# The Valuation Channel of External Adjustment

# Technical Appendix

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## A Model Details

### A.1 Households and Governments

Let $V_t^z$ ($V_t^{z^*}$) denote the nominal price of shares in home (foreign) firm $z$ ($z^*$) during period $t$, $D_t^z$ ($D_t^{z^*}$) denote nominal dividends issued by the firm, and $x_{t+1}^z$ ($x_{t+1}^{z^*}$) denote the representative household's holdings of shares in home firm $z$ (foreign firm $z^*$) entering period $t+1$. The budget constraint of the representative home household is:

$$\begin{aligned} & \int_0^a V_t^z x_{t+1}^z dz + \mathcal{E}_t \int_a^1 V_t^{z^*} x_{t+1}^{z^*} dz^* + P_t C_t + P_t T_t \\ = & \int_0^a \left(V_t^z + D_t^z\right) x_t^z dz + \mathcal{E}_t \int_a^1 \left(V_t^{z^*} + D_t^{z^*}\right) x_t^{z^*} dz^* + W_t L_t, \end{aligned} \tag{36}$$

where $T_t$ is lump-sum taxation, $\mathcal{E}_t$ is the nominal exchange rate (units of home currency per unit of foreign), and $W_t$ is the nominal wage. The foreign household's budget constraint is similar. Equation (1) follows from (36) by imposing the balanced budget constraint of the government ($T_t = G_t$, where $G_t$ is aggregate per capita home government spending), symmetry of firm behavior in equilibrium (implying equal share prices and dividends across firms in each country), dividing by the price level and denoting real variables by lower case letters, and using PPP and the following definitions:

$$\begin{aligned} \int_0^a x_{t+1}^z dz &= a x_{t+1}^z \equiv x_{t+1} = \text{ share of home equity held by the representative home household,} \\ \int_a^1 x_{t+1}^{z^*} dz^* &= (1-a)\, x_{t+1}^{z^*} \equiv x_{t+1}^* = \text{ share of foreign equity held by the representative home household.} \end{aligned}$$

Equation (2) follows similarly from the foreign household's budget constraint, defining:

$$\begin{aligned} \int_0^a x_{*t+1}^z dz &= a x_{*t+1}^z = x_{*t+1} = \text{ share of home equity held by the representative foreign household,} \\ \int_a^1 x_{*t+1}^{z^*} dz^* &= (1-a)\, x_{*t+1}^{z^*} = x_{*t+1}^* = \text{ share of foreign equity held by the representative foreign household.} \end{aligned}$$

### A.2 Firms

Home firm $z$ produces output with linear technology using labor as the only input:

$$Y_t^{Sz} = Z_t L_t^z,$$

where $Z_t$ is aggregate home productivity.

Home firm $z$ faces demand for its output given by:

$$Y_t^{Dz} = \left(\frac{p_t(z)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\omega} Y_t^W = \left(RP_t^z\right)^{-\theta} \left(RP_t\right)^{\theta-\omega} Y_t^W,$$

where $RP_t^z \equiv p_t(z)/P_t$ is the price of good $z$ in units of the world consumption basket, $RP_t \equiv P_{Ht}/P_t$ is the price of the home sub-basket of goods in units of the world consumption basket, and $Y_t^W$ is aggregate world demand of the consumption basket.

Firm profit maximization results in the pricing equation:

$$RP_t^z = \frac{\theta}{\theta - 1} \frac{w_t}{Z_t}.$$

Since $RP_t^z = RP_t$ at an optimum, labor demand is determined by

$$L_t^z = L_t = RP_t^{-\omega} \frac{Y_t^W}{Z_t}.$$

## B Model Solution

We solve the model by using the technique developed by Devereux and Sutherland (2011) and Tille and van Wincoop (2010). The technique combines a second-order approximation of the portfolio optimality conditions with first-order approximation of the rest of the model to obtain the optimal steady-state portfolio composition. Since we impose no cost of adjusting the net foreign asset position or other stationarity inducing device for transparency of results, there is no restriction in the model to pin down endogenously the steady-state level of overall net foreign assets. As customary in this situation, we assume an initial, symmetric steady state with zero net foreign assets. In this steady state, $RP = RP^* = 1$, and $y = y^* = L = L^* = 1$ by appropriate choice of $\chi$. With $nfa = 0$ and $R^D = 0$, it follows immediately that steady-state consumption is $C = C^* = 1 - G$. Income distribution is such that: $d = d^* = 1/\theta$ and $wL = w^*L^* = w = w^* = (\theta - 1)/\theta$. Hence, Euler equations for equity holdings imply $v = v^* = \beta/\left[(1-\beta)\theta\right]$.

Now, log-linearizing (10) around the symmetric steady state with zero net foreign assets, we have:

$$n\hat{f}a_{t+1} = \frac{1}{\beta} n\hat{f}a_t + \hat{\xi}_t + \frac{1-a}{1-G}\hat{y}_t^D - (1-a)\hat{C}_t^D - \frac{(1-a)G}{1-G}\hat{G}_t^D, \tag{37}$$

where:

$$\hat{\xi}_t \equiv \frac{\alpha}{\beta(1-G)}\hat{R}_t^D.$$

From equation (14), it follows that GDP and consumption differentials are related by:

$$\hat{y}_t^D = \frac{(1+\varphi)(\omega-1)}{\omega+\varphi}\hat{Z}_t^D - \frac{\varphi(\omega-1)}{\sigma(\omega+\varphi)}\hat{C}_t^D. \tag{38}$$

Hence, (37) becomes:

$$\begin{aligned} n\hat{f}a_{t+1} &= \frac{1}{\beta}n\hat{f}a_t + \hat{\xi}_t + \frac{(1-a)(1+\varphi)(\omega-1)}{(1-G)(\omega+\varphi)}\hat{Z}_t^D \\ &\quad -(1-a)\left[1+\frac{\varphi(\omega-1)}{\sigma(1-G)(\omega+\varphi)}\right]\hat{C}_t^D - \frac{(1-a)G}{1-G}\hat{G}_t^D. \end{aligned} \tag{39}$$

Log-linear versions of Euler equations for home and foreign consumption imply that the consumption differential is such that:

$$E_t\hat{C}_{t+1}^D = \hat{C}_t^D. \tag{40}$$

Since we are introducing no adjustment cost in net foreign assets or other stationarity-inducing device, the consumption differential is subject to familiar random walk behavior.

Equations (39) and (40) have solution:

$$\hat{C}_t^D = \eta_{C^D a}n\hat{f}a_t + \eta_{C^D Z^D}\hat{Z}_t^D + \eta_{C^D G^D}\hat{G}_t^D + \eta_{C^D \xi}\hat{\xi}_t, \tag{41}$$

$$n\hat{f}a_{t+1} = n\hat{f}a_t + \eta_{aZ^D}\hat{Z}_t^D + \eta_{aG^D}\hat{G}_t^D + \eta_{a\xi}\hat{\xi}_t, \tag{42}$$

where we guess that the elasticity of net foreign assets entering $t+1$ to net foreign assets at the start of period $t$ is $\eta_{aa} = 1$ because the model features no mechanism to generate stationarity of net foreign assets. (A convex cost of adjusting net foreign assets, or other stationarity-inducing devices, would pin down a unique, deterministic steady-state level of net foreign assets by making expected growth of the marginal utility of consumption a function of net foreign assets in the Euler equations. This would also imply $0 < \eta_{aa} < 1$, ensuring stationary net foreign asset dynamics in response to temporary shocks. See Ghironi, 2006, for more details.)

The elasticities $\eta$ in (41) and (42) can be obtained with the method of undetermined coefficients. This yields:

$$\begin{aligned}
\eta_{C^D a} &= \frac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{C^D Z^D} &= \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{C^D G^D} &= -\frac{\sigma(1-\beta)G(\omega+\varphi)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{C^D \xi} &= \frac{\sigma(1-\beta)(1-G)(\omega+\varphi)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{a Z^D} &= \frac{\beta(1-\phi_Z)(1-a)(\omega-1)(1+\varphi)}{(1-\beta\phi_Z)(1-G)(\omega+\varphi)}, \\
\eta_{a G^D} &= -\frac{\beta(1-\phi_G)G(1-a)}{(1-\beta\phi_G)(1-G)}, \\
\eta_{a\xi} &= \beta.
\end{aligned}$$

Now, log-linearizing the Euler equations for holdings of home and foreign shares in each country and taking the difference yields:

$$\hat{v}_t^D = E_t\left[\beta\hat{v}_{t+1}^D + (1-\beta)\hat{d}_{t+1}^D\right]. \tag{43}$$

Recall that income distribution with Dixit-Stiglitz preferences is such that dividends are a proportion $1/\theta$ of GDP in each country. Therefore, $d_t/d_t^* = y_t/y_t^*$, and equation (38) determines also the dividend differential in log-linear form. It follows that:

$$\hat{v}_t^D = \beta E_t\hat{v}_{t+1}^D + \frac{\phi_Z(1-\beta)(1+\varphi)(\omega-1)}{\omega+\varphi}\hat{Z}_t^D - \frac{\varphi(\omega-1)(1-\beta)}{\sigma(\omega+\varphi)}\hat{C}_t^D. \tag{44}$$

The solution for $\hat{v}_t^D$ then takes the form in equation (30). Substituting the guess (30) and its $t+1$ version into (44), using the solutions for $\hat{C}_t^D$ and $n\hat{f}a_{t+1}$, and applying the method of undetermined coefficients yields the elasticities in (30):

$$\begin{aligned}
\eta_{v^D a} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{v^D Z^D} &= \frac{(1-\beta)(1+\varphi)(\omega-1)[\sigma\phi_Z(1-G)(\omega+\varphi)-\varphi(\omega-1)(1-\phi_Z)]}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{v^D G^D} &= \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{v^D \xi} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}.
\end{aligned}$$

Given the solution for $\hat{C}_t^D$ and the log-linear equation for $\hat{d}_t^D$ implied by (38) and $\hat{d}_t^D = \hat{y}_t^D$, we also have the solution for $\hat{d}_t^D$:

$$\hat{d}_t^D = \eta_{d^D a} n\hat{f}a_t + \eta_{d^D Z^D}\hat{Z}_t^D + \eta_{d^D G^D}\hat{G}_t^D + \eta_{d^D \xi}\hat{\xi}_t, \tag{45}$$

with:

$$
\begin{aligned}
\eta_{d^D a} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} = \eta_{v^D a},\\
\eta_{d^D Z^D} &= \frac{(1+\varphi)(\omega-1)[\beta\varphi(\omega-1)(1-\phi_Z)+\sigma(1-\beta\phi_Z)(1-G)(\omega+\varphi)]}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]},\\
\eta_{d^D G^D} &= \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} = \eta_{v^D G^D},\\
\eta_{d^D \xi} &= -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]} = \eta_{v^D \xi}.
\end{aligned}
$$

The next step in solving for the steady-state portfolio consists of showing that the excess return $\hat{R}_t^D$ is a linear function of innovations to relative productivity and government spending. For this purpose, recall that

$$
\hat{R}_t^D = -\beta\hat{v}_t^D - (1-\beta)\hat{d}_t^D + \hat{v}_{t-1}^D = -\left[\beta\hat{v}_t^D + (1-\beta)\hat{d}_t^D\right] + \hat{v}_{t-1}^D.
$$

However, the following results hold:

$$
\begin{aligned}
\beta\eta_{v^D a} + (1-\beta)\eta_{d^D a} &= \eta_{v^D a} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]},\\
\beta\eta_{v^D Z^D} + (1-\beta)\eta_{d^D Z^D} &= \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]},\\
\beta\eta_{v^D G^D} + (1-\beta)\eta_{d^D G^D} &= \eta_{v^D G^D} = \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]},\\
\beta\eta_{v^D \xi} + (1-\beta)\eta_{d^D \xi} &= \eta_{v^D \xi} = -\frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}.
\end{aligned}
$$

Hence,

$$
\hat{R}_t^D = -\eta_{v^D a} n\hat{f}a_t - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\hat{Z}_t^D - \eta_{v^D G^D}\hat{G}_t^D - \eta_{v^D \xi}\hat{\xi}_t + \hat{v}_{t-1}^D.
$$

Or:

$$
\hat{R}_{t+1}^D = -\eta_{v^D a} n\hat{f}a_{t+1} - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\hat{Z}_{t+1}^D - \eta_{v^D G^D}\hat{G}_{t+1}^D - \eta_{v^D \xi}\hat{\xi}_{t+1} + \hat{v}_t^D.
$$

Taking (30) into account, it follows that:

$$
\begin{aligned}
\hat{R}_{t+1}^D &= -\eta_{v^D a} n\hat{f}a_{t+1} + \eta_{v^D a} n\hat{f}a_t - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\hat{Z}_{t+1}^D + \eta_{v^D Z^D}\hat{Z}_t^D\\
&\quad -\eta_{v^D G^D}\hat{G}_{t+1}^D + \eta_{v^D G^D}\hat{G}_t^D - \eta_{v^D \xi}\hat{\xi}_{t+1} + \eta_{v^D \xi}\hat{\xi}_t\\
&= -\eta_{v^D a}\left(n\hat{f}a_{t+1} - n\hat{f}a_t\right) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}\hat{Z}_{t+1}^D + \eta_{v^D Z^D}\hat{Z}_t^D\\
&\quad -\eta_{v^D G^D}\left(\hat{G}_{t+1}^D - \hat{G}_t^D\right) - \eta_{v^D \xi}\left(\hat{\xi}_{t+1} - \hat{\xi}_t\right).
\end{aligned}
$$

Now, use the solution for net foreign assets (42) and the assumptions

$$\hat{Z}_{t+1}^{D} = \phi_Z \hat{Z}_t^D + \varepsilon_{t+1}^{Z^D}, \quad \hat{G}_{t+1}^{D} = \phi_G \hat{G}_t^D + \varepsilon_{t+1}^{G^D}.$$

Then:

$$\begin{aligned}
\hat{R}_{t+1}^{D} &= -\eta_{v^D a}(\eta_{aZ^D}\hat{Z}_t^D + \eta_{aG^D}\hat{G}_t^D + \eta_{a\xi}\hat{\xi}_t) - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\varepsilon_{t+1}^{Z^D} \\
&\quad - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)\phi_Z}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\hat{Z}_t^D + \eta_{v^D Z^D}\hat{Z}_t^D \\
&\quad - \eta_{v^D G^D}\left[\varepsilon_{t+1}^{G^D} - (1-\phi_G)\hat{G}_t^D\right] - \eta_{v^D\xi}\left(\hat{\xi}_{t+1} - \hat{\xi}_t\right).
\end{aligned}$$

Next, straightforward algebra shows that:

$$\begin{aligned}
-\eta_{v^D a}\eta_{aZ^D} + \eta_{v^D Z^D} - \frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)\phi_Z}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]} &= 0, \\
-\eta_{v^D a}\eta_{aG^D} + \eta_{v^D G^D}(1-\phi_G) &= 0, \\
-\eta_{v^D a}\eta_{a\xi} + \eta_{v^D\xi} &= 0,
\end{aligned}$$

leaving us with:

$$\hat{R}_{t+1}^{D} = -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\varepsilon_{t+1}^{Z^D} - \eta_{v^D G^D}\varepsilon_{t+1}^{G^D} - \eta_{v^D\xi}\hat{\xi}_{t+1},$$

or:

$$\begin{aligned}
\hat{R}_{t+1}^{D} &= -\frac{\sigma(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\varepsilon_{t+1}^{Z^D} \\
&\quad - \frac{G\varphi(\omega-1)(1-\beta)}{(1-\beta\phi_G)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\varepsilon_{t+1}^{G^D} \\
&\quad + \frac{\varphi(\omega-1)(1-\beta)(1-G)}{(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\hat{\xi}_{t+1}.
\end{aligned}$$

But now recall that

$$\hat{\xi}_{t+1} \equiv \frac{\alpha}{\beta(1-G)}\hat{R}_{t+1}^{D}.$$

Hence, we can solve for the excess return $\hat{R}_{t+1}^{D}$ as:

$$\hat{R}_{t+1}^{D} = \eta_{R^D\varepsilon^{Z^D}}\varepsilon_{t+1}^{Z^D} + \eta_{R^D\varepsilon^{G^D}}\varepsilon_{t+1}^{G^D}, \tag{46}$$

with

$$\begin{aligned}
\eta_{R^D\varepsilon^{Z^D}} &= -\frac{\beta\sigma(1-a)(1-\beta)(\omega-1)(1+\varphi)(1-G)}{(1-\beta\phi_Z)\left\{\beta(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right] - \alpha\varphi(\omega-1)(1-\beta)\right\}}, \\
\eta_{R^D\varepsilon^{G^D}} &= -\frac{\beta G\varphi(1-a)(1-\beta)(\omega-1)}{(1-\beta\phi_G)\left\{\beta(1-a)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right] - \alpha\varphi(\omega-1)(1-\beta)\right\}}.
\end{aligned}$$

Note:

1. The solution for $\hat{R}^D_{t+1}$ is such that $E_t \hat{R}^D_{t+1} = 0$, as expected from Devereux and Sutherland (2011).

2. The elasticities $\eta_{R^D \varepsilon^{ZD}}$ and $\eta_{R^D \varepsilon^{GD}}$ depend on the steady-state portfolio holding $\alpha$, which we aim to solve for.

3. If labor supply is inelastic ($\varphi = 0$), $\eta_{R^D \varepsilon^{GD}} = 0$ and

$$\eta_{R^D \varepsilon^{ZD}} = -\frac{(1-\beta)(\omega - 1)}{(1-\beta\phi_Z)\,\omega}.$$

The excess return does not depend on government spending when labor supply is inelastic because government spending does not affect equilibrium profits in this case.

Now recall the no-arbitrage condition between home and foreign equity for home households:

$$E_t\left(C_{t+1}^{-\frac{1}{\sigma}} R_{t+1}\right) = E_t\left(C_{t+1}^{-\frac{1}{\sigma}} R^*_{t+1}\right).$$

A similar condition holds for foreign households:

$$E_t\left(C_{t+1}^{*-\frac{1}{\sigma}} R_{t+1}\right) = E_t\left(C_{t+1}^{*-\frac{1}{\sigma}} R^*_{t+1}\right).$$

Taking second-order approximations to these conditions and considering the difference of the resulting equations yields:

$$E_t\left(\hat{C}_{t+1}\hat{R}^D_{t+1}\right) - E_t\left(\hat{C}^*_{t+1}\hat{R}^D_{t+1}\right) = 0,$$

or:

$$E_t\left(\hat{C}^D_{t+1}\hat{R}^D_{t+1}\right) = 0. \tag{47}$$

Using (41) at $t+1$ and (46), this becomes:

$$E_t\left[\left(\eta_{C^D a} n\hat{f}a_{t+1} + \eta_{C^D Z^D}\hat{Z}^D_{t+1} + \eta_{C^D G^D}\hat{G}^D_{t+1} + \eta_{C^D \xi}\hat{\xi}_{t+1}\right)\left(\eta_{R^D \varepsilon^{ZD}}\varepsilon^{Z^D}_{t+1} + \eta_{R^D \varepsilon^{GD}}\varepsilon^{G^D}_{t+1}\right)\right] = 0,$$

or, using (42) and the assumptions on $\hat{Z}^D_{t+1}$ and $\hat{G}^D_{t+1}$:

$$E_t\left[\begin{array}{c} \eta_{C^D Z^D}\varepsilon^{Z^D}_{t+1}(\eta_{R^D \varepsilon^{ZD}}\varepsilon^{Z^D}_{t+1}) + \eta_{C^D G^D}\varepsilon^{G^D}_{t+1}(\eta_{R^D \varepsilon^{GD}}\varepsilon^{G^D}_{t+1}) \\ +\eta_{C^D \xi}\frac{\alpha}{\beta(1-G)}\left(\eta_{R^D \varepsilon^{ZD}}\varepsilon^{Z^D}_{t+1} + \eta_{R^D \varepsilon^{GD}}\varepsilon^{G^D}_{t+1}\right)^2 \end{array}\right] =$$

$$\eta_{C^D Z^D}\eta_{R^D \varepsilon^{ZD}}\sigma^2_{\varepsilon^{ZD}} + \eta_{C^D G^D}\eta_{R^D \varepsilon^{GD}}\sigma^2_{\varepsilon^{GD}} + \frac{\alpha}{\beta(1-G)}\eta_{C^D \xi}\eta^2_{R^D \varepsilon^{ZD}}\sigma^2_{\varepsilon^{ZD}} + \frac{\alpha}{\beta(1-G)}\eta_{C^D \xi}\eta^2_{R^D \varepsilon^{GD}}\sigma^2_{\varepsilon^{GD}} = 0, \tag{48}$$

where we used the assumption that $\varepsilon^{Z^D}_{t+1}$ is distributed independently from $\varepsilon^{G^D}_{t+1}$.

Substituting the solutions for the elasticities obtained above into (48) and rearranging yields the solution for the steady-state portfolio discussed in the main text:

$$\alpha = \frac{\beta(1-a)}{1-\beta}\left[1 - \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right].$$

## B.1 The Determinants of the Steady-State Portfolio

Define

$$\Omega \equiv \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}.$$

Then,

$$\alpha = \frac{\beta(1-a)}{1-\beta}(1-\Omega).$$

Assume $\omega > 1$ unless otherwise noted. The results below follow immediately from inspection of $\Omega$ and $\alpha$:

$$\frac{\partial\alpha}{\partial a} < 0, \quad \frac{\partial\alpha}{\partial\sigma} > 0, \quad \frac{\partial\alpha}{\partial\sigma^2_{\varepsilon GD}} < 0, \quad \frac{\partial\alpha}{\partial\phi_G} < 0, \quad \frac{\partial\alpha}{\partial\sigma^2_{\varepsilon ZD}} > 0, \quad \frac{\partial\alpha}{\partial\phi_Z} > 0.$$

The derivative of $\alpha$ with respect to $G$ is negatively proportional to the derivative of $G^2/(1-G)$. It is:

$$\frac{\partial G^2/(1-G)}{\partial G} = \frac{G(2-G)}{(1-G)^2} > 0,$$

since $0 \leq G < 1$. Hence, $\partial\alpha/\partial G < 0$.

The derivative of $\alpha$ with respect to $\beta$ is determined by:

$$\begin{aligned}\frac{\partial\alpha}{\partial\beta} &= (1-a)\left[\frac{\partial\beta/(1-\beta)}{\partial\beta}(1-\Omega) - \frac{\beta}{1-\beta}\frac{\partial\Omega}{\partial\beta}\right] \\ &= (1-a)\left[\frac{1-\Omega}{(1-\beta)^2} - \frac{\beta}{1-\beta}\frac{\partial\Omega}{\partial\beta}\right].\end{aligned}$$

Plausible parameter values imply $\Omega < 1$. Note also that the derivative of $\Omega$ with respect to $\beta$ is proportional to the derivative of $(1-\beta\phi_Z)^2/(1-\beta\phi_G)^2$. It is:

$$\frac{\partial(1-\beta\phi_Z)^2/(1-\beta\phi_G)^2}{\partial\beta} = -\frac{2(\phi_Z-\phi_G)[1-\beta\phi_Z(1-\beta\phi_G)]}{(1-\beta\phi_G)^4} \leq 0$$

under the plausible assumption $\phi_Z \geq \phi_G$. Therefore, under plausible assumptions on parameter values, $\partial\Omega/\partial\beta \leq 0$, and $\partial\alpha/\partial\beta > 0$.

The derivative of $\alpha$ with respect to $\varphi$ is negatively proportional to the derivative of $(\omega+\varphi)\varphi/(1+\varphi)^2$. It is:

$$\frac{\partial(\omega+\varphi)\varphi/(1+\varphi)^2}{\partial\varphi} = \frac{2\varphi-\omega(\varphi-1)}{(1+\varphi)^3},$$

which is positive if $2\varphi/(\varphi-1) > \omega$. This restriction is satisfied for plausible parameter values, implying that $\partial\alpha/\partial\varphi < 0$ for values of $\varphi$ that do not violate the restriction.

Finally, the derivative of $\alpha$ with respect to $\omega$ is negatively proportional to the derivative of $(\omega+\varphi)/(\omega-1)$. It is:

$$\frac{\partial(\omega+\varphi)/(\omega-1)}{\partial\omega} = -\frac{(1+\varphi)}{(\omega-1)^2} < 0.$$

Hence, $\partial\alpha/\partial\omega > 0$.

### B.2 The Labor Effort Differential

The solution for relative labor effort can be recovered easily from $\hat{L}_t^D = \hat{y}_t^D - T\hat{O}T_t - \hat{Z}_t^D$ by using $\hat{y}_t^D = \hat{d}_t^D$, the solution for $\hat{d}_t^D$ in (45), the log-linear version of (17):

$$T\hat{O}T_t = -\frac{1+\varphi}{\omega+\varphi}\hat{Z}_t^D + \frac{\varphi}{\sigma(\omega+\varphi)}\hat{C}_t^D,$$

and the solution for $\hat{C}_t^D$ in (41). Tedious but straightforward algebra yields:

$$\hat{L}_t^D = \eta_{L^D a} n\hat{f}a_t + \eta_{L^D Z^D}\hat{Z}_t^D + \eta_{L^D G^D}\hat{G}_t^D + \eta_{L^D \xi}\hat{\xi}_t,$$

with:

$$\begin{aligned}
\eta_{L^D a} &= -\frac{\omega\varphi(1-\beta)(1-G)}{\beta(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{L^D Z^D} &= \frac{\varphi(\omega-1)\left\{\begin{array}{c}\sigma(1-G)(\omega+\varphi)(1-\beta\phi_Z) \\ +(\omega-1)[\beta(1+\varphi)(1-\phi_Z)-(1-\beta\phi_Z)]-(1+\varphi)(1-\beta)\end{array}\right\}}{(1-\beta\phi_Z)(\omega+\varphi)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{L^D G^D} &= \frac{G\omega\varphi(1-\beta)}{(1-\beta\phi_G)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}, \\
\eta_{L^D \xi} &= -\frac{\omega\varphi(1-\beta)(1-G)}{(1-a)[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)]}.
\end{aligned}$$

# C Steady-State Equity Holdings

The symmetric steady state described above is such that $v = v^* = \beta/[(1-\beta)\theta]$. Given $\alpha \equiv v^* x^*$ and the solution for $\alpha$, it follows that $x^* = \alpha(1-\beta)\theta/\beta$, and $x_*^*$ is determined by $ax^* + (1-a)x_*^* = 1-a$. Next, combining $nfa = 0$ in steady state with $nfa \equiv v^*x^* - [(1-a)/a]vx_*$, $\alpha \equiv v^*x^*$, and the solution for $v = v^*$, we have $x_* = a\alpha(1-\beta)\theta/[(1-a)\beta]$, and $x$ is determined by $ax + (1-a)x_* = a$.

If $G = 0$ or $\varphi = 0$, these steady-state equity holdings reduce to:

$$\begin{aligned}
x^* &= (1-a)\theta, \quad x = a - (1-a)(\theta-1), \\
x_* &= a\theta, \quad x_*^* = 1 - a\theta. \qquad (49)
\end{aligned}$$

The smaller the share of income distributed as profit (the higher $\theta$), the smaller the share of home equity that home households should hold under this allocation, and the larger the share of foreign equity. Given $\theta > 1$, $x > 0$ if and only if $\theta < 1 + a/(1-a)$. If $a = 1/2$ (symmetric country size), the planner's equity allocation implies going short in domestic equity whenever $\theta > 2$ (that is, whenever less than half of income is distributed as profit). The planner's allocation always requires holding a positive amount of foreign equity ($\theta/2$ if $a = 1/2$).

# D Productivity Insurance

To verify that the constant portfolio $\alpha = \beta(1-a)/(1-\beta)$ (or the constant equity holdings in (49)) provide perfect insurance against productivity shocks, observe that, using equity market equilibrium and the proportionality of dividends and labor incomes to GDP, we can write the difference between the home and foreign budget constraints (1) and (2) as:

$$\begin{aligned}
&\frac{v_t}{1-a}(x_{t+1}-x_t) + \frac{v_t^*}{1-a}(x_{t+1}^*-x_t^*) + C_t^D + G_t^D \\
= &\left[\left(\frac{x_t}{1-a} - \frac{a}{1-a}\right)\frac{1}{\theta} + \frac{\theta-1}{\theta}\right] y_t + \left[\left(\frac{x_t^*}{1-a} - 1\right)\frac{1}{\theta} - \frac{\theta-1}{\theta}\right] y_t^*.
\end{aligned}$$

Straightforward substitutions show that $x_{t+1} = x_t = a - (1-a)(\theta-1)$ and $x_{t+1}^* = x_t^* = (1-a)\theta$ (that is, the equity allocation in (49)) imply $C_t^D = 0$ for every possible realization of $y_t$ and $y_t^*$ (that is, for every possible realization of $Z_t$ and $Z_t^*$) if $G_t^D = 0$. Thus, (49) is the allocation of equity that ensures perfect risk sharing in response to productivity shocks. As we showed in the main text, this is the allocation of equity chosen by households if labor supply is inelastic and/or steady-state government spending is zero.

# E Obtaining Equation (31)

First-differencing (30) and using the lagged version of (42) yields:

$$\begin{aligned}
\Delta\hat{v}_t^D &= \eta_{v^D a}\left(\eta_{aZ^D}\hat{Z}_{t-1}^D + \eta_{aG^D}\hat{G}_{t-1}^D + \eta_{a\xi}\hat{\xi}_{t-1}\right) + \eta_{v^D Z^D}\Delta\hat{Z}_t^D + \eta_{v^D G^D}\Delta\hat{G}_t^D + \eta_{v^D\xi}\Delta\hat{\xi}_t \\
&= \eta_{v^D Z^D}\hat{Z}_t^D + \eta_{v^D G^D}\hat{G}_t^D + \eta_{v^D\xi}\hat{\xi}_t - \left(\eta_{v^D Z^D} - \eta_{v^D a}\eta_{aZ^D}\right)\hat{Z}_{t-1}^D \\
&\quad - \left(\eta_{v^D G^D} - \eta_{v^D a}\eta_{aG^D}\right)\hat{G}_{t-1}^D - \left(\eta_{v^D\xi} - \eta_{v^D a}\eta_{a\xi}\right)\hat{\xi}_{t-1}. \qquad (50)
\end{aligned}$$

It is possible to verify that the following equalities hold:

$$\begin{aligned}
\eta_{v^D\xi} &= \eta_{v^D a}\eta_{a\xi}, \\
\eta_{v^D G^D}(1-\phi_G) &= \eta_{v^D a}\eta_{aG^D}, \\
\eta_{v^D Z^D} - \eta_{v^D a}\eta_{aZ^D} &= \frac{(1-\beta)(\omega-1)(1+\varphi)(1-G)\sigma\phi_Z}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}.
\end{aligned}$$

Using these results and our assumption on the relative government spending process, we can rewrite (50) as:

$$\Delta\hat{v}_t^D = \eta_{v^D Z^D}\hat{Z}_t^D - \frac{(1-\beta)(\omega-1)(1+\varphi)(1-G)\,\sigma\phi_Z}{(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\hat{Z}_{t-1}^D + \eta_{v^D G^D}\varepsilon_t^{G^D} + \eta_{v^D\xi}\hat{\xi}_t. \quad (51)$$

Next, note that (33) implies:

$$\hat{\xi}_t = \frac{\eta_{R^D\varepsilon^{ZD}}\alpha}{\beta(1-G)}\varepsilon_t^{Z^D} + \frac{\eta_{R^D\varepsilon^{GD}}\alpha}{\beta(1-G)}\varepsilon_t^{G^D}. \quad (52)$$

Using this result and the assumption on the relative productivity process, equation (51) becomes:

$$\begin{aligned}\Delta\hat{v}_t^D &= \left[\eta_{v^D Z^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{ZD}}\frac{\alpha}{\beta(1-G)}\right]\varepsilon_t^{Z^D} \\ &\quad - \frac{(1-\beta)(1+\varphi)(\omega-1)\,\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)}\hat{Z}_{t-1}^D + \left[\eta_{v^D G^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{GD}}\frac{\alpha}{\beta(1-G)}\right]\varepsilon_t^{G^D},\end{aligned}$$

that is, equation (31), where

$$\eta_{\Delta v^D\varepsilon^{ZD}} \equiv \eta_{v^D Z^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{ZD}}\frac{\alpha}{\beta(1-G)} \quad \text{and} \quad \eta_{\Delta v^D\varepsilon^{GD}} \equiv \eta_{v^D G^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{GD}}\frac{\alpha}{\beta(1-G)}.$$

Finally, substituting the expressions for $\alpha$ and the elasticities $\eta$'s obtained above in the definitions of $\eta_{\Delta v^D\varepsilon^{ZD}}$ and $\eta_{\Delta v^D\varepsilon^{GD}}$ yields:

$$\eta_{\Delta v^D\varepsilon^{ZD}} = \frac{(1-\beta)(1+\varphi)(\omega-1)\left[\sigma\phi_Z(1-G)(\omega+\varphi) - \frac{\varphi^2G^2(\omega+\varphi)(1-\phi_Z)(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}{(\omega+\varphi)(1-\beta\phi_Z)\left[\sigma(1-G)(\omega+\varphi) + \frac{\varphi^2G^2(\omega+\varphi)(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}$$

and

$$\eta_{\Delta v^D\varepsilon^{GD}} = \frac{\sigma\varphi(1-\beta)(\omega-1)}{(1-\beta\phi_G)\left[\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)\right]}\left\{1 - \frac{1}{1 - \frac{\sigma(1-G)(\omega+\varphi)+\varphi(\omega-1)}{\varphi(\omega-1)\left[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}}\right\}.$$

# F Obtaining Equation (28)

Given equations (23) and (31), and the solution for net foreign assets in (42), we can obtain the solution for portfolio rebalancing from:

$$\Delta\hat{x}_{t+1}^D = \Delta\hat{v}_t^D + \frac{1-G}{\alpha}\Delta n\hat{f}a_{t+1}.$$

It is:

$$\begin{aligned}\Delta\hat{x}^D_{t+1} &= \left[\eta_{v^D Z^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{ZD}}\frac{\alpha}{\beta(1-G)}\right]\varepsilon^{Z^D}_t - \frac{(1-\beta)(1+\varphi)(\omega-1)\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)}\hat{Z}^D_{t-1} \\ &+ \left[\eta_{v^D G^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{GD}}\frac{\alpha}{\beta(1-G)}\right]\varepsilon^{G^D}_t \\ &+ \frac{1-G}{\alpha}\left[\eta_{aZ^D}\hat{Z}^D_t + \eta_{aG^D}\hat{G}^D_t + \eta_{a\xi}\left(\frac{\eta_{R^D\varepsilon^{ZD}}\alpha}{\beta(1-G)}\varepsilon^{Z^D}_t + \frac{\eta_{R^D\varepsilon^{GD}}\alpha}{\beta(1-G)}\varepsilon^{G^D}_t\right)\right],\end{aligned}$$

where we used (52). Using $\eta_{a\xi} = \beta$ and rearranging this equation yields:

$$\Delta\hat{x}^D_{t+1} = \eta_{\Delta x^D\varepsilon^{ZD}}\varepsilon^{Z^D}_t + \eta_{\Delta x^D Z^D_{-1}}\hat{Z}^D_{t-1} + \eta_{\Delta x^D\varepsilon^{GD}}\varepsilon^{G^D}_t + \eta_{\Delta x^D G^D_{-1}}\hat{G}^D_{t-1},$$

where:

$$\begin{aligned}\eta_{\Delta x^D\varepsilon^{ZD}} &\equiv \eta_{v^D Z^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{ZD}}\frac{\alpha}{\beta(1-G)} + \frac{(1-G)\eta_{aZ^D}}{\alpha} + \eta_{R^D\varepsilon^{ZD}}, \\ \eta_{\Delta x^D Z^D_{-1}} &\equiv \frac{(1-G)\eta_{aZ^D}\phi_Z}{\alpha} - \frac{(1-\beta)(1+\varphi)(\omega-1)\phi_Z(1-\phi_Z)}{(\omega+\varphi)(1-\beta\phi_Z)}, \\ \eta_{\Delta x^D\varepsilon^{GD}} &\equiv \eta_{v^D G^D} + \eta_{v^D\xi}\eta_{R^D\varepsilon^{GD}}\frac{\alpha}{\beta(1-G)} + \frac{(1-G)\eta_{aG^D}}{\alpha} + \eta_{R^D\varepsilon^{GD}}, \\ \eta_{\Delta x^D G^D_{-1}} &\equiv \frac{(1-G)\eta_{aG^D}\phi_G}{\alpha}.\end{aligned}$$

Tedious algebra shows that:

$$\eta_{\Delta x^D Z^D_{-1}} = \phi_Z\eta_{\Delta x^D\varepsilon^{ZD}} \quad \text{and} \quad \eta_{\Delta x^D G^D_{-1}} = \phi_G\eta_{\Delta x^D\varepsilon^{GD}},$$

with:

$$\begin{aligned}\eta_{\Delta x^D\varepsilon^{ZD}} &= \frac{\beta(1-a)(1-\phi_Z)(\omega-1)(1+\varphi)}{\alpha(1-\beta\phi_Z)(\omega+\varphi)}\left[1 - \frac{(1-\beta)\alpha}{\beta(1-a)}\right], \\ \eta_{\Delta x^D\varepsilon^{GD}} &= -\frac{G(1-\phi_G)\beta(1-a)}{\alpha(1-\beta\phi_G)}.\end{aligned}$$

Hence,

$$\begin{aligned}\Delta\hat{x}^D_{t+1} &= \eta_{\Delta x^D\varepsilon^{ZD}}\left(\varepsilon^{Z^D}_t + \phi_Z\hat{Z}^D_{t-1}\right) + \eta_{\Delta x^D\varepsilon^{GD}}\left(\varepsilon^{G^D}_t + \phi_G\hat{G}^D_{t-1}\right) \\ &= \eta_{\Delta x^D Z^D}\hat{Z}^D_t + \eta_{\Delta x^D G^D}\hat{G}^D_t,\end{aligned}$$

that is, equation (28), where we conveniently redefined $\eta_{\Delta x^D Z^D} \equiv \eta_{\Delta x^D\varepsilon^{ZD}}$ and $\eta_{\Delta x^D G^D} \equiv \eta_{\Delta x^D\varepsilon^{GD}}$.

Finally, substituting the expressions for $\alpha$ and the elasticities $\eta$'s obtained above in the definitions of $\eta_{\Delta x^D Z^D}$ and $\eta_{\Delta x^D G^D}$ yields:

$$\eta_{\Delta x^D Z^D} = \frac{(1-\phi_Z)(1-\beta)(1+\varphi)(\omega-1)}{(\omega+\varphi)(1-\beta\phi_Z)} \left[ \frac{\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}}{1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}} \right]$$

and

$$\eta_{\Delta x^D G^D} = -\frac{G(1-\phi_G)(1-\beta)}{(1-\beta\phi_G)\left[1-\frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}\right]}.$$

# G The Determinants of the Valuation Share

## G.1 Productivity Shocks

We begin by studying the determinants of the valuation share for periods that follow the impact period of a shock, that is, $v\hat{a}l^S_{t\geq 1}$.

Recall the definition of $\Omega$ in the steady-state portfolio $\alpha$ obtained in Appendix B:

$$\Omega \equiv \frac{G^2(\omega+\varphi)\varphi(1-\beta\phi_Z)^2\sigma^2_{\varepsilon GD}}{\sigma(\omega-1)(1+\varphi)^2(1-G)(1-\beta\phi_G)^2\sigma^2_{\varepsilon ZD}}.$$

Then,

$$\eta_{\Delta x^D Z^D} = \frac{(1-\phi_Z)(1-\beta)(1+\varphi)(\omega-1)\Omega}{(\omega+\varphi)(1-\beta\phi_Z)(1-\Omega)},$$

and it is straightforward to verify that:

$$v\hat{a}l^S_{t\geq 1} = 1-\Omega = \frac{(1-\beta)\alpha}{\beta(1-a)}.$$

Assuming $\omega > 1$ throughout, the results on the steady-state portfolio in Appendix B imply that:

$$\frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\sigma} > 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\sigma^2_{\varepsilon GD}} < 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\phi_G} < 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\sigma^2_{\varepsilon ZD}} > 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\phi_Z} > 0.$$

$$\frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial G} < 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\beta} \geq 0, \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\varphi} < 0, \quad \text{and} \quad \frac{\partial v\hat{a}l^S_{t\geq 1}}{\partial\omega} > 0,$$

where the last three results hold for plausible parameter values.

Next, we prove that the share of valuation in net foreign asset adjustment in the impact period ($v\hat{a}l^S_0$) is smaller if substitutability between home and foreign goods ($\omega$) rises.

Recall that $v\hat{a}l_0^S = \left(1 - \eta_{\Delta x^D Z^D} / \eta_{\Delta v^D \varepsilon Z D}\right)^{-1}$. Using the definition of $\Omega$ and the expressions for $\eta_{\Delta x^D Z^D}$ and $\eta_{\Delta v^D \varepsilon Z D}$, we can write:

$$\frac{\eta_{\Delta x^D Z^D}}{\eta_{\Delta v^D \varepsilon Z D}} = \Gamma\Lambda,$$

where:

$$\begin{aligned}
\Gamma &\equiv \frac{\Omega\left(1 - \phi_Z\right)}{\left(1 - \Omega\right)\phi_Z}, \\
\Lambda &\equiv \frac{\sigma\left(1 - G\right)\left(\omega + \varphi\right) + \varphi\left(\omega - 1\right)\Omega}{\sigma\left(1 - G\right)\left(\omega + \varphi\right) - \varphi\left(\omega - 1\right)\Omega\frac{1-\phi_Z}{\phi_Z}}.
\end{aligned}$$

Tedious algebra shows that $\partial\Lambda/\partial\omega = 0$. (To verify this, we use the result from Appendix B that the derivative of $\Omega$ with respect to $\omega$ is proportional to $-\left(1 + \varphi\right) / \left(\omega - 1\right)^2$, and, in particular, $\partial\Omega/\partial\omega = -\Omega\left(1 + \varphi\right) / \left[\left(\omega - 1\right)\left(\omega + \varphi\right)\right]$.) It follows that:

$$\begin{aligned}
\frac{\partial\frac{\eta_{\Delta x^D Z^D}}{\eta_{\Delta v^D \varepsilon Z D}}}{\partial\omega} &= \Lambda\frac{\partial\Gamma}{\partial\omega} = \Lambda\frac{\left(1 - \phi_Z\right)}{\phi_Z\left(1 - \Omega\right)^2}\frac{\partial\Omega}{\partial\omega} \\
&= -\frac{\Lambda\left(1 - \phi_Z\right)\Omega\left(1 + \varphi\right)}{\phi_Z\left(1 - \Omega\right)^2\left(\omega - 1\right)\left(\omega + \varphi\right)}.
\end{aligned}$$

Hence, assuming $\omega > 1$ and $\Omega \neq 1$, $\Lambda > 0$ is necessary and sufficient for $\partial\left(\eta_{\Delta x^D Z^D} / \eta_{\Delta v^D \varepsilon Z D}\right) / \partial\omega \leq 0$. Given $\omega > 1$, the condition $\Lambda > 0$ is satisfied if and only if:

$$\frac{\sigma\left(1 - G\right)\left(\omega + \varphi\right)\phi_Z}{\varphi\left(\omega - 1\right)\left(1 - \phi_Z\right)} > \Omega.$$

This holds for plausible parameter values (for instance, with the parameters in our numerical exercise, $\Omega = .084$ and the left-hand side of the inequality is equal to 11.4). Hence, for parameters in a plausible range, $\partial\left(\eta_{\Delta x^D Z^D} / \eta_{\Delta v^D \varepsilon Z D}\right) / \partial\omega \leq 0$. Therefore, $\partial v\hat{a}l_0^S / \partial\omega \leq 0$.

## G.2 Government Spending Shocks

The share of valuation in net foreign adjustment to government spending shocks is zero in all periods but the impact one. We verify here that the share in the impact period is an increasing function of substitutability between home and foreign goods ($\omega$).

Recall that, in response to a relative government spending shock, $v\hat{a}l_0^S = \left(1 - \eta_{\Delta x^D G^D} / \eta_{\Delta v^D \varepsilon G D}\right)^{-1}$. Using the definition of $\Omega$ and the expressions for $\eta_{\Delta x^D G^D}$ and $\eta_{\Delta v^D \varepsilon G D}$, we can write:

$$v\hat{a}l_0^S = \frac{G\left(1 - \phi_G\right)\left(1 - \Psi\right)}{\sigma},$$

where:

$$\Psi \equiv \frac{\sigma\left(1 - G\right)\left(\omega + \varphi\right) + \varphi\left(\omega - 1\right)}{\varphi\left(1 - \Omega\right)\left(\omega - 1\right)}.$$

Tedious but straightforward algebra shows that:

$$\frac{\partial \Psi}{\partial \omega} = -\frac{\sigma\left(1-G\right)\varphi\left(1-\Omega\right)\left(1+\varphi\right) + \frac{\varphi\Omega(1+\varphi)}{\omega+\varphi}\left[\sigma\left(1-G\right)\left(\omega+\varphi\right)+\varphi\left(\omega-1\right)\right]}{\left[\varphi\left(1-\Omega\right)\left(\omega-1\right)\right]^2}.$$

The assumptions $\omega > 1$ and $\Omega < 1$ (satisfied for all parameter values we experimented with) are sufficient for $\partial\Psi/\partial\omega < 0$. It follows that $\partial\left(\eta_{\Delta x^D G^D}/\eta_{\Delta v^D \varepsilon^{GD}}\right)/\partial\omega > 0$ and, therefore, $\partial v\hat{a}l_0^S/\partial\omega > 0$ when we consider adjustment to relative government spending shocks.

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