{E})$
c. $P(\overline{E} \mid F)$
d. $P(\overline{E} \mid \overline{F})$
e. $P(\overline{F} \mid E)$

9. In the town of Barlow, Alaska, there are three types of restaurants: American (A), French (F), and Greek (G). There are equal numbers of the three types of restaurants. Additionally, half the restaurants in town have a liquor license (L), whereas others do not ($\overline{L}$). We also know the following:

$$p(L \cap A) = \tfrac{1}{3}$$

$$p(\overline{L}|G) = 1$$

Now compute:

a. $p(F \mid L)$
b. $p(L \mid A)$
c. $p(L \mid \overline{F})$
d. $p(F \mid \overline{L})$
e. $p(\overline{A} \mid L)$

10. Suppose two dice, a black die and a white die, are thrown. Two outcomes are defined as follows:

$$A = \{\text{white die comes up a 5 or a 6}\}$$

$$B = \{\text{black die comes up a 6}\}$$

Compute the following.
(*Hint:* Make a contingency table.)

a. $p(A)$
b. $p(B)$
c. $p(A \cap B)$
d. $p(A \cup B)$
e. $p(A \mid B)$
f. $p(B \mid A)$

g. $p(B \mid \bar{A})$
h. $p(\bar{A} \mid B)$
i. $p(\bar{A} \mid \bar{B})$

Suppose we define a third event in addition to the first two:

$$C = \{\text{black die comes up a 1}\}$$

Compute the following:

j. $p(A \cap C)$
k. $p(B \cap C)$
l. $p[(B \cup C)|A]$

11. Suppose that we have two dice, die A and die B, and we throw them both together. Compute the following probabilities:
   a. At least one 6 is obtained.
   b. A double 6 is obtained.
   c. A 2 and a 1 are obtained.
   d. Die A turns up 6 and die B turns up 1.
   e. A "double" is obtained (double 1, double 2, and so on).
   f. The number on die A and the number on die B differ by 1.
   g. The number on die A is 1 greater than the number on die B.
   h. No 1s or 2s are obtained.

12. Suppose that I flip a coin and draw a card from a deck.
   a. What is the probability that the coin comes up heads *and* that I get a diamond?
   b. What is the probability that the coin comes up heads *or* I get a diamond, but not both?
   c. What is the probability that the coin comes up heads or tails or that I get a spade (or both)?
   d. What is the probability that the coin comes up heads or tails and that I get a spade?

13. Joe Smith is drinking beer after a baseball game. To decide how many beers to drink, he plans to use the following procedure. To decide on the first beer, he throws a die. If the die comes up any number from 1 to 5, he drinks a beer; otherwise he goes home. If he gets to drink his first beer, he decides on the second beer by throwing the die again when he finishes. This time if the die comes up 1, 2, 3, or 4 he drinks a second beer; otherwise he goes home. If he gets to drink his second beer, he decides on a third beer by throwing the die again when he finishes. This time, if the die comes up 1, 2, or 3, he drinks a third beer; otherwise he goes home. He will definitely go home after the third beer.

   What is the probability that Joe drinks:
   a. No beers—he just goes home?
   b. One beer and then he goes home?
   c. Two beers and then he goes home?
   d. Three beers?

14. Suppose that I teach a seminar with five students, three females and two males. I decide to pick one student randomly to lead this week's seminar and one student to lead next week's seminar. I do this by putting the five names into a hat, drawing out one name for this week and then, not replacing the name in the hat, drawing another name for next week. What are the following probabilities?
   a. This week's seminar is led by a female.
   b. This week's seminar is led by a male and next week's seminar is led by a female.
   c. This week's and next week's seminars are led by members of the same sex.
   d. At least one of the seminars is led by a female.

15. Consider two dice, die C and die D. When thrown together, the two dice are completely normal except for one strange thing. If die C comes up 4 (which, of course, happens with probability $\frac{1}{6}$), it casts a cosmic force field over die D, which in turns causes the numbers 1 through 6 on die D to have the following probability of occurrence.

$$p(1) = \tfrac{1}{2}$$
$$p(2) = \tfrac{1}{8}$$
$$p(3) = \tfrac{1}{8}$$
$$p(4) = \tfrac{1}{8}$$
$$p(5) = \tfrac{1}{16}$$
$$p(6) = \tfrac{1}{16}$$

Compute the probabilities of the following outcomes of throwing two dice:
   a. Die $C = 1$ and die $D = 5$.
   b. Die $C = 1$ or 2 or 3, and die $D = 1$ or 2.
   c. Die $C = 3$ or 4, and die $D = 1$.
   d. Die $C = 4$ and die $D = 3$.

16. A peculiar deck of cards has only red jacks. (The black suits have 11s instead of jacks.) Define:

H: as "draw a heart"

J: as "draw a jack"

Note that

$$p(H) = \tfrac{1}{4}$$

$$p(J) = \tfrac{2}{52} = \tfrac{1}{26}$$

I claim that drawing the jack of hearts is equivalent to the event $H \cap J$, and the probability of drawing the jack of hearts is

$$p(H \cap J) = p(H)p(J) = (\tfrac{1}{4})(\tfrac{1}{26}) = \tfrac{1}{104}$$

My friend, however, claims that since there are 52 cards and one jack of hearts, the probability of drawing the jack of hearts is $\frac{1}{52}$. Who is right? What error has been committed?

17. Joe Smith is interested in the appearance of unidentified flying objects (UFOs). Checking the records, he discovers that there have been many more reported UFO sightings between the hours of noon and 6 P.M. than between the hours of midnight and 6 A.M. He therefore concludes that UFO pilots prefer flying around in daylight rather than during the night. What is wrong with Joe's reasoning?

18. Al Jones is trying to decide on a career and is choosing between being a lion-tamer or a lumberjack. Al discovers that more lumberjacks than lion-tamers are killed every year. Therefore, Al decides to be a lion-tamer, concluding that he is less likely to get killed that way. Is this a reasonable thing for Al to do? Prove (by making up an example) that even though the fact noted above is correct, Al's probability of being killed can be *greater* if he is a lion-tamer than if he is a lumberjack. (*Hint:* Consider the universal set made up *only* of people who are lion-tamers or lumberjacks. You may assume that nobody is both a lion-tamer and a lumberjack.)

19. Show that if A and B are independent, $\bar{A}$ and $\bar{B}$ are also independent. [*Hint:* Remember that $(A - B)(C - D) = AC - BC - AD + BD$.]

20. Suppose that you have a sample space S and two events A and B. Suppose further that

$$p(A|B) > p(A)$$

Prove that $p(A|\bar{B}) < p(A)$. (*Hint:* This should make sense in intuitive terms. Using a familiar example, suppose that the unconditional probability of passing an exam is 0.75, but the conditional probability of passing given that you have done the reading is 0.85. It should then make sense that the probability of passing given that you *didn't* do the reading would be less than 0.75.)

21. Suppose that there are two events A and B and

$$p(A|B) = p(A)$$

Prove $p(A|\bar{B}) = p(A)$. (*Hint:* Do *not* try to prove this just by making up an example. Your proof must be for the general case.)