



The role of net foreign assets in a New Keynesian small open economy model

Fabio Ghironi^{a,b,c,*}

^a*Boston College, USA*

^b*National Bureau of Economic Research, USA*

^c*Euro Area Business Cycle Network, Belgium*

Received 5 April 2004; accepted 15 May 2007

Available online 14 July 2007

Abstract

This paper develops a small open economy, sticky-price model that determines a unique, stable long-run asset position for households as function of their incentive to anticipate or postpone consumption and labor effort across periods. This is accomplished by adopting an overlapping-generations structure in which new households with no assets enter the economy in each period. The same characteristics of household behavior that determine long-run assets are also important determinants of the model's responses to shocks. Stabilizing producer prices results in a milder recession following a drop in world demand than stabilizing consumer prices because it prevents the markup in the pricing of goods from increasing. In addition, given an initial foreign debt, allowing consumer prices to rise causes a decrease in the *ex post* real interest rate on impact, lowering the interest burden of the initial debt. The differences across policy rules generated by the initial asset position are robust to changes in the latter as long as these are brought about by changes in parameter values that do not alter the fundamental characteristics of household (and firm) behavior that are also the key determinants of long-run assets.

© 2007 Elsevier B.V. All rights reserved.

JEL classification: F41

Keywords: Markup; Model stationarity; Net foreign assets; Small open economy; Sticky prices

*Corresponding author at Department of Economics, Boston College, 140 Commonwealth Avenue, Chestnut Hill, MA 02467-3859, USA. Tel.: +1 617 552 3686; fax: +1 617 552 2308.

E-mail address: Fabio.Ghironi@bc.edu

1. Introduction

What determines long-run net foreign asset positions? How does asset accumulation affect the propagation of shocks under alternative specifications of monetary policy? This paper develops a small open economy, sticky-price model that addresses these questions.

The determination of long-run net foreign asset positions is an important question in international macroeconomics for empirical and theoretical reasons. Lane and Milesi-Ferretti (2001, 2002a, b) provide evidence of non-zero, long-run net foreign assets for a number of countries. Therefore, it is important to understand the determinants of such positions, and how these persistent imbalances and their determinants may influence dynamics in response to shocks. From a theoretical standpoint, it is common practice in international macroeconomics to solve models by log-linearizing them around the deterministic steady state – of which net asset holdings are a central component. However, absent appropriate modifications, familiar representative agent models fail to pin down a unique steady-state level of net foreign assets. Once log-linearized around an initial position that is usually chosen as a matter of convenience, these models result in non-stationary dynamics following temporary shocks, with unfavorable consequences for the reliability of the log-linearization and the feasibility of stochastic analysis.¹

This paper develops a small open economy, sticky-price model that determines a unique, stable long-run asset position for the economy by changing the demographic structure relative to the familiar representative agent framework. The model follows Weil (1989a, b) in assuming that the world economy is populated by distinct, infinitely lived households that come into being on different dates and are born owning no assets. This demographic structure, combined with the assumption that newly born agents have no financial wealth, generates a unique, endogenously determined steady state to which the world economy returns over time following non-permanent shocks. The model, which extends the Weil setup to allow for endogenous labor supply and differences in income across agents of different generations at each point in time, makes it possible to provide a structural interpretation of the determination of long-run asset positions based on the incentives of individual households to anticipate or postpone consumption and labor effort across periods. If the world real interest rate, the subjective discount factor of home households, and other characteristics of the period utility function are such that the steady-state consumption and labor supply profiles of individual home

¹The best known example of this issue is perhaps Obstfeld and Rogoff's (1995) model of macroeconomic interdependence. The current account plays a crucial role in the transmission of shocks in that model. But the failure to pin down a unique steady state causes the consumption differential between countries implied by the model to follow a random walk. So do net foreign assets. The level of asset holdings that materializes in the period immediately following a shock becomes the new long-run position. In Obstfeld and Rogoff's sticky-price model, this results in long-run non-neutrality of money. As Schmitt-Grohé and Uribe (2003) point out, in stochastic models, the unconditional variances of endogenous variables are infinite, even if exogenous shocks are bounded.

households display upward and downward tilts, respectively, home households accumulate a positive steady-state asset balance.²

To illustrate the functioning of the model, I set structural parameters to values that are common in the literature and show that the model delivers plausible predicted long-run properties. I then analyze how a decrease in world demand is transmitted to the home economy under alternative price stability rules.³ The exercise highlights the role of both asset holdings and markup dynamics in the transmission of shocks. Stabilizing producer prices results in a milder recession following a drop in world demand than a rule that stabilizes consumer prices for two reasons. First, it prevents a markup increase that has unfavorable effects on factor demands when the policymaker targets consumer prices. Second, given an initial foreign debt, stabilizing producer prices and allowing consumer prices to rise after the shock ameliorates the consequences of the recession by causing a decrease in the *ex post* real interest rate on impact, thus lowering the interest burden of the initial debt in the period of the shock. Studying dynamics for different initial asset holdings shows that the differences across price stability rules generated by the initial asset position are robust to changes in the latter as long as these are brought about by changes in parameter values that do not alter the fundamental characteristics of household (and firm) behavior that are also the key determinants of long-run assets.

By including monopolistic competition and sticky prices, this paper contributes to the recent literature on New Keynesian, open economy models, much of which de-emphasizes the role of asset accumulation in favor of analytical tractability, with the analysis of a New Keynesian model in which asset accumulation plays an important role. Scholars of international macroeconomics had soon recognized the indeterminacy/non-stationarity problem of the standard representative agent model as developed in Obstfeld and Rogoff's (1995) seminal article. Determinacy of the steady state and stationarity fail in the model because the average rate of growth of consumption implied by the Euler equation does not depend on average holdings of net foreign assets. Hence, setting consumption to be constant does not pin down steady-state asset holdings. This makes the choice of the economy's initial position for the purpose of analyzing the consequences of a shock a matter of convenience, with the unfavorable consequences mentioned above. Some scholars decided to dismiss the issue.⁴ Others tried to finesse it in various ways. For example, Corsetti and Pesenti (2001) build on insights in Cole and Obstfeld (1991) and develop a version of the Obstfeld–Rogoff model in which the intratemporal elasticity of substitution between domestic and foreign goods in consumption is equal to one.

²The world real interest rate is exogenous to the small open economy. Ghironi et al. (2008) extend the model to the case of heterogeneous discount factors across countries of comparable size and endogenous world interest rate. There, period utility is logarithmic. This paper shows that the general properties of the Weil setup hold under a more general, non-separable specification of utility, which allows for a time-varying consumption-to-wealth ratio and is more appealing on quantitative grounds.

³Results for additional shocks and policy rules are available on request.

⁴See Lane (2001) for a survey of the initial literature following Obstfeld and Rogoff's article.

Under this assumption, the current account does not react to shocks if the initial net foreign asset position is zero, and thus net foreign asset accumulation plays no role in the international business cycle. The dynamics of the terms of trade are the centerpiece of international adjustment in Corsetti and Pesenti's model.

Nevertheless, the Corsetti–Pesenti setup shares the indeterminacy of the steady state with the original Obstfeld–Rogoff model. There too, setting consumption to be constant does not pin down steady-state asset holdings, for the same reason mentioned above. The choice of a zero-asset initial equilibrium, combined with the assumption on the elasticity of substitution between domestic and foreign goods, allows Corsetti and Pesenti to (*de facto*) shut off the current account channel. This makes stochastic analysis possible in a highly tractable framework, but at a cost in terms of realism. Any initial asset position that differs from zero brings the non-stationarity back to the surface. But the assumption that fluctuations happen around a steady state with non-zero assets is reasonable given the evidence in Lane and Milesi-Ferretti (2001, 2002a, b). In addition, the trade literature abounds with estimates significantly above one for the elasticity in question (Feenstra, 1994; Harrigan, 1993; Lai and Trefler, 2002; Shiells et al., 1986).

An alternative way of dealing with the non-stationarity problem by de-emphasizing the role of net foreign asset dynamics in the transmission of shocks consists of assuming that financial markets are internationally complete. With complete markets, power utility, and unitary elasticity of substitution between domestic and foreign goods, the current account does not react to shocks in two-country models with zero initial net wealth. If the elasticity of substitution between domestic and foreign goods differs from one, the current account moves in response to output differences (even though perfect risk sharing ensures that the cross-country consumption differential is zero if purchasing power parity (PPP) holds). However, history independence of the equilibrium allocation ensures that net foreign assets are determined residually and their dynamics play no active role in shock transmission.⁵ Like the Corsetti–Pesenti specification, market completeness yields highly tractable models suitable for stochastic analysis at a cost in terms of realism. As pointed out in Obstfeld and Rogoff (2001), the complete markets assumption is at odds with empirical evidence. Benigno and Thoenissen (2007), Corsetti et al. (2008), and Duarte and Stockman (2005), among others, argue that market incompleteness is a necessary ingredient of models that aim to explain important puzzles in international finance.

Entry of new households with no assets in each period solves the determinacy/non-stationarity problem under incomplete markets by introducing a connection between aggregate per capita consumption growth and asset holdings through the discrepancy between the assets of newborn agents (zero) and those of older households. Schmitt-Grohé and Uribe (2003) survey alternative solutions to the issue that rely on representative agent models while preserving a role for the current account. I discuss these approaches below. The main advantages of the approach in this paper are that it does not require any assumption on the functional form of a

⁵For instance, see Benigno, G. (2004), Benigno and Benigno (2007), and Galí and Monacelli (2005).

cost of adjusting asset holdings or an endogenous discount factor and it provides a fully structural interpretation for the determination of long-run asset positions, with implications also for off-steady-state dynamics.

The structure of the paper is as follows. Section 2 presents the model. Section 3 analyzes the determination of steady-state assets. Section 4 illustrates model dynamics in response to a decrease in world demand. Section 5 concludes.

2. The model

The world consists of two countries, home and foreign. I denote foreign variables with an asterisk and world variables with a superscript W. In each period t , the world economy is populated by a continuum of distinct, infinitely lived households between 0 and N_t^W . Each of these households consumes, supplies labor, and holds bonds and shares in firms. Following Weil (1989a, b), I assume that households come into being on different dates and are born owning no financial assets. The number of households in the home economy (N_t) grows over time at the exogenous rate n , so that $N_{t+1} = (1+n)N_t$. I normalize the size of a household to 1, so that the number of households alive at each point in time is also the economy's population. Foreign population grows at the same rate as home. I assume that the ratio N_t/N_t^* is sufficiently small that home's population is small relative to the rest-of-the-world's. The world economy has existed since the infinite past. I normalize world population at time 0 so that $N_0^W = 1$.

At time 0, the number of households in the world economy equals the number of goods that are supplied. A continuum of goods $i \in [0, 1]$ is produced in the world by monopolistically competitive, infinitely lived firms, each producing a single differentiated good. Over time, the number of households grows, but the commodity space remains unchanged. Thus, as time goes, the ownership of firms spreads over a larger number of households. Profits are distributed to consumers as dividends. The structure of the market for each good is given. The domestic economy produces goods in the interval $[0, a]$, which is also the size of the home population at time 0. The foreign economy produces goods in the range $(a, 1]$. The constant ratio N_t/N_t^* equals $a/(1-a)$. Thus, the assumption that N_t/N_t^* is small is sufficient to ensure that home produces a small share of the goods available for consumption in each period.

For simplicity, and consistent with evidence of home bias in equity markets, I assume that only home (foreign) households hold shares of home (foreign) firms. Nominal, uncontingent bonds are the only internationally traded assets. Each country issues bonds denominated in units of the country's currency.

2.1. Households

Agents have perfect foresight, though they can be surprised by initial unexpected shocks. Households have identical preferences over consumption, C , and leisure, $1-L$, where I normalize the endowment of time in each period to one and L denotes

labor effort supplied in a competitive labor market.⁶ At time t_0 , the representative home consumer j born in period $v \in [-\infty, t_0]$ maximizes the intertemporal utility function:

$$U_{t_0}^{v,j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} [C_t^{v,j} \rho (1 - L_t^{v,j})^{1-\rho}]^{1-(1/\sigma)} / \left(1 - \frac{1}{\sigma}\right), \tag{1}$$

where β and ρ are strictly between zero and one, and $\sigma > 0$.⁷

The consumption basket for the representative domestic consumer born in period v is

$$C_t^{v,j} = \left(\int_0^a c_t^{v,j}(i)^{\frac{\theta-1}{\theta}} di + \int_a^1 c_{t^*}^{v,j}(i)^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \tag{2}$$

with $\theta > 1$. $c_t^{v,j}(i)$ ($c_{t^*}^{v,j}(i)$) is consumption of good i produced in the home (foreign) country. Since a is small, the share of domestic goods in the consumption basket is small. Foreign agents have identical preferences for consumption.⁸

The assumptions that the domestic population is small relative to the rest-of-the-world's, the number of goods produced in the home economy is small, and the relative weight of foreign goods in the consumption basket is large – combined with free international borrowing and lending – are equivalent to the assumption that home is a small open economy, which has a negligible impact on the rest of the world.

Letting $p_t(i)$ ($p_t^*(i)$) be the home (foreign) currency price of good i , the consumption-based price indexes (CPIs) at home and abroad are, respectively:

$$P_t = \left(\int_0^1 p_t(i)^{1-\theta} di \right)^{1/(1-\theta)}, \quad P_t^* = \left(\int_0^1 p_t^*(i)^{1-\theta} di \right)^{1/(1-\theta)}. \tag{3}$$

There are no impediments to trade. Firms have no incentive to price discriminate across markets, and the law of one price holds for each individual good.⁹ Letting ε denote the domestic currency price of one unit of foreign currency, it is $p_t(i) = \varepsilon_t p_t^*(i)$. The law of one price and identical consumption preferences across countries imply consumption-based PPP, i.e., $P_t = \varepsilon_t P_t^*$.¹⁰

⁶Since monetary policy is conducted by setting the nominal interest rate below, I do not model the demand for cash currency, and I assume a cashless economy as in Woodford (2003).

⁷Among others, Sbordone (2001) has demonstrated the importance of non-separable preferences over consumption and leisure for quantitative business cycle models.

⁸I assume the same elasticity of substitution, θ , across all goods, domestic and foreign, in both countries. Corsetti and Pesenti (2001) assume unitary elasticity of substitution between domestic and foreign sub-baskets to remove current account fluctuations. The empirical trade literature finds values as high as 12 for the elasticity of substitution between US imports and exports. This upper bound is reasonable for the elasticity that determines the steady-state markup of prices over marginal cost in the numerical exercise below. Hence, I assume the same elasticity of substitution between all goods to simplify the analysis, given the intent to keep a role for net foreign asset dynamics.

⁹The assumption that consumers born at different points in time have the same preferences ensures that firms have no incentives to price discriminate across consumers of different ages.

¹⁰Much literature following Obstfeld and Rogoff (1995) focused on the law of one price and PPP as the main weakness of the setup and extended it to allow for deviations. For example, see Benigno, G. (2004) and Betts and Devereux (2000). I retain PPP to keep the analysis relatively simple for the purposes of this paper. (PPP is also a standard, albeit implicit assumption of small open economy, international real business cycle models, such as those in Schmitt-Grohé and Uribe, 2003.)

Let V_t^i denote the date t price of a claim to the representative domestic firm i 's entire future profits (starting on date $t+1$) in units of home currency and D_t^i be the nominal dividends the firm issues on date t . Let $x_{t+1}^{v,ji}$ denote the share of the representative domestic firm i owned by the representative domestic consumer j born in period v at the end of period t . The consumer enters period t holding nominal bonds issued in the two countries and shares purchased during $t-1$. He or she receives interest and dividends on the assets, earns capital gains or losses on shares, earns labor income, and consumes. Savings are divided between increases in bond holdings and the value of shares to be carried into the next period. Letting $A_{t+1}^{v,j}$ ($A_{t+1}^{v,j*}$) denote holdings of domestic (foreign) bonds entering time $t+1$, the period budget constraint in units of domestic currency is

$$\begin{aligned} A_{t+1}^{v,j} - A_t^{v,j} + \varepsilon_t \left(A_{t+1}^{v,j*} - A_t^{v,j*} \right) + \int_0^a \left(V_t^i x_{t+1}^{v,ji} - V_{t-1}^i x_t^{v,ji} \right) di \\ = i_t A_t^{v,j} + \varepsilon_t i_t^* A_t^{v,j*} + \int_0^a D_t^i x_t^{v,ji} di + \int_0^a \left(V_t^i - V_{t-1}^i \right) x_t^{v,ji} di + W_t L_t^{v,j} - P_t C_t^{v,j}, \end{aligned} \quad (4)$$

where i_t (i_t^*) is the nominal interest rate on bonds denominated in home (foreign) currency between $t-1$ and t (determined at $t-1$). W_t is the nominal wage paid for one unit of labor, taken as given by workers. Newly born individuals are not linked by altruism to individuals born in previous periods. Hence, individuals are born owning no financial wealth or cash balances (although they are born owning the present discounted value of their labor income): $A_v^{v,j} = A_{v*}^{v,j} = x_v^{v,ji} = 0$. This assumption is crucial to ensure that the model has an endogenously determined steady state, to which the economy returns following temporary shocks.

Dropping the j superscript, because symmetric agents make identical equilibrium choices, optimal supply of labor is determined by a standard labor-leisure tradeoff equation:

$$L_t^v = 1 - (1 - \rho) C_t^v / (\rho W_t / P_t). \quad (5)$$

Euler equations for domestic and foreign bonds yield uncovered interest parity (UIP):

$$1 + i_{t+1} = (1 + i_{t+1}^*) \varepsilon_{t+1} / \varepsilon_t. \quad (6)$$

Letting r_{t+1} (r_{t+1}^*) denote the home (foreign) consumption-based real interest rate between t and $t+1$, Fisher parity conditions imply $1 + i_{t+1} = (1 + r_{t+1}) P_{t+1} / P_t$ and $1 + i_{t+1}^* = (1 + r_{t+1}^*) P_{t+1}^* / P_t^*$. Thus, UIP and PPP yield real interest rate equalization: $r_{t+1} = r_{t+1}^*$. Because home is small, i_{t+1}^* and the world *ex ante* real interest rate r_{t+1} are exogenous to the home economy.

The Euler equation for holdings of domestic bonds reduces to

$$C_t^v = \frac{1}{\beta^\sigma (1 + r_{t+1})^\sigma} C_{t+1}^v \left(\frac{W_t / P_t}{W_{t+1} / P_{t+1}} \right)^{(1-\rho)(1-\sigma)}, \quad v \leq t. \quad (7)$$

Unless $\sigma = 1$, in which case period utility is additively separable in consumption and leisure, consumption growth depends on real wage growth.

Consumers are indifferent between bonds and shares so long as the gross rate of return on shares equals the gross real interest rate:¹¹

$$1 + r_{t+1} = \frac{V_{t+1}^i + D_{t+1}^i}{V_t^i} \frac{P_t}{P_{t+1}}. \tag{8}$$

2.2. Firms

2.2.1. Output supply

Production requires labor and physical capital, K . Capital is a composite good, defined as the consumption bundle. Output supplied by the representative domestic firm i at time t is

$$Y_t^{Si} = Z_t K_t^{f^i} (E_t L_t^i)^{1-\gamma}. \tag{9}$$

It is not necessary to index production and factor demands by a “date of birth” because all firms have existed since the infinite past. Z_t is an economy-wide, exogenous productivity shock. E_t is exogenous, worldwide, labor-augmenting technological progress, such that $E_t = (1 + g)E_{t-1}$ (g will be the steady-state rate of growth of aggregate per capita output). I assume $1 + r > (1 + n)(1 + g)$, where r is the steady-state world real interest rate, to ensure stability.

2.2.2. Output demand and price stickiness

Output demand comes from several sources. Maximizing C^v subject to a spending constraint yields the demands of goods produced in the two countries by the representative home consumer born in period v : $c_t^v(h) = (p_t(h)/P_t)^{-\theta} C_t^v$ and $c_t^v(f) = (p_t(f)/P_t)^{-\theta} C_t^v$, respectively. Identity of preferences implies analogous expressions for foreign consumers’ demands.

At time t , total demand for home good i coming from the aggregate of domestic consumers alive in that period is

$$\begin{aligned} c_t(i) &= a \left[\dots + \frac{n}{(1+n)^{t+1}} c_t^{-t}(i) + \dots + \frac{n}{(1+n)^2} c_t^{-1}(i) + \frac{n}{1+n} c_t^0(i) \right] \\ &\quad + n c_t^1(i) + n(1+n) c_t^2(i) + \dots + n(1+n)^{t-1} c_t^t(i) \\ &= \left(\frac{p_t(i)}{P_t} \right)^{-\theta} [a(1+n)^t C_t], \end{aligned} \tag{10}$$

¹¹As usual, first-order conditions and the period budget constraint must be combined with appropriate transversality conditions (omitted) to ensure optimality.

where C_t is aggregate per capita home consumption of the composite basket.¹²

$$C_t \equiv \frac{a \left[\dots + \frac{n}{(1+n)^{t+1}} C_t^{-t} + \dots + \frac{n}{(1+n)^2} C_t^{-1} + \frac{n}{1+n} C_t^0 + n C_t^1 + n(1+n) C_t^2 + \dots + n(1+n)^{t-1} C_t^t \right]}{a(1+n)^t} \tag{11}$$

Similarly, total demand for home good i by foreign consumers is

$$c_t^*(i) = \left(\frac{p_t(i)}{P_t} \right)^{-\theta} [(1-a)(1+n)^t C_t^*], \tag{12}$$

where C_t^* is foreign aggregate per capita consumption.

Capital accumulation obeys

$$K_{t+1}^i - K_t^i = I_t^i - \delta K_t^i. \tag{13}$$

I_t^i is investment and $\delta \in (0,1)$ is the rate of depreciation. Investment is a composite basket of all goods produced in the world economy, defined as the consumption basket.

Adjusting the capital stock is costly. I assume that the firm must purchase materials in the amount $CAC_t^i = \eta I_t^i / (2K_t^i)$, $\eta > 0$, to install new capital and make it operational. As usual, the cost is convex in the amount of investment and allows for the fact that larger firms (firms with a larger capital stock) can absorb a given amount of new capital at a lower cost.¹³

Changing the output price is another source of costs. I follow Rotemberg (1982) and assume a quadratic cost of output price inflation volatility around a steady-state level $\bar{\pi}$. The real cost for firm i is $PAC_t^i = \phi [(p_t(i)/p_{t-1}(i)) - 1 - \bar{\pi}]^2 K_t^i / 2$, $\phi \geq 0$. We can think of PAC_t^i as the amount of marketing materials that the firm must purchase when implementing a price change, which increases with the size of the firm and the size of the price change.¹⁴

Total demand of good i produced in the home country follows from adding the demands for that good originating in the two countries. It is:

$$Y_t^{Di} = (p_t(i)/P_t)^{-\theta} Y_t^{AWD}, \tag{14}$$

¹²To understand the aggregation formula, note that vintage $v = 0$ of home consumers, born at time 0, has size a . Therefore, total home population in period 1 is $a(1+n)$, of which an individuals are new-born. In period 2, population contains $N_2 - N_1 = an(1+n)$ individuals born in that period. Continuing with this reasoning shows that generation t consists of $an(1+n)^{t-1}$ households. Going back in time from $t = 0$, population at time -1 is $a/(1+n)$. Hence, generation 0 consists of $an/(1+n)$ households. And so on. Vintage $-t$ consists of $an/(1+n)^{t+1}$ households.

¹³I assume it is equally costly to replace depreciated capital as to install additional new capital. The demographic structure of the model ensures that this does not pose problems for existence and stability of the steady state. The assumption has no important consequence for the key properties of the model.

¹⁴Carré and Collard (2003), Hairault and Portier (1993), and Ireland (2001) adopt a similar specification.

where Y_t^{AWD} is *aggregate* (as opposed to *aggregate per capita*) world demand of the composite good – the sum of aggregate world consumption, investment, and the costs of adjusting capital and prices: $Y_t^{AWD} \equiv C_t^{AW} + I_t^{AW} + CAC_t^{AW} + PAC_t^{AW}$.

2.2.3. *Optimality conditions*

The real dividends issued by the representative domestic firm in period t are equal to revenues minus costs:

$$\frac{D_t^i}{P_t} = \frac{p_t(i)}{P_t} Y_t^i - \left[\frac{W_t}{P_t} L_t^i + I_t^i + \frac{\eta (I_t^i)^2}{2 K_t^i} + \frac{\phi}{2} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 - \bar{\pi} \right)^2 K_t^i \right].$$

At time t , the firm chooses the domestic currency price of its product, labor, investment, and capital to maximize the present discounted value of current and future dividends subject to constraints (9), (13), (14), and the market clearing condition $Y_t^{Si} = Y_t^{Di}(= Y_t^i)$.¹⁵ The firm takes the wage, the aggregate price index, Z , E , and world aggregates as given.

Let λ_t^i denote the Lagrange multiplier on the market clearing constraint $Y_t^{Si} = Y_t^{Di}(= Y_t^i)$. The first-order condition with respect to $p_t(i)$ returns the pricing equation:

$$p_t(i) = \Psi_t^i P_t \lambda_t^i. \tag{15}$$

The price of good i equals the product of a markup (Ψ_t^i) times the nominal shadow value of an extra unit of output (nominal marginal cost, $P_t \lambda_t^i$). Symmetric home firms make identical choices in equilibrium. Therefore, $p_t(i)$ is also the home economy’s producer price index (PPI).¹⁶

The markup Ψ_t^i depends on output demand and the impact of today’s pricing decision on today’s and tomorrow’s cost of adjusting the output price:

$$\Psi_t^i \equiv \theta Y_t^i \left\{ (\theta - 1) Y_t^i + \phi \frac{P_t}{p_t(i)} \left[\begin{array}{c} K_t^i \frac{p_t(i)}{p_{t-1}(i)} \left(\frac{p_t(i)}{p_{t-1}(i)} - 1 - \bar{\pi} \right) \\ - \frac{K_{t+1}^i}{(1 + r_{t+1})} \frac{p_{t+1}(i)}{p_t(i)} \left(\frac{p_{t+1}(i)}{p_t(i)} - 1 - \bar{\pi} \right) \end{array} \right] \right\}^{-1}. \tag{16}$$

If $\phi = 0$ (if prices are flexible), Ψ_t^i reduces to $\theta/(\theta-1)$. Introducing price rigidity ($\phi > 0$) generates endogenous markup fluctuations as firms find it optimal to smooth price changes and absorb changes in marginal costs through changes in the markup. However, if θ is very large, the markup tends to 1, and prices tend to reflect marginal costs.

¹⁵The objective of the firm follows from forward iteration of Eq. (8) to obtain the real value of the firm as the present discounted value of future dividends. The firm maximizes this expression plus current dividends.

¹⁶I keep the superscript i for individual firm variables in this section because later I denote aggregate per capita levels of some of these variables by dropping the superscript.

Once numerator and denominator of the markup expression in (16) are written in terms of detrended, aggregate per capita quantities (which converge to well defined steady-state levels – see below), log-linearizing Eq. (16) around the steady state with inflation $\bar{\pi}$ yields a New Keynesian Phillips curve that relates the current markup to current and future PPI inflation. In a closed economy, PPI and CPI coincide in equilibrium, and the New Keynesian Phillips curve can be rewritten in terms of current marginal cost and current and future inflation using the fact that the markup is the reciprocal of marginal cost.¹⁷ In an open economy, the New Keynesian Phillips curve can be rewritten in terms of PPI inflation, marginal cost, and the relative price of the representative domestic good, $RP_t \equiv p_t(i)/P_t$, here a measure of the economy's terms of trade.¹⁸ Note that monopoly power of home firm i over its product implies that, even if home is a small open economy, the terms of trade are endogenous to domestic economic developments.

Combining the first-order condition for L_t^i with Eq. (16) yields

$$\frac{W_t}{P_t} = \frac{p_t(i)}{P_t \Psi_t^i} (1 - \gamma) \frac{Y_t^i}{L_t^i}. \quad (17)$$

Monopoly power causes firms to increase the value of the marginal product of labor above the real wage and demand less labor than under perfect competition. The wedge between real wage and value of marginal product then fluctuates with changes in the markup due to price stickiness.

The first-order condition for I_t^i implies that firm i 's investment is positive if and only if the shadow value of an extra unit of capital in place at the end of period t (q_t^i) is larger than 1:

$$I_t^i = K_t^i (q_t^i - 1) / \eta. \quad (18)$$

In turn, q_t^i obeys the difference equation:

$$q_t^i = \left(\frac{1}{1 + r_{t+1}} \right) \left[q_{t+1}^i (1 - \delta) + \frac{W_{t+1}}{P_{t+1}} \frac{\gamma}{1 - \gamma} \frac{L_{t+1}^i}{K_{t+1}^i} + \frac{\eta}{2} \left(\frac{I_{t+1}^i}{K_{t+1}^i} \right)^2 - \frac{\phi}{2} \left(\frac{p_{t+1}(i)}{p_t(i)} - 1 - \bar{\pi} \right)^2 \right]. \quad (19)$$

The shadow price of a unit of capital in place at the end of period t is the discounted sum of the shadow price of capital at time $t + 1$ net of depreciation, the shadow value of the incremental output generated by capital at $t + 1$, and the marginal contribution of capital in place at the end of period t to the costs of installing capital and changing the price of the firm's output at time $t + 1$.

¹⁷This establishes the isomorphism found by Roberts (1995) between the Rotemberg (1982) specification of price stickiness and Yun's (1996) rendition of the Calvo (1983) model.

¹⁸The terms of trade are actually given by $p_t(i)/(e_t p_t^*(f))$, where $p_t^*(f)$ is the foreign PPI. Under the assumptions of this paper, the fraction of domestic goods in the world consumption bundle is negligible. Hence, $p_t^*(f)$ is only negligibly different from P_t^* . Because of purchasing power parity, $p_t(i)/(e_t P_t^*) = p_t(i)/P_t$. See also Benigno and Benigno (2007).

The solution for q can be written as

$$q_t^i = \left[\frac{V_t^i}{P_t} + \sum_{s=t+1}^{\infty} R_{t,s} \left(\frac{1}{\Psi_s^i} - 1 \right) \frac{P_s(i)}{P_s} Y_s^i \right] / K_{t+1}^i, \tag{20}$$

where $R_{t,s} \equiv 1/\prod_{u=t+1}^s (1 + r_u)$. This result mirrors that of Hayashi (1982). The ratio of real equity value to capital, $(V_t^i/P_t)/K_{t+1}^i$, is the so-called average q . Under perfect competition (when θ is infinite), the markup reduces to 1, and marginal and average q coincide. The shadow value of an additional unit of capital at the end of period t is smaller than average q under monopolistic competition because a larger capital stock causes production to increase and the output price to decrease. This conflicts with a monopolist’s incentive to keep the price higher and supply less output than in the absence of monopoly power. Markup fluctuations affect investment decisions by generating fluctuations in the discrepancy between average and marginal q .

2.3. Monetary policy and markup dynamics

Monetary policy is conducted by setting the nominal interest rate i_{t+1} according to interest setting rules discussed below. Different rules generate different CPI inflation. Because firms react to CPI dynamics in their price setting (Eq. (15)), different CPI inflation translates into different producer prices and markup (Eq. (16)). Changes in the latter affect labor demand and investment (Eqs. (17), (18), and (20)). In turn, labor market equilibrium and the labor-leisure tradeoff tie labor demand to consumption. Hence, inflation affects consumption via its impact on the markup. Put differently, alternative policy rules can affect the real economy by causing differences in the behavior of the relative price of the representative domestic good, RP_t . As observed in Goodfriend and King (1997), markup movements could be removed completely by stabilizing producer price inflation at the steady state.

2.4. Monetary policy and ex post asset returns

Markup fluctuations are the main channel through which different monetary regimes have different implications for business cycles and welfare in a small open economy in which the domestic *ex ante* real interest rate is tied to the world rate, which is exogenous to the home economy. However, there is an additional channel through which domestic monetary policy can affect the economy in the model of this paper, associated to the effects of unexpected variation in the *ex post* real interest rate at the time of a shock.

The home *ex ante* real interest rate is tied to the foreign one in all periods. Thus, in all periods *after* an initial shock, *ex ante* and *ex post* real interest rates coincide, as there is no further unexpected shock. However, *ex post* real returns can differ across countries in the period of an unexpected shock. Suppose the shock happens at time 0, after which no new shock happens. Let a hat denote percentage deviations from steady state below. For inflation, depreciation, and interest rates, the hat denotes the percentage deviation of gross rates from the steady state. Then, real interest rates are

such that $\hat{r}_0 = \hat{i}_0 - \hat{\pi}_0^{\text{CPI}}$ and $\hat{r}_0^* = \hat{i}_0^* - \hat{\pi}_0^{\text{CPI}*}$, where \hat{i}_0 and \hat{r}_0 are the home nominal and real interest rates between periods -1 and 0 , respectively, and the same timing notation holds for foreign rates. From the perspective of period -1 (*ex ante*), real interest rates at home and abroad are equal: $\hat{r}_0 = \hat{r}_0^* = 0$. *Ex post*, the realized real returns at time 0 are such that $\hat{r}_0^{\text{EP}} - \hat{r}_0^{\text{EP}*} = -(\hat{\pi}_0^{\text{CPI}} - \hat{\pi}_0^{\text{CPI}*}) = -\hat{e}_0 = -\hat{e}_0$, where \hat{e} and \hat{e} denote the percentage deviations of gross depreciation and the exchange rate from the steady state, respectively, and the superscript EP stands for *ex post*. Since nominal interest rates between periods -1 and 0 are determined at time -1 , before any shock happens, they are set at the respective steady-state levels. It follows that $\hat{i}_0 = \hat{i}_0^* = 0$, and *ex post* real returns at time 0 at home and abroad equal the negative of the respective CPI inflation rates. PPP then implies that the *ex post* real interest rate differential is equal to the negative of depreciation at time 0 (or the negative of the deviation of the exchange rate from the steady state at time 0). In a nutshell, real interest rate equalization follows from UIP and PPP, but UIP can be violated *ex post* at the time of an unexpected shock. This *ex post* real interest rate differential disappears after period 0 , since no new unexpected shock happens, and the *ex post* real interest rate coincides with the *ex ante* rate.

Albeit short-lived, the time 0 difference in *ex post* real returns across countries provides an additional channel through which home's monetary policy can affect the economy, as the initial movement of the home exchange rate is determined by nominal interest rate setting from time 0 on at home and abroad via UIP. For instance, if a shock causes home CPI inflation to increase on impact and the home currency depreciates, the time 0 domestic, *ex post* real interest rate is below the steady state. If the country's steady-state net foreign asset position is negative, a lower than expected real interest rate decreases the real interest burden of existing foreign debt. Importantly, this channel for real effects of monetary policy operates also if goods prices are flexible, and it is at work in the log-linearized model if the initial steady-state asset position is different from zero.¹⁹

3. Steady-state determinacy and model stationarity

Steady-state determinacy and model stationarity fail for an open economy whenever the equilibrium rate of aggregate per capita consumption growth is independent of the economy's aggregate per capita net foreign assets. In that case, the requirement that consumption be constant in steady state does not determine a unique steady state for net foreign assets. The steady state of the model presented here is determined endogenously by the structural parameters and is stable, if appropriate conditions are satisfied. Aggregate per capita consumption growth depends on aggregate per capita net foreign assets because of the discrepancy between the financial wealth of the newly born (zero) and the aggregate per capita financial wealth of those already alive.

¹⁹If initial assets are zero, the effect of *ex post* return or interest burden variation drops from the log-linearized model.

In this section, I focus on the determination of the constant steady-state level of home consumers' detrended, aggregate per capita, real, net asset holdings, $b_t \equiv B_t / (E_t P_{t-1})$. (B_{t+1} denotes aggregate per capita household assets entering $t + 1$, obtained by aggregating individual household assets, $B_{t+1}^v \equiv A_{t+1}^v + \varepsilon_t A_{t+1}^{v*} + a V_t x_{t+1}^v$, across generations and dividing by population.) The analysis clarifies how the demographic structure of the model and the assumption that newly born households have no financial wealth (so that Ricardian equivalence does not hold) play a crucial role for determinacy of the steady state.²⁰

The derivation of a law of motion for consumers' aggregate per capita assets that takes the optimal path of consumption into account is more complicated in this model, in which labor supply is governed by a labor-leisure tradeoff equation, than in Weil's (1989a, b) or Obstfeld and Rogoff's (1996, Chap. 3.7), where labor income is exogenous.

If income is exogenous, one can assume that agents of different generations have identical income at each point in time.²¹ Under this hypothesis, aggregate per capita income at each point in time is equal to each individual household's income. But assuming identical incomes for agents of different ages would be wrong here. Given that all agents face the same wage rate, the assumption would imply that agents of different generations are supplying the same amount of labor. By the labor-leisure tradeoff condition, this would require agents born at different dates to have identical consumption levels, which cannot be true in general, given that agents of different generations have accumulated different amounts of assets. The impossibility of constant labor income across generations complicates the solution of the model. This notwithstanding, the complications can be dealt with by using the consumption-Euler equation and the labor-leisure tradeoff condition. Combining these equations with the intertemporal budget constraint for the representative household of vintage v yields

$$C_t^v + (1 - L_t^v)W_t/P_t = \Theta_t^{-1} \left[(1 + r_t)B_t^v/P_{t-1} + \sum_{s=t}^{\infty} R_{t,s}W_s/P_s \right], \tag{21}$$

where $\Theta_t \equiv \sum_{s=t}^{\infty} \beta^{\sigma(s-t)} (R_{t,s})^{1-\sigma} [(W_t/P_t)/(W_s/P_s)]^{(1-\rho)(\sigma-1)}$ and $R_{t,t} \equiv 1$. Θ_t^{-1} can thus be interpreted as a generalized propensity to consume goods and leisure out of the agent's resources: assets entering period t , interest income, and human wealth, defined as the present discounted value of the household's remaining endowment of time in terms of the real wage.²²

Eq. (21) and the intertemporal budget constraint for the representative household of generation v imply

$$B_{t+1}^v/P_t = (1 + r_t)(1 - \Theta_t^{-1})B_t^v/P_{t-1} + W_t/P_t - \Theta_t^{-1} \sum_{s=t}^{\infty} R_{t,s}W_s/P_s. \tag{22}$$

²⁰I assume that the process for the nominal interest rate i converges to a steady state and that productivity (Z_t) is stationary around a steady-state value of 1.

²¹This is Weil's (1989a, b) and Blanchard's (1985) assumption, as well as Obstfeld and Rogoff's (1996, Chap. 3.7).

²²Note that $\Theta_t = 1/(1-\beta)$ in the case of logarithmic utility ($\sigma = 1$).

This equation expresses asset accumulation by the representative household born in generation v as a function of the paths of the real wage and the real interest rate, which do not depend on the household's date of birth. Eq. (22) extends the results in Weil (1989a, b), Obstfeld and Rogoff (1996, Chap. 3.7), and Blanchard (1985) to the case of endogenous labor income and time-varying consumption-to-wealth ratio.

Applying the aggregation procedure in Section 2.2.2 to Eq. (22) (recalling that households are born with no assets) and dividing both sides by trend productivity E_t , yields the law of motion for detrended, aggregate per capita, real assets held by home consumers:

$$b_{t+1} = \frac{(1+r_t)(1-\Theta_t^{-1})}{(1+g)(1+n)} b_t + \frac{w_t - \Theta_t^{-1} \sum_{s=t}^{\infty} R_{t,s}(1+g)^{s-t} w_s}{(1+g)(1+n)}, \tag{23}$$

where Θ_t has been re-defined as

$$\Theta_t \equiv \sum_{s=t}^{\infty} \beta^{\sigma(s-t)} (R_{t,s})^{1-\sigma} (1+g)^{(1-\rho)(1-\sigma)(s-t)} (w_t/w_s)^{(1-\rho)(\sigma-1)}$$

and $w_t \equiv W_t/(E_t P_t)$. If the (time-varying) slope coefficient in (23) is smaller than 1 and the forcing function (which depends on the path of real interest rate and wage) converges to a finite value, home consumers' assets, b_{t+1} , converge to a steady-state level starting from any initial position.²³

The steady state of the home economy is characterized by a constant detrended real wage, determined by labor market clearing, and a constant real interest rate r , determined abroad. Assuming $[\beta(1+r)]^\sigma (1+g)^{(1-\rho)(1-\sigma)} < (1+n)(1+g)$, the steady-state level of detrended, aggregate per capita assets accumulated by home consumers as a function of the steady-state real wage and interest rate is

$$\bar{b} = \left\{ \frac{\beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)} - (1+g)}{(r-g)[(1+n)(1+g) - \beta^\sigma (1+r)^\sigma (1+g)^{(1-\rho)(1-\sigma)}]} \right\} \bar{w}. \tag{24}$$

Home consumers are net creditors in detrended, aggregate per capita terms if $[\beta(1+r)]^\sigma (1+g)^{(1-\rho)(1-\sigma)} > 1+g$. They are net debtors if $[\beta(1+r)]^\sigma (1+g)^{(1-\rho)(1-\sigma)} < 1+g$. To gain intuition on this result, consider the case in which $\sigma = 1$ and $g = 0$. Suppose also that the rest-of-the-world economy has already completed the transition to a steady-state position when the situation at home is taken into consideration, i.e., the world real interest rate is constant and equal to r along the path to home's steady state.²⁴ To simplify the argument further, suppose that the real wage is already constant at its steady-state level. The law of motion of home

²³I assume that the conditions ensuring convergence are satisfied.

²⁴ r is determined by the structural characteristics of the foreign economy. The assumption that the latter is already in steady state, whereas home is not, is not innocuous in general. It can be made here because the disparity in the size of the economies ensures that changes in domestic variables over time have no impact on foreign ones. If the economies were of comparable size, it would be necessary to analyze the simultaneous convergence of the two economies to the steady state, because home variables would affect foreign ones.

consumers' aggregate per capita assets reduces to

$$B_{t+1}/P_t = [\beta(1+r)/(1+n)]B_t/P_{t-1} + \{[\beta(1+r) - 1]/[r(1+n)]\}\bar{w}. \quad (25)$$

If $\beta(1+r)/(1+n) < 1$, a steady-state level of real aggregate per capita assets exists and is stable. For this steady-state level to be positive, the intercept of the linear relation between B_{t+1}/P_t and B_t/P_{t-1} must be positive. Under the assumptions of the special case we are considering, $\beta(1+r)$ is the slope of the time path of individual consumption. When $\beta(1+r) > 1$, individual consumption is increasing over time. If income were exogenous (as in Blanchard, 1985; Weil, 1989a, b; and Obstfeld and Rogoff, 1996, Chap. 3.7), one could assume that agents of different generations have the same income at each point in time. Under the assumption of constant individual labor income, for agents' consumption to be increasing over time, it must be the case that households are accumulating financial assets. Hence, the steady state (existence and uniqueness of which are ensured by population growth and the assumption that newborn agents have no financial assets) must be characterized by positive aggregate per capita consumer assets, since no individual has negative asset holdings.

In a framework in which labor income is endogenous, individual labor income is *not* constant even when aggregate per capita income is, because agents of different generations supply different amounts of labor. When income is not constant, one can think of situations in which individual consumption increases over time while assets are being decumulated, for example, depending on the agent's age. However, this is not the case in the steady state of the model. In fact, taking into account the Euler equation for labor supply (obtained by combining the consumption Euler equation with the labor-leisure tradeoff) removes the (direct) dependence of an agent's accumulation of assets on the quantity of labor supplied (which depends on the individual's date of birth) and shows that equilibrium asset accumulation is a function of the real wage alone (which does not depend on the individual's age). When the economy is in steady state, individual asset accumulation obeys

$$\bar{B}_{t+1}^v/\bar{P}_t = \beta(1+r)\bar{B}_t^v/\bar{P}_{t-1} + \{[\beta(1+r) - 1]/r\}\bar{w}, \quad (26)$$

which shows that $\beta(1+r) > 1$ is sufficient to ensure that the household's assets are increasing over time regardless of the household's date of birth. The intuition is clear if we look at the Euler equation for labor supply. The rate of change of an individual's supply of labor between any two periods during which the economy is in steady state is

$$(\bar{L}_{t+1}^v - \bar{L}_t^v)/\bar{L}_t^v = -[\beta(1+r) - 1](1 - \bar{L}_t^v)/\bar{L}_t^v, \quad (27)$$

which is *negative* if $\beta(1+r) > 1$. Because labor income is declining over time in steady state, the household accumulates assets to sustain an increasing consumption. The individual consumption and labor supply tilt factor $\beta(1+r) - 1$ determines whether or not the country's consumers are creditors or debtors in steady state. If individual consumption (labor effort) is increasing (decreasing) over time, consumers are net creditors in the long run. Else, they run a debt.

The result is robust to the adoption of a more general isoelastic utility function, in which σ is different from 1, and the introduction of productivity growth. The slope

coefficient of a household's consumption path is $[(\beta^\sigma(1+r)^\sigma(1+g)^{(1-\rho)(1-\sigma)})]/(1+g)$ under the assumption that the real wage and the real interest rate are constant. As in the simpler case, this expression determines also the tilt of labor supply. Again, a household's consumption can increase over time in steady state (and labor supply can decrease) only if the household is accumulating assets. The slope coefficient of the law of motion for detrended, aggregate per capita, real asset holdings is $[\beta^\sigma(1+r)^\sigma(1+g)^{(1-\rho)(1-\sigma)}]/[(1+n)(1+g)]$ in this case. If this coefficient is smaller than 1, i.e., if population growth is sufficiently fast, new households with no assets are entering the economy sufficiently quickly that detrended, aggregate per capita assets reach a stable steady state. This involves a positive level of assets because $[\beta(1+r)]^\sigma(1+g)^{(1-\rho)(1-\sigma)} > 1+g$ implies that there are no households with negative assets. If it were $[\beta(1+r)]^\sigma(1+g)^{(1-\rho)(1-\sigma)} < 1+g$, all households would be dissaving, and steady-state, detrended, aggregate per capita assets would be negative. The existence/stability condition

$$\frac{\beta^\sigma(1+r)^\sigma(1+g)^{(1-\rho)(1-\sigma)}}{(1+n)(1+g)} < 1$$

determines the sign of the denominator of \bar{b} , while the individual consumption and labor supply tilt factor determines the sign of the numerator.²⁵

Given steady-state asset holdings, aggregate per capita consumption and labor supply can be obtained easily. Steady-state aggregate per capita labor supply is vertical in the (\bar{L}, \bar{w}) space. In steady state, employment is determined by the amount of labor that is supplied, and the real wage adjusts to clear the market.²⁶

Once the steady state is determined, the equations that govern the dynamics of detrended aggregate per capita variables can be log-linearized around it knowing that the transition dynamics following temporary shocks will bring the economy back to the original position over time under plausible assumptions on parameter values.

3.1. Antecedents and alternative approaches

Several scholars have used various versions of the overlapping-generations model to deal with the issue of steady-state determinacy and model stationarity in open economies. The most widely used variants assume finite lifetime (Buiter, 1981; Finn, 1990) or a positive probability of death as in Blanchard's (1985) model (Cardia,

²⁵To determine whether the country *as a whole* is a debtor or a creditor, one needs to account for the fact that shares, which are assets from the consumers' perspective, are a liability for firms. Letting a_{t+1} be the country's detrended, aggregate per capita, real net foreign assets (aggregating consumers and firms) entering period $t+1$, it is

$$(1+n)(1+g)a_{t+1} = (1+r_t)a_t + y_t - c_t - inv_t - (\eta/2)inv_t^2/k_t - (\phi/2)[(p_t(i)/p_{t-1}(i)) - 1 - \bar{\pi}]^2k_t,$$

where $c_t \equiv C_t/E_t$, $inv_t \equiv I_t/E_t$, $k_t \equiv K_t/E_t$, y_t is detrended, aggregate per capita GDP in units of the consumption basket ($y_t = RP_t Z_t k_t^\alpha L_t^{1-\alpha}$), and I_t , K_t , and L_t are in aggregate per capita terms.

²⁶Solutions for the steady-state levels of variables other than consumer assets are available on request.

1991).²⁷ Weil (1989b) uses a continuous-time version of the setup in this paper, with exogenous endowment income that is identical across generations, to generalize Buiter's (1981) results.²⁸ Extending the Weil model to allow for positive probability of death as in Blanchard (1985) is straightforward and does not add to the main results of this paper. The Blanchard–Weil specification is an ingredient of the Quarterly Projection Model (QPM) and the Canadian Policy Analysis Model (CPAM) of the Bank of Canada.²⁹ Following this paper, Smets and Wouters (2002) and Devereux (2003) put forth small open economy models that rely on the Blanchard and Weil specifications, respectively. This paper contributes to this literature by demonstrating the properties of the Weil assumption under a general specification of preferences, which results in endogenous differences in labor income across generations and a time-varying consumption-to-wealth ratio.

Other scholars have pursued different ways to generate steady-state determinacy and model stationarity under incomplete markets, without relying on the dynamics of population. Correia et al. (1995) use a particular form of non-separability between consumption and labor effort in utility, first introduced by Greenwood et al. (1988), such that the marginal rate of substitution between consumption and labor effort is a function only of the latter. This implies that the household chooses effort independently of its consumption decisions. Using this assumption, Correia et al. develop a representative agent model of a small open economy in which a stable steady state exists for employment and the ratios capital/employment, consumption/capital, and net foreign assets/capital.³⁰ The approach pursued here has the advantage of generating a steady state for the components of these ratios.

Mendoza (1991) and Obstfeld (1990) obtain stationarity by assuming that the rate of time preference depends on consumption and, through this, on net foreign assets, an approach originally proposed by Uzawa (1968) and initially introduced in continuous-time, open economy models by Obstfeld (1981a, b).³¹ Smets and Wouters (1999) assume that agents derive utility from asset holdings. Heathcote and Perri (2002) and Neumeier and Perri (2005) introduce a convex cost of adjusting bond holdings in the consumers' period budget constraint, an approach originally followed by Turnovsky (1985).³² Kollmann (2002), Mendoza and Uribe (2000),

²⁷Frenkel and Razin (1987) develop a two-country model that relies on the Blanchard assumption, but they do so “to conduct a meaningful analysis of budget deficits in the absence of distortions” (p. 311) rather than with the explicit purpose of generating stationary dynamics.

²⁸Exchange rate indeterminacy results in indeterminacy of the current account in Weil (1989b). This problem can be resolved by designing monetary policy at home and abroad appropriately.

²⁹See Poloz et al. (1994) on QPM and Black and Rose (1997) on CPAM. References therein provide detailed information on the two models.

³⁰See also Devereux et al. (1992) and Pierdzioch (2003).

³¹Corsetti et al. (2008), Hirose (2003), Kim and Kose (2003), McDonald and Guest (2001), Schmitt-Grohé (1998), and Uribe (1997) are more recent examples of the same approach.

³²Benigno, P. (2001) and Laxton and Pesenti (2003) make a similar assumption. The cost of adjusting financial positions is the approach that has been adopted in the IMF's new Global Economy Model (GEM).

Schmitt-Grohé and Uribe (2001), and Senhadji (2003) obtain stationarity of their small open economy models by assuming that the interest rate at which the home economy can borrow internationally is given by the world interest rate plus a premium that increases in the country's stock of foreign debt.

All these assumptions ensure that the equilibrium rate of consumption growth depends on asset holdings, so that setting consumption to be constant pins down a steady-state distribution of net foreign assets. Models are then solved numerically around the steady state, as in this paper. The main advantage of the approach pursued here is that it makes it possible to provide a structural interpretation of the determination of steady-state asset holdings that does not hinge on any special assumption about the functional form of the discount factor, utility from asset holdings, cost of bond holdings, or the determination of the interest premium. For instance, costs of adjusting bond holdings or debt-elastic interest rate premium usually determine steady-state assets as the exogenous centering of the adjustment cost or premium function. In the Weil-world of this paper, steady-state assets are determined endogenously by the parameters of preferences and technology. As illustrated below, this has implications also for off-steady-state dynamics and their interpretation. Each individual household is modeled as the familiar representative agent of most intertemporal macroeconomic models, including Obstfeld and Rogoff's (1995). Aggregate per capita assets are stationary, individual household's are not.³³

Schmitt-Grohé and Uribe (2003) compare five different versions of the flexible-price, small open economy model (Uzawa preferences, cost of portfolio adjustment, debt elastic premium, standard non-stationary setup, complete markets) and conclude that all versions deliver similar dynamics at business cycle frequency (though consumption is smoother under complete markets) when they are parameterized to match the behavior of the Canadian economy. This finding should not come as a complete surprise, at least as far as stationary, incomplete markets models are concerned. Different solutions to non-stationarity under incomplete markets should deliver similar results if they are parameterized to match a given economy. The similarity of results across the stationary, incomplete markets results, the non-stationary model, and the complete markets world is more striking.³⁴ Home and foreign goods are perfect substitutes in Schmitt-Grohé and Uribe (2003). This removes any role for terms of trade dynamics. Ghironi (2006) shows that differences in results across stationary, incomplete markets economies,

³³Searching for a determinate non-stochastic steady state and log-linearizing around it is not the only approach to macroeconomic interdependence under incomplete markets. Ljungqvist and Sargent (2004) describe a number of models in which a stationary equilibrium is defined in terms of a stationary probability distribution of asset-holdings/state-of-nature pairs. They discuss conditions under which this distribution exists and is unique. Clarida (1990) and Devereux and Saito (1997) use a similar approach. However, this is different from the more traditional approach in international business cycle analysis, which typically relies on the linearization (or higher-order approximation) of the model around a deterministic steady state.

³⁴Though supported also by results in Baxter and Crucini (1995), Chari et al. (2002), Heathcote and Perri (2002), and Kehoe and Perri (2002).

the non-stationary case, and the complete markets world are sensitive to the degree of substitutability between home and foreign goods and to the persistence of exogenous shocks.³⁵

4. Net foreign assets, monetary policy, and the transmission of shocks

This section analyzes the role of asset accumulation in the transmission of shocks in the model of this paper under alternative price stability rules for the domestic central bank. To this purpose, I set parameters at values that are common in the literature and show that the model delivers plausible implications for the steady state. I then log-linearize the model around the steady state and study the transmission of a world demand shock under different policy rules that stabilize consumer or producer prices. I conclude the section by verifying that the key results are robust to different levels of steady-state net foreign assets.

4.1. Parameterization and properties of the steady state

I interpret periods as quarters and consider the following benchmark parameterization: $\beta = .995$ (slightly higher than the standard .99, but still a conventional value), $\sigma = .5$ (implying a coefficient of relative risk aversion equal to 2), $\rho = .37$ (with the other parameters, this choice for the relative weight of consumption versus leisure in utility implies that households work approximately 1/3 of the time in steady state), $\theta = 10$ (following Chari et al., 2000, and implying an 11 percent steady-state markup in the pricing of goods), $\gamma = .33$ (a conventional choice for the elasticity of output to capital), $\delta = .025$ (implying the standard 10 percent yearly rate of capital depreciation). I set the scaling of price adjustment costs $\phi = 16.96$. With the other parameters, this ensures that the markup coefficient in the New Keynesian Phillips curve for domestic PPI inflation is the same as would be implied by a benchmark Calvo (1983)–Yun (1996) model with four-quarter average duration of prices.³⁶ I set the scaling parameter for the cost of adjusting capital $\eta = 20$. Together

³⁵In a recent paper, Bodenstein (2006) criticizes costs of adjusting asset holdings, debt-elastic interest rate premia, endogenous discounting, and overlapping generations as solutions to the issue of steady-state determinacy and model stationarity on the ground that they cannot pin down a unique steady state (or ensure uniqueness of dynamics) if preferences are such that there are multiple steady states for relative prices under financial autarky. It should be noted, however, that these devices were proposed to pin down the asset position under international trade in an incomplete menu of assets, not to address a relative price indeterminacy that arises under financial autarky.

³⁶In the benchmark Calvo–Yun model with which most readers are familiar, the markup coefficient in the New Keynesian Phillips curve is equal to $(1-\alpha)(1-\alpha\beta)/\alpha$ in absolute value, where $1-\alpha$ is the probability of price adjustment in each period. When combined with the other assumptions of this paper, Calvo–Yun price rigidity would result in a more complicated expression for the coefficient. I use the benchmark expression to calibrate ϕ to facilitate interpretation. This results in a markup coefficient of .0846 in absolute value – well within the range of empirical estimates of the New Keynesian Phillips curve and, if anything, erring on the side of price flexibility. (Ireland, 2001, estimates a scaling coefficient of 77 for the US in a model with quadratic cost of price adjustment.)

with the other parameter choices, this implies an average cost of adjusting capital of 6.6 percent of GDP. This choice ensures that the impact response of aggregate per capita investment to productivity, foreign interest rate, or world demand shocks is at most approximately four times that of GDP in absolute value.³⁷ I set $\bar{Z} = 1$, the rate of exogenous trend productivity growth $g = .0025$ (implying a 1 percent yearly average growth rate of GDP per capita), and the rate of entry of new households in each period $n = .005$. This is a relatively high value (for instance, the average rate of quarterly population growth for the US has been .0025 between 1973:1 and 2000:3). However, a more complicated version of the model including probability of death as in Blanchard (1985) would make it possible to replicate the properties of this parameterization with lower n and the appropriate probability of death. Thus, I use $n = .005$ to mimic the properties of the more general, but largely isomorphic model.

I normalize steady-state aggregate per capita foreign output to 1, assume zero foreign CPI inflation in steady state, and set the world real interest rate r to the standard level .0101.³⁸ I assume that the steady-state domestic nominal interest rate is such that $\bar{i} = \bar{i}^* = r$, so that UIP implies zero exchange rate depreciation between any periods t and $t + 1$ in which the economy is in steady state. In turn, PPP implies zero steady-state CPI (and – by assumption of equality in steady state – PPI) inflation. Since UIP is a forward-looking relation, the additional assumption that there is no unexpected exchange rate jump in steady state ensures that the exchange rate is constant in all periods in steady state.

Table 1 summarizes the benchmark parameter values and shows the implied steady-state properties of the model. Parameter values that are common in the literature result in empirically plausible properties. Consumption and investment are 72.36 and 20.39 percent of GDP, respectively. The fraction of GDP that is distributed to labor is 60.3 percent and dividends are 12.68 percent. Positive steady-state dividends and the computation of share prices as present discounted value of dividends over the infinite future imply that the home country's equity is worth 40.96 units of consumption. Although consumer assets are positive by virtue of the aggregation of domestic equity and net foreign bond holdings, the latter are negative, i.e., the home country runs a steady-state net foreign debt, equal to 59.71 percent of annualized GDP. The steady-state current account, given by $r\bar{a} + \bar{y} - \bar{c} - i\bar{n}v - (\eta/2)i\bar{n}v^2/\bar{k}$, is – 1.79 percent of GDP on a quarterly basis, with a trade surplus of .62 percent and an interest burden of foreign debt of 2.41 percent of GDP per

³⁷Setting $\eta = 10$ and adjusting ϕ to 14.77 to keep constant the average price duration in the benchmark Calvo–Yun interpretation of the model lowers the average cost of capital adjustment to 3.8 percent of GDP and has no major qualitative consequence, but it causes investment to respond to some shocks in more than four-fold fashion.

³⁸If we assume that the world interest rate is pinned down by a foreign household discount factor β^* so that $\beta^*(1+r) = 1$, this is equivalent to assuming $\beta^* = .99$. Setting $\beta(1+r) > 1$ makes it possible to generate non-zero steady-state consumer assets also in the often-studied case in which $\sigma = 1$ (log utility) and $g = 0$ (no exogenous productivity growth). As shown in Section 3, the overlapping-generations structure of this paper ensures that the model has a well-defined steady state for aggregates per capita even when consumer patience differs across countries – a point originally made by Buiter (1981) and re-examined in the context of the discrete-time Weil model in Ghironi et al. (2008).

Table 1
Benchmark parameter values and steady-state properties

a. *Parameter values and steady-state levels of exogenous variables*

Household discount factor: $\beta = .99$
 Elasticity of intertemporal substitution: $\sigma = .5$
 Relative weight of consumption in utility: $\rho = .37$
 Elasticity of substitution across goods: $\theta = 10$
 Elasticity of output to capital: $\gamma = .33$
 Rate of capital depreciation: $\delta = .025$
 Scaling of price adjustment cost: $\phi = 16.96$
 Scaling of capital adjustment cost: $\eta = 20$
 Trend labor productivity growth: $g = .0025$
 Rate of entry of new households: $n = .005$
 Steady-state total productivity: $\bar{Z} = 1$
 Steady-state foreign (world) aggregate per capita output: $\bar{y}^W = 1$
 Steady-state foreign CPI inflation: $\bar{\pi}^{\text{CPI}^*} = 0$
 Steady-state world real interest rate: $r = .0101$
 Steady-state nominal interest rates: $\bar{i} = \bar{i}^* = r$

b. *The steady state*

CPI inflation and currency depreciation: $\bar{\pi}^{\text{CPI}} = \bar{e} = 0$
 Labor effort: $\bar{L} = .3286$
 Detrended real wage: $\bar{w} = 1.5344$
 Detrended aggregate per capita GDP: $\bar{y} = .8362$
 Share of labor income in GDP = .6030
 Detrended aggregate per capita consumption: $\bar{c} = .6051$
 Share of consumption in GDP = .7236 (1)
 Detrended aggregate per capita investment: $\bar{i}n = .1705$
 Share of investment in GDP = .2039 (2)
 Cost of capital adjustment as share of GDP = .0663 (3)
 Detrended aggregate per capita capital: $\bar{k} = 5.2443$
 Detrended aggregate per capita real dividends: $\bar{d} = .1060$
 Share of dividends in GDP = .1268
 Detrended aggregate per capita real equity value: $\bar{v} = 40.9573$
 Detrended aggregate per capita consumer assets: $\bar{b} = 38.9601$
 Detrended aggregate per capita net foreign debt: $\bar{a} = -1.9972$
 Net foreign debt as share of annualized GDP = $-.5971$
 Current account as share of GDP = $-.0179$ (4)
 Interest burden of foreign debt as share of GDP = $-.0241$ (5)
 Trade surplus as share of GDP = .0062

Note: $1 - (1) - (2) - (3) + (5) = (4) = [(1 + n)(1 + g) - 1]\bar{a}/\bar{y}$.

quarter.³⁹ Lane and Milesi-Ferretti (2001) document that 9 percent of industrial countries had net foreign liabilities in excess of 20 percent of GDP in 1997, and approximately 18 percent of developing countries had net foreign liabilities above 40

³⁹The model does not require the current account to be zero in the steady state with constant detrended, aggregate per capita net foreign assets, as one can see from the net foreign asset equation in footnote 25.

percent of GDP. They show that several countries have maintained permanently negative and quite large net foreign asset positions in the period 1970–1998. Therefore, a steady-state net foreign debt of approximately 60 percent of annualized GDP is not an unrealistic benchmark when one wants to study shock propagation in the presence of external asset imbalances.

4.2. Monetary policy

To illustrate the dynamic properties of the model, I consider the impulse responses of the log-linearized setup to a decrease in world demand for the consumption basket under alternative price stability rules for the domestic central bank.

There are three foreign variables that enter the system determining domestic dynamics: world consumption demand, \hat{y}_t^W (which is exogenous to the small open economy); the nominal interest rate, \hat{i}_{t+1}^* ; and foreign CPI inflation, $\hat{\pi}_t^{\text{CPI}^*}$. In general, these variables are determined jointly by the dynamics of the world economy. For simplicity, when considering a shock to world demand, I assume that world demand behaves in *AR*(1) fashion as $\hat{y}_t^W = \varphi_{y,w} \hat{y}_{t-1}^W \forall t > t_0 = 0$, where $t_0 = 0$ is the period of an initial one-percent change in \hat{y}_t^W and $0 \leq \varphi_{y,w} < 1$, while $\hat{i}_{t+1}^* = \hat{\pi}_t^{\text{CPI}^*} = 0 \forall t \geq t_0 = 0$.⁴⁰

I compare the responses to the world demand shock under two alternative strict inflation targeting (*SIT*) rules. I assume that the central bank sets the nominal interest rate to keep inflation in producer or consumer prices at the steady-state level in *all* periods, including when an unexpected shock happens: $\hat{\pi}_t^v = 0 \forall t \geq t_0 = 0$, $v = \text{PPI or CPI}$.⁴¹ A simple interest rate rule that implements this policy regime for any shock in the model has the central bank set the interest rate to react proportionally to the foreign interest rate, expected exchange rate depreciation, and current inflation: $\hat{i}_{t+1} = \hat{i}_{t+1}^* + \hat{e}_{t+1} + \hat{\pi}_t^v$, $v = \text{PPI or CPI}$. Combining this rule with UIP, $\hat{i}_{t+1} - \hat{i}_{t+1}^* = \hat{e}_{t+1}$, yields $\hat{\pi}_t^v = 0 \forall t \geq t_0 = 0$, $v = \text{PPI or CPI}$.⁴²

The *SIT*–*CPI* regime in which CPI inflation is zero in all periods is equivalent to a fixed exchange rate regime under the assumptions of this paper. Because of PPP, the *SIT*–*CPI* regime requires the central bank to engineer a rate of

⁴⁰This is the same approach as in Galí and Monacelli (2005). Results from modeling the three foreign variables using a three-variable VAR are available on request.

⁴¹Given zero steady-state inflation, strict inflation targeting is equivalent to complete price stability.

⁴²If it were useful to increase credibility, implicitly assumed perfect in this paper, the central bank could announce a version of the rule that allows for a more than proportional reaction to inflation as in Taylor (1993). Combining the modified rule with UIP would still result in $\hat{\pi}_t^v = 0 \forall t \geq t_0 = 0$, $v = \text{PPI or CPI}$, in equilibrium. Of course, there are other possible interest rate rules that would implement the *SIT* regime. (Given equilibrium determinacy, all rules that implement the chosen regime result in identical dynamics.) For instance, *SIT* could also be accomplished by following a rule of the form $\hat{i}_{t+1} = \alpha \hat{\pi}_t^v$, $\alpha > 1$, $v = \text{PPI or CPI}$, with $\alpha \rightarrow \infty$. But this would be impractical and quite risky in reality: As observed by Svensson (2003), policy errors would be immensely costly.

depreciation such that $\hat{e}_t = -\hat{\pi}_t^{\text{CPI}^*}$. Since I assume $\hat{\pi}_t^{\text{CPI}^*} = 0$, *SIT–CPI* results in $\hat{e}_t = \hat{e}_t - \hat{e}_{t-1} = 0 \forall t \geq t_0 = 0$. It follows that the exchange rate is fixed at its steady-state level in all periods, $\hat{e}_t = \hat{e}_{-1} = 0 \forall t \geq t_0 = 0$, under *SIT–CPI*.⁴³

Finally, it should be noted that the *SIT–PPI* regime $\hat{\pi}_t^{\text{PPI}} = 0 \forall t \geq t_0$ corresponds to mimicking the flexible-price equilibrium by keeping the markup at the steady state in all periods.⁴⁴ Mimicking the flexible-price equilibrium is the optimal monetary policy under commitment in many closed economy, sticky-price models with monopolistic competition because it removes the distortion associated with sticky prices. Woodford (2003) reviews the argument in detail. Benigno and Benigno (2003) demonstrate that the policy of mimicking the flexible price equilibrium is optimal for open economies only under special assumptions, which are not satisfied in this paper. Nevertheless, markup dynamics play such a key role in the model, and the policy of markup stability has received so much attention in the literature, that it is interesting to understand how such policy would influence the dynamics of the model.⁴⁵

4.4. Impulse responses

Fig. 1 presents the responses to a 1 percent negative impulse to world demand at time 0, with persistence $\varphi_{y,w} = .9$. Circle markers denote the *SIT–PPI* responses, while cross markers denote the *SIT–CPI* responses. Quantity responses are those of detrended, aggregate per capita variables (except for labor effort, which does not need detrending). Hats are omitted from the labels. The percentage deviation from the steady state is on the vertical axis, scaled so that .3, for instance, denotes .3 percent, and not 30 percent. Periods are interpreted as quarters, and the number of years after the initial impulse is on the horizontal axis.

The shock causes a recession in the small open economy, with employment (*L*), investment (denoted *I* instead of *inv* in the figure for brevity), GDP (*y*), and consumption (*c*) falling below the steady state. The recession is significantly more severe if the central bank is targeting CPI inflation. The intuition is simple and reflects the two channels through which monetary policy operates in the model – markup and relative price dynamics generated by nominal rigidity, and unexpected variation in *ex post* real interest rates at the time of the shock.

When the central bank targets CPI inflation, PPI inflation (π^{PPI} in the figure) falls on impact as home firms reduce output prices in an effort to sustain demand.

⁴³In general, given UIP, the rule $\hat{i}_{t+1} = \hat{i}_{t+1}^* + \tau \hat{e}_t$, $\tau > 0$, would implement a fixed exchange rate.

⁴⁴Log-linearizing the markup Eq. (16) after expressing numerator and denominator in terms of detrended, aggregate per capita variables yields

$$\hat{\psi}_t = -\frac{\phi(1 + \bar{\pi})\bar{k}}{(\theta - 1)\bar{y}} \left[\hat{\pi}_t^{\text{PPI}} - \frac{(1 + n)(1 + g)}{1 + r} \hat{\pi}_{t+1}^{\text{PPI}} \right].$$

Thus, the markup is constant at its steady-state level if and only if producer price inflation is.

⁴⁵Results for policy rules that do not completely stabilize inflation and other shocks are available on request.

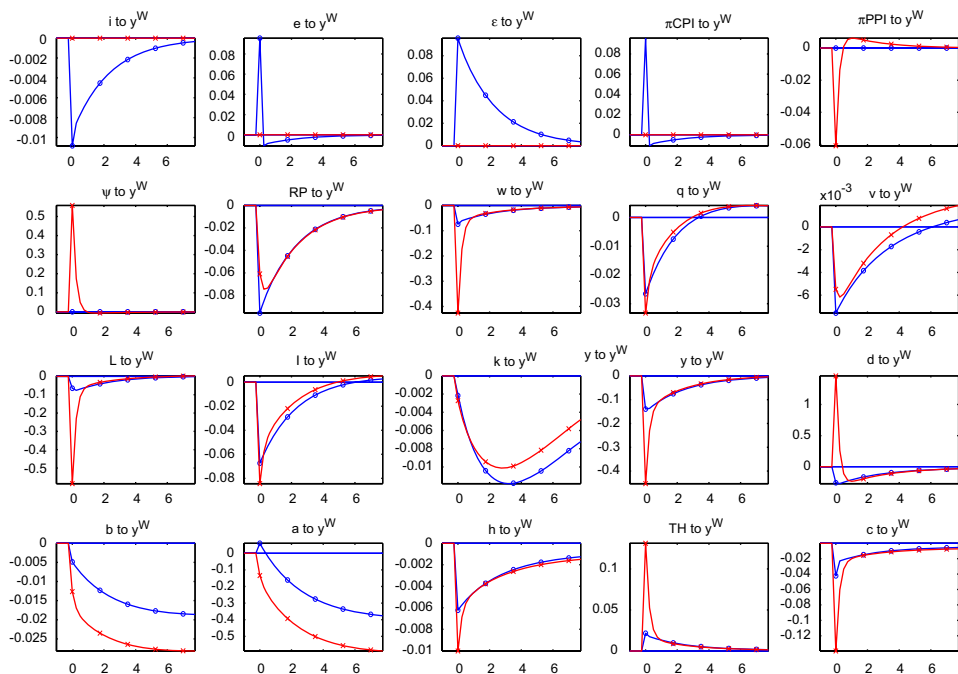


Fig. 1. Impulse responses, world demand shock.

Associated with an initial drop in PPI inflation is the fact that firms increase the markup component of prices (Ψ) to preserve profitability. The larger markup induces a larger fall in labor demand, GDP, the real wage (w), and therefore consumption. The initial upward spike in the markup under CPI inflation targeting causes dividends (d) to increase initially. However, both the share price (v) and the shadow value of installed capital (q) fall in response to the shock. The movement in the shadow value of capital is fairly similar across policy scenarios, resulting in less pronounced investment and capital (k) response differences across policy rules. The real asset position of households (b) deteriorates, and households increase their foreign borrowing (a), except initially in the case of PPI targeting, in an effort to sustain consumption. The worsening of the asset position combines with lower human wealth (h) and propensity to consume to generate the drop in consumption.⁴⁶

Since the foreign interest rate and the exchange rate do not move, there is no movement in the home nominal interest rate under CPI inflation targeting. There is no time 0, *ex post* real interest rate shock under this rule, and markup and relative price variation is the only channel through which monetary policy operates. Instead, when the central bank targets PPI inflation, interest rate easing to prevent PPI inflation from falling results in initial exchange rate depreciation and an upward

⁴⁶Human wealth is re-defined as: $h_t \equiv \sum_{s=t}^{\infty} R_{t,s}(1+g)^{s-t} w_s$. An increase in TH corresponds to a lower value of θ_t^{-1} in Eq. (21).

spike in CPI inflation. As discussed in Section 2, this CPI inflation movement implies a surprise downward movement in the *ex post* home real interest rate at time 0. Given an initial foreign debt position, this reduces the real interest burden of debt during period 0, contributing to mitigate the recession and the drop in consumption under PPI inflation targeting relative to CPI targeting.⁴⁷

4.5. Robustness

The example above highlights the contribution of the initial asset position in generating differences in the responses to a given shock across different specifications of domestic monetary policy.⁴⁸ A natural question is whether these differences across policy rules are sensitive to the size of the economy's net foreign asset position. For instance, does the result that fully stabilizing PPI inflation contributes to dampening the recession in response to a drop in world demand by generating a favorable change in the time 0 *ex post* interest burden of the steady-state net foreign debt position depend on the size of the latter?

In a model in which the steady-state level of net foreign assets is indeterminate (and thus chosen as a matter of convenience) or pinned down simply by the centering of an adjustment cost or interest premium function, it is possible to study this question without changing any of the parameters of preferences and technology. Alternatively, it is possible to explore the consequences of changes in parameter values holding the steady-state level of assets constant. This is not possible in the model of this paper, where the steady state is uniquely determined as a function of structural parameters. Thus, when analyzing dynamics around a different steady-state level of assets, observed differences may be due to the change in the steady state or to an effect of parameter changes that would alter dynamics in similar fashion also for unchanged steady state.

Fortunately, the non-linearity of the steady state as a function of parameter values is helpful here, as very small changes in some parameter values have a large effect on steady-state asset levels. One can thus be reasonably confident that changes in results that may be observed are driven by the sizable change in the steady state rather than by significant effects that would arise after large changes in parameter values for unchanged steady state.

To verify the robustness of results to changes in the long-run level of net foreign assets, I consider three alternative parameterizations that yield different steady-state net foreign asset positions by changing the degree of consumer patience relative to the benchmark calibration. I increase β very slightly to .99505 in Alternative A. Due

⁴⁷Intuitively, the initial *ex post* interest rate shock plays a relatively larger role in affecting dynamics after a world demand shock when the markup channel of monetary transmission is less active. It is possible to verify that a policy rule of the form $\hat{i}_{t+1} = \alpha \hat{\pi}_t^{\text{CPI}}$, $\alpha > 1$ but finite, yields identical dynamics to *SIT-CPI* following a world demand shock (and a productivity shock) under the assumptions of this paper. The responses of most domestic variables to the world demand shock under the rule $\hat{i}_{t+1} = 1.5 \hat{\pi}_t^{\text{PPI}}$ are similar to those under the *SIT-PPI* regime.

⁴⁸Responses to world interest rate and domestic productivity shocks confirm the role of initial assets for results.

to non-linearity, this very small increase in consumer patience is sufficient to reduce the home country's steady-state foreign debt position to 10.54 percent of annualized GDP and the current account to $- .32$ percent of GDP on a quarterly basis. Further increasing β to .9951 (Alternative B) causes the home country to have positive steady-state assets equal to 39.7 percent of annualized GDP and a current account surplus of 1.19 percent of quarterly GDP. Alternative C ($\beta = .9948$) is an extreme scenario in which the home country runs a steady-state debt of over 246 percent of annualized GDP and a quarterly steady-state current account deficit of 7.4 percent of GDP. Importantly, all alternatives leave the sign of the steady-state individual consumption and labor supply tilt factors discussed in Section 3 unchanged. In all scenarios, individual consumption (labor supply) profiles are tilted upward (downward), ensuring positive aggregate per capita consumer assets. Individual consumption and labor supply profiles in Alternative B are steep enough that consumer assets exceed the value of equity, so that the country has positive net foreign assets in steady state.

I re-compute the impulse responses under the benchmark parameter values and the alternatives holding the monetary policy regime constant. I perform the exercise under the policy regime that maximizes the size of the *ex post* real interest rate shock at time 0. For the world demand shock, this is the *SIT-PPI* rule. The rationale is that this is the case in which changes in the initial asset position are most likely to have a large impact on results.

Fig. 2 presents the impulse responses for the world GDP shock (circle markers for the benchmark parameterization, cross markers for Alternative A, square markers for Alternative B, star markers for Alternative C; the responses of PPI inflation and the markup – zero in all periods – are omitted). As it is apparent from the figure, changing the initial asset position even substantially has small effects on the dynamics of most variables around the steady state for *given* policy rule. As demonstrated in Ghironi et al. (2008), along with determining steady-state household assets, the tilt in steady-state consumption and labor supply profiles of individual households is also a crucial determinant of the responses to shocks around the steady state in this kind of model.⁴⁹ As mentioned above, the changes in the discount factor β that induce the different steady-state asset positions in Fig. 2 crucially do not change the direction of tilts in steady-state individual behavior. Thus, they do not result in significantly different dynamics around the respective steady state. In Alternative C, with a very large steady-state foreign debt, the favorable effect of the initial increase in CPI inflation becomes substantially more pronounced, allowing households to increase their assets initially (b rises above the

⁴⁹In Ghironi et al.'s (2008) two-country model with heterogeneous discount factors, the fact that steady-state consumption (labor supply) profiles of the relatively impatient country are tilted downward (upward) is crucial for the result that a permanent worldwide productivity increase induces the country to increase its borrowing, with an initial aggregate consumption increase that overshoots the new long-run position and decreases toward it over time. Steady-state household behavior in the relatively patient country displays opposite tilts. Consistently, aggregate consumption in this country increases by less than the long-run response on impact and converges to the latter from below. No movement in net foreign assets happens absent these steady-state tilts, and consumption jumps immediately to the new long-run level in both countries.

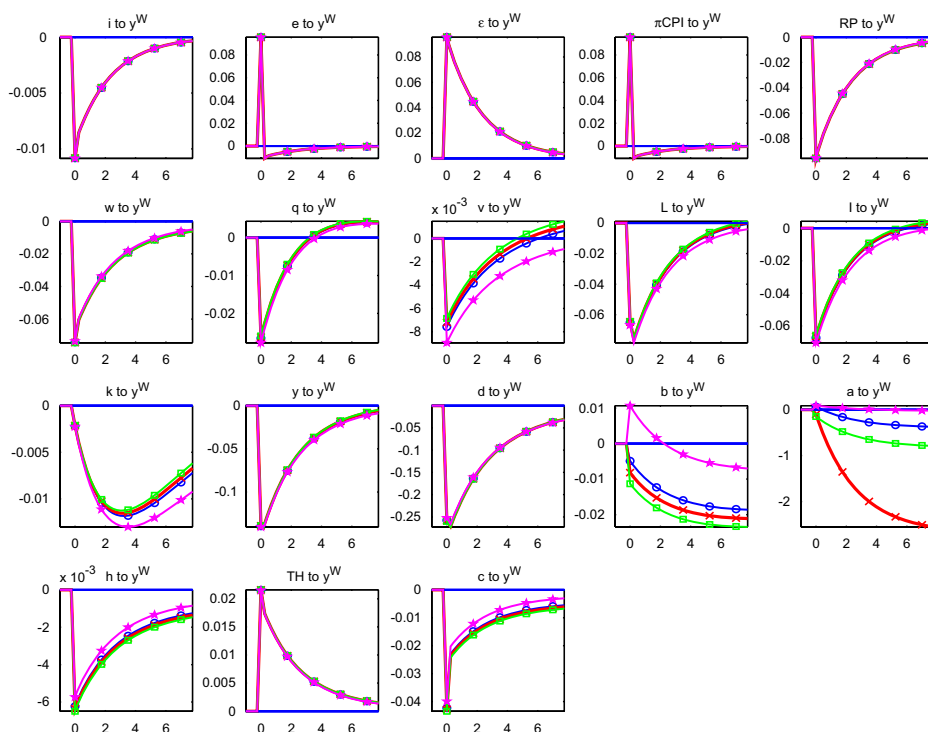


Fig. 2. Impulse responses, world demand shock, *SIT-PPI*, alternative steady-state asset positions.

steady state), and somewhat dampening the drop in consumption, which is then reabsorbed relatively more quickly. In Alternative B, when the country is a creditor, higher CPI inflation has an unfavorable wealth effect, inducing households to run down their assets more aggressively (relative to the benchmark and Alternative A) in an effort to sustain consumption. Similar conclusions obtain for world interest rate and productivity shocks – and for different policy rules. It follows that the differences *across* policy rules implied by the effects of the starting asset position are robust to changes in the latter as long as these changes do not follow from altering the fundamental characteristics of optimal behavior of individual households.⁵⁰

5. Conclusions

This paper developed a small open economy model with incomplete asset markets that solves the problem of steady state determinacy and model non-stationarity by changing the demographic structure from the familiar representative agent frame-

⁵⁰Repeating the exercise of Fig. 1 for different asset positions confirms this conclusion. Benigno, P. (2006) shows how the effects of unanticipated price changes on external positions highlighted in this paper can have important implications for the desirability of exchange rate movements.

work to an overlapping-generations structure as in Weil (1989a, b), in which new infinitely lived households enter the economy at each point in time and are born owning no financial assets. The model extends the original Weil setup (and other work on related overlapping-generations models) to a more general class of preferences, which allows for differences in endogenous labor income across agents of different generations and a time-varying consumption-to-wealth ratio. The main advantage over alternative approaches to the issue of steady state determinacy and model non-stationarity is that the model of this paper provides a structural interpretation of the determination of long-run asset positions that does not hinge on special assumptions about costs of adjusting bond holdings, an endogenous discount factor, or the determination of a debt elastic interest premium. By determining long-run assets uniquely as function of the parameters of preferences and technology, the model highlights the connection between characteristics of the steady state and off-steady-state dynamics. Changes in parameter values that alter the long-run asset position even significantly do not imply significant changes in the dynamics of most variables around the respective steady state if they do not alter the fundamental characteristics of optimal agent behavior (such as the agents' desire to anticipate or postpone consumption and labor effort).

By incorporating nominal rigidity, this paper also complements the recent literature on New Keynesian, open economy models that de-emphasize the role of net foreign asset dynamics. The analysis of shock transmission highlights the role of asset positions and markup dynamics in generating different dynamics under alternative monetary policy rules.

Acknowledgments

This paper was circulated previously under the title: “Understanding Macroeconomic Interdependence: Do We Really Need to Shut Off the Current Account?” I thank Wouter den Haan, two anonymous referees, many colleagues, and participants in several seminars and conferences for helpful comments. Work on this paper was partly funded by a Jean Monnet Fellowship in the General Programme of the Robert Schuman Centre for Advanced Studies at the European University Institute. Andrei Levchenko and Irina Telyukova provided outstanding research assistance. Remaining errors are my responsibility.

References

- Baxter, M., Crucini, M., 1995. Business cycles and the asset structure of foreign trade. *International Economic Review* 36, 821–853.
- Benigno, G., 2004. Real exchange rate persistence and monetary policy rules. *Journal of Monetary Economics* 51, 473–502.
- Benigno, G., Benigno, P., 2003. Price stability in open economies. *Review of Economic Studies* 70, 743–764.
- Benigno, G., Benigno, P., 2007. Exchange rate determination under interest rate rules. *Journal of International Money and Finance*, forthcoming.

- Benigno, G., Thoenissen, C., 2007. Consumption and real exchange rates with incomplete financial markets and non-traded goods. *Journal of International Money and Finance*, forthcoming.
- Benigno, P., 2001. Price stability with imperfect financial integration. CEPR Discussion Paper 2854.
- Benigno, P., 2006. Are valuation effects desirable from a global perspective? NBER Working Paper 12219.
- Betts, C., Devereux, M.B., 2000. Exchange rate dynamics in a model of pricing to market. *Journal of International Economics* 50, 215–244.
- Black, R., Rose, D., 1997. Canadian policy analysis model: CPAM. Bank of Canada Working Paper 97-16.
- Blanchard, O.J., 1985. Debt, deficits, and finite horizons. *Journal of Political Economy* 93, 223–247.
- Bodenstein, M., 2006. Closing open economy models. Manuscript, Board of Governors of the Federal Reserve System.
- Buiter, W.H., 1981. Time preference and international lending and borrowing in an overlapping-generations model. *Journal of Political Economy* 89, 769–797.
- Calvo, G.A., 1983. Staggered prices in a utility maximizing framework. *Journal of Monetary Economics* 12, 983–998.
- Cardia, E., 1991. The dynamics of a small open economy in response to monetary, fiscal, and productivity shocks. *Journal of Monetary Economics* 28, 411–434.
- Carré, M., Collard, F., 2003. Monetary union: a welfare based approach. *European Economic Review* 47, 521–552.
- Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2000. Sticky price models of the business cycles: can the contract multiplier solve the persistence problem? *Econometrica* 68, 1151–1179.
- Chari, V.V., Kehoe, P.J., McGrattan, E.R., 2002. Can sticky price models generate volatile and persistent real exchange rates? *Review of Economic Studies* 69, 533–563.
- Clarida, R.H., 1990. International lending and borrowing in a stochastic, stationary equilibrium. *International Economic Review* 31, 543–558.
- Cole, H.L., Obstfeld, M., 1991. Commodity trade and international risk sharing: how much do financial markets matter? *Journal of Monetary Economics* 28, 3–24.
- Correia, I., Neves, J.C., Rebelo, S., 1995. Business cycles in a small open economy. *European Economic Review* 39, 1089–1113.
- Corsetti, G., Dedola, L., Leduc, S., 2008. International risk-sharing and the transmission of productivity shocks. *Review of Economic Studies*, forthcoming.
- Corsetti, G., Pesenti, P., 2001. Welfare and macroeconomic interdependence. *Quarterly Journal of Economics* 116, 421–446.
- Devereux, M.B., 2003. A tale of two currencies: the Asian crisis and the exchange rate regimes of Hong Kong and Singapore. *Review of International Economics* 11, 38–54.
- Devereux, M.B., Gregory, A.W., Smith, G.W., 1992. Realistic cross-country consumption correlations in a two-country, equilibrium, business cycle model. *Journal of International Money and Finance* 11, 3–16.
- Devereux, M.B., Saito, M., 1997. Growth and risk-sharing with incomplete international assets markets. *Journal of International Economics* 42, 453–481.
- Duarte, M., Stockman, A.C., 2005. Rational speculation and exchange rates. *Journal of Monetary Economics* 52, 3–29.
- Feenstra, R.C., 1994. New product varieties and the measurement of international prices. *American Economic Review* 84, 157–177.
- Finn, M.G., 1990. On savings and investment dynamics in a small open economy. *Journal of International Economics* 29, 1–21.
- Frenkel, J.A., Razin, A., 1987. *Fiscal Policies and the World Economy*. MIT Press, Cambridge.
- Galí, J., Monacelli, T., 2005. Monetary policy and exchange rate volatility in a small open economy. *Review of Economic Studies* 72, 707–734.
- Ghironi, F., 2006. Macroeconomic interdependence under incomplete markets. *Journal of International Economics* 70, 428–450.
- Ghironi, F., İřcan, T.B., Rebucci, A., 2008. Net foreign asset positions and consumption dynamics in the international economy. *Journal of International Money and Finance*, forthcoming.

- Goodfriend, M.S., King, R.G., 1997. The new neoclassical synthesis and the role of monetary policy. In: Bernanke, B.S., Rotemberg, J.J. (Eds.), *NBER Macroeconomics Annual 1997*. MIT Press, Cambridge, MA, pp. 231–282.
- Greenwood, J., Hercowitz, Z., Huffman, G.W., 1988. Investment, capacity utilization and the real business cycle. *American Economic Review* 78, 402–417.
- Hairault, J.-O., Portier, F., 1993. Money, New-Keynesian macroeconomics and the business cycle. *European Economic Review* 37, 1533–1568.
- Harrigan, J., 1993. OECD imports and trade barriers in 1983. *Journal of International Economics* 35, 91–111.
- Hayashi, F., 1982. Tobin's marginal q and average q : a neoclassical interpretation. *Econometrica* 50, 213–224.
- Heathcote, J., Perri, F., 2002. Financial autarky and international business cycles. *Journal of Monetary Economics* 49, 601–627.
- Hirose, K., 2003. Exchange rate and current account dynamics with capital accumulation: a dynamic optimization approach. Manuscript, Hitotsubashi University.
- Ireland, P., 2001. Sticky-price models of the business cycle: specification and stability. *Journal of Monetary Economics* 47, 3–18.
- Kehoe, P.J., Perri, F., 2002. International business cycles with endogenous incomplete markets. *Econometrica* 70, 907–928.
- Kim, S.H., Kose, M.A., 2003. Dynamics of open economy business cycle models: understanding the role of the discount factor. *Macroeconomic Dynamics* 7, 263–290.
- Kollmann, R., 2002. Monetary policy rules in the open economy: effects on welfare and business cycles. *Journal of Monetary Economics* 49, 989–1015.
- Lai, H., Trefler, D., 2002. The gains from trade with monopolistic competition: specification, estimation, and mis-specification. NBER Working Paper 9169.
- Lane, P.R., 2001. The new open economy macroeconomics: a survey. *Journal of International Economics* 54, 235–266.
- Lane, P.R., Milesi-Ferretti, G.M., 2001. The external wealth of nations: measures of foreign assets and liabilities for industrial and developing countries. *Journal of International Economics* 55, 263–294.
- Lane, P.R., Milesi-Ferretti, G.M., 2002a. Long-term capital movements. In: Bernanke, B.S., Rogoff, K. (Eds.), *NBER Macroeconomics Annual 2001*. MIT Press, Cambridge, MA, pp. 73–116.
- Lane, P.R., Milesi-Ferretti, G.M., 2002b. External wealth, the trade balance, and the real exchange rate. *European Economic Review* 46, 1049–1071.
- Laxton, D., Pesenti, P., 2003. Monetary rules for small, open, emerging economies. *Journal of Monetary Economics* 50, 1109–1146.
- Ljungqvist, L., Sargent, T.J., 2004. *Recursive Macroeconomic Theory*, Second Ed. MIT Press, Cambridge, MA.
- McDonald, I.M., Guest, R.S., 2001. How Uzawa preferences improve the simulation properties of the small open economy model. *Journal of Macroeconomics* 23, 417–440.
- Mendoza, E.G., 1991. Real business cycles in a small open economy. *American Economic Review* 81, 797–818.
- Mendoza, E.G., Uribe, M., 2000. Devaluation risk and the business-cycle implications of exchange-rate management. *Carnegie-Rochester Conference Series on Public Policy* 53, 239–296.
- Neumeyer, P.A., Perri, F., 2005. Business cycles in emerging economies: the role of interest rates. *Journal of Monetary Economics* 52, 345–380.
- Obstfeld, M., 1981a. Macroeconomic policy, exchange-rate dynamics, and optimal asset accumulation. *Journal of Political Economy* 89, 1142–1161.
- Obstfeld, M., 1981b. Capital mobility and devaluation in an optimizing model with rational expectations. *American Economic Review (Papers and Proceedings)* 71, 217–221.
- Obstfeld, M., 1990. Intertemporal dependence, impatience, and dynamics. *Journal of Monetary Economics* 26, 45–75.
- Obstfeld, M., Rogoff, K., 1995. Exchange rate dynamics redux. *Journal of Political Economy* 103, 624–660.

- Obstfeld, M., Rogoff, K., 1996. *Foundations of International Macroeconomics*. MIT Press, Cambridge, MA.
- Obstfeld, M., Rogoff, K., 2001. The six major puzzles in international macroeconomics: is there a common cause? In: Bernanke, B.S., Rogoff, K. (Eds.), *NBER Macroeconomics Annual 2000*. MIT Press, Cambridge, MA, pp. 339–390.
- Pierdzioch, C., 2003. Non-separable consumption-labor choice and the international transmission of monetary policy shocks: a note. *International Economic Journal* 17, 55–64.
- Poloz, S., Rose, D., Tetlow, R., 1994. The Bank of Canada's new Quarterly Projection Model: an introduction. *Bank of Canada Review*, Autumn, pp. 23–38.
- Roberts, J.M., 1995. New Keynesian economics and the Phillips curve. *Journal of Money, Credit, and Banking* 27, 975–984.
- Rotemberg, J.J., 1982. Monopolistic price adjustment and aggregate output. *Review of Economic Studies* 49, 517–531.
- Sbordone, A.M., 2001. An optimizing model of US wage and price dynamics. Working Paper 2001-10, Department of Economics, Rutgers University.
- Schmitt-Grohé, S., 1998. The international transmission of economic fluctuations: effects of US business cycles on the Canadian economy. *Journal of International Economics* 44, 257–287.
- Schmitt-Grohé, S., Uribe, M., 2001. Stabilization policy and the costs of dollarization. *Journal of Money, Credit, and Banking* 33, 482–509.
- Schmitt-Grohé, S., Uribe, M., 2003. Closing small open economy models. *Journal of International Economics* 61, 163–185.
- Senhadji, A.S., 2003. External shocks and debt accumulation in a small open economy. *Review of Economic Dynamics* 6, 207–239.
- Shiells, C.R., Stern, R.M., Deardorff, A.V., 1986. Estimates of the elasticities of substitution between imports and home goods for the United States. *Weltwirtschaftliches Archiv* 122, 497–519.
- Smets, F., Wouters, R., 1999. The exchange rate and the monetary transmission mechanism in Germany. *De Economist* 147, 489–521.
- Smets, F., Wouters, R., 2002. Openness, imperfect exchange rate pass-through and monetary policy. *Journal of Monetary Economics* 49, 947–981.
- Svensson, L.E.O., 2003. What is wrong with Taylor rules? Using judgment in monetary policy through targeting rules. *Journal of Economic Literature* 41, 426–477.
- Taylor, J.B., 1993. Discretion vs. policy rules in practice. *Carnegie-Rochester Conference Series on Public Policy* 39, 195–214.
- Turnovsky, S.J., 1985. Domestic and foreign disturbances in an optimizing model of exchange rate determination. *Journal of International Money and Finance* 4, 151–171.
- Uribe, M., 1997. Exchange rate based inflation stabilization: the initial real effects of credible plans. *Journal of Monetary Economics* 39, 197–221.
- Uzawa, H., 1968. Time preference, the consumption function and optimum asset holdings. In: Wolfe, J.N. (Ed.), *Value, Capital and Growth: Papers in Honor of Sir John Hicks*. Edinburgh University Press, Edinburgh, pp. 485–504.
- Weil, P., 1989a. Overlapping families of infinitely-lived agents. *Journal of Public Economics* 38, 183–198.
- Weil, P., 1989b. Money, time preference, and external balance. *European Economic Review* 33, 564–572.
- Woodford, M., 2003. *Interest and Prices: Foundations of a Theory of Monetary Policy*. Princeton University Press, Princeton, NJ.
- Yun, T., 1996. Nominal price rigidity, money supply endogeneity, and business cycles. *Journal of Monetary Economics* 37, 345–370.