ECON 425
Topics in Monetary Economics:
The International Monetary System
from the Gold Standard to Globalization
Homework Assignment 5

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Due in class on Wednesday, May 29, 2019

This homework assignment requires you to answer some essay questions on the Bretton Woods system and work on a simple model of policy interactions.

Here are the essay questions (hint: Topic 5 slides):

1. Explain Robert Triffin's dilemma associated with the international liquidity problem of the Bretton Woods system.
2. What solutions where proposed for the Bretton Woods system's liquidity problem.
3. What were the Special Drawing Rights and what issues were associated with them?
4. What solutions were proposed for the confidence problem of the Bretton Woods system?
5. Explain the $n-1$ balance of payments problem studied by Robert Mundell in 1968.

And here is the model exercise:

1 Policy Interactions under Flexible Exchange Rates

Consider the following simplified version of the Ghironi-Giavazzi model we studied in class.

The world consists of two countries, called home and foreign. All variables (except interest rates) are in logs and measure deviations from zero-shock equilibrium levels. Foreign variables are denoted with an asterisk.

Output in each country (y for home, y^* for foreign) is a function of employment (n, n^*) and a worldwide productivity shock x:

$$y = (1-\alpha) n - x, \tag{1}$$

$$y^* = (1-\alpha) n^* - x, \tag{2}$$

where $0 < \alpha < 1$ and x is identically and independently distributed with zero mean.

Labor demand in each country is determined by optimality conditions for firm behavior that equate real wages to the marginal product of labor. In logs, these conditions are:

$$w - p = -\alpha n - x, \tag{3}$$

$$w^* - p^* = -\alpha n^* - x \tag{4}$$

where w and w^* are the nominal wages, and p and p^* are product prices.

Consumer price levels (q, q^*) are given by:

$$q = ap + (1-a)(e + p^*) = p + (1-a) z, \tag{5}$$

$$q^* = a(p - e) + (1-a) p^* = p^* - az, \tag{6}$$

where a is the share of spending on the home good by consumers in each country ($0 < a < 1$), e is the nominal exchange rate (units of home currency per unit of foreign), and z is the terms of trade, defined by $z \equiv e + p^* - p$ (units of the home good per unit of the foreign good).

Expenditure equilibrium conditions for the home and foreign goods are:

$$y = \delta (1-a) z + \varepsilon [ay + (1-a) y^*] - \nu [ar + (1-a) r^*], \tag{7}$$

$$y^* = -\delta az + \varepsilon [ay + (1-a) y^*] - \nu [ar + (1-a) r^*], \tag{8}$$

where $0 < \delta < 1$, $0 < \varepsilon < 1$, and $0 < \nu < 1$, and r and r^* are the *ex ante* real interest rates.

Denoting nominal interest rates with i and i^*, r and r^* are determined by:

$$r = i - (E(q_{+1}) - q), \tag{9}$$

$$r^* = i^* - \left(E\left(q^*_{+1}\right) - q^*\right), \tag{10}$$

where $E(q_{+1})$ $(E(q_{+1}^*))$ is the expected value of the home (foreign) CPI one period ahead based on the currently available information.

Optimal bond holding behavior in the two countries implies uncovered interest rate parity (UIP):

$$i - i^* = E(e_{+1}) - e. \tag{11}$$

- Proceeding as in class, use the *ex ante* real interest rate equations (9) and (10), UIP (11), the CPI equations (5) and (6), and the definition of the terms of trade $z \equiv e + p^* - p$ to show that $r = r^*$, so that equations (7) and (8) can be rewritten as:

$$y = \delta(1-a)z + \varepsilon[ay + (1-a)y^*] - \nu r, \tag{12}$$

$$y^* = -\delta az + \varepsilon[ay + (1-a)y^*] - \nu r. \tag{13}$$

Denoting money demand (equal to money supply in equilibrium) with m at home and m^* in the foreign country, money market equilibrium in each country requires:

$$m = p + y, \tag{14}$$

$$m^* = p^* + y^*. \tag{15}$$

Note: Relative to Ghironi-Giavazzi, we simplify the model by removing the effect of interest rates on money demand. The money market equilibrium conditions above are thus analogous to those in Eichengreen's model of policy interactions under the interwar Gold Standard.

- Proceeding as in class, show that prices and employment in each country are such that:

$$p = w + \alpha n + x, \tag{16}$$

$$p^* = w^* + \alpha n^* + x, \tag{17}$$

and

$$n = m - w, \tag{18}$$

$$n^* = m^* - w^*. \tag{19}$$

Assume that firms and workers in each country agree to wages set at the end of the previous period to minimize the expected squared deviation of employment from the zero shock equilibrium in each country. In other words, w is chosen to minimize $E_{-1}\left(n^2\right)/2$ and w^* is chosen to minimize $E_{-1}\left(n^{*2}\right)/2$, where E_{-1} denotes the expectation conditional on information available at the end of the previous period.

Assume that the exchange rate is flexible, and central banks use the respective money supplies as their policy instruments. Central banks choose money supplies to minimize loss functions that depend on the squared deviations of CPI inflation and employment from their zero shock levels. In other words, policymakers have no motive to move their money supplies other than responding to shocks:

$$L^{CB} = \frac{1}{2}\left[\gamma q^2 + (1-\gamma) n^2\right], \tag{20}$$

$$L^{CB*} = \frac{1}{2}\left[\gamma q^{*2} + (1-\gamma) n^{*2}\right], \tag{21}$$

where $0 < \gamma < 1$. Assume that central banks care more about inflation than employment, i.e. $\gamma > 1/2$.

- Proceeding as in class, show that the assumptions we are making imply that wage setting results in $w = w^* = 0$.

- Use a superscript D to denote the difference between home and foreign variables (for instance, $m^D \equiv m - m^*$). Use the money market equations (14) and (15), the expenditure equations (12) and (13), equations (16)-(19), and the result about wage setting above to show that the exchange rate is determined by:

$$e = \frac{1-\alpha(1-\delta)}{\delta} m^D.$$

- Why didn't we have to use the UIP equation (11) as part of exchange rate determination like we had done in class? (Hint: Think about the money market equations and compare the exchange rate solution above to the one we obtained in class.)

- Given the result about wage setting, our simplified model immediately gives us the reduced form solutions for home and foreign employment as $n = m$ and $n^* = m^*$. Next, you should find the reduced form solutions for q and q^* as functions of m, m^*, and x. (Hint: Note that the price equations (16) and (17), the results about wage setting, and the reduced forms for home and foreign employment immediately give you the reduced forms for p and p^*. The price equations (16) and (17), the results about wage setting, and the reduced forms for home and foreign employment also allow you to immediately have the reduced form solution for p^D as function of m^D. Given $z \equiv e + p^* - p = e - p^D$ and the solutions for e and p^D, you can easily find the solution for z. Then, you can use the solutions for p, p^*, and z in equations (5) and (6) to obtain the desired reduced form solutions for q and q^*. At this stage, use $m^D \equiv m - m^*$ to write q and q^* as functions of m, m^*, and x by appropriately collecting terms.) If you do things right, you should find:

$$q = \frac{\alpha\delta + (1-a)(1-\alpha)}{\delta} m - \frac{(1-a)(1-\alpha)}{\delta} m^* + x, \tag{22}$$

$$q^* = \frac{\alpha\delta + a(1-\alpha)}{\delta} m^* - \frac{a(1-\alpha)}{\delta} m + x. \tag{23}$$

Note that these equations can be rewritten as:

$$q = \alpha m + x + \frac{(1-a)(1-\alpha)}{\delta} m^D = p + (1-a) z,$$

$$q^* = \alpha m^* + x - \frac{a(1-\alpha)}{\delta} m^D = p^* - az.$$

If you cannot complete the derivation of (22) and (23), just take them as given in what follows.

- Given the reduced form solutions for CPI and employment in each country, find the first-order conditions for each central bank's optimal choice of its money supply under non-cooperation. For the home central bank, write the first-order condition with m as a function of m^* and x; for the foreign central bank, write it with m^* as a function of m and x.

- Comment on these reaction functions:

  - How do central banks respond to a shock $x > 0$? Why?

  - How does the home (foreign) central bank respond to foreign (home) policy? What is the intuition for your answer?

- Assume $a = 1/2$ (equal country size) and $x > 0$ and plot the central banks' reaction functions in a diagram with m^* on the horizontal axis and m on the vertical axis.

Continue to assume $a = 1/2$ for the rest of this exercise

- Solve the system of the two central banks' first-order conditions for the Nash equilibrium values of m and m^* as functions of x.

- Given the solutions for m and m^*, find the implied solutions for n, n^*, q, q^*, e, and z as functions of x.

Now assume that the central banks act cooperatively and jointly choose m and m^* to minimize:

$$\frac{1}{4}\left[\gamma q^2 + (1-\gamma) n^2\right] + \frac{1}{4}\left[\gamma q^{*2} + (1-\gamma) n^{*2}\right].$$

- Find the first-order conditions for the optimal choices of m and m^* under cooperation.

- If you write the first-order condition with respect to m as a reaction function for m as a function of m^* and x and the first-order condition with respect to m^* as a reaction function for m^* as a function of m and x, how do these reaction functions differ from those obtained under non-cooperation?

- What is the crucial difference in central bank behavior between cooperation and non-cooperation?

- Solve the system of the two first-order conditions under cooperation for the cooperative values of m and m^* as functions of x. How do these solutions differ from those under non-cooperation? What is the intuition for this difference?

- Find the implied solutions for n, n^*, q, q^*, e, and z as functions of x under cooperation and compare them to those under non-cooperation.

- Are central banks better off when they cooperate? Why?