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Macroeconomic interdependence under incomplete markets

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Abstract

This paper uses a two-country, flexible-price model with overlapping generations of infinitely lived households to study the role of net foreign asset dynamics in the propagation of productivity shocks. Absence of Ricardian equivalence ensures existence of a unique steady-state level of net foreign assets, to which the economy returns following temporary shocks. Model dynamics are significantly different from those of a setup in which terms of trade movements perform all the international adjustment and net foreign assets do not move. The difference relative to a complete markets economy in which net foreign asset movements play no role in shock transmission is smaller. It is amplified if the substitutability across goods rises and if shocks are permanent.

Keywords: Current account; Incomplete markets; Net foreign assets; Ricardian equivalence; Stationarity

JEL classification: F41; G15

1. Introduction

This paper uses a two-country, flexible-price model with overlapping generations of infinitely lived households and incomplete asset markets to study the role of net foreign asset dynamics in the propagation of productivity shocks.

Changes in net foreign assets play a role in the international transmission of shocks in representative agent, open economy models with incomplete asset markets through the history

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dependence of the equilibrium allocation encoded in a country's asset stock.[1] For instance, the current account plays a central role in Obstfeld and Rogoff's (1995a) model that sparked much literature in recent years. But the inability of the model to pin down a unique, endogenously determined steady state and the implied non-stationarity of the framework caused much of the subsequent literature to de-emphasize changes in net foreign assets as an important mechanism for the propagation of shocks across countries and over time.[2]

Determinacy of the (non-stochastic) steady state and stationarity fail in incomplete market models that do not address the issue in some way because consumption growth does not depend on net foreign assets in the Euler equation for bond holdings. Hence, setting consumption to be constant does not pin down steady-state assets.[3] This makes the choice of the economy's initial position for the purpose of analyzing the consequences of shocks a matter of convenience, with unfavorable consequences for the results of standard log-linearization.[4]

A possible way of addressing the problem is to assume that the elasticity of substitution between domestic and foreign goods in consumption is equal to one and the initial net foreign asset position is zero. These are the central assumptions in Corsetti and Pesenti's (2001) rendition of the Obstfeld-Rogoff model, building on insights in Cole and Obstfeld (1991). Under these assumptions (henceforth, the CO-CP model), the current account does not react to shocks, and thus it plays no role in the international business cycle. The dynamics of the terms of trade are the centerpiece of international adjustment.

Nevertheless, the steady state remains indeterminate in the CO-CP model. The choice of a zero-asset initial equilibrium, combined with the assumption on the elasticity of substitution between domestic and foreign goods, de facto shuts off the current account channel. This yields a highly tractable framework at a cost in terms of realism. Any initial position that differs from zero assets brings model non-stationarity back to the surface. In addition, the trade literature abounds with estimates significantly above one for the elasticity in question (for instance, see Lai and Trefler, 2002, and references therein).

An alternative way of dealing with the non-stationarity problem by de-emphasizing the role of net foreign asset dynamics consists of assuming that financial markets are internationally complete. With complete markets, power utility, and unitary elasticity of substitution between domestic and foreign goods, the current account does not react to shocks in two-country models with zero initial net wealth that are popular in the literature. If the elasticity of substitution between domestic and foreign goods differs from one, the current account moves in response to output differences (even though perfect risk sharing ensures that the cross-country consumption differential is zero if purchasing power parity holds). However, history independence of the equilibrium allocation ensures that net foreign assets are determined residually and their dynamics play no active role in shock transmission. Like the CO-CP specification, market completeness yields highly tractable models suitable for stochastic analysis at a cost in terms of realism.[5]

[1] See Ljungqvist and Sargent (2004) on the properties of incomplete market economies.

[2] All shocks (including temporary ones) have permanent consequences on the consumption differential between countries in Obstfeld and Rogoff's model. Asset holdings change permanently to a new level, which depends on the initial one and becomes the new steady state until the next shock happens.

[3] This result dates back at least to Becker (1980).

[4] As Schmitt-Grohé and Uribe (2003) point out, in a stochastic environment, the unconditional variances of endogenous variables are infinite, even if exogenous shocks are bounded. In such an environment, one is left wondering about the sustainability of foreign debt.

[5] As pointed out in Obstfeld and Rogoff (2001), the complete markets assumption is at odds with empirical evidence. Several other studies have pointed out that market incompleteness is necessary to explain important puzzles in international finance.

In a recent article, Schmitt-Grohé and Uribe (2003) – henceforth, SGU – compare alternative solutions to the non-stationarity issue that rely on representative agent models while preserving a role for net foreign asset dynamics in the transmission of shocks.[6] The model of this paper follows Weil (1989a,b) in assuming that the world economy is populated by distinct, infinitely lived households that come into being on different dates and are born owning no assets. The departure from Ricardian equivalence implied by the assumption that newly born agents have no financial wealth pins down a unique steady state (determined by tilts in individual household consumption and labor income profiles) to which the world economy returns over time following non-permanent shocks.[7] In the spirit of SGU – and Baxter and Crucini (1995), henceforth, BC – I compare the dynamics of the Weil model after productivity shocks to those of the non-stationary, representative agent case, the CO-CP model, and the complete markets model with zero initial net foreign assets.

I find that models that rely on the CO-CP assumption of unitary elasticity of substitution between domestic and foreign goods can miss quantitatively significant features of shock transmission after non-permanent shocks if the true value of the elasticity of substitution differs from one. The difference relative to a complete markets benchmark is smaller. A similar conclusion holds with respect to the non-stationary, incomplete markets case, especially over the first two–three years after a shock. The differences are more significant if the substitutability between domestic and foreign goods rises (but remains finite). This can generate non-negligible differences in dynamics over the short and medium term, in contrast to SGU's findings. The small open economy of their model produces and consumes the same good as the rest of the world. Similarly, both countries produce and consume the same good in BC. This removes terms of trade dynamics from the model. I find that terms of trade movements are important to generate differences across incomplete markets specifications. As in BC, the difference between incomplete and complete markets is amplified substantially if productivity shocks are permanent.

The structure of the paper is as follows. Section 2 presents the model. Section 3 analyzes the role of net foreign asset dynamics in the log-linearized model. Section 4 concludes.

2. The model

The world consists of two countries, home and foreign. In each period t, the world economy is populated by a mass N_t^W of atomistic, infinitely lived households. (A superscript W denotes world variables. Foreign variables are starred.) Each household consumes, supplies labor, and holds bonds. Households in both countries trade a riskless real bond denominated in units of the world consumption basket domestically and internationally. Following Weil (1989a,b), I assume that households are born on different dates *owning no assets*, but they own the present discounted value of their labor income.[8] The number of households in the home economy, N_t, grows over time at the exogenous rate n, i.e., $N_{t+1}=(1+n)N_t$. I normalize the size of a household to one, so

[6] These include introducing a cost of adjusting bond holdings, an endogenous discount factor, or a debt-elastic interest rate premium. All these mechanisms operate by introducing a link between consumption growth and asset holdings in the Euler equation for bonds.

[7] Buiter (1981), Cardia (1991), and Finn (1990) also use overlapping generations to pin down the steady state and deliver stationary model dynamics. Obstfeld and Rogoff (1995b, 1996 Ch. 3) discuss the properties of the Weil model. This paper extends it to incorporate endogenous labor supply and differences in labor income across generations. Cavallo and Ghironi (2002) use a monetary version of the model in this paper to study the role of net foreign assets in exchange rate dynamics. See also Devereux (2003), Ganelli (2005), and Smets and Wouters (2002).

[8] Blanchard (1985) combines this assumption with a positive probability of not surviving until the next period.

that the number of households alive at each point in time is the economy's population. Foreign population (N_t^*) grows at the same rate as home population. The world economy has existed since the infinite past. I normalize world population at time $t=0$ so that $N_0^W=1$.

There are two goods in the world economy. Each country is fully specialized in the production of a country-specific good, performed by a continuum of atomistic, perfectly competitive, infinitely lived firms. Home firms, producing the home good, occupy the interval [0, a], which is also the size of the home population at time zero; foreign firms, producing the foreign good, are in the range (a, 1].[9]

2.1. Households

Agents have perfect foresight, though they can be surprised by initial unexpected shocks. Consumers have identical preferences over a real consumption index (C) and leisure ($1-L$, where L is labor effort supplied in a competitive labor market, and I normalize the endowment of time in each period to one). At any time t_0, the representative home consumer j born in period $\upsilon \in [-\infty, t_0]$ maximizes the intertemporal utility function:[10]

$$U_{t_0}^{\upsilon j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0}\left[\rho \log C_t^{\upsilon j} + (1-\rho)\log\left(1-L_t^{\upsilon j}\right)\right], \quad \text{with } 0<\beta, \rho<1. \tag{1}$$

The consumption index is $C_t^{\upsilon j} = \left[a^{\frac{1}{\omega}}\left(C_{Ht}^{\upsilon j}\right)^{\frac{\omega-1}{\omega}} + (1-a)^{\frac{1}{\omega}}\left(C_{Ft}^{\upsilon j}\right)^{\frac{\omega-1}{\omega}}\right]^{\frac{\omega-1}{\omega}}$, where $\omega>0$ is the elasticity of substitution between consumption of domestic and foreign goods (C_H and C_F, respectively). Foreign agents consume an identical basket of goods. Trade in goods is free. There are no transportation and transaction costs.

The consumer enters a period holding bonds purchased in the previous period. He or she receives interest on these bond holdings, earns labor income, consumes, and purchases new bonds with which he or she will enter the next period. Letting $B_{t+1}^{\upsilon j}$ denote the consumer's holdings of bonds entering $t+1$, the period budget constraint is:

$$B_{t+1}^{\upsilon j} = (1+r_t)B_t^{\upsilon j} + w_t L_t^{\upsilon j} - C_t^{\upsilon j}, \tag{2}$$

where r_t is the risk-free world real interest rate between $t-1$ and t, and w_t is the real wage, both in units of the consumption basket.[11]

The representative home consumer born in period υ maximizes the intertemporal utility function (1) subject to the constraint (2). Dropping the j superscript (because symmetric agents

[9] See Ghironi (2000b) for a version of the model with monopolistic competition. Flexible prices and monopoly offsetting subsidies imply that the relevant equilibrium conditions are identical to those in this paper.

[10] I assume logarithmic utility for simplicity. The properties of the Weil setup for steady state determinacy and stationarity of open economy models hold also for more general, non-separable utility functions. See Ghironi (2000a) for the proof in a small open economy model with monopolistic competition, sticky prices, and physical capital accumulation.

[11] Given that individuals are born owning no financial wealth, because they are not linked by altruism to individuals born in previous periods, $B_\upsilon^{\upsilon j}=0$. As noted before, however, individuals are born owning the present discounted value of their labor income.

within a generation make identical choices in equilibrium), the first-order conditions for optimal labor supply and bond holdings yield:[12]

$$L_t^{\upsilon} = 1 - \frac{1-\rho}{\rho}\frac{C_t^{\upsilon}}{w_t}, \tag{3}$$

$$C_t^{\upsilon} = \frac{1}{\beta(1+r_{t+1})}C_{t+1}^{\upsilon}, \text{ for all } v \leq t. \tag{4}$$

Foreign consumers maximize a similar intertemporal utility function, with identical parameters, and are subject to an analogous budget constraint as home consumers. Similar optimality conditions hold.

2.2. Firms

Output supplied at time t by the representative home firm i is a linear function of labor demanded by the firm: $Y_t^{Si} = Z_t L_t^i$, where Z_t is exogenous, country-wide productivity.[13] Production by the representative foreign firm is a linear function of L_t^{i*}, with productivity Z_t^*.

Output demand comes from domestic and foreign consumers. The demand for firm i by the representative domestic household born in period υ is $C_{Hit}^{\upsilon} = (RP_t)^{-\omega} C_t^{\upsilon}$. I denote with RP_t the price of the home good in units of consumption. Atomistic, competitive home firms take this price as given. Aggregating across home households alive at time t, *total demand* for firm i coming from domestic consumers is $C_{Hit} = (RP_t)^{-\omega} a(1+n)^t c_t$, where

$$c_t \equiv \frac{a\left[\cdots \frac{n}{(1+n)^{t+1}}C_t^{-t} + \cdots + \frac{n}{(1+n)^2}C_t^{-1} + \frac{n}{1+n}C_t^0 + nC_t^1 + n(1+n)C_t^2 + \cdots + n(1+n)^{t-1}C_t^t\right]}{a(1+n)^t}$$

is *aggregate per capita* home consumption of the composite consumption basket.

Given identity of preferences across countries, demand for firm i by foreign consumers is $C_{Hit}^* = (RP_t)^{-\omega}(1-a)(1+n)^t c_t^*$, where c_t^* is aggregate per capita foreign consumption, defined similarly to c_t. (Absence of transportation and transaction costs implies that the price of the home good in units of consumption is the same at home and abroad.)

Total demand for home firm i is obtained by adding the demands originating in the two countries: $Y_t^{Di} = (RP_t)^{-\omega}\hat{c}_t^W$, where $\hat{c}_t^W$ is *aggregate* (as opposed to aggregate per capita) world demand of the composite good: $\hat{c}_t^W \equiv N_t c_t + N_t^* c_t^*$.[14]

Perfect competition results in prices equal to marginal costs, so that $RP_t = \frac{w_t}{Z_t}$. Using the market clearing conditions $Y_t^{Si} = Y_t^{Di}$, $\hat{c}_t^W = \hat{Y}_t^{SW} = \hat{Y}_t^{DW}$ $(= \hat{Y}_t^W)$, and the expressions for firm i's supply and demand, labor demand can be written as $L_t^i = (RP_t)^{-\omega}\frac{\hat{Y}_t^W}{Z_t}$.[15] Optimal behavior by foreign firms results in similar price and labor demand equations.

[12] As usual, these conditions and the period budget constraint must be combined with the transversality condition (omitted) to ensure optimality.

[13] Because all firms in the world economy are born at $t = -\infty$, after which no new firms appear, it is not necessary to index output and labor demands by the firms' date of birth.

[14] Where necessary for clarity, I use a "hat" to differentiate the aggregate level of a variable from the aggregate per capita level.

[15] Although all firms in each country demand the same amount of labor in equilibrium, I leave the i superscript on labor demand to differentiate labor employed by an individual firm from aggregate per capita employment, which will be denoted by dropping the superscript.

2.3. Aggregation

2.3.1. Households

I present only the equations for the home economy. Equations for foreign are similar, except for variables being starred.

Aggregate per capita labor supply equations are obtained by aggregating labor–leisure trade-off equations across generations and dividing by total population. This yields:

$$L_t = 1 - \frac{1-\rho}{\rho}\frac{c_t}{w_t}. \tag{5}$$

The consumption Euler equation in aggregate per capita terms features an adjustment for consumption by the newborn generation at time $t+1$:

$$c_t = \frac{1+n}{\beta(1+r_{t+1})}\left(c_{t+1} - \frac{n}{1+n}C_{t+1}^{t+1}\right). \tag{6}$$

The adjustment for consumption of the newborn generation at $t+1$ in Eq. (6) is central to ensure determinacy of the steady state of the model.

Newborn households hold no assets, but they own the present discounted value of their labor income. Using the Euler Eq. (4) and a newborn household's intertemporal budget constraint, it is possible to show that the household's consumption in the first period of its life is a fraction of its human wealth, h:

$$C_t^t = \rho(1-\beta)h_t, \tag{7}$$

where h is defined as the present discounted value of the households' remaining lifetime in terms of the real wage: $h_t \equiv \sum_{s=t}^{\infty} R_{t,s}w_s$, with $R_{t,s} \equiv \left[\prod_{u=t+1}^{s}(1+r_u)\right]^{-1}$, $R_{t,t} \equiv 1$. The dynamics of h are determined by:

$$h_t = \frac{h_{t+1}}{1+r_{t+1}} + w_t. \tag{8}$$

Aggregating the budget constraint (2) across living generations yields the law of motion for home's aggregate per capita net foreign assets:

$$(1+n)B_{t+1} = (1+r_t)B_t + w_tL_t - c_t. \tag{9}$$

A similar equation holds for B_{t+1}^*. For the bond market to be in equilibrium, aggregate home assets (liabilities) must equal aggregate foreign liabilities (assets), i.e., it must be $\hat{B}_t + \hat{B}_t^* = 0 \ \forall t$. In aggregate per capita terms, it must be $aB_t + (1-a)B_t^* = 0$.

2.3.2. Firms

Aggregate per capita output in units of consumption (y_t) is obtained by expressing each firm's production in units of the world basket, multiplying by the number of firms, and dividing by population. This yields $y_t = RP_tZ_tL_t$. For given employment and productivity, each country's real GDP rises with the relative price of the good produced in that country, as this is worth more units of the consumption basket. Marginal cost pricing ($RP_t = \frac{w_t}{Z_t}$) and $y_t = RP_tZ_tL_t$ imply that GDP is equal to labor income, or $y_t = w_tL_t$.

Aggregate per capita labor demand is $L_t = (RP_t)^{-\omega}\frac{y_t^W}{Z_t}$, where y_t^W is aggregate per capita world production of the composite good, equal to aggregate per capita world consumption, c_t^W. It is $y_t^W = ay_t + (1-a)y_t^*$ and $c_t^W = ac_t + (1-a)c_t^*$; $y_t^W = c_t^W$ to ensure market clearing.[16]

3. The role of asset accumulation

The model of Section 2 has a unique steady state as long as $n>0$.[17] The solution for the steady state is described in Appendix A.[18] To summarize, assuming initial steady-state levels of domestic and foreign productivity $\overline{Z}=\overline{Z}^*=1$, the steady state of the model is such that $\overline{B} = \overline{B}^* = 0$, $\overline{r} = \frac{1-\beta}{\beta}$, $\overline{w} = \overline{RP} = \overline{w}^* = \overline{RP}^* = 1$, $\overline{h} = \overline{h}^* = \frac{1}{1-\beta}$, $\overline{y} = \overline{c} = \overline{C}_v^v = \overline{L} = \overline{y}^* = \overline{c}^* = \overline{C}_v^{v*} = \overline{L}^* = \overline{y}^W = \overline{c}^W = \rho$.

This section analyzes dynamics in a neighborhood of the steady state by solving the log-linear version of the model. In what follows, sans-serif variables denote percent deviations from steady-state levels. It is convenient to solve the log-linearized model for cross-country differences ($\mathsf{x}_t^D \equiv \mathsf{x}_t - \mathsf{x}_t^*$ for any pair of variables x and x^*) and world aggregates ($\mathsf{x}_t^W \equiv a\mathsf{x}_t + (1-a)\mathsf{x}_t^*$). Solutions for individual country variables can then be recovered easily.

3.1. Country differences

Appendix B shows that we can reduce the log-linear model for cross-country differences to a system of two first-order difference equations for relative human wealth (h_t^D) and the stock of net foreign assets entering period $t+1$ (B_{t+1}, where $\mathsf{B}_{t+1} \equiv \frac{dB_{t+1}}{\overline{c}}$) as functions of relative human wealth at time $t+1$, the predetermined stock of net foreign assets at time t, and the productivity differential (Z_t^D). The Appendix also shows that the CO-CP result holds in the model of this paper: If the elasticity of substitution between home and foreign goods (ω) is equal to one, given zero initial net foreign assets, home and foreign GDPs and consumption levels are equal regardless of productivity, and net foreign assets do not move in response to shocks.

We can write:

$$\begin{bmatrix} \mathsf{h}_{t+1}^D \\ \mathsf{B}_{t+1} \end{bmatrix} = M_1 \begin{bmatrix} \mathsf{h}_t^D \\ \mathsf{B}_t \end{bmatrix} + M_2 \mathsf{Z}_t^D, \tag{10}$$

where:

$$M_1 \equiv \begin{bmatrix} \frac{\rho\omega + \beta(1-\rho)}{\beta[1+\rho(\omega-1)]} & -\frac{\rho(1-\rho)(1-\beta)^2}{\beta^2(1-a)[1+\rho(\omega-1)]} \\ -\frac{\omega(1-a)}{(1+n)[1+\rho(\omega-1)]} & \frac{1+\rho(\omega\beta-1)}{\beta(1+n)[1+\rho(\omega-1)]} \end{bmatrix}, \quad M_2 \equiv \begin{bmatrix} -\frac{\rho(1-\beta)(\omega-1)}{\beta[1+\rho(\omega-1)]} \\ \frac{(\omega-1)(1-a)}{(1+n)[1+\rho(\omega-1)]} \end{bmatrix}.$$

[16] Substituting $y_t = w_t L_t$ into Eq. (9) and using the resulting equation and its foreign counterpart in conjunction with asset market equilibrium yields $y_t^w = c_t^W$. Consistent with Walras' Law, asset market equilibrium implies goods market equilibrium, and vice versa.

[17] It is easy to verify that the steady state is indeterminate if $n=0$. In this case, the model reduces to the familiar representative agent framework and setting consumption to be constant in the Euler equation provides no restriction to pin down international bond holdings.

[18] The main difference between the overlapping generations mechanism and the approaches to pin down the steady state reviewed in SGU is that the former provides a structural interpretation for the determination of long-run assets that does not hinge on assumptions on the functional form of a cost of adjusting asset holdings, an endogenous discount factor, or the determination of a debt-elastic interest rate premium — see footnote 20.

If $n=0$, the matrix M_1 has eigenvalues $e_1 = \frac{1}{\beta}$ and $e_2=1$. Since $\beta<1$, the number of eigenvalues outside the unit circle equals the number of non-predetermined variables, ensuring that the system in Eq. (10) has a determinate solution (Blanchard and Kahn, 1980). The second eigenvalue exactly equal to one is consistent with non-stationary net foreign asset dynamics in the case $n=0$.[19] If n becomes strictly positive, this eigenvalue is "pulled" inside the unit circle, delivering stationarity of net foreign assets. Graphing the characteristic polynomial for the matrix M_1 shows that the solution remains determinate when $n>0$.[20]

I assume $\mathsf{Z}_t = \phi \mathsf{Z}_{t-1}$, $\mathsf{Z}^*_t = \phi \mathsf{Z}^*_{t-1}$, $\forall t>0$, $0 \leq \phi \leq 1$ ($t=0$ is the time of an initial, surprise impulse below). Hence, $\mathsf{Z}^D_t = \phi \mathsf{Z}^D_{t-1}$. If $\phi=1$, impulses to productivity have permanent consequences, causing the economy to move to a new steady state because of non-stationarity of the productivity process.

The stock of net foreign assets and the levels of exogenous productivities describe the state of the economy in each period. The solution of the system (10) can be written as:

$$\mathsf{B}_{t+1} = \eta_{BB}\mathsf{B}_t + \eta_{BZ^D}\mathsf{Z}^D_t, \tag{11}$$

$$\mathsf{h}^D_t = \eta_{h^D B}\mathsf{B}_t + \eta_{h^D Z^D}\mathsf{Z}^D_t. \tag{12}$$

The elasticities η as functions of the structural parameters of the model can be obtained with the method of undetermined coefficients.[21] Stationarity of net foreign assets requires $\eta_{BB}<1$. As mentioned above, this happens whenever $n>0$. For plausible parameter values, a favorable shock to relative domestic productivity causes domestic agents to accumulate net foreign assets to smooth consumption dynamics if $\phi<1$ (see below). Hence, $\phi_{BZ^D}>0$ in this case ($\eta_{BZ^D}=0$ if $\phi=1$). Plausible parameter values ensure that $\eta_{h^D B}$ and $\eta_{h^D Z^D}$ are positive too.[22]

Given any pair of endogenous, non-state variables x and x^* the solution for their difference can be written in a similar fashion to Eq. (12) as $\mathsf{x}^D_t = \eta_{x^D B}\mathsf{B}_t + \eta_{x^D Z^D}\mathsf{Z}^D_t$, and the relevant elasticities can be obtained with the method of undetermined coefficients.

3.1.1. Impulse responses

The solution of the model can be used to trace the response of cross-country differences to productivity shocks and illustrate the importance of some key parameters.

I start with the following benchmark parameter values: $\beta=.99$, $\rho=.33$, $\omega=1.2$, $a=.5$, and $n=.01$. Periods are interpreted as quarters. The choice of n is higher than realistic if one has

[19] In this case, any shock that changes the consumption differential today has permanent consequences. Asymmetric shocks cause assets to change permanently by generating initial differences in consumption and GDP. Consistent with CO-CP, the effect of this unit root is removed if $\omega=1$ and the initial level of net foreign assets is chosen to be zero.

[20] The intuition for stationary asset dynamics with $n>0$ is easily understood in a partial equilibrium, small open economy version of the model, with constant world interest rate r and domestic wage w. In that case, the law of motion for net foreign assets can be written as $B_{t+1} = \frac{\beta(1+r)}{1+n}B_t + \frac{\beta(1+r)-1}{r(1+n)}w$. If $\frac{\beta(1+r)}{1+n}<1$, a unique steady state for home aggregate per capita net foreign assets exists and is stable. Entry of new households with no assets eventually "wipes out" the consequences of shocks. Given the existence/stability condition $\frac{\beta(1+r)}{1+n}<1$, the individual consumption/labor income tilt factor $\beta(1+r)-1$ determines whether the country runs a positive or negative long-run asset position. Ghironi (2000a) proves that the same results and intuition hold for a non-separable specification of utility with intertemporal elasticity of substitution that can differ from one.

[21] See Ghironi (2000b) for details.

[22] If $\omega=1$, $\eta_{h^D Z^D}=\eta_{BZ^D}=0$: Asset accumulation does not react to shocks ($\mathsf{B}_t = 0$) and $\mathsf{h}^D_t = 0$ $\forall t$.

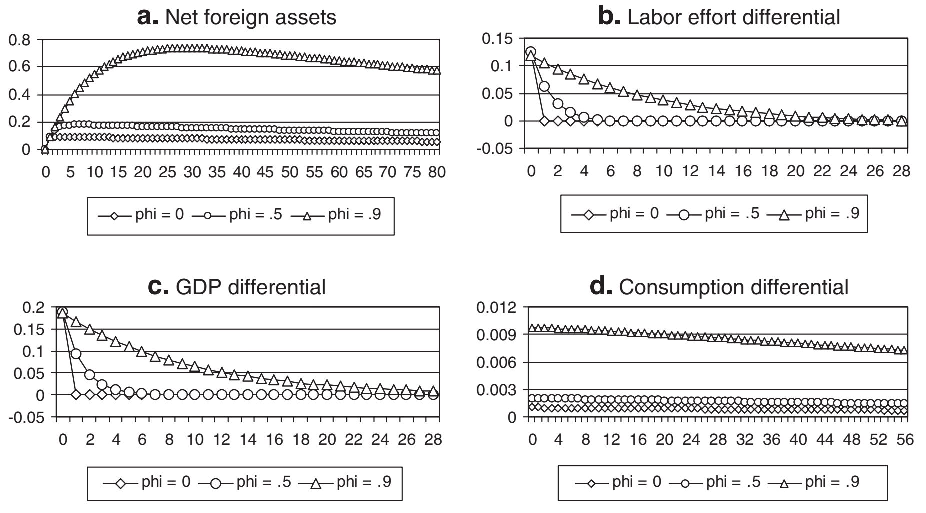

Fig. 1. Impulse responses, net foreign assets and cross-country differences.

developed economies in mind.[23] I use $n = .01$ as a benchmark example and discuss the implications of lower (and higher) values in what follows.[24] The value of ω is consistent with the international real business cycle (RBC) literature. Estimates from the trade literature suggest that values significantly above one would be reasonable. I explore the consequences of higher ω below.

Fig. 1 shows the responses (percent deviations from steady state) of aggregate per capita real net foreign assets and the labor effort, GDP, and consumption differentials following a one percent increase in relative domestic productivity.[25] I consider three values of ϕ in the figure ($\phi = 0$, .5, and .9) and omit (but mention below) the responses for $\phi = 1$.

When $\phi < 1$, the home economy accumulates net foreign assets following the shock to smooth the effect of the latter on consumption. When the shock is purely temporary ($\phi = 0$), net foreign assets decrease monotonically after the initial accumulation. A more persistent increase in productivity (with $\phi < 1$) causes the home economy to continue accumulating assets for several quarters before settling on the downward path to the steady state. In all cases, the speed at which net foreign assets return to the steady state once the productivity differential has died out is very slow, as implied by a low rate of entry of new households in the economy. This is consistent with the evidence of persistence in net foreign asset changes documented in several studies.

The sign of $\eta_{L^D B}$ and $\eta_{y^D B}$ (the elasticities of the employment and real GDP differentials to net assets) is opposite the sign of $\eta_{c^D B}$ (the elasticity of the consumption differential to net assets). For plausible parameter values, it is $\eta_{c^D B} > 0$. Intuitively, accumulation of net foreign assets allows the home economy to sustain a higher consumption path than foreign. It follows that $\eta_{L^D B} < 0$ and $\eta_{y^D B} < 0$: Ceteris paribus, accumulation of net foreign assets causes domestic agents to supply less

[23] The average rate of quarterly population growth for the U.S. between 1973:1 and 2000:3 has been .0025.

[24] Extending the model to incorporate probability of death as in Blanchard (1985) would make it possible to reproduce the dynamics generated by $n = .01$ with a lower rate of entry of new households by choosing the appropriate value of the probability of death. The choice of $n = .01$ thus mimics the behavior of a more complicated, yet largely isomorphic setup.

[25] The responses are scaled so that, for instance, .3 on the vertical axis denotes .3%.

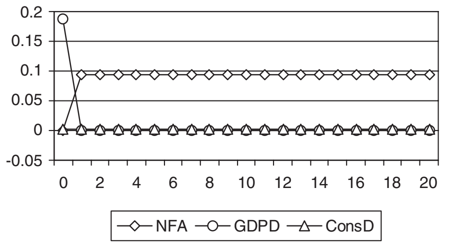

Fig. 2. The non-stationary case.

labor than foreign, resulting in higher domestic real wage and relative price than abroad (the terms of trade – the ratio $\frac{RP_t}{RP_t^*}$ – appreciate in response to asset accumulation), and in lower domestic GDP relative to foreign. (It is $\eta_{y^D B}=0$ if $\omega=1$, but I assume $\omega>1$ unless explicitly stated.)

On impact, a favorable shock to domestic productivity causes domestic employment and GDP to rise above foreign (because $\eta_{L^D Z^D}$ and $\eta_{y^D Z^D}$, the elasticities of the employment and GDP differentials to relative productivity, are positive).[26] Domestic employment and GDP are higher than foreign for a longer time the more persistent the shock.[27] Once this has died out, $\eta_{L^D B}<0$ and $\eta_{y^D B}<0$ cause the employment and GDP differentials to return to zero from (slightly) below. Instead, the consumption differential is positive throughout the transition (and larger the more persistent the shock). Note that the deviations of the employment and GDP differentials from the steady state become very small once the productivity shock has died out, since $\eta_{L^D B}$ and $\eta_{y^D B}$ are very small for the parameter values in this example. Cross-country differences caused by net foreign asset accumulation are quantitatively small when ω is close to one. Consistent with intertemporal optimization, the consumption differential is smoother than the GDP differential. The deviation of the consumption differential from the steady state is substantially smaller than that of the employment and GDP differentials even in the first few periods after the shock.

The home economy accumulates no assets if the shock is permanent ($\phi=1$). In this case, domestic GDP and consumption rise permanently above foreign exactly by the same amount in the period of the shock. Instead, there is no employment differential.

To further illustrate the role of n, Figs. 2 and 3 show the responses of net assets (NFA), GDP, and consumption differential to a one-time productivity impulse with no persistence ($\phi=0$) in the cases $n=0$ and $n=.5$, respectively. The temporary shock has permanent consequences on assets and the consumption and GDP differentials when $n=0$. As the case $n=.5$ shows, raising n to an unrealistically large value makes the speed of convergence of asset holdings to the steady state much faster (in this case, $\eta_{BB}=.67$ as opposed to $\eta_{BB}=.994$ with $n=.01$).[28]

Finally, Fig. 4 displays the responses of assets and the GDP and consumption differentials to a zero-persistence shock when $n=.01$ but $\omega=4$. The range of variation of endogenous variables in

[26] The positive elasticity of equilibrium employment to productivity is related to the negative elasticity of the relative price differential. Domestic equilibrium employment rises above foreign because a lower domestic real price in response to higher productivity generates more demand for home goods.

[27] Though $\eta_{L^D Z^D}=0$ if $\phi=1$.

[28] In principle, the Blanchard-Weil setup can thus generate fast convergence to the steady state. But a central issue is whether or not this is desirable. The evidence on time series properties of net foreign assets suggests that slow convergence is empirically appealing.

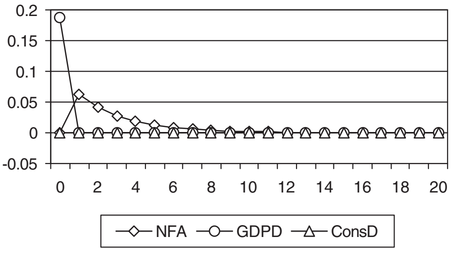

Fig. 3. Fast convergence.

this case is an order of magnitude larger than with $\omega=1.2$. Cross-country differences caused by asymmetric shocks are bigger if goods are more highly substitutable across countries.

3.2. World averages

Completing the solution of the model requires solving for the levels of world aggregate variables, to be used in conjunction with cross-country differences to determine individual country variables. The solution for world variables is $\mathsf{RP}_t^W = \mathsf{L}_t^W = 0,\ \mathsf{w}_t^W = \mathsf{y}_t^W = \mathsf{c}_t^W = \mathsf{Z}_t^W$ regardless of the value of ω (see Appendix C). World productivity changes translate into equal changes in GDP, consumption, and the real wage. These changes have offsetting effects on pricing and employment, which remain insulated from changes in productivity. Finally, the world real interest rate between t and $t+1$ must equal world productivity growth between the two periods: $\mathsf{r}_{t+1} = \mathsf{Z}_{t+1}^W - \mathsf{Z}_t^W$. From these results, it follows that a purely temporary productivity shock ($\phi=0$) has only transitory effects on world aggregates and the real interest rate.

3.3. Complete asset markets

Suppose now that the elasticity of substitution between domestic and foreign goods differs from one, but agents at home and abroad have access to complete asset markets in one-period, contingent securities. Assume that the asset market opens before the time of initial, surprise productivity shocks. In this case, net foreign holdings of contingent securities (net foreign assets) play no role in the solution for the dynamics of other endogenous variables. To see this, observe first that the solution for world aggregates is left unaffected by the change in the menu of assets available to agents. The solution for cross-country differences is instead affected as follows.

Starting from a symmetric steady state, perfect "risk" sharing in complete asset markets implies that the consumption differential is zero in all periods: $\mathsf{c}_t^D = 0\ \forall t$.[29] It is then possible to verify that $\mathsf{L}_t^D = \gamma(1-\rho)\mathsf{Z}_t^D,\ \mathsf{w}_t^D = \gamma\rho\mathsf{Z}_t^D,\ \mathsf{RP}_t^D = -\frac{\gamma}{\omega-1}\mathsf{Z}_t^D$ and $\mathsf{y}_t^D = \gamma\mathsf{Z}_t^D$, where $\gamma \equiv \frac{\omega-1}{1+\rho(\omega-1)}$. At no point in the solution does one have to use the law of motion for net foreign assets. When asset markets are complete and $\omega \neq 1$, net foreign holdings of contingent securities do move over time in response to relative GDP movements. However, these net foreign asset changes are determined residually. They do not affect the dynamics of other endogenous

[29] See Ljungqvist and Sargent (2004) on the properties of complete markets.

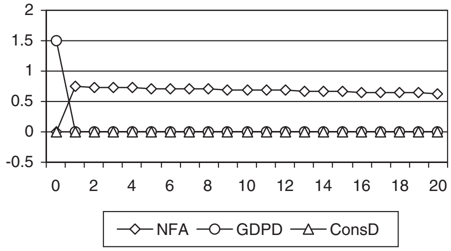

Fig. 4. Higher substitutability.

variables, which are independent of history as represented by the stock of net foreign assets accumulated by each country.[30]

3.4. Country dynamics and asset accumulation

Given solutions for differences and world averages, it is easy to obtain solutions for individual country variables. For any pair of variables x and x^*, it is $\mathsf{x} = \mathsf{x}^W + (1-a)\mathsf{x}^D$ and $\mathsf{x}^* = \mathsf{x}^W - a\mathsf{x}^D$.

I perform a simple experiment to gauge the role of net foreign asset accumulation for individual country dynamics. Suppose reality is such that markets are incomplete, $\omega = \omega^R \neq 1$, and convergence of net foreign assets to the steady state after non-permanent, asymmetric productivity shocks is slow. Our interest is in evaluating what is missed by approximating reality with the following alternatives: (a) Assuming that markets are incomplete, $\omega = \omega^R$, and net foreign assets return to the steady state quickly after non-permanent shocks (an unrealistically high value of n); (b) Assuming incomplete markets in a non-stationary, representative agent setup in which $\omega = \omega^R$, and net foreign assets never return to the steady state ($n = 0$); (c) Assuming that markets are incomplete, but $\omega = 1$; (d) Assuming $\omega = \omega^R$ with complete asset markets.

The results of the previous sub-sections yield a number of qualitative conclusions. Assuming $\omega = 1$ implies that consumption and employment levels in the two countries are equalized regardless of productivity and that consumption is tied to GDP, so that net foreign assets do not move after shocks. In this case, there is no endogenous persistence – i.e., persistence beyond that of exogenous shocks – in the changes in consumption and other variables triggered by changes in productivity. If the true elasticity of substitution between domestic and foreign goods differs from one, the paths of variables following shocks are different. Countries run current account imbalances following asymmetric shocks. If asset markets are incomplete, cross-country differences in aggregate per capita levels of consumption and other variables persist (beyond the persistence of the shock, if this is not permanent) until net foreign assets return to the steady state.[31] If asset markets are complete and $\omega \neq 1$, the cross-country consumption differential is zero

[30] If $\omega = 1$, the complete and incomplete markets solutions coincide.

[31] The fact that net foreign asset dynamics play a role in shock transmission when $\omega \neq 1$ does not depend on the particular mechanism that is used to pin down the steady state and induce stationarity under incomplete markets. All the mechanisms explored in the literature (including overlapping generations) operate by introducing a link between consumption growth and asset holdings in the Euler equation for aggregate per capita consumption. But, as can be seen in Appendix B, the properties of the scenario $\omega = 1$ (or $\omega \neq 1$) with respect to existence of consumption and GDP differentials and a role for net foreign assets in shock transmission hold irrespective of the form of the model's Euler equation.

in all periods, and there is no endogenous persistence in employment, GDP, wage, and relative price differentials.

Section 3.2 showed that $\mathsf{y}_t^W = \mathsf{c}_t^W$ and Z_t^W and $\mathsf{L}_t^W = 0$ regardless of ω and the structure of asset markets. If $\omega = 1$, $\mathsf{y}_t = \mathsf{y}_t^* = \mathsf{c}_t = \mathsf{c}_t^* = \mathsf{w}_t = \mathsf{w}_t^* = \mathsf{Z}_t^W$ and $\mathsf{L}_t^D = 0$. But $\mathsf{L}_t^W = \mathsf{L}_t^D = 0$ implies $\mathsf{L}_t = \mathsf{L}_t^* = 0$. In contrast to the case $\omega \neq 1$, employment levels in individual countries do not react to productivity under the assumption of unitary intratemporal elasticity of substitution between domestic and foreign goods. If the true elasticity of substitution between domestic and foreign goods differs from one, the description of the economy provided by the CO-CP assumption generates errors in the characterization of the impact effects of a productivity change and misses the transition dynamics caused by asset accumulation. If asset markets are incomplete in reality, assuming complete markets erroneously removes the dependence of the equilibrium on the history of asset accumulation.

The open question is how significant the differences across specifications are from a quantitative perspective. The impulse responses for the benchmark parameterization of Section 3.1.1 suggest that errors caused by the assumption $\omega=1$ (alternative (c) above) may be more significant in the characterization of the path of employment than consumption, the reason being the absence of a reaction of employment to productivity in the case $\omega=1$. Consumption differentials generated by asset accumulation are likely to be small because of the small elasticity of the consumption differential to net foreign assets for plausible parameter values. Nevertheless, we expect the difference in the consumption response generated by alternatives (c) and (d) to become larger if the true ω is further away from one and if productivity shocks are more persistent (if ϕ is higher).

Fig. 5 presents the responses of home consumption, the terms of trade (RP^D), employment, and GDP to a one percent impulse to home productivity for the benchmark economy and alternatives (a)–(d).[32] I assume the same benchmark parameter values as in Section 3.1.1, repeated here for convenience: $\beta=.99$, $\rho=.33$, $\omega=\omega^R=1.2$, $a=.5$, and $n=.01$. I assume shock persistence $\phi=.9$ — at the low end of the range in the RBC literature. I set $n=.5$ in alternative (a) (stationary model, fast convergence). The value of n is irrelevant in the $\omega=1$ and complete markets cases. Fig. 6 repeats the exercise for $\omega^R=4$. The diagrams on the left side of each figure are the impulse responses, the diagrams on the right present the differences between the benchmark response and the responses under alternatives (a)–(d) as percentage of the steady-state level of the variable under consideration. Where appropriate, the difference between the benchmark and alternative (c) in the right-side diagrams is measured on the secondary vertical axis.

The home economy consumes and produces the same good as the rest of the world in BC and SGU. This amounts to $\omega \rightarrow \infty$: Home and foreign goods are perfect substitutes. When this happens, the terms of trade are one in all periods ($\mathsf{RP}_t^D = 0$). For comparison with BC and SGU, Fig. 7 repeats the exercise of Figs. 5 and 6 for $\omega^R \rightarrow \infty$ (approximated by $\omega^R=1{,}000{,}000$).

The $\omega=1$ and complete markets scenarios (alternatives (c) and (d), respectively) generate identical paths of consumption that are smoother than those for the other cases in Fig. 5. Similarly, SGU finds that complete markets generate a smoother response of consumption to productivity shocks than alternative model specifications. When $\omega^R=1.2$, differences in consumption dynamics across specifications are negligible at least for approximately three years after the shock. Differences widen significantly when $\omega^R=4$ (Fig. 6). As for the case $\omega^R=1.2$, the widest

[32] Impulse responses for the foreign economy are available upon request.

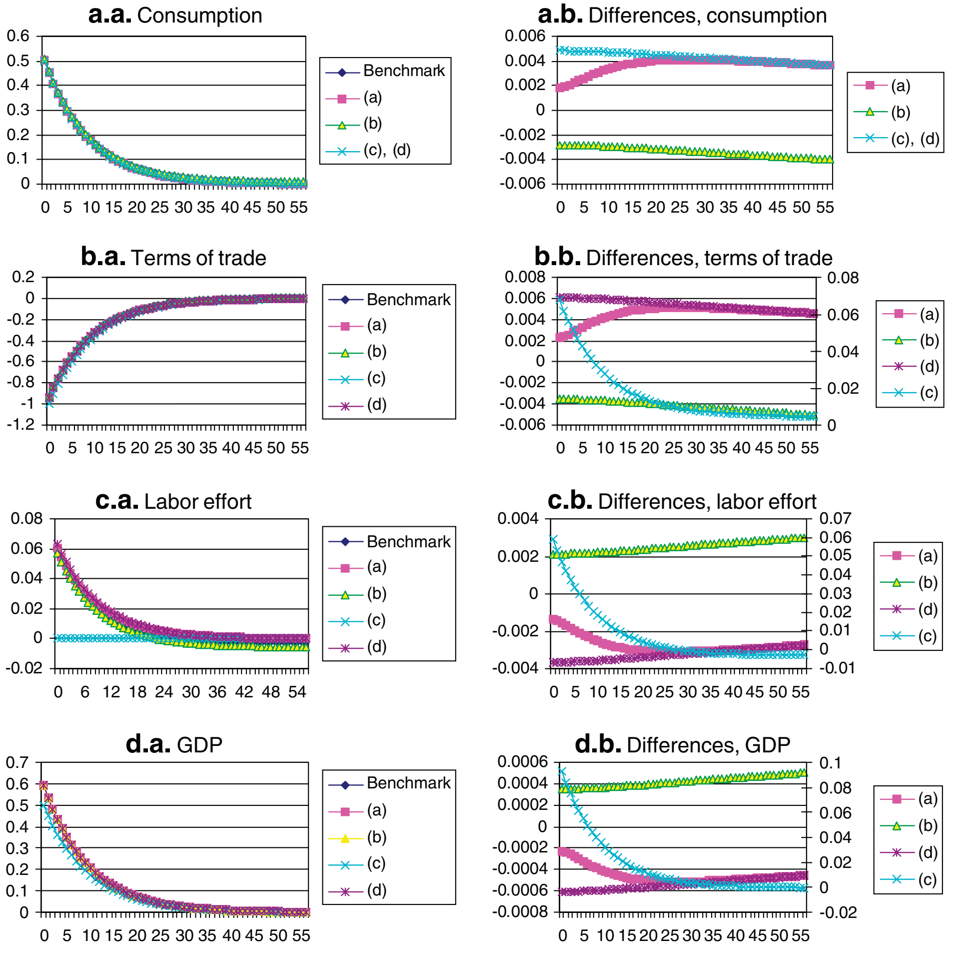

Fig. 5. Model comparison, $\phi=.9$, $\omega^R=1.2$.

difference is between the non-stationary model (alternative (b)) and the $\omega=1$ and complete markets cases (alternatives (c) and (d)). An elasticity of substitution between domestic and foreign goods in line with the trade literature results in short- and medium-run differences across consumption paths for different model specifications that remain small, but harder to classify as negligible.

If both countries produce the same good and there is no scope for terms of trade dynamics (Fig. 7), the difference between the benchmark and alternatives (c) and (d) increases. Increasing the substitutability across domestic and foreign goods from close to one to a value in the range estimated by the trade literature causes the difference to increase significantly. The effect of going all the way to perfect substitutability is smaller. Consistent with BC and SGU, the difference between consumption paths under the benchmark and complete markets is noticeable, but small.[33] Interestingly, the difference between the benchmark economy and the non-stationary

[33] Heathcote and Perri (2002) obtain a similar result.

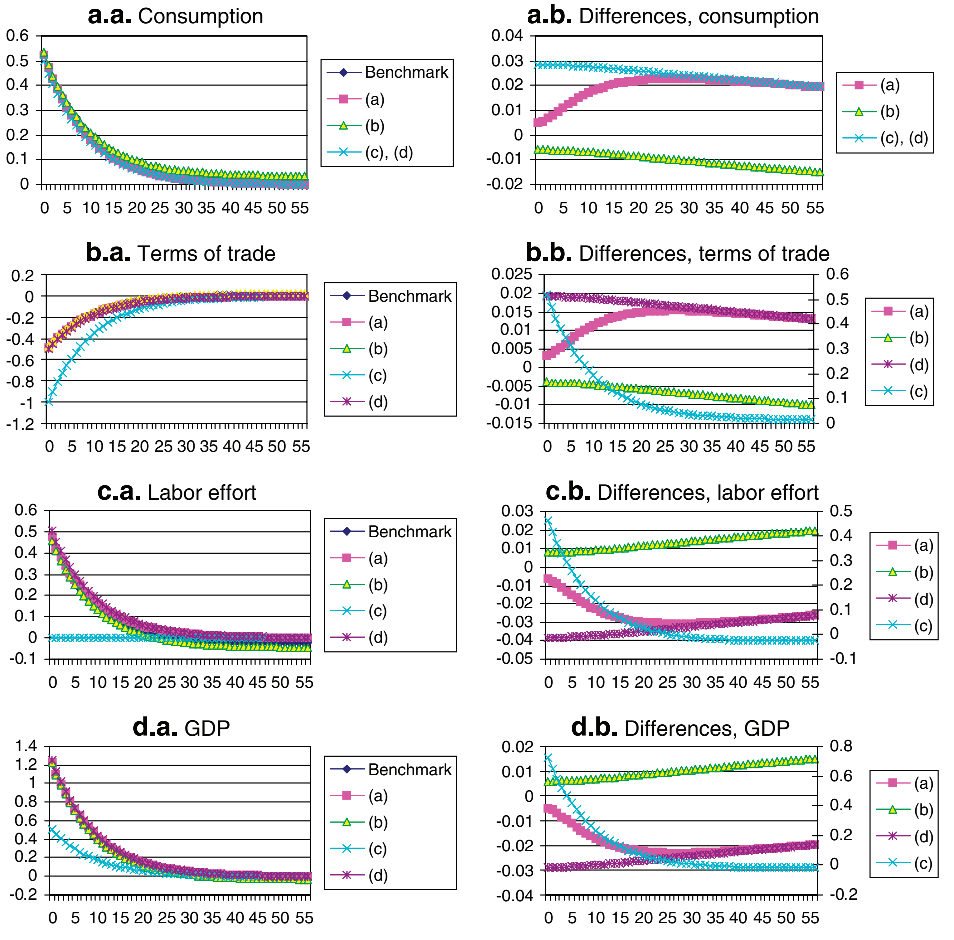

Fig. 6. Model comparison, $\phi = .9$, $\omega^R = 4$.

alternative is now extremely small for the first eight quarters or so of the exercise, significantly smaller than for the previous values of ω^R. SGU's result that the difference in short-run dynamics between a stationary model with plausible speed of convergence to the steady state and a non-stationary economy is negligible emerges when home and foreign goods are perfect substitutes. This suggests that the absence of terms of trade dynamics implied by perfect substitutability is important to explain SGU's findings. Once domestic and foreign products differ and the terms of trade respond to shocks, differences between incomplete markets specifications become more pronounced also in the short run. Put differently, terms of trade movements are important to generate noticeable short-run differences in responses across plausible incomplete markets scenarios.[34]

[34] Home's terms of trade deteriorate as a consequence of a favorable productivity shock that increases the supply of domestic goods when ω is finite. In the case of a non-permanent shock, this results in higher labor effort at home than abroad (except if $\omega = 1$). It should be noted that differences across scenarios cannot follow from differences in the dynamics of the world real interest rate, as this is a function of world productivity only, which does not change across scenarios.

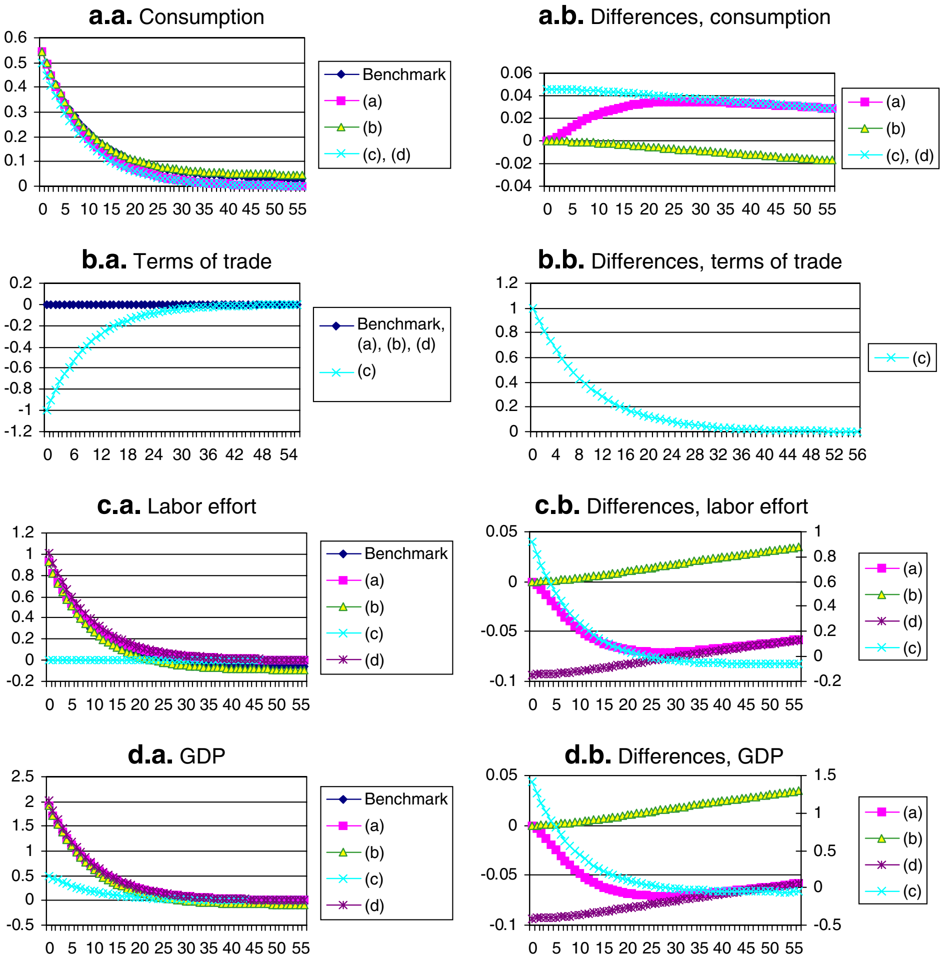

Fig. 7. Model comparison, $\phi=.9$, $\omega^R=1{,}000{,}000$.

Results on employment are similar. Differences in responses are small. They are negligible if $\omega^R=1.2$, except relative to the case $\omega=1$. They become wider if $\omega^R=4$. As suggested above, the CO-CP assumption of unitary elasticity of substitution between domestic and foreign goods can cause more significant errors in the description of labor effort dynamics by implying that employment does not react to productivity shocks. The assumption $\omega=1$ (alternative (c)) results in a sizable difference when $\omega^R=4$. (The labor effort and GDP differences between the benchmark and the case $\omega=1$ are measured on the secondary vertical axis.) If the true ω is significantly different from one (but finite), the difference in employment dynamics between the benchmark and the complete markets model (d) is somewhat larger than the difference in consumption responses. The same result emerges when domestic and foreign goods are perfect substitutes, in contrast to SGU's conclusion that only differences in consumption dynamics are noticeable in this case. The wealth effect of net foreign asset accumulation on the supply of labor effort at home and abroad in a two-country, rather than small open economy, setup motivates this

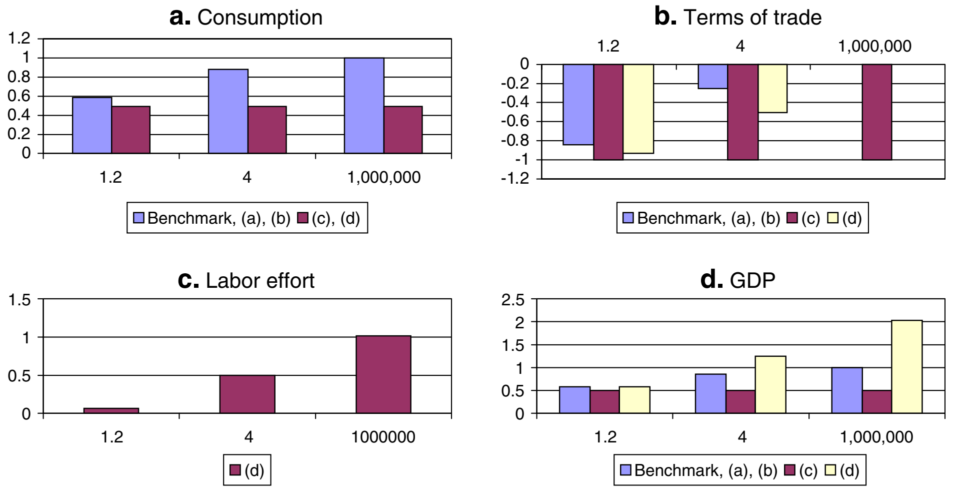

Fig. 8. Model comparison, $\phi=1$.

difference in results. However, SGU's result that there is a negligible difference between the (plausible) stationary case and the non-stationary one in the short run emerges also with respect to labor effort when home and foreign goods are perfect substitutes.

Approximation errors caused by alternatives (a), (b), and (d) are negligible when one considers GDP in the case $\omega^R=1.2$. As for employment, the assumption $\omega=1$ causes the largest error, amplified by higher substitutability across goods. The inaccuracy of the complete markets model is of the same size as for consumption. Again in contrast to SGU, perfect substitutability between domestic and foreign goods and the resulting absence of terms of trade movements cause a wider difference between GDP responses relative to consumption responses under the benchmark and alternative (d). However, $\omega^R\rightarrow\infty$ reproduces SGU's finding of negligible differences in GDP dynamics between the benchmark specification and the non-stationary alternative.[35]

3.4.1. Permanent shocks

BC finds that the difference in impulse responses between complete and incomplete markets is amplified if shocks are permanent ($\phi=1$). For comparison of results, Fig. 8 presents the responses of home consumption, the terms of trade, labor effort, and GDP to a permanent domestic productivity shock with $\omega^R=1.2$, 4, and 1,000,000. Since endogenous, non-predetermined variables jump immediately to the new permanent position when $\phi=1$, the panels in Fig. 8 have ω^R on the horizontal axis and the response level on the vertical axis.

When $\phi=1$, consumption and GDP increase permanently by the same amount. Since agents do not accumulate assets in response to permanent shocks, the responses of consumption, labor effort, and GDP are identical regardless of the value of n, including the non-stationary case $n=0$.

As in the case of a non-permanent shock, complete markets and the assumption $\omega=1$ yield the same response of consumption to a permanent shock regardless of the value of ω^R. The response reflects the impact of the shock on world productivity, i.e., .5% in all cases. The response under

[35] In general, the difference between the benchmark and the non-stationary alternative is very small in the early portion of the response to shocks, and it increases (in absolute value) toward a permanent level as time progresses and the benchmark economy returns to the steady state.

complete markets or $\omega=1$ is smaller than in the benchmark economy. The difference ranges between .0833 when $\omega^R=1.2$ and .5 when $\omega^R\to\infty$. As in BC, permanent productivity shocks result in much larger differences in responses between complete and incomplete markets.

Labor effort does not respond to the productivity shock in the benchmark economy or if $\omega=1$. (For this reason, Fig. 8 shows only the response for the complete markets economy with $\omega=\omega^R$.) Thus, the assumption $\omega=1$ does not result in any error in the characterization of labor effort if the shock is permanent. In both the benchmark economy and the scenario $\omega=1$, agents consume the real value of the permanent increase in productivity in all periods without altering their labor effort. Consumption rises by less if $\omega=1$ because home's terms of trade deteriorate by more, which causes the real value of domestic output to increase by less as a consequence of the shock.[36]

Agents increase their supply of labor effort under complete markets. They do so by more the higher the value of ω^R. To understand this, observe that higher substitutability across domestic and foreign goods makes smaller terms of trade movements necessary to keep international goods market equilibrium after shocks. Thus, the higher ω^R, the smaller the response of the terms of trade to the shock. At the same time, the higher ω^R, the larger the positive impact of any given terms of trade deterioration on domestic labor demand. When ω^R is close to one, the complete markets economy behaves very similarly to the scenario $\omega=1$. Relative price movements are such that labor effort moves by very little. Regardless of ω^R, domestic and foreign consumption levels are equal to average world productivity, as when $\omega=1$. But $\omega^R\neq 1$ implies that employment and GDP levels now differ across countries — and consumption and GDP can differ in each country. To keep consumption constant at the same level as in the case $\omega=1$ in both countries in a situation in which goods are more highly substitutable, domestic GDP must increase by more, foreign GDP must increase by less (or even decrease if ω^R is sufficiently high). For this reason, domestic labor demand increases after the shock, foreign labor demand falls, the more so the higher the value of ω^R. Thus, the difference between the complete markets case and the benchmark in the response of employment to a permanent shock, which is larger than for a non-permanent shock even for ω^R close to one, becomes substantially wider as goods become better substitutes across countries. Similar conclusions hold for GDP responses: The scenario $\omega=1$ underestimates the response of GDP to the shock. Complete markets overestimate the response. The errors are larger than for non-permanent shocks and become more significant if ω^R increases. Finally, it should be noted that, if $\omega^R\to\infty$, no deterioration in the terms of trade following the shock and no adjustment in labor effort imply that, under the benchmark specification, consumption and GDP increase by the same amount as the productivity shock.

4. Conclusions

This paper studied the role of net foreign asset dynamics in the international propagation of productivity shocks using a model with overlapping generations of infinitely lived households and incomplete asset markets. The results show that models that de-emphasize international transmission through net foreign asset changes (by assuming unitary elasticity of substitution between domestic and foreign goods in consumption and/or complete asset markets) can result in

[36] It is possible to verify that the impact deterioration of the terms of trade in the case $\omega=1$ is always equal to the size of the shock (one percent), regardless of its persistence. When the shock is permanent, RP^{D} falls permanently by one percent. Terms of trade adjustment is the centerpiece of international transmission in the CO-CP model.

non-negligible errors in the description of the economy if this is characterized by incomplete financial markets and an elasticity of substitution that differs from one.[37] The results of this paper call for re-thinking the findings of studies on optimal policy in open economies based on models that de-emphasize asset accumulation.[38]

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Appendix A. The steady state

This Appendix solves for the unique steady state of the model with $n>0$. The consumption Euler equation in aggregate per capita terms (Eq. (6)) features consumption by the newborn generation at time $t+1$. Because households are born with no assets, their consumption in the first period of life is a function only of their human wealth (Eq. (7)). But this also determines a household's initial choice of asset accumulation. Hence, a link is introduced between aggregate per capita consumption growth and asset accumulation in the Euler Eq. (6). The link operates through the discrepancy between assets of agents already alive at each point in time and assets of newborn agents (zero). It is through this link that a unique steady-state level of aggregate per capita net foreign assets is determined.

To see the mechanism at work, focus on the home economy, and set aggregate per capita consumption to be constant in Eq. (6). It is:

$$\bar{c}\left[1-\frac{\beta(1+\bar{r})}{1+n}\right]=\frac{n}{1+n}\bar{C}_{\upsilon}^{\upsilon}, \tag{13}$$

where $\bar{C}_{\upsilon}^{\upsilon}$ is steady-state consumption by a newborn generation in the first period of its life.

From Eq. (7) and the definition of h, it is $\bar{C}_{\upsilon}^{\upsilon}=\rho(1-\beta)\frac{1+\bar{r}}{\bar{r}}\bar{w}$. Hence, aggregate per capita consumption as a function of the steady-state real wage and interest rate is:

$$\bar{c}=\frac{n\rho(1-\beta)(1+\bar{r})}{\bar{r}[1+n-\beta(1+\bar{r})]}\bar{w}. \tag{14}$$

Using $\bar{y}=\bar{w}\bar{L}$, Eq. (14), and steady-state versions of Eqs. (5) and (9) yields:

$$\bar{B}=\frac{1}{\bar{r}-n}\left\{\frac{n(1-\beta)(1+\bar{r})-\bar{r}[1+n-\beta(1+\bar{r})]}{\bar{r}[1+n-\beta(1+\bar{r})]}\right\}\bar{w}. \tag{15}$$

[37] The role of net foreign assets is further enhanced if long-run assets are not zero, consistent with evidence in Lane and Milesi-Ferretti (2001) and other studies by the same authors. See Ghironi, İşcan, and Rebucci (2003).

[38] See Benigno (2001) for a step in this direction.

In turn, substituting Eq. (15) and its foreign counterpart in the asset market equilibrium condition, $a\bar{B}+(1-a)\bar{B}^*=0$, implies:

$$\frac{1}{\bar{r}-n}\left\{\frac{n(1-\beta)(1+\bar{r})-\bar{r}[1+n-\beta(1+\bar{r})]}{\bar{r}[1+n-\beta(1+\bar{r})]}\right\}[a\bar{w}+(1-a)\bar{w}^*]=0. \tag{16}$$

Given non-zero real wages at home and abroad, the only admissible level of the interest rate that satisfies the market clearing condition is $\bar{r}=\frac{1-\beta}{\beta}$.[39] Substituting $\bar{r}=\frac{1-\beta}{\beta}$ into Eq. (15) and its foreign counterpart yields steady-state levels of domestic and foreign net foreign assets $\bar{B}=\bar{B}^*=0$. Consistent with the fact that the two economies are structurally symmetric in per capita terms, the long-run net foreign asset position is zero.

Given $\bar{r}=\frac{1-\beta}{\beta}$, steady-state consumption in Eq. (14) becomes $\bar{c}=\rho\bar{w}$. Hence, the steady-state version of Eq. (5) implies immediately that $\bar{L}=\rho$. Steady-state aggregate per capita employment is vertical in the real wage–employment space. It is determined by the relative importance of consumption and leisure in utility. The more important consumption, the more labor is supplied to generate income that is necessary to finance consumption. Now assume $\bar{Z}=\bar{Z}^*=1$. From $\bar{y}=\overline{RP}\rho=\bar{w}\rho$ and $\bar{y}^*=\overline{RP}^*\rho=\bar{w}^*\rho$, we have $\frac{\bar{y}}{\bar{y}^*}=\frac{\bar{w}}{\bar{w}^*}$. Steady-state labor demands are: $\bar{L}=\overline{RP}^{-\omega}\bar{y}^W=\bar{w}^{-\omega}\bar{y}^W$ and $\bar{L}^*=\overline{RP}^{*-\omega}\bar{y}^W=\bar{w}^{*-\omega}\bar{y}^W$. Imposing labor market clearing and taking the ratio of these equations shows that steady-state real wages (and relative prices) are equal in the two countries ($\bar{w}=\bar{w}^*$), which implies that consumption and GDP levels are also equalized.

To complete the solution for the steady state, we need to determine the level of the real wage. From the previous results, $\bar{y}=\bar{c}=\bar{y}^*=\bar{c}^*=\bar{y}^W=\bar{c}^W$. Hence, labor demand at home can be written as $\bar{L}=\bar{w}^{-\omega}\bar{c}$. Imposing labor market clearing and using $\bar{c}=\rho\bar{w}$ makes it possible to conclude that $\bar{w}=1$ $(=\bar{w}^*)$ if $\omega\neq 1$. (If $\omega=1$, steady-state real wages and relative prices are pinned down by the fact that $\overline{RP}=\overline{RP}^*$ implies a unitary value of the steady-state terms of trade between home and foreign — the ratio $\frac{\overline{RP}}{\overline{RP}^*}$. In this case, one can verify that a Cobb–Douglas consumption basket of the form $C=\frac{C_H^a C_F^{1-a}}{a^a(1-a)^{1-a}}$ implies $\overline{RP}=\overline{RP}^*=1=\bar{w}=\bar{w}^*$.) Thus, consumption equals employment in steady state: In the absence of asset holdings, it is optimal to have equal consumption and effort levels, given that labor is transformed into consumption goods at a unitary rate.[40]

Appendix B. Cross-country differences

In log-linear terms, the difference between domestic and foreign relative prices (the terms of trade) equals the difference between marginal costs: $\mathsf{RP}_t^D=\mathsf{w}_t^D-\mathsf{Z}_t^D$. Hence, relative employment and GDP are, respectively:

$$\mathsf{L}_t^D=-\omega\mathsf{RP}_t^D-\mathsf{Z}_t^D=-\omega\mathsf{w}_t^D+(\omega-1)\mathsf{Z}_t^D, \tag{17}$$

$$\mathsf{y}_t^D=\mathsf{RP}_t^D+\mathsf{L}_t^D+\mathsf{Z}_t^D=-(\omega-1)(\mathsf{w}_t^D-\mathsf{Z}_t^D). \tag{18}$$

[39] To rule out $\bar{r}=n$, observe that this would result in identical tilts in the steady-state consumption and labor supply profiles of home and foreign individual households. (The steady-state version of the Euler Eq. (4) is $\bar{C}_t^v=[\beta(1+\bar{r})]^{-1}\bar{C}_{t+1}^v$. If $\bar{r}\neq\frac{1-\beta}{\beta}$, the steady-state consumption – and labor supply – profiles of individual households are tilted, even if aggregate per capita consumption is constant.) Given constant wages, this would imply that *all* households at home and abroad are either accumulating positive assets or accumulating debt, which would violate asset market clearing.

[40] In steady state, a newborn household's consumption is also equal to ρ. Because $\beta(1+\bar{r})=1$, each household's consumption remains constant at that level as long as the economy is in steady state.

If $\omega=1$, domestic and foreign GDPs are equal ($\mathsf{y}_t = \mathsf{y}_t^*$) regardless of relative productivity.

The labor–leisure tradeoff in Eq. (5) and its foreign counterpart determine the real wage differential as a function of consumption and employment differentials:

$$\mathsf{w}_t^D = \mathsf{c}_t^D + \frac{\rho}{1-\rho}\mathsf{L}_t^D. \tag{19}$$

Combining Eqs. (17) and (19) yields:

$$\mathsf{L}_t^D = -\frac{1-\rho}{1+\rho(\omega-1)}[\omega\mathsf{c}_t^D-(\omega-1)\mathsf{Z}_t^D]. \tag{20}$$

If $\omega=1$, $\mathsf{L}_t^D = -\mathsf{c}_t^D$. In this case, Eqs. (17) and (20) imply $\mathsf{c}_t^D = \mathsf{w}_t^D$. Substituting this result into Eq. (19) yields $\mathsf{L}_t^D = 0$. Thus, it must be the case that, if $\omega=1$, $\mathsf{c}_t = \mathsf{c}_t^*$, $\mathsf{w}_t = \mathsf{w}_t^*$, and $\mathsf{L}_t = \mathsf{L}_t^*$. Consistent with CO-CP, unitary elasticity of substitution ensures that domestic and foreign consumption, the real wage, and employment are equal regardless of productivity.

It is possible to verify that consumption is determined by $c_t=\rho(1-\beta)[(1+r_t)B_t+h_t]$. Log-linearizing this consumption function and its foreign counterpart, taking the difference of the resulting equations, and imposing asset market equilibrium ($\mathsf{B}_t^* = -\frac{a}{1-a}\mathsf{B}_t$ yields:

$$\mathsf{c}_t^D = \frac{\rho(1-\beta)}{\beta(1-a)}\mathsf{B}_t + \mathsf{h}_t^D. \tag{21}$$

The consumption differential in each period reflects the net foreign asset position of the two economies and the differential between domestic and foreign human wealth.

Relative human wealth obeys $\mathsf{h}_t^D = \beta\mathsf{h}_{t+1}^D + (1-\beta)\mathsf{w}_t^D$. Because $\mathsf{w}_t = \mathsf{w}_t^*$ when $\omega=1$, unitary intratemporal elasticity of substitution implies $\mathsf{h}_t = \mathsf{h}_t^*$. Using Eqs. (19), (20), and (21), we obtain the equation for human wealth in the system (10).

Finally, log-linearizing the laws of motion for domestic and foreign net foreign assets yields:

$$\mathsf{B}_{t+1} = \frac{1}{1+n}\left(\frac{1}{\beta}\mathsf{B}_t + \mathsf{y}_t-\mathsf{c}_t\right),\ \mathsf{B}_{t+1}^* = \frac{1}{1+n}\left(\frac{1}{\beta}\mathsf{B}_t^* + \mathsf{y}_t^*-\mathsf{c}_t^*\right). \tag{22}$$

As steady-state assets are zero, changes in the real interest rate have no direct impact on asset accumulation. If $\omega=1$, $\mathsf{y}_t = \mathsf{y}_t^*$ and $\mathsf{c}_t = \mathsf{c}_t^*$. Hence, to preserve asset market equilibrium, it must be $\mathsf{B}_t = \mathsf{B}_t^* = 0\ \forall t$: If $\omega=1$, consumption levels are equalized across countries and current accounts are always zero: $\mathsf{y}_t = \mathsf{c}_t$ and $\mathsf{y}_t^*=\mathsf{c}_t^*$, as in CO-CP. Using Eqs. (18), (19), (20), (21), and asset market equilibrium, the difference between the laws of motion in Eq. (22) yields the net foreign asset equation in Eq. (10).

Appendix C. World aggregates

World aggregates are defined as weighted averages of individual country variables, with weights a for home and $1-a$ for foreign, respectively. The following equations hold:

$$\begin{aligned}&\mathsf{RP}_t^W = \mathsf{W}_t^W-\mathsf{Z}_t^W,\ \mathsf{L}_t^W = -\omega\mathsf{RP}_t^W + \mathsf{y}_t^W-\mathsf{Z}_t^W,\ \mathsf{y}_t^W = \mathsf{RP}_t^W + \mathsf{L}_t^W + \mathsf{Z}_t^W,\\ &\mathsf{y}_t^W = \mathsf{c}_t^W,\ \mathsf{w}_t^W = \mathsf{c}_t^W + \frac{\rho}{1-\rho}\mathsf{L}_t^W.\end{aligned} \tag{23}$$

The equations in (23) constitute a system of five equations in five unknowns (RP_t^W, w_t^W, L_t^W, y_t^W, and c_t^W) as functions of exogenous world productivity, Z_t^W. Since RP_t^W

(the price of a unit of world consumption in units of world consumption) is equal to one in all periods, the solution is (regardless of the value of ω): $\mathsf{RP}_t^W = \mathsf{L}_t^W = 0$, $\mathsf{w}_t^W = \mathsf{y}_t^W = \mathsf{c}_t^W = \mathsf{Z}_t^W$.

Finally, the world interest rate is determined as follows. Using the labor–leisure tradeoff Eq. (5) and the consumption function $c_t = \rho(1-\beta)[(1+r_t)B_t + h_t]$, we can write the law of motion for home net foreign assets as

$$B_{t+1} = \frac{\beta(1+r_t)}{1+n} B_t + \frac{w_t - (1-\beta)h_t}{1+n}.$$

Multiplying this equation by a and its foreign counterpart by $1-a$, adding the resulting equations, and imposing asset market equilibrium yields $w_t^W = (1-\beta)h_t^W$. However, from Eq. (8), $h_t^W = \frac{h_{t+1}^W}{1+r_{t+1}} + w_t^W$. The last two equations together imply $1 + r_{t+1} = \frac{w_{t+1}^W}{\beta w_t^W}$. In log-linear terms, $\mathsf{r}_{t+1} = \mathsf{w}_{t+1}^W - \mathsf{w}_t^W$, or, using $\mathsf{w}_t^W = \mathsf{Z}_t^W$, $\mathsf{r}_{t+1} = \mathsf{Z}_{t+1}^W - \mathsf{Z}_t^W$.

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