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Net foreign assets and the exchange rate: Redux revived☆

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Abstract

We revisit Obstfeld and Rogoff's (1995) results on exchange rate dynamics in a two-country, monetary model with incomplete asset markets, stationary net foreign assets, and endogenous monetary policy. The nominal exchange rate exhibits a unit root. Under flexible prices, it also depends on the stock of real net foreign assets. With sticky prices, the exchange rate depends on the past GDP differential, along with net foreign assets. Endogenous monetary policy and asset dynamics have consequences for exchange rate overshooting. A persistent relative productivity shock results in delayed overshooting under both flexible and sticky prices. A persistent relative interest rate shock generates undershooting under flexible prices.

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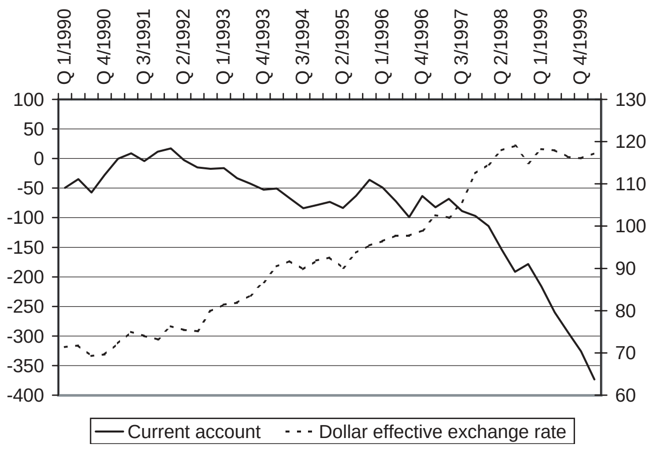

Fig. 1. The dollar and the U.S. current account.

1. Introduction

Obstfeld and Rogoff's (1995) "Exchange Rate Dynamics Redux" was originally written to put forth a model of exchange rate determination with an explicit role for current account imbalances. The non-stationarity of the model led most of the subsequent literature in the so-called "new open economy macroeconomics" to develop in different directions and "forget" the insights of the model on the dynamic relation between the exchange rate and net foreign asset accumulation by de-emphasizing the role of the latter.[1]

Fig. 1 shows two well known stylized facts: the persistent and growing U.S. current account deficit over the 1990s and the likewise persistent appreciation of the dollar.[2] It is a commonly held view that the advent of the "new economy" has been the most significant exogenous shock to affect the position of the U.S. economy relative to the rest of the world in recent years. We can interpret this shock as a (persistent) favorable relative productivity shock. A story that one could tell about the stylized facts in Fig. 1 is that the shock caused the U.S. to borrow from the rest of the world and the capital inflow generated exchange rate appreciation. This story could be reconciled with models of exchange rate determination developed in the 1970s and early 1980s. Among others, examples are Dornbusch and Fischer (1980) and Branson and Henderson (1985). If the shock is taken as permanent, the story can also be reconciled with Obstfeld and Rogoff's original model. Nevertheless, the

[1] This is achieved either by assuming unitary intratemporal elasticity of substitution between domestic and foreign goods in consumption as in Corsetti and Pesenti (2001) or by combining the assumptions of complete markets and power utility. Kollmann (2001) is a recent exception to the trend, although he uses a non-stationary model. For a survey of the literature, see Lane (2001).

[2] Source: National Accounts and Federal Reserve, respectively. Effective dollar rate: Broad exchange rate weighted average. Current account unit: billions of dollars.

argument cannot be reconciled with the overwhelming majority of new generation models that followed.

In this paper, we go back to the original intent of Obstfeld and Rogoff's work and develop a two-country model of exchange rate determination in which stationary net foreign asset dynamics play an explicit role. We deal with indeterminacy of the steady state and non-stationarity of the original incomplete markets setup by adopting the overlapping generations framework illustrated in Ghironi (2000). If exogenous shocks are stationary, the departure from Ricardian equivalence generated by the birth of new households with no assets in all periods is sufficient to ensure existence of a determinate steady state and stationarity of real variables. Unexpected temporary shocks cause countries to run current account imbalances, which are re-absorbed over time as the world economy returns to the original steady state.[3]

Differently from Obstfeld and Rogoff (1995) (and in line with the more recent literature on monetary policy), we allow for endogenous monetary policy in the form of interest rate reaction functions for the two countries. We consider familiar interest setting rules as in Taylor (1993). Interest rates react to the deviations of CPI inflation and GDP from their steady-state levels. They are also subject to exogenous shocks to allow for the possibility of exogenous changes in monetary policy.[4]

We solve the model with the method of undetermined coefficients illustrated in Campbell (1994). We rely on Uhlig's (1999) implementation of the method when solving the model numerically. The method has the advantage of delivering a process equation for the exchange rate with straightforward quantitative and empirical implications.

We are able to solve a benchmark model with purchasing power parity and flexible prices analytically. The solution for the nominal exchange rate exhibits a unit root, consistent with the empirical findings of Meese and Rogoff (1983). However, today's exchange rate also depends on the stock of real net foreign assets accumulated in the previous period. The intuition is as follows. No-arbitrage ensures that uncovered interest parity holds in our model: expected exchange rate depreciation equals the nominal interest rate differential. To the extent that interest rates react to variables that are affected by net foreign assets (namely, GDP, through the wealth effect on labor supply), net foreign assets affect the exchange rate too. Thus, the model implies that asset holdings help predict the nominal exchange rate. Consistent with the evidence for the U.S., *ceteris paribus*, a decrease in asset holdings—a current account

[3] Alternative approaches to the non-stationarity issue that preserve a role for net foreign asset dynamics under incomplete markets rely on introducing a cost of bond holdings, an endogenous discount factor, or a debt-sensitive risk premium. See Ghironi (2000) and Schmitt-Grohé and Uribe (2001) for references and discussion. (Schmitt-Grohé and Uribe compare the quantitative performance of these approaches in a small open economy setup.) Net foreign asset dynamics do not hinge on assumptions about a bond holding cost function or a non-standard discount factor in our model. Each individual household in the economy behaves as the representative agent of the original Obstfeld-Rogoff setup. Aggregate per capita assets are stationary, individual household's are not. Devereux (2002) and Smets and Wouters (2002) are recent studies that use a setup similar to ours.

[4] Benigno and Benigno (2001) study the consequences of endogenous interest setting for exchange rate dynamics in a sticky-price model with no net foreign asset accumulation.

deficit/capital inflow—generates an appreciation of the domestic currency for reasonable parameter values. The response of the exchange rate to shocks is more different from that of a simple random walk the slower the convergence of net foreign assets to the steady state and the higher the degree of substitutability between domestic and foreign goods in consumption.

The exchange rate overshoots its new long-run level following a temporary (relative) productivity shock. If the shock is persistent, endogenous monetary policy and asset dynamics generate delayed overshooting. Endogenous monetary policy is responsible for exchange rate *under*shooting after persistent (relative) interest rate shocks. (“Persistent” does *not* mean “permanent” throughout the paper. When we consider permanent shocks, we say so explicitly.)

Next, we analyze exchange rate and asset dynamics in a sticky-price world. We introduce price stickiness by assuming that it is costly to change output prices over time as in Rotemberg (1982). It is harder to solve the model analytically in this case. We investigate the effect of nominal and real shocks using a plausible calibration of the model. When prices are sticky, the exchange rate still exhibits a unit root under the Taylor rule, as in Benigno and Benigno (2001). The current level of the exchange rate depends on the past GDP differential, along with net foreign assets. Temporary shocks to relative productivity result in delayed overshooting. So do persistent shocks. Temporary relative interest rate shocks cause immediate overshooting. No overshooting may happen when interest rate shocks are persistent.

Our results on exchange rate overshooting contrast with Obstfeld and Rogoff's (1995), who obtain no overshooting following monetary and/or productivity shocks in their benchmark setup. We show that price stickiness is not necessary to generate overshooting once asset dynamics and endogenous monetary policy are accounted for. This brings a new perspective to bear on a topic that has been at the center of theoretical and empirical research on exchange rates since Dornbusch's (1976) seminal paper. Our model has the potential for reconciling the evidence in favor of delayed overshooting in Clarida and Galí (1994) and Eichenbaum and Evans (1995) with rational behavior and uncovered interest parity.

As far as the empirical performance is concerned, the model delivers exchange rate appreciation following a favorable shock to relative productivity in an environment in which monetary policy follows the Taylor rule. However, the model does not generate accumulation of net foreign debt following the shock, unless the latter is permanent and prices are sticky. The reason is that consumption smoothing is the only motive for asset accumulation in the model. If the relative productivity shock is permanent and prices are sticky, the new long-run level of domestic GDP relative to foreign is above the short-run differential, which causes domestic agents to borrow from abroad to smooth consumption. It remains to be seen whether the advent of the “new economy” has shifted U.S. productivity permanently above foreign. If one believes that the shock has been persistent, but not permanent, the model can explain only part of the dynamics in Fig. 1. Inclusion of capital accumulation and investment appears a promising way of completing the theory. On more rigorous grounds, the model yields straightforward, empirically testable implications for exchange rate dynamics.

The rest of the paper is organized as follows. Section 2 presents the model. Section 3 illustrates the log-linear equations that determine exchange rate and asset dynamics. Section 4 discusses the relation between net foreign assets and the exchange rate under flexible prices. Section 5 extends the analysis to the case of sticky prices. Section 6 concludes.

2. The model

The model is a monetary version of the setup in Ghironi (2000). The world consists of two countries, *home* and *foreign*. In each period t, the world economy is populated by a continuum of infinitely lived households between 0 and N_t^{W}. Each household consumes, supplies labor, and holds financial assets. As in Weil (1989), we assume that households are born on different dates *owning no assets*, but they own the present discounted value of their labor income.[5] The number of households in the home economy, N_t, grows over time at the exogenous rate $n>0$, i.e., $N_{t+1} = (1+n)N_t$. We normalize the size of a household to 1, so that the number of households alive at each point in time is the economy's population. Foreign population (N_t^*) grows at the same rate as home population. The world economy has existed since the infinite past. It is useful to normalize world population at time 0 to the continuum between 0 and 1, so that $N_0^{\mathrm{W}} = 1$.

A continuum of goods $i \in [0,1]$ are produced in the world by monopolistically competitive, infinitely lived firms, each producing a single differentiated good. Firms have existed since the infinite past. At time 0, the number of goods that are supplied in the world economy is equal to the number of households. The latter grows over time, but the commodity space remains unchanged. Thus, as time goes, the ownership of firms spreads across a larger number of households. Profits are distributed to consumers via dividends, and the structure of the market for each good is taken as given. We assume that the domestic economy produces goods in the interval $[0,a]$, which is also the size of the home population at time 0, whereas the foreign economy produces goods in the range $(a,1]$.

The asset menu includes nominal, uncontingent bonds denominated in units of domestic and foreign currency, money balances, and shares in firms. Private agents in both countries trade the bonds domestically and internationally. Shares in home (foreign) firms and domestic (foreign) currency balances are held only by home (foreign) residents.

2.1. Households

Agents have perfect foresight, though they can be surprised by initial unexpected shocks. Consumers have identical preferences over a real consumption index (C),

[5] Blanchard (1985) combines this assumption with a positive probability of not surviving until the next period. This is advantageous for calibration purposes (see below), besides being plausible. We adopt the Weil setup here because it is relatively simpler to illustrate.

labor effort supplied in a competitive market (L), and real money balances (M/P, where M denotes nominal money holdings and P is the consumption-based price index—CPI). We normalize the endowment of time in each period to 1. At any time t_0, the representative home consumer j born in period $\upsilon \in [-\infty, t_0]$ maximizes the intertemporal utility function:

$$U_{t_0}^{\upsilon j} = \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[\rho \log C_t^{\upsilon j} + (1-\rho)\log(1-L_t^{\upsilon j}) + \chi \log \frac{M_t^{\upsilon j}}{P_t} \right], \tag{1}$$

with $0<\rho<1$.[6]

The consumption index for the representative domestic consumer is a standard CES aggregator of foreign and domestic sub-indexes:

$$C_t^{\upsilon j} = [a^{1/\omega}(C_{\mathrm{H}t}^{\upsilon j})^{(\omega-1)/\omega} + (1-a)^{1/\omega}(C_{\mathrm{F}t}^{\upsilon j})^{(\omega-1)/\omega}]^{\omega/(\omega-1)},$$

where $\omega > 0$ is the intratemporal elasticity of substitution between domestic and foreign goods. The consumption sub-indexes that aggregate individual domestic and foreign goods are, respectively,

$$C_{\mathrm{H}t}^{\upsilon j} = \left[\left(\frac{1}{a}\right)^{1/\theta} \int_0^a c_t^{\upsilon j}(i)^{(\theta-1)/\theta}\, \mathrm{d}i \right]^{\theta/(\theta-1)}$$

and

$$C_{\mathrm{F}t}^{\upsilon j} = \left[\left(\frac{1}{1-a}\right)^{1/\theta} \int_a^1 c_{*t}^{\upsilon j}(i)^{(\theta-1)/\theta}\, \mathrm{d}i \right]^{\theta/(\theta-1)},$$

where $c_{*t}^{\upsilon j}(i)$ denotes time t consumption of good i produced in the foreign country, and $\theta > 1$ is the elasticity of substitution between goods produced inside each country.

The CPI is $P_t = [aP_{\mathrm{H}t}^{1-\omega} + (1-a)P_{\mathrm{F}t}^{1-\omega}]^{1/(1-\omega)}$, where P_{H} (P_{F}) is the price sub-index for home (foreign)-produced goods—both expressed in units of the home currency. Letting $p_t(i)$ be the home currency price of good i, we have $P_{\mathrm{H}t} = (\frac{1}{a}\int_0^a p_t(i)^{1-\theta}\, \mathrm{d}i)^{1/(1-\theta)}$ and $P_{\mathrm{F}t} = (\frac{1}{1-a}\int_a^1 p_t(i)^{1-\theta}\, \mathrm{d}i)^{1/(1-\theta)}$.

We assume that there are no impediments to trade and that firms do not engage in local currency pricing (i.e., pricing in the currency of the economy where goods are sold). Hence, the law of one price holds for each individual good and $p_t(i) = \varepsilon_t p_t^*(i)$, where ε_t is the exchange rate (units of domestic currency per unit of foreign) and $p_t^*(i)$ is the foreign currency price of good i. This hypothesis and identical intratemporal consumer preferences across countries ensure that consumption-based purchasing power parity (PPP) holds, i.e., $P_t = \varepsilon_t P_t^*$.

[6] We focus on domestic households. Foreign agents maximize an identical utility function. They consume the same basket of goods as home agents, with identical parameters, and they are subject to similar constraints. We will sometimes refer to the representative consumer of generation υ simply as the "representative consumer" below. It is understood that consumers of different generations can behave differently in our model.

The representative consumer enters a period holding bonds, money balances, and shares purchased in the previous period. She or he receives interests and dividends on these assets, may earn capital gains or incur losses on shares, earns labor income, is taxed, and consumes.

Denote the date t price (in units of domestic currency) of a claim to the representative domestic firm i's entire future profits (starting on date $t+1$) by V_t^i. Let $x_{t+1}^{\upsilon ji}$ be the share of the representative domestic firm i owned by the representative domestic consumer j born in period υ at the end of period t. D_t^i denotes the nominal dividends firm i issues on date t. Then, letting $A_{t+1}^{\upsilon j}(A_{t+1}^{*\upsilon j})$ be the home consumer's holdings of domestic (foreign) currency denominated bonds entering time $t+1$, the period budget constraint expressed in units of domestic currency is:

$$\begin{aligned} P_t C_t^{\upsilon j} + P_t T_t^{\upsilon} + A_{t+1}^{\upsilon j} + \varepsilon_t A_{t+1}^{*\upsilon j} + \int_0^a V_t^i x_{t+1}^{\upsilon ji}\,\mathrm{d}i + M_t^{\upsilon j} \\ = (1+i_t)A_t^{\upsilon j} + \varepsilon_t(1+i_t^*)A_t^{*\upsilon j} + \int_0^a (V_t^i + D_t^i)x_t^{\upsilon ji}\,\mathrm{d}i + M_{t-1}^{\upsilon j} + W_t L_t^{\upsilon j}, \end{aligned} \tag{2}$$

where i_t (i_t^*) is the nominal interest rate on holdings of domestic (foreign) bonds between $t-1$ and t, W_t is the nominal wage, $M_{t-1}^{\upsilon j}$ denotes the agent's holdings of nominal money balances entering period t, and T_t^{υ} is a lump-sum net real transfer, which is identical across members of generation υ. Given that individuals are born owning no financial wealth, because not linked by altruism to individuals born in previous periods, $A_{\upsilon}^{\upsilon j} = A_{\upsilon}^{*\upsilon j} = x_{\upsilon}^{\upsilon ji} = M_{\upsilon-1}^{\upsilon j} = 0$.

The representative domestic consumer born in period υ maximizes the intertemporal utility function (1) subject to the constraint (2). Dropping the j superscript (because symmetric agents make identical choices in equilibrium), optimal labor supply is given by:

$$L_t^{\upsilon} = 1 - \frac{1-\rho}{\rho}\frac{C_t^{\upsilon}}{w_t}, \tag{3}$$

which equates the marginal cost of supplying labor with the marginal utility of consumption generated by the corresponding increase in labor income (w_t denotes the real wage, W_t/P_t).

Making use of this equation, the first-order condition for optimal holdings of domestic currency bonds yields the Euler equation:

$$C_t^{\upsilon} = \left[\beta(1+i_{t+1})\left(\frac{P_t}{P_{t+1}}\right)\right]^{-1} C_{t+1}^{\upsilon} \tag{4}$$

for all $\upsilon \leqslant t$.

Demand for home currency real balances is:

$$\frac{M_t^{\upsilon}}{P_t} = \frac{\chi}{\rho}\frac{1+i_{t+1}}{i_{t+1}} C_t^{\upsilon}. \tag{5}$$

Real domestic currency balances increase with consumption and decrease with the opportunity cost of holding money.

Condition (4) can be combined with the first-order condition for holdings of foreign bonds to yield a no-arbitrage condition between domestic and foreign currency bonds for domestic agents. Absence of unexploited arbitrage opportunities requires:

$$1 + i_{t+1} = (1 + i^*_{t+1})\frac{\varepsilon_{t+1}}{\varepsilon_t}. \tag{6}$$

The consumption-based real interest rate between t and $t+1$ is defined by the familiar Fisher parity condition

$$1 + r_{t+1} = (1 + i_{t+1})\frac{P_t}{P_{t+1}} = \frac{1 + i_{t+1}}{1 + \pi^{CPI}_{t+1}}, \tag{7}$$

where π^{CPI}_{t+1} is CPI inflation ($\pi^{CPI}_{t+1} \equiv (P_{t+1}/P_t) - 1$). PPP ensures that $1 + \pi^{CPI}_t = (1 + e_t)(1 + \pi^{CPI*}_t)$, where $1 + \pi^{CPI*}_t \equiv P^*_t/P^*_{t-1}$ and $1 + e_t \equiv \varepsilon_t/\varepsilon_{t-1}$. Combining (7) with (6) and making use of PPP shows that $1 + r_{t+1} = 1 + r^*_{t+1} = (1 + i^*_{t+1})P^*_t/P^*_{t+1}$: real interest rates are equal across countries in the absence of unexpected shocks that may cause no-arbitrage conditions to fail ex post.

Absence of arbitrage opportunities between bonds and shares in the domestic economy requires $1 + i_{t+1} = (D^i_{t+1} + V^i_{t+1})/V^i_t$. Letting $d^i_t \equiv D^i_t/P_t$ and $v^i_t = V^i_t/P_t$, we can re-write this no-arbitrage condition as

$$1 + r_{t+1} = \frac{d^i_{t+1} + v^i_{t+1}}{v^i_t}. \tag{8}$$

As usual, first-order conditions and the period budget constraint must be combined with appropriate transversality conditions to ensure optimality.

2.2. Firms

Output supplied at time t by the representative domestic firm i is a linear function of labor demanded by the firm:

$$Y^{Si}_t = Z_t L^i_t. \tag{9}$$

Z_t is an exogenous economy-wide productivity parameter. Production by the representative foreign firm is a linear function of L^{i*}_t, with productivity parameter Z^*_t.[7]

Output demand comes from several sources: domestic and foreign consumers and domestic and foreign firms. The demand for home good i by the representative home consumer born in period υ is $c^\upsilon_t(i) = (p_t(i)/P_{Ht})^{-\theta}(P_{Ht}/P_t)^{-\omega}C^\upsilon_t$, obtained by maximizing C^υ subject to a spending constraint. *Total demand* for home good i

[7] Because all firms in the world economy are born at $t = -\infty$, after which no new goods appear, it is not necessary to index output and factor demands by the firms' date of birth. As for consumers, we focus on domestic firms below. Foreign firms are symmetric in all respects.

coming from domestic consumers is

$$c_t(i) = a\begin{bmatrix} \cdots\frac{n}{(1+n)^{t+1}}c_t^{-t}(i) + \cdots + \frac{n}{(1+n)^2}c_t^{-1}(i) + \frac{n}{1+n}c_t^0(i) \\ +nc_t^1(i) + n(1+n)c_t^2(i) + \cdots + n(1+n)^{t-1}c_t^t(i) \end{bmatrix}$$
$$= \left(\frac{p_t(i)}{P_{\mathrm{H}t}}\right)^{-\theta}\left(\frac{P_{\mathrm{H}t}}{P_t}\right)^{-\omega} a(1+n)^t c_t, \tag{10}$$

where

$$c_t \equiv \frac{a\begin{bmatrix} \cdots\frac{n}{(1+n)^{t+1}}C_t^{-t} + \cdots + \frac{n}{(1+n)^2}C_t^{-1} + \frac{n}{1+n}C_t^0 \\ +nC_t^1 + n(1+n)C_t^2 + \cdots + n(1+n)^{t-1}C_t^t \end{bmatrix}}{a(1+n)^t} \tag{11}$$

is aggregate per capita home consumption.

Given identity of intratemporal preferences, *total* demand for the same good by foreign consumers is $c_t^*(i) = (p_t(i)/P_{\mathrm{H}t})^{-\theta}(P_{\mathrm{H}t}/P_t)^{-\omega}(1-a)(1+n)^t c_t^*$, where c_t^* is aggregate per capita foreign consumption.

Changing the price of its output is costly for the firm, which generates nominal rigidity. Specifically, we assume that the real cost (measured in units of the composite good) of output-price inflation volatility around a steady-state level of inflation equal to 0, is $PAC_t^i \equiv (\kappa/2)[(p_t(i)/p_{t-1}(i)) - 1]^2(p_t(i)/P_t)Y_t^i$. When the firm changes the price of its output, a set of material goods—e.g., new catalogs, price tags, etc.—need to be purchased. The price adjustment cost (PAC^i) captures the amount of marketing materials that must be purchased to implement a price change. Because the amount of these materials is likely to increase with firm size, PAC^i increases with revenues ($(p_t(i)/P_t)Y_t^i$), which are taken as a proxy for size. The cost is convex in inflation; faster price movements are more costly to the firm. We assume $\kappa \geqslant 0$. When $\kappa = 0$, prices are flexible. The quadratic specification for the cost of adjusting prices, first introduced by Rotemberg (1982), yields dynamics for the aggregate economy that are similar to those resulting from staggered price setting as in Calvo (1983).

Total demand for good i produced in the home country is obtained by adding the demands for that good originating in the two countries. Making use of the results above, it is

$$Y_t^{\mathrm{D}i} = \left(\frac{p_t(i)}{P_{\mathrm{H}t}}\right)^{-\theta}\left(\frac{P_{\mathrm{H}t}}{P_t}\right)^{-\omega} Y_t^{\mathrm{DW}}. \tag{12}$$

Y_t^{DW} is aggregate world demand of the composite good, defined as $Y_t^{\mathrm{DW}} \equiv C_t^{\mathrm{W}} + PAC_t^{\mathrm{W}}$. $C_t^{\mathrm{W}} \equiv (1+n)^t[ac_t + (1-a)c_t^*]$ and $PAC_t^{\mathrm{W}} \equiv aPAC_t^i + (1-a)PAC_t^{*i}$ denote aggregate world consumption and the world aggregate cost of adjusting prices, respectively.[8]

[8] The expression for the world aggregate cost of adjusting prices derives from the assumption that the number of firms is constant. In the expression for PAC_t^{W}, we have already made use of the fact that symmetric firms make identical equilibrium choices. Keeping the i superscript for individual firms' variables allows us to denote several aggregate per capita variables referring to firms by dropping the superscript below.

At time t_0, firm i maximizes the present discounted value of dividends to be paid from t_0 on: $v^i_{t_0} + d^i_{t_0} = \sum_{s=t_0}^{\infty} R_{t_0,s} d^i_s$, where

$$R_{t_0,s} \equiv \frac{1}{\prod_{u=t_0+1}^{s}(1+r_u)}, \quad R_{t_0,t_0} = 1.$$

Firm revenues are taxed at a constant, proportional rate τ. In addition, firms receive a lump-sum transfer (or tax) from the government, T^{fi}_t. At each point in time, dividends are given by real revenues, net of taxes, plus the lump-sum transfer, minus costs: $d^i_t = (1-\tau)(p_t(i)/P_t)Y^i_t + T^{fi}_t - \{(W_t/P_t)L^i_t + (\kappa/2)[(p_t(i)/p_{t-1}(i)) - 1]^2(p_t(i)/P_t)Y^i_t\}$. The firm chooses the price of its product and the amount of labor demanded in order to maximize the present discounted value of its current and future profits subject to constraints (9) and (12), and the market clearing condition $Y^i_t = Y^{Si}_t = Y^{Di}_t$. Firm i takes the aggregate price indexes, the wage rate, Z_t, world aggregates, and taxes and transfers as given.

Let λ^i_t denote the Lagrange multiplier on the constraint $Y^{Si}_t = Y^{Di}_t$. Then, λ^i_t is the shadow price of an extra unit of output to be sold in period t, or the marginal cost of time t sales. The first-order condition with respect to $p_t(i)$ yields the pricing equation:

$$P_t(i) = \Psi^i_t P_t \lambda^i_t, \tag{13}$$

which equates the price charged by firm i to the product of the (nominal) shadow value of one extra unit of output—the (nominal) marginal cost ($P_t\lambda^i_t$)—and a markup (Ψ^i_t). The latter depends on output demand as well as on the impact of today's pricing decision on today's and tomorrow's costs of adjusting the output price:

$$\Psi^i_t \equiv \theta Y^i_t \left\{ (\theta-1) Y^i_t \left[1 - \tau - \frac{\kappa}{2}\left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right)^2 \right] + \kappa \Upsilon^i_t \right\}^{-1}, \tag{14}$$

where

$$\Upsilon^i_t \equiv Y^i_t \frac{p_t(i)}{p_{t-1}(i)}\left(\frac{p_t(i)}{p_{t-1}(i)} - 1 \right) - \frac{Y^i_{t+1}}{1+r_{t+1}} \frac{P_t}{P_{t+1}} \left(\frac{p_{t+1}(i)}{p_t(i)} \right)^2 \left(\frac{p_{t+1}(i)}{p_t(i)} - 1 \right)$$

reflects the firm's incentive to smooth price changes over time.

If $\kappa = 0$, i.e., if prices are fully flexible, $\Psi^i_t = \theta/[(\theta-1)(1-\tau)]$, the familiar constant-elasticity markup. If $\kappa \neq 0$, price rigidity generates endogenous fluctuations of the markup. Firms react to CPI dynamics in their pricing decisions. Changes in monetary policy generate changes in CPI inflation. Hence, they affect producer prices and the markup. Through this channel, they generate different dynamics of relative prices and the real economy.

The first-order condition for the optimal choice of L^i_t yields

$$\frac{W_t}{P_t} = \lambda^i_t Z_t. \tag{15}$$

Today's real wage must equal the shadow value of an extra unit of labor in production.

Using the market clearing conditions $Y^{Si}_t = Y^{Di}_t$ and $Y^{DW}_t = Y^{SW}_t = Y^W_t$, the expressions for supply and demand of good i, and recalling that symmetric firms make identical equilibrium choices (so that $p_t(i) = P_{Ht}$ and $p_t(i)$ is the producer price index, PPI) yields

$$L_t^i = \left(\frac{p_t(i)}{P_t}\right)^{-\omega} \frac{Y_t^W}{Z_t}. \tag{16}$$

Firm i's labor demand is a decreasing function of the relative price of good i and of labor productivity. It is an increasing function of world demand of the composite good. Henceforth, we denote the relative price of good i by $RP_t^i \equiv p_t(i)/P_t$.

2.3. The government

We assume that governments in both countries run balanced budgets. The government taxes firm revenues at a rate that compensates for monopoly power in a zero-inflation steady state and removes the markup over marginal cost charged by firms in a flexible-price world. The tax rate is determined by $1 - \tau = \theta/(\theta - 1)$, which yields $\tau = -1/(\theta - 1)$. Because the tax rate is negative, firms receive a subsidy on their revenues and pay lump-sum taxes determined by $T_t^{fi} = \tau RP_t^i Y_t^i$. In addition, the government injects money into the economy through lump-sum transfers of seignorage revenues to households: $P_t T_t^v = -(M_t^{vj} - M_{t-1}^{vj})$. Similarly for the foreign government.

2.4. Aggregation and equilibrium

2.4.1. Households

Aggregate per capita consumption and labor supply are obtained by aggregating consumption and labor supply across generations and dividing by total population at each point in time. Aggregate per capita labor supplies follow from aggregating the labor–leisure tradeoffs in the two economies

$$L_t = 1 - \frac{1-\rho}{\rho}\frac{c_t}{w_t}, \quad L_t^* = 1 - \frac{1-\rho}{\rho}\frac{c_t^*}{w_t^*}. \tag{17}$$

Labor supply rises with the real wage and decreases with consumption.

Consumption Euler equations in aggregate per capita terms contain an adjustment for consumption by the newborn generation at time $t+1$:

$$\begin{aligned} c_t &= \frac{1+n}{\beta(1+r_{t+1})}\left(c_{t+1} - \frac{n}{1+n} C_{t+1}^{t+1}\right), \\ c_t^* &= \frac{1+n}{\beta(1+r_{t+1})}\left(c_{t+1}^* - \frac{n}{1+n} C_{t+1}^{t+1*}\right). \end{aligned} \tag{18}$$

Newborn households hold no assets, but they own the present discounted value of their labor income. We define human wealth, h_t, as the present discounted value of the household's lifetime endowment of time in terms of the real wage: $h_t \equiv \sum_{s=t}^{\infty} R_{t,s} w_s$, $h_t^* \equiv \sum_{s=t}^{\infty} R_{t,s} w_s^*$. The dynamics of h and h^* are described by the following forward-looking difference equations:

$$h_t = \frac{h_{t+1}}{1+r_{t+1}} + w_t, \quad h_t^* = \frac{h_{t+1}^*}{1+r_{t+1}} + w_t^*. \tag{19}$$

Using the labor–leisure tradeoff (3), the Euler equation (4), and a newborn household's intertemporal budget constraint, it is possible to show that the household's consumption in the first period of its life is a fraction of the household's human wealth at birth:

$$C_{t+1}^{t+1} = \rho(1-\beta)h_{t+1}, \quad C_{t+1}^{t+1*} = \rho(1-\beta)h_{t+1}^*. \tag{20}$$

Aggregate per capita real money demands in the two economies are:

$$m_t \equiv \frac{M_t}{P_t} = \frac{\chi}{\rho}\frac{1+i_{t+1}}{i_{t+1}}c_t, \quad m_t^* \equiv \frac{M_t^*}{P_t^*} = \frac{\chi}{\rho}\frac{1+i_{t+1}^*}{i_{t+1}^*}c_t^*. \tag{21}$$

In the absence of arbitrage opportunities between bonds and shares, the aggregate per capita equity values of the home and foreign economies entering period $t+1$ must evolve according to:

$$v_t = \frac{1+n}{1+r_{t+1}}v_{t+1} + \frac{d_{t+1}}{1+r_{t+1}}, \quad v_t^* = \frac{1+n}{1+r_{t+1}}v_{t+1}^* + \frac{d_{t+1}^*}{1+r_{t+1}}. \tag{22}$$

where $v_t \equiv aV_t^i/(P_tN_{t+1})$, $v_t^* \equiv aV_t^{*i}/(P_t^*N_{t+1}^*)$, and d_t and d_t^* denote aggregate per capita real dividends, equal to $(1-\tau)y_t + T_t^f - w_tL_t - pac_t$ and $(1-\tau^*)y_t^* + T_t^{f*} - w_t^*L_t^* - pac_t^*$, respectively (note that $\tau = \tau^*$). y_t (y_t^*) denotes domestic (foreign) aggregate per capita, real GDP, defined below. pac_t (pac_t^*) is the aggregate per capita cost of nominal rigidity at home (abroad).

The law of motion of aggregate per capita net foreign assets is obtained by aggregating an equilibrium version of the budget constraint (2) across generations alive at each point in time. It is:

$$\begin{aligned}(1+n)B_{t+1} &= (1+r_t)B_t + w_tL_t + d_t - c_t,\\ (1+n)B_{t+1}^* &= (1+r_t)B_t^* + w_t^*L_t^* + d_t^* - c_t^*,\end{aligned} \tag{23}$$

where

$$B_{t+1} \equiv \frac{A_{t+1} + \varepsilon_t A_{t+1}^*}{P_t} \quad \text{and} \quad B_{t+1}^* \equiv \frac{\frac{A_{*t+1}}{\varepsilon_t} + A_{*t+1}^*}{P_t^*}$$

denote domestic and foreign net bond holdings (A_* denotes foreign households' holdings of home bonds, A_*^* denotes their holdings of foreign bonds). A country's net foreign assets and net foreign bond holdings coincide in a world in which all shares are held domestically.[9]

[9] Strictly speaking, these equations hold in all periods *after* the initial one. No-arbitrage conditions may be violated between time $t_0 - 1$ and t_0 if an unexpected shock surprises agents at the beginning of period t_0. Using log-linear versions of these equations to determine asset accumulation in the initial period is harmless if one is willing to assume that the steady-state levels of A, A^*, A_*, and A_*^* are all zero. (As we show below, the model pins down the steady-state levels of B and B^* endogenously. Because domestic and foreign bonds are perfect substitutes once no-arbitrage conditions are met, the model does not pin down the levels of A, A^*, A_*, and A_*^*.)

Because $d_t = y_t - w_t L_t - pac_t$ and $d_t^* = y_t^* - w_t^* L_t^* - pac_t^*$ in equilibrium, the equations in (23) become

$$\begin{aligned}(1+n)B_{t+1} &= (1+r_t)B_t + y_t - c_t - pac_t,\\ (1+n)B_{t+1}^* &= (1+r_t)B_t^* + y_t^* - c_t^* - pac_t^*.\end{aligned} \tag{24}$$

2.4.2. Firms

Aggregate per capita, real GDP in each economy is obtained by expressing production of each differentiated good in units of the composite basket, multiplying by the number of firms, and dividing by population. In equilibrium, $RP_t^i = RP_t$ and similarly for foreign firms. Thus:

$$y_t = RP_t Z_t L_t, \quad y_t^* = RP_t^* Z_t^* L_t^*. \tag{25}$$

For given employment and productivity, real GDP rises with the relative price of the representative good produced, as this is worth more units of the consumption basket.

Aggregate per capita labor demand is:

$$L_t = RP_t^{-\omega} \frac{y_t^W}{Z_t}, \quad L_t^* = RP_t^{*^{-\omega}} \frac{y_t^W}{Z_t^*}, \tag{26}$$

where y_t^W is aggregate per capita world production of the composite good, equal to aggregate per capita world consumption plus the aggregate per capita resource cost of price changes, $c_t^W + pac_t^W$. It is $y_t^W = ay_t + (1-a)y_t^*$, $c_t^W = ac_t + (1-a)c_t^*$, $pac_t^W = apac_t + (1-a)pac_t^*$. Market clearing requires $y_t^W = c_t^W + pac_t^W$.

Domestic and foreign relative prices are equal to markups over marginal costs:

$$RP_t = \Psi_t \frac{w_t}{Z_t}, \quad RP_t^* = \Psi_t^* \frac{w_t^*}{Z_t^*}. \tag{27}$$

2.4.3. International equilibrium

For international asset markets to be in equilibrium, net, aggregate home assets (liabilities) must equal net, aggregate foreign liabilities (assets). In terms of aggregates per capita, it must be $aB_t + (1-a)B_t^* = 0$. Using this condition, the equations in (24) reduce to $y_t^W = c_t^W + pac_t^W$: consistent with Walras' Law, asset market equilibrium implies goods market equilibrium, and vice versa.

2.5. The steady state

2.5.1. Real variables

The procedure for finding the steady-state levels of real variables follows the same steps as in Ghironi (2000). As described there, the departure from Ricardian equivalence caused by entry of new households with no assets in each period generates dependence of aggregate per capita consumption growth on the stock of aggregate per capita net foreign assets. This yields determinacy of steady-state real net foreign asset holdings, and thus of the steady-state levels of other real variables in the model.

We denote steady-state levels of variables with overbars. A subscript -1 indicates that the steady state described below is going to be the position of the economy up to and including period $t = -1$ in our exercise.[10] Unexpected shocks can surprise agents at the beginning of period 0, generating the dynamics we describe in the following sections.

Given initial steady-state levels of productivity ($\bar{Z}_{-1} = \bar{Z}^*_{-1} = 1$) and inflation ($\bar{\pi}^{PPI}_{-1} = \bar{\pi}^{PPI*}_{-1} = \bar{\pi}^{CPI}_{-1} = \bar{\pi}^{CPI*}_{-1} = 0$, where $\pi^{PPI}_t \equiv (p_t(i) - p_{t-1}(i))/p_{t-1}(i)$ and π^{PPI*}_t is defined similarly), real variables are stationary, in the sense that they return to the initial position determined below following non-permanent shocks to productivity and/or inflation. (The restriction that inflation shocks ought not to be permanent for real variables to return to the steady state described below applies to the general case in which prices are sticky ($\kappa > 0$). If prices are flexible ($\kappa = 0$), real variables return to the steady state below also after permanent changes in inflation. When $\kappa > 0$, permanent deviations of domestic or foreign inflation from zero impose permanent resource costs on the economy. These costs generate a different long-run equilibrium for real variables.)

The model determines the steady state as follows. Consider the home economy, and set aggregate per capita consumption to be constant. It is:

$$\bar{c}_{-1}\left[1 - \frac{\beta(1+\bar{r}_{-1})}{1+n}\right] = \frac{n}{1+n}\bar{C}^{\upsilon}_{\upsilon_{-1}},$$

where $\bar{C}^{\upsilon}_{\upsilon}$ is steady-state consumption by a newborn generation in the first period of its life. It must be $\beta(1+\bar{r}_{-1})/(1+n) < 1$ for steady-state consumption to be positive. As we shall see, this holds as long as $n > 0$. Now, from Eq. (20) and the definition of h, it is $\bar{C}^{\upsilon}_{\upsilon_{-1}} = \rho(1-\beta)[(1+\bar{r}_{-1})/\bar{r}_{-1}]\bar{w}_{-1}$. Hence, aggregate per capita consumption as a function of the steady-state interest rate and real wage is

$$\bar{c}_{-1} = \frac{n\rho(1-\beta)(1+\bar{r}_{-1})}{\bar{r}_{-1}[1+n-\beta(1+\bar{r}_{-1})]}\bar{w}_{-1}. \tag{28}$$

Under the assumption that $\bar{Z}_{-1} = 1$, steady-state GDP is $\bar{y}_{-1} = \overline{RP}_{-1}\bar{L}_{-1}$. From the pricing equation, $\overline{RP}_{-1} = \bar{w}_{-1}$, because the monopolistic distortion is removed by the subsidy τ. It follows that

$$\bar{y}_{-1} = \bar{w}_{-1}\bar{L}_{-1}. \tag{29}$$

Using Eqs. (28), (29), and steady-state versions of the domestic labor supply in (17) and of the law of motion for home's net foreign assets yields

$$\bar{B}_0 = \frac{1}{\bar{r}_{-1} - n}\left\{\frac{n(1-\beta)(1+\bar{r}_{-1}) - \bar{r}_{-1}[1+n-\beta(1+\bar{r}_{-1})]}{\bar{r}_{-1}[1+n-\beta(1+\bar{r}_{-1})]}\right\}\bar{w}_{-1}. \tag{30}$$

The subscript for initial steady-state asset holdings is 0 rather than -1 because time-0 asset holdings are determined at time -1. Foreign steady-state assets ($\bar{B}^*_0$) are given by a similar expression, function of $\bar{r}_{-1}$ and $\bar{w}^*_{-1}$. Substituting for $\bar{B}_0$ and $\bar{B}^*_0$ in the asset market equilibrium condition, $a\bar{B}_0 + (1-a)\bar{B}^*_0 = 0$, yields

[10] There are two reasons for time indexes for steady-state levels of variables. On one side, when we consider non-stationary exogenous shocks, these will cause the economy to settle at a new long-run position. On the other side, we shall see that the levels of nominal variables may exhibit a unit root regardless of stationarity of the exogenous shocks.

$$\frac{1}{\bar{r}_{-1}-n}\left\{\frac{n(1-\beta)(1+\bar{r}_{-1})-\bar{r}_{-1}[1+n-\beta(1+\bar{r}_{-1})]}{\bar{r}_{-1}[1+n-\beta(1+\bar{r}_{-1})]}\right\} \times [a\bar{w}_{-1}+(1-a)\bar{w}^{*}_{-1}]=0. \tag{31}$$

Given non-zero real wages at home and abroad, the only admissible level of the interest rate that satisfies the market clearing condition is such that $\beta(1+\bar{r}_{-1})=1$, or $\bar{r}_{-1}=(1-\beta)/\beta$. Substituting this result into the expressions for $\bar{B}_0$ and $\bar{B}^{*}_0$ yields steady-state levels of domestic and foreign net foreign assets $\bar{B}_0=\bar{B}^{*}_0=0$. Consistent with the fact that the two economies are structurally symmetric in per capita terms, the long-run net foreign asset position is a zero equilibrium. Differently from Obstfeld and Rogoff (1995), this position is pinned down *endogenously* by the model.

Given these results, it is easy to verify that steady-state levels of endogenous variables other than real balances are:

$$\bar{w}_{-1}=\overline{RP}_{-1}=\bar{w}^{*}_{-1}=\overline{RP}^{*}_{-1}=1, \quad \bar{h}_{-1}=\bar{h}^{*}_{-1}=\frac{1}{1-\beta},$$

$$\bar{y}_{-1}=\bar{c}_{-1}=\bar{C}^{\upsilon}_{\upsilon_{-1}}=\bar{L}_{-1}=\bar{y}^{*}_{-1}=\bar{c}^{*}_{-1}=\bar{C}^{\upsilon *}_{\upsilon_{-1}}=\bar{L}^{*}_{-1}=\bar{y}^{\mathrm{W}}_{-1}=\bar{c}^{\mathrm{W}}_{-1}=\rho,$$

$$\bar{\Psi}_{-1}=\bar{\Psi}^{*}_{-1}=1, \quad \overline{pac}_{-1}=\bar{d}_{-1}=\bar{v}_{-1}=\overline{pac}^{*}_{-1}=\bar{d}^{*}_{-1}=\bar{v}^{*}_{-1}=0.$$

2.5.2. Real money balances and nominal variables

Given steady-state consumption, domestic steady-state real balances are determined by $\bar{m}_{-1}=\chi(1+\bar{i}_{-1})/(\bar{i}_{-1})$. Similarly for foreign. In a zero-inflation steady state, nominal interest rates at home and abroad are equal to the steady-state real interest rate: $\bar{i}_{-1}=\bar{i}^{*}_{-1}=(1-\beta)/\beta$. It follows that domestic and foreign real balances are, respectively: $\bar{m}_{-1}=\bar{m}^{*}_{-1}=\frac{\chi}{1-\beta}$.

Nominal money balances at home and abroad are determined by, respectively: $\bar{M}_{-1}=[\chi/(1-\beta)]\bar{P}_{-1}$, $\bar{M}^{*}_{-1}=[\chi/(1-\beta)]\bar{P}^{*}_{-1}$. Taking the ratio of $\bar{M}_{-1}$ to $\bar{M}^{*}_{-1}$ and using PPP yields $\bar{\varepsilon}_{-1}=\bar{M}_{-1}/\bar{M}^{*}_{-1}$. The steady-state exchange rate equals the ratio of money supplies. In the analysis below, we assume that monetary policy is conducted by setting the nominal interest rate. In order to pin down the initial steady-state level of the exchange rate, we assume that the initial level of money supplies was set by the domestic and foreign central banks at $\bar{M}_{-1}=\bar{M}^{*}_{-1}=\chi/(1-\beta)$. Structural symmetry of the two economies implies that the central banks' optimal choice of steady-state money supplies would satisfy $\bar{M}_{-1}=\bar{M}^{*}_{-1}$ if the two authorities had identical objectives. The level $\chi/(1-\beta)$ conveniently implies $\bar{\varepsilon}_{-1}=\bar{P}_{-1}=\bar{P}^{*}_{-1}=\bar{p}_{-1}(h)=\bar{p}^{*}_{-1}(f)=1$ ($\bar{p}_{-1}(h)$ and $\bar{p}^{*}_{-1}(f)$ are the steady-state levels of the domestic and foreign PPIs, respectively, which follow from $\overline{RP}_{-1}=\bar{p}_{-1}(h)/\bar{P}_{-1}=\overline{RP}^{*}_{-1}=\bar{p}^{*}_{-1}(f)/\bar{P}^{*}_{-1}=1$). The model does not pin down the steady-state levels of all nominal variables endogenously as functions of the structural parameters only. As a consequence, monetary policy may generate the presence of a unit root in the dynamics of price levels, the exchange rate, and nominal money balances. Steady-state levels of nominal variables may change as a consequence of temporary shocks depending on the nature of monetary policy.

3. The log-linear system

The equations that determine domestic and foreign variables can be log-linearized around the steady state. We use sans serif fonts to denote percentage deviations from the steady state. Percentage deviations of inflation, depreciation, and interest rates from the steady state refer to gross rates. From now on, π denotes the percentage deviation of the corresponding (gross) inflation rate from the steady state. It is convenient to solve the model for cross-country differences ($\mathsf{x}_t^{\mathrm{D}} \equiv \mathsf{x}_t - \mathsf{x}_t^*$ for any variable x) and world aggregates ($\mathsf{x}_t^{\mathrm{W}} \equiv a\mathsf{x}_t + (1-a)\mathsf{x}_t^*$). The levels of individual country variables can be recovered given solutions for differences and world aggregates. Because the focus of this paper is on the relation between the exchange rate and asset accumulation, which are determined by cross-country difference in our setup, we report only the main log-linear equations for cross-country differences.

3.1. No-arbitrage conditions

PPP implies that the CPI inflation differential equals exchange rate depreciation:

$$\pi_t^{CPI^{\mathrm{D}}} = \mathsf{e}_t, \tag{32}$$

where $\mathsf{e}_t \equiv \epsilon_t - \epsilon_{t-1}$ and ϵ denotes the percentage deviation of ε from the steady state.

Uncovered interest parity (UIP) implies

$$\mathsf{i}_{t+1}^{\mathrm{D}} = \epsilon_{t+1} - \epsilon_t. \tag{33}$$

3.2. Households

The relative labor–leisure tradeoff is

$$\mathsf{w}_t^{\mathrm{D}} = \mathsf{c}_t^{\mathrm{D}} + \frac{\rho}{1-\rho}\mathsf{L}_t^{\mathrm{D}}. \tag{34}$$

Log-linear Euler equations and consumption functions for newborn households imply that the consumption differential obeys

$$\mathsf{c}_t^{\mathrm{D}} = (1+n)\mathsf{c}_{t+1}^{\mathrm{D}} - n\mathsf{h}_{t+1}^{\mathrm{D}}, \tag{35}$$

where h is the deviation of human wealth from the steady state. The ex ante real interest rate has no effect, because agents in both countries face identical real rates. The random walk result of the standard Obstfeld and Rogoff (1995) model for real variables is transparent here. If $n = 0$, i.e., if no new agents with zero assets enter the economy, the consumption differential between the two countries follows a random walk. Any shock that causes a consumption differential today has permanent consequences on the relative level of consumption. When $n > 0$, the Euler equation is adjusted for consumption of a newborn generation in the first period of its life ($\mathsf{C}_t^{t\mathrm{D}} = \mathsf{h}_t^{\mathrm{D}}$). The human wealth differential, h^{D}, is determined by

$$h_t^{\mathrm{D}} = \beta h_{t+1}^{\mathrm{D}} + (1-\beta) w_t^{\mathrm{D}}. \tag{36}$$

3.3. Firms

The GDP differential obeys

$$y_t^{\mathrm{D}} = RP_t^{\mathrm{D}} + L_t^{\mathrm{D}} + Z_t^{\mathrm{D}}. \tag{37}$$

The relative price differential reflects relative markup and marginal cost dynamics

$$RP_t^{\mathrm{D}} = \psi_t^{\mathrm{D}} + w_t^{\mathrm{D}} - Z_t^{\mathrm{D}}, \tag{38}$$

where ψ denotes the percentage deviation of the markup (Ψ) from the steady state.[11] Similarly, the difference between domestic and foreign labor demand depends on the markup differential and on relative marginal cost and productivity:

$$L_t^{\mathrm{D}} = -\omega(\psi_t^{\mathrm{D}} + w_t^{\mathrm{D}} - Z_t^{\mathrm{D}}) - Z_t^{\mathrm{D}}. \tag{39}$$

Substituting Eqs. (38) and (39) into (37) yields an expression for the GDP differential as a function of relative markup and cost dynamics:

$$y_t^{\mathrm{D}} = -(\omega - 1)(\psi_t^{\mathrm{D}} + w_t^{\mathrm{D}} - Z_t^{\mathrm{D}}). \tag{40}$$

Combining labor demand (39) with the labor–leisure tradeoff (34) yields the equilibrium real wage differential:

$$w_t^{\mathrm{D}} = \frac{1}{1+\rho(\omega-1)}[(1-\rho)c_t^{\mathrm{D}} - \rho\omega\psi_t^{\mathrm{D}} + \rho(\omega-1)Z_t^{\mathrm{D}}]. \tag{41}$$

From firms' optimal pricing (Eq. (13) for domestic firms and the analogous equation for foreign), the PPI inflation differential depends positively on the CPI inflation differential and on relative markup and marginal cost growth:

$$\pi_t^{PPI^{\mathrm{D}}} = \pi_t^{CPI^{\mathrm{D}}} + \psi_t^{\mathrm{D}} - \psi_{t-1}^{\mathrm{D}} + w_t^{\mathrm{D}} - w_{t-1}^{\mathrm{D}} - (Z_t^{\mathrm{D}} - Z_{t-1}^{\mathrm{D}}). \tag{42}$$

Alternatively, the PPI inflation differential can be written as a function of nominal depreciation and relative real GDP growth, if $\omega \neq 1$:

$$\pi_t^{PPI^{\mathrm{D}}} = \epsilon_t - \epsilon_{t-1} - \frac{1}{\omega-1}(y_t^{\mathrm{D}} - y_{t-1}^{\mathrm{D}}). \tag{43}$$

Finally, using $1-\tau = 1-\tau^* = \theta/(\theta-1)$ and the definitions of domestic and foreign markups, relative markup dynamics depend on current and future pricing decisions:

$$\psi_t^{\mathrm{D}} = -\frac{\kappa}{\theta}[\pi_t^{PPI^{\mathrm{D}}} - \beta(1+n)\pi_{t+1}^{PPI^{\mathrm{D}}}]. \tag{44}$$

[11] We define the domestic terms of trade following Obstfeld and Rogoff (1995) as $p(h)/(\varepsilon p^*(f))$, where $p(h)$ ($p^*(f)$) is the producer currency price of the representative home (foreign) good. It is easy to verify that RP^{D} is the percentage deviation of the terms of trade from the steady state.

3.4. Asset accumulation

Log-linearizing the laws of motion for the real net foreign assets of domestic and foreign households, subtracting the resulting equation for foreign assets from that for home assets, and imposing the log-linear bond market equilibrium condition, $aB_t + (1-a)B_t^* = 0$, yields

$$B_{t+1} = \frac{1}{1+n}\left[\frac{1}{\beta}B_t + (1-a)(y_t^{\mathrm{D}} - c_t^{\mathrm{D}})\right]. \tag{45}$$

Accumulation of aggregate per capita domestic net foreign assets is faster (slower) the larger (smaller) the GDP (consumption) differential. (Because $\bar{B}_0 = \bar{B}_0^* = 0$, B and B^* are defined as percentage deviations of B and B^* from the steady-state level of domestic and foreign consumption, respectively.)

The dynamics of the relative equity value (relative stock market dynamics) reflect the relative behavior of the markup in the two economies (see Cavallo and Ghironi, 2001, for details):

$$v_t^{\mathrm{D}} = \beta(1+n)v_{t+1}^{\mathrm{D}} + \beta\psi_{t+1}^{\mathrm{D}}. \tag{46}$$

3.5. Monetary policy

We assume that central banks set interest rates according to simple Taylor-type rules of the form

$$i_{t+1} = \alpha_1 y_t + \alpha_2\pi_t^{CPI} + \xi_t, \quad i_{t+1}^* = \alpha_1 y_t^* + \alpha_2\pi_t^{CPI*} + \xi_t^* \tag{47}$$

with $\alpha_1 \geqslant 0$, $\alpha_2 > 1$. (Recall that i_{t+1} and i_{t+1}^* are set at time t.) The reaction coefficients to GDP and inflation are identical at home and abroad. Because the two economies are identical in all structural features, if central banks with identical objectives independently chose the optimal values of α_1 and α_2, they would choose identical reaction coefficients. ξ and ξ^* are exogenous interest rate shocks. We assume $\xi_t = \mu\xi_{t-1}$, $\xi_t^* = \mu\xi_{t-1}^*$, $\forall t > 0$ ($t = 0$ being the time of an initial, surprise impulse in our exercise), $0 \leqslant \mu \leqslant 1$. Hence, $\xi_t^{\mathrm{D}} = \mu\xi_{t-1}^{\mathrm{D}}$.

The interest rate rules in (47) yield $i_{t+1}^{\mathrm{D}} = \alpha_1 y_t^{\mathrm{D}} + \alpha_2\pi_t^{CPI^{\mathrm{D}}} + \xi_t^{\mathrm{D}}$. Because PPP implies $\pi_t^{CPI^{\mathrm{D}}} = \epsilon_t - \epsilon_{t-1}$, it is

$$i_{t+1}^{\mathrm{D}} = \alpha_1 y_t^{\mathrm{D}} + \alpha_2(\epsilon_t - \epsilon_{t-1}) + \xi_t^{\mathrm{D}}. \tag{48}$$

Before moving on, we stress that nominal interest rates react to the deviations of GDP from the steady state rather than to the output gap—the deviation of GDP from the flexible price equilibrium—in our benchmark policy specification. This is consistent with Taylor's (1993) original analysis. But a reaction to the output gap is the standard in the recent normative literature on monetary policy. We stick to the Taylor benchmark for essentially two reasons. First, this is a positive, rather than normative, paper. One of its purposes is to try and offer an explanation for dynamics that were observed in the 1990s, a period for which the Taylor-specification fits the

U.S. data fairly well. (Clarida and Gertler, 1997; Clarida et al., 1998, provide evidence for other countries.) Second, the normative claim that central banks should react to the output gap is borne out of representative agent models subject to rather stringent assumptions. It is not clear that the same result would hold here.

In addition to the assumptions about interest rate setting, we assume that speculative bubbles in prices or the exchange rate are ruled out by the commitment to fractional backing mechanisms as in Obstfeld and Rogoff (1983).

4. Net foreign assets and exchange rate dynamics under flexible prices

To understand better the impact of price stickiness on exchange rate dynamics in our setup, we start by analyzing exchange rate determination under flexible prices. If prices are flexible ($\kappa = 0$), a dichotomy exists between nominal and real variables in the model. Real variables affect nominal ones, but the converse is not true (except for real balances, which are a function of the nominal interest rate). There is no longer a time-varying, forward-looking markup. Equilibrium profits are zero in all periods, along with the equity value of both economies. The equations that describe firm behavior in the log-linear system for cross-country differences between real variables simplify, as $\psi_t^{\mathrm{D}} = d_t^{\mathrm{D}} = v_t^{\mathrm{D}} = 0\ \forall t$.

Given the simplified, flexible-price system, it is easy to show that $c_t^{\mathrm{D}} = w_t^{\mathrm{D}} = L_t^{\mathrm{D}} = y_t^{\mathrm{D}} = 0$ if $\omega = 1$. Unitary intratemporal elasticity of substitution ensures that domestic and foreign consumption, the real wage, employment, and GDP are equal regardless of productivity. Hence, to preserve bond market equilibrium, it must be $B_t = B_t^* = 0$ if $\omega = 1$. This is the result first obtained by Corsetti and Pesenti (2001). If the elasticity of substitution between domestic and foreign goods is one, accumulation of net foreign assets plays no role in the transmission of shocks, and current accounts are always zero: $y_t = c_t$ and $y_t^* = c_t^*$. The same result would arise with complete asset markets and $\omega = 1$. Assuming complete markets in one-period, contingent bonds with $\omega \neq 1$ would yield $c_t^{\mathrm{D}} = 0$ through perfect 'risk-sharing' between the domestic and the foreign economy. Net foreign assets would respond to relative GDP movements, but they would be determined residually.

To solve the model, observe that aggregating the consumption functions for individual domestic and foreign households and log-linearizing yields the following expression for the consumption differential:

$$c_t^{\mathrm{D}} = \frac{\rho(1-\beta)}{\beta(1-a)} B_t + h_t^{\mathrm{D}}. \tag{49}$$

The consumption differential in each period reflects the net foreign asset position of the two economies and the differential between the expected real wage paths from that period on.

Using (49) in conjunction with the flexible-price versions of (36), (40), (41), and (45) yields:

$$B_{t+1} = \gamma_1 B_t - \gamma_2 h_t^{\mathrm{D}} + \gamma_3 Z_t^{\mathrm{D}}, \tag{50}$$

$$h_t^{\mathrm{D}} = \gamma_4 h_{t+1}^{\mathrm{D}} + \gamma_5 B_t + \gamma_6 Z_t^{\mathrm{D}}, \tag{51}$$

where

$$\gamma_1 \equiv \frac{1+\rho(\omega\beta-1)}{\beta(1+n)[1+\rho(\omega-1)]}, \quad \gamma_2 \equiv \frac{\omega(1-a)}{(1+n)[1+\rho(\omega-1)]},$$
$$\gamma_3 \equiv \frac{(\omega-1)(1-a)}{(1+n)[1+\rho(\omega-1)]}, \quad \gamma_4 \equiv \frac{\beta[1+\rho(\omega-1)]}{\rho\omega+\beta(1-\rho)},$$
$$\gamma_5 \equiv \frac{\rho(1-\rho)(1-\beta)^2}{\beta(1-a)[\rho\omega+\beta(1-\rho)]}, \quad \gamma_6 \equiv \frac{\rho(1-\beta)(\omega-1)}{\rho\omega+\beta(1-\rho)}.$$

Eqs. (50) and (51) constitute a system of two equations in two unknowns (the endogenous state variable B and the forward-looking variable h^{D}) plus the exogenous relative productivity term Z^{D}. We assume $Z_t = \phi Z_{t-1}$, $Z_t^* = \phi Z_{t-1}^*$, $\forall t > 0$, $0 \leqslant \phi \leqslant 1$. Hence, $Z_t^{\mathrm{D}} = \phi Z_{t-1}^{\mathrm{D}}$. The stock of net foreign assets and the levels of the exogenous productivity parameters describe the state of the (real) economy in each period. We assume that the restrictions on structural parameter values such that the solution of system (50)–(51) exists and is unique are satisfied. The solution can be written as

$$B_{t+1} = \eta_{BB} B_t + \eta_{BZ^{\mathrm{D}}} Z_t^{\mathrm{D}}, \tag{52}$$

$$h_t^{\mathrm{D}} = \eta_{h^{\mathrm{D}}B} B_t + \eta_{h^{\mathrm{D}}Z^{\mathrm{D}}} Z_t^{\mathrm{D}}, \tag{53}$$

where η_{BB} is the elasticity of time-$t+1$ assets to their time-t level, $\eta_{BZ^{\mathrm{D}}}$ is the elasticity of time-$t+1$ assets to the time-t productivity differential between home and foreign (Z_t^{D}), $\eta_{h^{\mathrm{D}}B}$ is the elasticity of h_t^{D} to time-t assets, and $\eta_{h^{\mathrm{D}}Z^{\mathrm{D}}}$ is the elasticity of h_t^{D} to Z_t^{D}. The values of the elasticities η as functions of the structural parameters of the model can be obtained with the method of undetermined coefficients as in Campbell (1994).

Given the solutions for real variables, the path of the nominal exchange rate can be determined by using the UIP condition (33) in conjunction with the interest setting rules for the domestic and foreign economy. Combining Eq. (48) with UIP and rearranging, we obtain

$$\epsilon_{t+1} - (1+\alpha_2)\epsilon_t + \alpha_2\epsilon_{t-1} = \alpha_1 y_t^{\mathrm{D}} + \xi_t^{\mathrm{D}}. \tag{54}$$

Now, the solution for the GDP differential is

$$y_t^{\mathrm{D}} = \eta_{y^{\mathrm{D}}B} B_t + \eta_{y^{\mathrm{D}}Z^{\mathrm{D}}} Z_t^{\mathrm{D}}, \tag{55}$$

where $\eta_{y^{\mathrm{D}}B}$ ($\eta_{y^{\mathrm{D}}Z^{\mathrm{D}}}$) is the elasticity of the GDP differential to the net foreign asset position (productivity differential).

Hence, the dynamics of real net foreign assets and the exchange rate are determined by the system (52), (54), (55), $Z_t^{\mathrm{D}} = \phi Z_{t-1}^{\mathrm{D}}$, and $\xi_t^{\mathrm{D}} = \mu \xi_{t-1}^{\mathrm{D}}$. The roots of the characteristic polynomial for the exchange rate equation are 1 and α_2. The assumption that $\alpha_2 > 1$ is sufficient to ensure determinacy. The presence of a root on the unit circle does not pose problems for determinacy of the solution given our assumptions on fractional backing.[12] Following Uhlig (1999), we conjecture a solution for the exchange rate of the form:

$$\epsilon_t = \eta_{\varepsilon\varepsilon}\epsilon_{t-1} + \eta_{\varepsilon B}\boldsymbol{B}_t + \eta_{\varepsilon Z^{\mathrm{D}}}\boldsymbol{Z}_t^{\mathrm{D}} + \eta_{\varepsilon\xi^{\mathrm{D}}}\xi_t^{\mathrm{D}}, \tag{56}$$

with elasticities $\eta_{\varepsilon\varepsilon}$, $\eta_{\varepsilon B}$, $\eta_{\varepsilon Z^{\mathrm{D}}}$, and $\eta_{\varepsilon\xi^{\mathrm{D}}}$.

The conjectured solution can be used to obtain expressions for the exchange rate elasticities with the method of undetermined coefficients. Substitute (55), (56), and the $t+1$-version of (56) into (54). Use (52) to substitute for $\boldsymbol{B}_{t+1}$ and $\phi\boldsymbol{Z}_t^{\mathrm{D}}$ for $\boldsymbol{Z}_{t+1}^{\mathrm{D}}$. Equating coefficients on ϵ_{t-1} on the left-hand side and on the right-hand side of the resulting equation yields $\eta_{\varepsilon\varepsilon}^2 - (1+\alpha_2)\eta_{\varepsilon\varepsilon} + \alpha_2 = 0$. This polynomial has roots 1 and α_2. Because $\alpha_2 > 1$, this root would yield unambiguously unstable dynamics for the exchange rate. Hence, we select $\eta_{\varepsilon\varepsilon} = 1$: the exchange rate exhibits a unit root. The intuition is simple: the reaction of interest rates to CPI inflation in an environment in which PPP holds at all points in time (including when an unexpected shock happens) causes today's interest setting to depend also on yesterday's level of the exchange rate. (At time 0, it is $\boldsymbol{i}_1^{\mathrm{D}} = \alpha_1\boldsymbol{y}_0^{\mathrm{D}} + \alpha_2\epsilon_0 + \xi_0^{\mathrm{D}}$, because the economy is assumed to be in steady state up to and including $t = -1$.) In turn, this causes today's exchange rate to depend on its past value. Stability imposes that the relevant root be 1.

It is important to note that validity of the Taylor principle ($\alpha_2 > 1$) is not necessary for the exchange rate to exhibit a unit root. When the Taylor principle holds, the solution we are describing is unique. If the Taylor principle does not hold, it is possible to prove that there exists *a* solution in which the exchange rate does not contain a unit root. However, indeterminacy of the solution when α_2 is smaller than 1 causes existence of sunspot equilibria that may well exhibit a unit root, including the solution described here. Instead, there would be no unit root in the exchange rate if central banks were setting interest rates to react to the *level* of the CPI rather than to CPI inflation with a strictly positive coefficient.

Equating coefficients on $\boldsymbol{B}_t$ in the undetermined coefficients equation and using $\eta_{\varepsilon\varepsilon} = 1$ yields $\eta_{\varepsilon B} + \eta_{\varepsilon B}\eta_{BB} - (1+\alpha_2)\eta_{\varepsilon B} = \alpha_1\eta_{y^{\mathrm{D}}B}$, from which $\eta_{\varepsilon B} = -\alpha_1\eta_{y^{\mathrm{D}}B}/(\alpha_2 - \eta_{BB})$. As $\alpha_2 > 1$ and $\eta_{BB} < 1$ (assets are stationary), $\alpha_2 - \eta_{BB} > 0$. Thus, the sign of $\eta_{\varepsilon B}$—the elasticity of the exchange rate to net foreign assets—is the opposite of that of $\eta_{y^{\mathrm{D}}B}$—the elasticity of the GDP differential to net foreign assets. If this elasticity is negative, accumulation of foreign debt (a capital inflow, $\boldsymbol{B}_t < 0$) results in an appreciation of the exchange rate below its steady-state level. We show that the sign of $\eta_{y^{\mathrm{D}}B}$ is the opposite of the sign of $\eta_{c^{\mathrm{D}}B}$ in the appendix of Cavallo and Ghironi (2001). For most plausible combinations of values of the structural parameters β, ρ, ω, ϕ, a, and n, it is $\eta_{c^{\mathrm{D}}B} > 0$. Intuitively, accumulation of net foreign assets allows the home economy to sustain a higher consumption path than foreign. It follows that $\eta_{y^{\mathrm{D}}B} < 0$: *ceteris paribus*, accumulation of net foreign assets causes domestic agents to

[12] Details are in the appendix of Cavallo and Ghironi (2001).

supply less labor than foreign, the domestic real wage and relative price are higher than abroad (i.e., the domestic terms of trade improve), and domestic GDP falls relative to foreign. Hence, $\eta_{\varepsilon B} > 0$.[13]

The positive elasticity of the exchange rate to net foreign assets is consistent with the empirical evidence for the U.S. Suppose that the domestic economy runs a current account deficit during period 0. This corresponds to a capital inflow in the model. Net foreign assets entering period 1 are negative. $\eta_{\varepsilon B} > 0$ implies that the exchange rate appreciates during period 1 as a consequence of the inflow of capital at time 0. The fact that the model delivers appreciation following a capital inflow under Taylor-type policies is consistent with the observed behavior of the dollar over the last few years. The experience of the U.S. in the 1990s has been one of current account deficits, capital inflows, accumulation of increasing net foreign debt, and appreciation of the dollar under monetary policy consistent with the Taylor rule. The model says that, *ceteris paribus*, accumulation of net foreign debt causes domestic GDP to rise above foreign because domestic agents have an incentive to supply more labor. The reaction of central banks to GDP movements widens the interest rate differential, which results in appreciation. The mechanism highlighted in the model is not inconsistent with the behavior of the U.S. economy in the 1990s.

If central banks do not react to GDP movements ($\alpha_1 = 0$), the exchange rate is not affected by asset accumulation ($\eta_{\varepsilon B} = 0$). The latter matters for the exchange rate because it generates a GDP differential across countries. If this differential has no impact on interest rate setting, it has no effect on the exchange rate either.

Equating coefficients on Z_t^{D} and using the previous results yields the elasticity of the exchange rate to productivity:

$$\eta_{\varepsilon Z^{\mathrm{D}}} = -\frac{\alpha_1[(\alpha_2 - \eta_{BB})\eta_{y^{\mathrm{D}}Z^{\mathrm{D}}} + \eta_{y^{\mathrm{D}}B}\eta_{BZ^{\mathrm{D}}}]}{(\alpha_2 - \phi)(\alpha_2 - \eta_{BB})}.$$

Our assumptions ensure that it is $\alpha_2 - \phi > 0$. The sign of $\eta_{\varepsilon Z^{\mathrm{D}}}$ depends on that of $(\alpha_2 - \eta_{BB})\eta_{y^{\mathrm{D}}Z^{\mathrm{D}}} + \eta_{y^{\mathrm{D}}B}\eta_{BZ^{\mathrm{D}}}$. A favorable shock to relative domestic productivity causes domestic agents to accumulate net foreign assets to smooth consumption dynamics for plausible parameter values if $\phi < 1$ (see below). Hence, $\eta_{BZ^{\mathrm{D}}} > 0$ and $\eta_{y^{\mathrm{D}}B}\eta_{BZ^{\mathrm{D}}} < 0$ if $\phi < 1$. Because $\eta_{y^{\mathrm{D}}Z^{\mathrm{D}}} > 0$ for the same combinations of parameters, a sufficiently aggressive reaction of the central banks to inflation (α_2 sufficiently large) ensures $\eta_{\varepsilon Z^{\mathrm{D}}} < 0$: ceteris paribus, a positive shock to relative domestic productivity generates an appreciation of the exchange rate.[14] If $\alpha_1 = 0$, the exchange rate does not react to relative productivity shocks ($\eta_{\varepsilon Z}\mathrm{D} = 0$). The intuition is similar to that for $\eta_{\varepsilon B}$.

Finally, equating coefficients on ξ_t^{D} and solving yields $\eta_{\varepsilon\xi^{\mathrm{D}}} = -1/(\alpha_2 - \mu)$. Because $\alpha_2 - \mu > 0$ under our assumptions, the elasticity of the exchange rate to the relative

[13] For example, interpreting periods as quarters, these results hold with $\beta = 0.99$ (a standard value of the discount factor at quarterly frequency), $\rho = 0.33$ (in steady state, agents spend one-third of their time working), $\omega = 1.2$ (a conservative choice for this parameter, the results still hold for higher, possibly more realistic values), $\phi = 0$ (no persistence in productivity, the results hold also for ϕ as high as 0.99), $a = 0.5$ (the two economies have equal size), and $n = 0.01$ (population grows by 1 percent per quarter).

[14] All the results in this paragraph hold for the parameter values mentioned above and with the standard Taylor-reaction of the interest rates to inflation, $\alpha_2 = 1.5$.

interest rate shock is negative: $\eta_{\varepsilon\xi^{\mathrm{D}}} < 0$. An exogenous increase in the domestic interest rate relative to foreign causes the domestic currency to appreciate. The appreciation is larger the smaller $\alpha_2 - \mu$. If central banks react aggressively to inflation (α_2 large), the appreciation triggered by the shock is smaller. To understand the mechanism, suppose $\mu = 0$. In this case, the exchange rate jumps instantly to its new long-run level. The depreciation rate ($\mathsf{e}_t = \epsilon_t - \epsilon_{t-1}$) is zero in all periods after the time of the shock ($t = 0$, during which the depreciation rate equals the initial jump of the exchange rate—$\mathsf{e}_0 = \epsilon_0$). On impact, domestic inflation falls relative to foreign by the extent of the initial appreciation. This causes the interest differential to fall endogenously by α_2 times the initial appreciation. In equilibrium, the interest rate differential must be zero at all points in time, because it must equal expected depreciation in the following period. (At time 0 agents expect no further exchange rate movement in future periods.) Given a 1 percent exogenous impulse to the interest rate differential, it follows that the initial appreciation that is required to keep the interest differential at zero at the time of the shock is smaller the larger α_2. If the interest rate shock is more persistent, i.e., $\mu \in (0, 1)$, the exchange rate appreciates by more: a persistent shock generates expectations of continuing appreciation that are incorporated in the initial movement of the exchange rate.

A permanent shock to the interest rate differential ($\mu = 1$) would cause the percentage deviation of the exchange rate from the steady state to increase (in absolute value) by a constant amount in all periods. This implies that the percentage deviation of the exchange rate from the steady state reaches -100 percent in finite time. But a constant deviation of the rate of depreciation from its steady-state level (zero) amounts to a constant, non-zero rate of depreciation (appreciation in this case). An exchange rate that appreciates at a constant rate becomes arbitrarily small, but never actually reaches zero. Thus, the case of a permanent shock raises the issue of the reliability of the log-linear approximation, which becomes less and less informative on the actual path of the exchange rate as its deviation from the steady state becomes larger. The zero-bound on the exchange rate is indeed a non-issue in the case of a constant rate of appreciation. The conclusion of the log-linear model for long-run exchange rate behavior in the case of permanent shocks should be taken with caution.

To summarize, a flexible price setup yields the following exchange rate equation:

$$\epsilon_t = \epsilon_{t-1} - \frac{\alpha_1 \eta_{y^{\mathrm{D}}B}}{\alpha_2 - \eta_{BB}} \mathsf{B}_t - \frac{\alpha_1[(\alpha_2 - \eta_{BB})\eta_{y^{\mathrm{D}}Z^{\mathrm{D}}} + \eta_{y^{\mathrm{D}}B}\eta_{BZ^{\mathrm{D}}}]}{(\alpha_2 - \phi)(\alpha_2 - \eta_{BB})} \mathsf{Z}_t^{\mathrm{D}} - \frac{1}{\alpha_2 - \mu} \xi_t^{\mathrm{D}}. \tag{57}$$

The nominal exchange rate contains a unit root, but the stock of aggregate per capita real net foreign assets helps predict the exchange rate if central banks react to GDP movements in setting the interest rate. If there is no such reaction (or if there are no productivity shocks that generate movements in real variables), the process for the exchange rate simplifies to

$$\epsilon_t = \epsilon_{t-1} - \frac{1}{\alpha_2 - \mu} \xi_t^{\mathrm{D}}, \tag{58}$$

which is exactly the random walk result of Meese and Rogoff (1983) if $\mu = 0$.

Eq. (58) describes the exchange rate process also if $\omega = 1$. In this case, it is $\eta_{y^{\mathrm{D}}B} = \eta_{y^{\mathrm{D}}Z^{\mathrm{D}}} = \eta_{BZ^{\mathrm{D}}} = 0$, so that $\eta_{\varepsilon B} = \eta_{\varepsilon Z^{\mathrm{D}}} = 0$ (and $B_t = 0\ \forall t$). If the intratemporal elasticity of substitution is equal to 1, productivity shocks do not affect the exchange rate regardless of whether or not interest rate setting is reacting to GDP movements. This suggests that models that assume $\omega = 1$ may be poorly suited to analyze the relation between the exchange rate and productivity.

Finally, a unit root in the exchange rate is associated with unit roots in price levels and nominal money balances. Taylor rules of the form (47) do not generate stationary levels of nominal variables. This is consistent with the empirical evidence in favor of unit roots in these variables.

4.1. Impulse response analysis

To evaluate the relevance of asset holdings for exchange rate dynamics under flexible prices, we calculate impulse responses to productivity and interest rate shocks for a plausible parameterization of the model. Periods are interpreted as quarters. We use the parameter values mentioned above: $\beta = 0.99$, $\rho = 0.33$, $\omega = 1.2$, $a = 0.5$, and $n = 0.01$. Our choice of n is higher than realistic, at least if one has developed economies in mind and n is interpreted strictly as the rate of growth of population.[15] However, we could reproduce the same speed of return to the steady state with slower population growth in a version of the model that incorporates probability of not surviving as in Blanchard (1985). We take $n = 0.01$ as a proxy for that situation. In contrast, we use a lower than realistic value of ω. Estimates from the trade literature suggest that values significantly above 1 would be reasonable. (For example, see Shiells et al., 1986.) Our conservative choice of benchmark allows us to show that even small departures from the $\omega = 1$-case generate quite different results. We point out important consequences of higher values of ω below. We assume $\alpha_1 = 0.5$ and $\alpha_2 = 1.5$, as in the interest rate rule popularized by Taylor (1993).

Figs. 2 and 3 show the dynamics of aggregate per capita real net foreign assets and the exchange rate following a 1 percent increase in relative domestic productivity. We consider three values of the persistence parameter ϕ in the figures (0, 0.5, and 0.75) and omit (but mention) the responses for $\phi = 1$. When $\phi < 1$, the shock causes domestic GDP to rise above foreign (not shown). The GDP differential is more persistent the higher ϕ and returns to the steady state monotonically after time 0. The home economy accumulates net foreign assets following the shock (Fig. 2). When the shock is temporary ($\phi = 0$), net foreign assets decrease monotonically in the periods after the initial one. A persistent increase in productivity ($0 < \phi < 1$) causes the home economy to continue accumulating assets for several quarters before settling on the downward path to the steady state. The home economy accumulates no assets if the shock is permanent ($\phi = 1$). Domestic GDP and consumption rise permanently above foreign exactly by the same amount in the

[15] The average rate of quarterly population growth for the U.S. between 1973:1 and 2000:3 has been 0.0025.

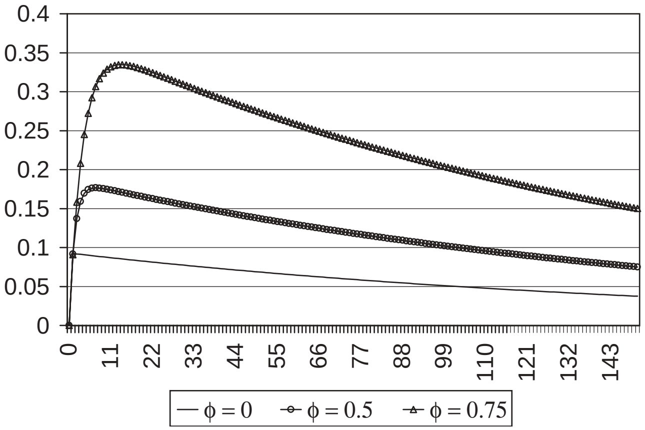

Fig. 2. Net foreign assets, productivity shock.

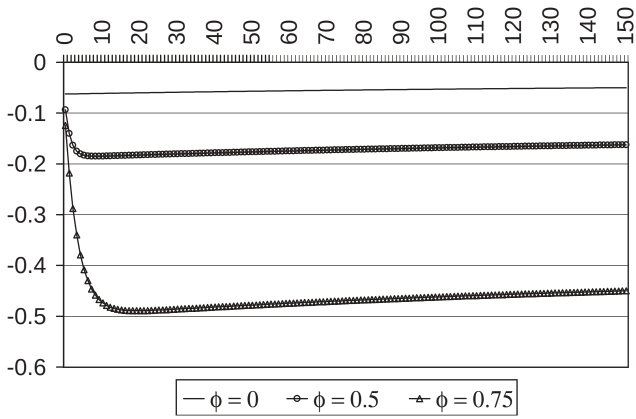

Fig. 3. Exchange rate, productivity shock.

period of the shock. Net foreign asset dynamics triggered by non-permanent shocks are extremely persistent. This is consistent with the evidence of persistence in net foreign assets in Kraay et al. (2000), whose regression results support an elasticity of net foreign assets at time $t+1$ to the time-t value that is very close to 1.[16]

The exchange rate appreciates on impact, the more so the more persistent the shock (Fig. 3). If $\phi = 0$, the path of the exchange rate is monotonic after the initial downward jump. The exchange rate overshoots its long-run level. Endogeneity of interest rate setting with $\alpha_1 > 0$ and asset dynamics generate overshooting with flexible prices. To understand this, observe that the exchange rate is determined by $\epsilon_t = \epsilon_{t-1} - [\alpha_1 \eta_{y^D B}/(\alpha_2 - \eta_{BB})]B_t$ in all periods after the initial shock. As B becomes positive after the initial period, the exchange rate climbs very slowly towards the new

[16] See also Ghironi (2000).

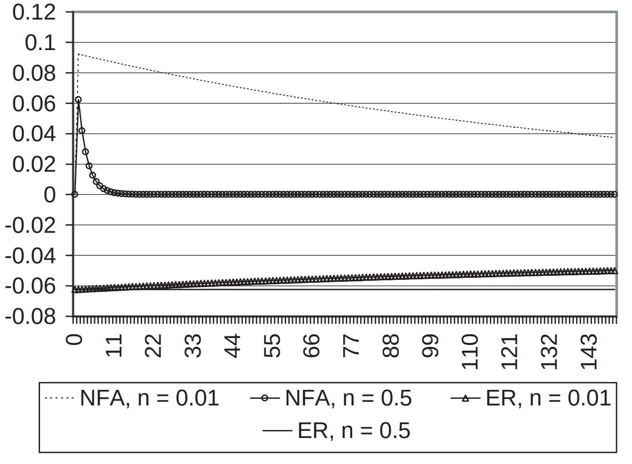

Fig. 4. Productivity shock, no persistence.

steady-state position (recall that $\eta_{\varepsilon B} > 0$). (The depreciation rate e_t becomes positive—albeit small—as the exchange rate starts moving towards its new steady-state level.) The new steady state is reached when net foreign assets have completed their transition back to zero. The transition is very slow because so is the speed of convergence of net foreign assets (determined by the rate at which new households enter the economy).

If $\phi \in (0, 1)$, delayed overshooting obtains. The initial jump is followed by further appreciation. A persistent (but not permanent) shock causes the stock of assets to increase until the shock has died out. That puts upward pressure on the exchange rate.[17] However, the shock generates appreciation beyond the initial jump as long as the productivity differential remains positive. As the shock dies out, the dynamics of asset holdings drive the exchange rate to its new long run level, between the initial response and the peak appreciation.

A permanent relative productivity shock ($\phi = 1$) causes no change in net foreign assets. The percentage deviation of the exchange rate from the steady state increases (in absolute value) by the same amount in all periods. The caveat we mentioned above about the reliability of the log-linearization for the path of the exchange rate following permanent shocks applies here.

To further investigate the relation between net foreign assets and the exchange rate, Fig. 4 shows their responses to a 1 percent domestic productivity shock with $\phi = 0$ for the benchmark value of n (0.01) and for an arbitrary, unrealistically large value (0.5), which delivers much faster convergence of real assets to the steady state.[18] The response of the exchange rate settles at its new long-run level much faster

[17] Note that changes in the persistence of shocks have no impact on the elasticity of other endogenous variables to asset holdings.

[18] With sticky prices, n is bounded above by $\bar{r}_{-1}$ for the log-linear system to be stable. We denote net foreign assets and the exchange rate with NFA and ER, respectively, in Figs. 4 and 5.

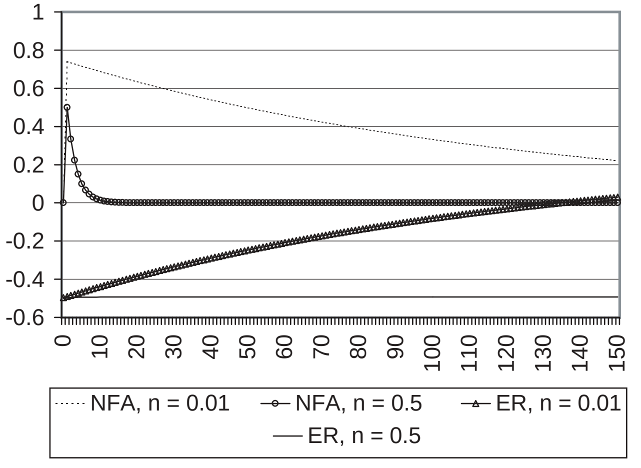

Fig. 5. Productivity shock, no persistence, $\omega = 4$.

when $n = 0.5$. Put differently, the response of the exchange rate is closer to that of a pure random walk the faster the speed of convergence of net foreign assets to the steady state. Even if the elasticity of the exchange rate to net foreign assets is very small for the parameter values we use ($\eta_{\varepsilon B} = 0.0014$ [0.0005] when $n = 0.01$ [0.5]), near non-stationary net foreign assets generate exchange rate dynamics that can be quite different from those of a random walk.

Fig. 5 repeats the exercise for a higher, more realistic value of ω ($\omega = 4$). The range of variation of net foreign assets and the exchange rate is one order of magnitude larger. Cross-country differences caused by asymmetric shocks are bigger if goods are more highly substitutable across countries. Even if $\eta_{\varepsilon B}$ remains small ($\eta_{\varepsilon B} = 0.0081$ [0.0041] when $n = 0.01$ [0.5]), the difference between the $n = 0.01$ and 0.5 cases becomes more pronounced.[19] The extent to which slow convergence to the steady state causes net foreign assets to affect exchange rate dynamics is more relevant the higher the degree of substitutability between domestic and foreign goods in consumption.

Eq. (58) determines the exchange rate in response to a 1 percent domestic interest rate shock (with $\xi_t^{\mathrm{D}} = \xi_t$). The response is non-stationary and no overshooting takes place. The cases $\mu = 0$ and 1 have been discussed above. When $\mu \in (0, 1)$, Eq. (58) implies that the exchange rate *under*shoots its new long-run level on impact. It continues to appreciate as the shock dies out and eventually settles at its new steady state.[20]

To summarize our analysis of the flexible price benchmark, the unit root in (57), combined with stationary real net foreign assets and shock processes, unambiguously delivers a non-stationary process for the nominal exchange rate. Since the

[19] In this case, the exchange rate actually *depreciates* in the long run when $n = 0.01$.

[20] The value of ω has of course no impact on the effect of interest rate shocks under flexible prices.

deviation of net foreign assets from the steady state becomes negligible in finite time following a non-permanent shock, the exchange rate eventually settles on a new long-run position if shocks are not permanent.[21] Notwithstanding the presence of a unit root in the exchange rate, impulse response analysis supports the idea that net foreign asset dynamics help predict the path of the nominal exchange rate to the extent that the elasticity of the latter to net foreign assets is different from zero. The quantitative relevance of net foreign assets for exchange rate behavior is enhanced if their law of motion is near non-stationary and if the elasticity of substitution between domestic and foreign goods is significantly above 1. Finally, the exercise of this section shows that price stickiness is not necessary to obtain exchange rate over- or undershooting following exogenous impulses. Endogenous interest rate setting and asset dynamics are sufficient.

5. Sticky prices

The exchange rate continues to be determined by Eq. (54). However, the dynamics of the real GDP differential (and of all other real variables) following productivity and interest rate shocks are now affected by the markup fluctuations generated by nominal rigidity.

It is possible to prove that $\omega = 1$ implies $B_{t+1} = 0 \ \forall t$ also under sticky prices regardless of other parameter values. Intuitively, Eq. (40) shows that the GDP differential is always zero regardless of productivity and interest rates if the elasticity of substitution between domestic and foreign goods is one. Because countries are starting off with zero net assets, identical GDP levels imply that the two economies have identical real resources to allocate to consumption in all periods. Thus, the utility maximizing choice entails $c_t^{\mathrm{D}} = 0 \ \forall t$.

The system on which we focus our attention for the general case $\omega \neq 1$ consists of Eqs. (35), (36), (40), (41), (43)–(46), and (54), combined with our assumptions on the shock processes, $\xi_t^{\mathrm{D}} = \mu \xi_{t-1}^{\mathrm{D}}$ and $Z_t^{\mathrm{D}} = \phi Z_{t-1}^{\mathrm{D}}$. It is hard to obtain an easily interpretable analytical solution for this system. Thus, we resort to Uhlig's (1999) numerical implementation of Campbell (1994). The endogenous state vector is $[B_{t+1}, \epsilon_t, \psi_t^{\mathrm{D}}, w_t^{\mathrm{D}}, y_t^{\mathrm{D}}, h_t^{\mathrm{D}}, v_t^{\mathrm{D}}]'$, the vector of other endogenous variables is $[\pi_t^{PPI^{\mathrm{D}}}, c_t^{\mathrm{D}}]'$, and the vector of exogenous driving forces is $[Z_t^{\mathrm{D}}, \xi_t^{\mathrm{D}}]'$. We include h_t^{D} and v_t^{D} in the state vector to avoid singularity problems in the solution. The method returns a unique stable solution for the parameter values we consider. We use the baseline parameterization above, which we repeat for convenience: $\beta = 0.99$, $\rho = 0.33$, $\omega = 1.2$, $a = 0.5$, $n = 0.01$, $\alpha_1 = 0.5$, and $\alpha_2 = 1.5$. We set κ to 77, the estimate in Ireland (2001), and θ to 6, consistent with Rotemberg and Woodford (1992). These values imply that PPI inflation of 1 percent would generate a resource cost of 0.385 percent

[21] If $n = 0$, a favorable shock to home productivity with $\phi = 0$ causes domestic net foreign assets to settle at a new (higher) steady-state level by the beginning of period 1. The exchange rate appreciates at $t = 0$, but it depreciates in all following periods, eventually shooting to infinity.

Table 1
The sticky-price solution, elasticities

Endogenous states to endogenous states

	B	ϵ	ψ^{D}	w^{D}	y^{D}	h^{D}	v^{D}
B	0.9944	0	0	0	0.3261	0	0
ϵ	0.0008	1	0	0	−0.4017	0	0
ψ^{D}	−0.0074	0	0	0	−5.3284	0	0
w^{D}	0.0096	0	0	0	1.9855	0	0
y^{D}	−0.0004	0	0	0	0.6686	0	0
h^{D}	0.0044	0	0	0	0.063	0	0
v^{D}	−0.0312	0	0	0	−10.676	0	0

Endogenous states to shocks

	$\phi=0$ Z^{D}	$\mu=0$ ξ^{D}	$\phi=0.5$ Z^{D}	$\mu=0.5$ ξ^{D}	$\phi=1$ Z^{D}	$\mu=1$ ξ^{D}
B	0.0076	−0.0435	0.0115	−0.0498	−0.0602	−0.0705
ϵ	−0.0093	−0.6131	−0.0217	−0.9067	−0.0989	−1.9899
ψ^{D}	1.3686	0.7104	1.3002	0.8265	0.8804	−0.0916
w^{D}	−0.4464	−0.2647	−0.4206	−0.3088	−0.1543	−0.12
y^{D}	0.0156	−0.0891	0.0241	−0.1035	0.0548	−0.0057
h^{D}	−0.0035	−0.0084	−0.0053	−0.0193	0.1607	0.0537
v^{D}	−0.2483	1.4235	0.5182	4.1245	3.8639	25.0141

Other endogenous variables to endogenous states

	B	ϵ	ψ^{D}	w^{D}	y^{D}	h^{D}	v^{D}
$\pi^{PPI^{\mathrm{D}}}$	0.003	0	0	0	1.2554	0	0
c^{D}	0.011	0	0	0	0.0098	0	0

Other endogenous variables to shocks

	$\phi=0$ Z^{D}	$\mu=0$ ξ^{D}	$\phi=0.5$ Z^{D}	$\mu=0.5$ ξ^{D}	$\phi=1$ Z^{D}	$\mu=1$ ξ^{D}
$\pi^{PPI^{\mathrm{D}}}$	−0.0871	−0.1674	−0.1421	−0.389	−0.3727	−1.9615
c^{D}	0.0002	−0.0013	0.0008	−0.0029	0.1764	0.1368

of aggregate per capita real GDP. Our choice of θ implies a steady-state markup (non-adjusted for the subsidy τ) of 20 percent.

Table 1 shows the solution for the relevant elasticities for different values of the persistence parameters ϕ and μ. A few facts emerge. The exchange rate continues to display a unit root. The latter emerges also under sticky prices as a consequence of endogenous interest rate setting with reactions to inflation rates.

Indeed, Table 1 suggests that nominal price rigidity implies that all variables in the endogenous state vector (with the exception of the exchange rate) can be written as functions of B_t, y^{D}_{t-1}, and the shocks only (i.e., that the values of all other elasticities are zero, at least for the benchmark parameterization and a number of plausible alternatives).

The solution for the exchange rate in Table 1 is of the form

$$\epsilon_t = \epsilon_{t-1} + \eta_{\varepsilon B} B_t + \eta_{\varepsilon y^{\mathrm{D}}} y_{t-1}^{\mathrm{D}} + \eta_{\varepsilon Z^{\mathrm{D}}} Z_t^{\mathrm{D}} + \eta_{\varepsilon \xi^{\mathrm{D}}} \xi_t^{\mathrm{D}}. \tag{59}$$

On empirical grounds, the sticky-price solution points to the past GDP differential as a determinant of the current exchange rate, along with net foreign assets accumulated in the previous period. As for the flexible-price case, $\eta_{\varepsilon B}$ is positive and small: a capital inflow generates exchange rate appreciation. Accumulation of net foreign debt is associated to higher domestic GDP than foreign because domestic agents supply more labor. The interest rate differential rises in response to a positive GDP differential, which leads to appreciation. The elasticity to the past GDP differential ($\eta_{\varepsilon y^{\mathrm{D}}}$) is negative for similar reasons. Sticky prices introduce persistence in the GDP process beyond its dependence on assets accumulated in the previous period. (Recall that GDP is measured in units of the consumption basket, i.e., by multiplying production of domestic goods by their relative price. Stickiness in the latter introduces stickiness in GDP.) As a consequence of GDP persistence, a positive GDP differential yesterday translates into a higher interest rate differential today and, hence, into appreciation. The negative elasticity to productivity ($\eta_{\varepsilon Z^{\mathrm{D}}}$) reflects the fact that higher domestic productivity generates higher domestic output directly, as in the flexible-price world. Again, this leads to a higher domestic interest rate and appreciation.[22] Consistent with uncovered interest parity, an exogenous increase in $\eta_{\varepsilon\xi^{\mathrm{D}}}$ causes the domestic currency to appreciate on impact.

As in the flexible-price world, changes in shock persistence affect only the elasticities to the shocks themselves. The elasticity of assets to productivity ($\eta_{BZ^{\mathrm{D}}}$) is an increasing function of the persistence of productivity shocks for $\phi < 1$. Intuitively, the more persistent a (non-permanent) favorable productivity shock, the stronger the incentive of households to accumulate assets to smooth consumption ($\eta_{y^{\mathrm{D}} Z^{\mathrm{D}}}$ is unambiguously increasing in ϕ). Interestingly, $\eta_{BZ^{\mathrm{D}}}$ is negative in the special case $\phi = 1$. The domestic country accumulates debt if there is a permanent favorable shock to relative productivity. The reason is that a permanent favorable productivity shock causes the new long-run level of domestic GDP to be above the initial jump following the shock (GDP stickiness through relative price persistence is responsible for this; see also below). As a consequence, optimal consumption smoothing dictates that domestic agents borrow in the anticipation of permanently higher income in the future. The relation between $\eta_{B\xi^{\mathrm{D}}}$ and the persistence of interest rate shocks (μ) is non-monotonic and less easily interpretable. The elasticity of the exchange rate to the productivity differential is larger (in absolute value) the more persistent the latter. As under flexible prices, a more persistent productivity differential generates anticipation of a more persistent interest rate differential, and hence a larger movement of the exchange rate on impact. As expected, the elasticity of the exchange rate to the relative interest rate shock increases with the persistence of the latter.

[22] Not surprisingly, if $\alpha_1 = 0$ (interest rates do not react to GDP movements), it is $\eta_{\varepsilon B} = \eta_{\varepsilon y^{\mathrm{D}}} = \eta_{\varepsilon Z^{\mathrm{D}}} = 0$ also under sticky prices.

5.1. Impulse response analysis

5.1.1. Productivity shock

Fig. 6 shows impulse responses to a 1 percent favorable shock to relative domestic productivity for the values of ϕ in Table 1.[23] Consider the $\phi = 0$—case. Under flexible prices, the exchange rate overshoots its new long-run equilibrium on impact. It then converges monotonically to the new steady state. The home country accumulates assets in the initial period, which it decumulates back to the steady state along the transition dynamics. Sticky prices cause a hump-shaped response of net foreign assets and delayed overshooting, as we observed under flexible prices when the productivity shock was persistent. The reason for this difference is exactly that price-stickiness imparts persistence in the dynamics of the GDP differential as described above. Domestic firms initially lower prices more than foreign firms, though the domestic markup rises relative to foreign to preserve profitability. Labor demand falls (not shown), and so does the real wage. However, these movements are quickly reversed. A more persistent favorable GDP differential causes home agents to continue accumulating assets in the first periods after the shock. As under flexible prices, continuing asset accumulation puts upward pressure on the exchange rate. But the added persistence in the GDP differential acts as persistence in productivity under flexible prices, causing further appreciation. When the GDP differential is close to zero, the upward pressure on the exchange rate from asset holdings above the steady state kicks in, and assets and the exchange rate converge slowly to their steady-state levels. Interestingly, the stock market value of the domestic economy relative to foreign is below the steady state throughout the transition, reflecting the fact that the markup is below the steady state for most of the time. As expected, domestic consumption is (slightly) above foreign.

When the productivity shock is more persistent—$\phi \in (0, 1)$—the dynamics of net foreign assets and the exchange rate are similar, with a hump-shaped response of assets and delayed exchange rate overshooting. Persistence in productivity introduces a hump in the response of GDP, consistent with the persistence effect of price rigidity and markup dynamics. (GDP rises with labor demand as the markup falls after the initial increase.) The markup is above the steady state for longer, which causes an initial stock market expansion relative to foreign.

If $\phi = 1$, the domestic country borrows from abroad for the reasons described above. Domestic GDP climbs to a permanently higher level than foreign over time due to price stickiness and markup dynamics. (Consumption smoothing causes the domestic economy to run a debt after a permanent favorable shock to relative productivity also in Obstfeld and Rogoff, 1995. The shock has no short-run effect on GDP in their setup.) The exchange rate appreciates in all periods in Fig. 6 (appreciation reaches 20 percent 30 years after the shock, though the caveat mentioned above applies here). The relative markup is permanently above the initial

[23] In Figs. 6 and 7, net foreign assets in each period (B1) are net foreign assets at the end of that period. The exchange rate and the PPI inflation and markup differentials are denoted by eps, ppiiD, and mkupD, respectively. The relative interest rate shock is csiD.

steady state (albeit by very little) and relative PPI inflation is permanently lower. The real wage differential falls initially, as higher productivity and the initial relative markup movement depress domestic labor demand relative to foreign. However, higher long-run GDP at home than abroad results in a positive long-run real wage

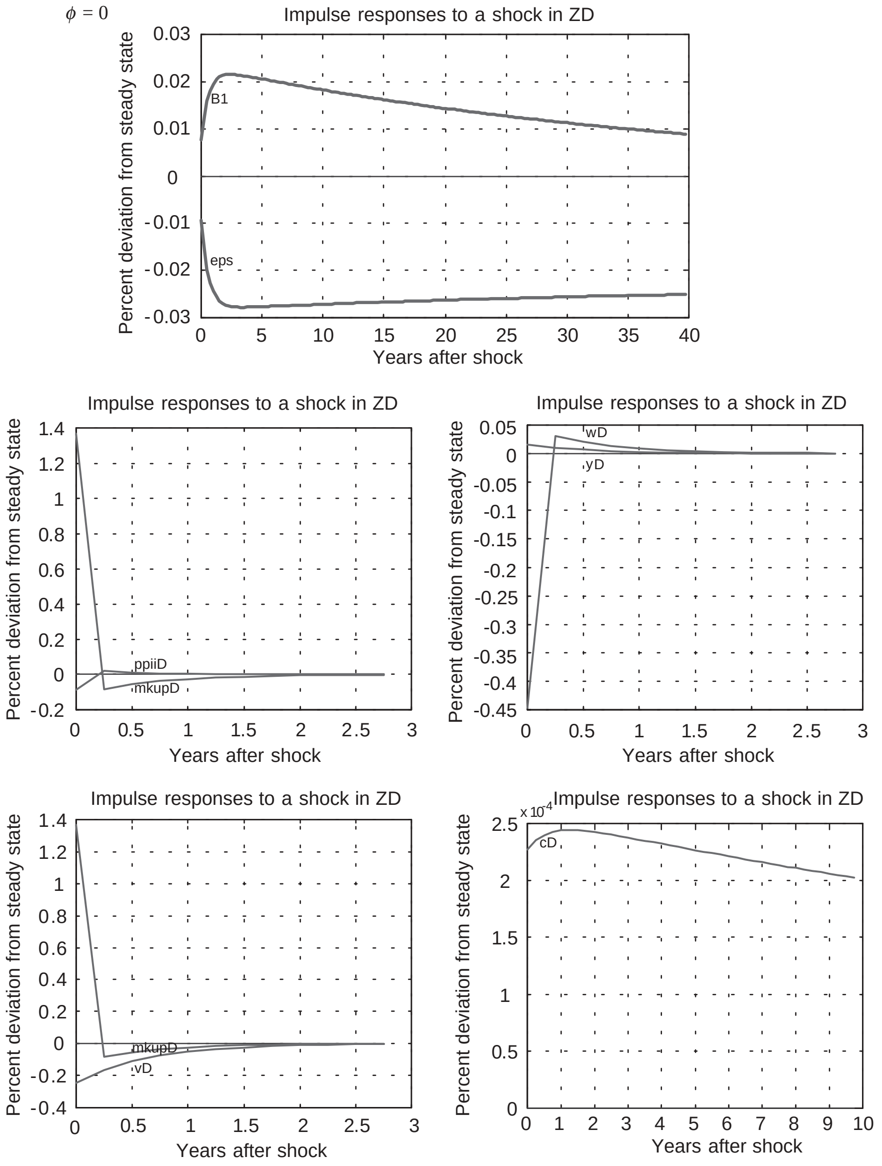

Fig. 6. Productivity shock, sticky prices.

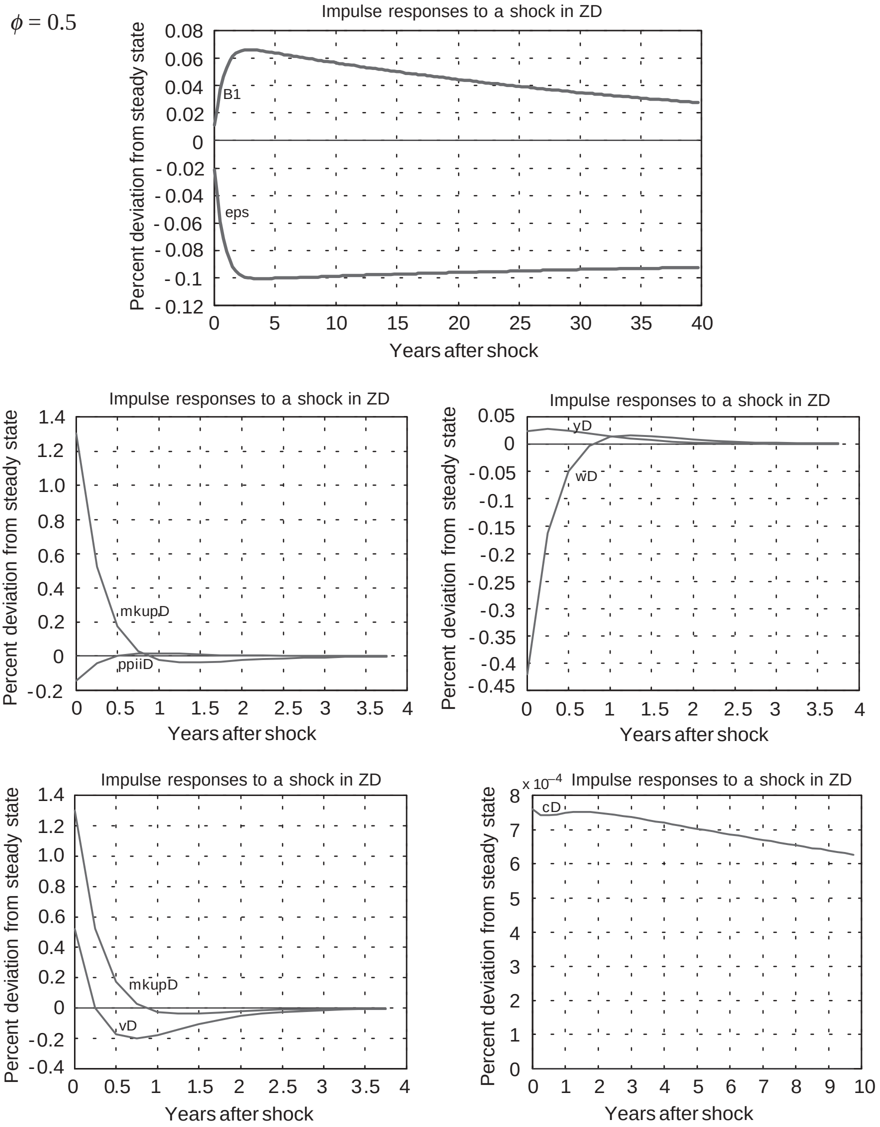

Fig. 6 (*continued*).

differential. A permanently higher markup causes the domestic stock market value to rise permanently above foreign. So does domestic consumption.

If one believes that the advent of the 'new economy' has shifted U.S. productivity permanently above foreign, our model provides a qualitative account of empirical observations of the past few years. The model also delivers much richer exchange rate dynamics following a productivity shock under sticky prices than its antecedent

$\phi = 1$

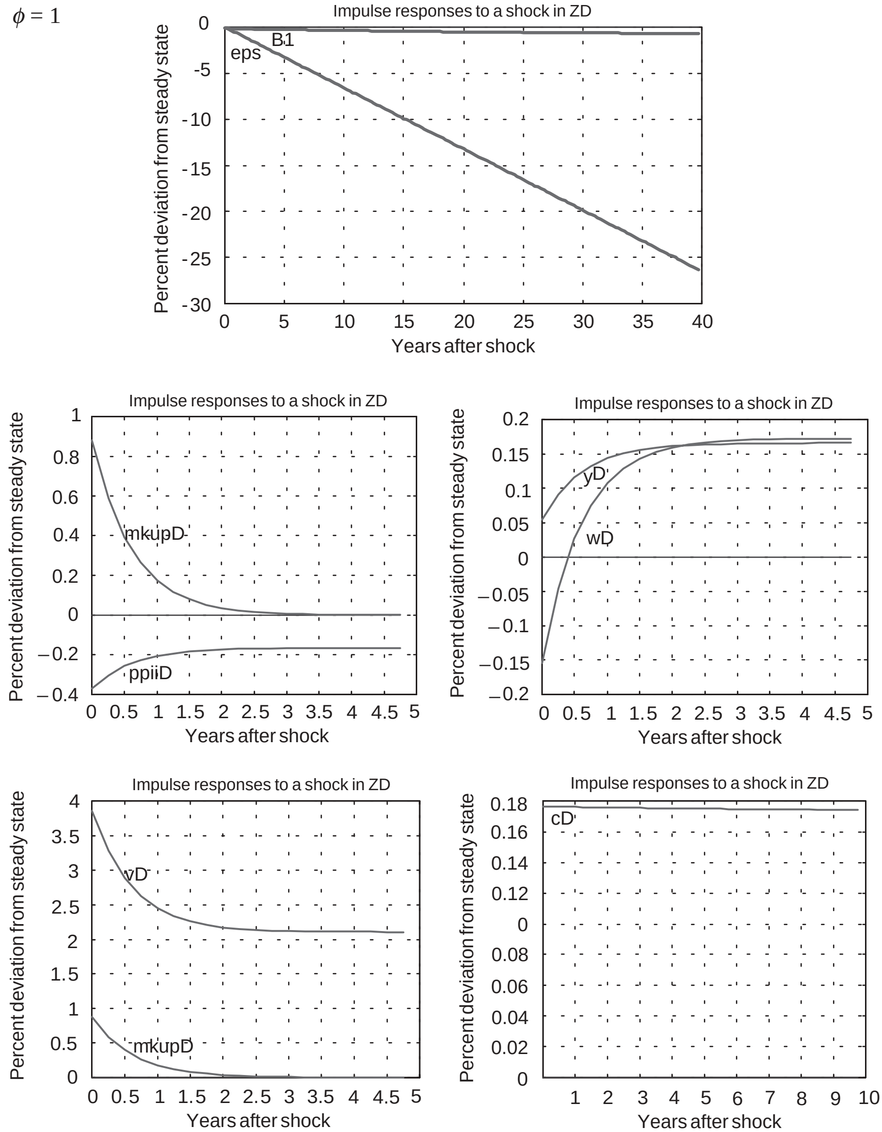

Fig. 6 (*continued*).

by Obstfeld and Rogoff (1995). The case of a temporary productivity shock is trivial in that model. Supply equations are not binding in the short run, and the shock has no output effect. A permanent shock causes the exchange rate to appreciate permanently in the period of the shock. No further dynamics happen and no overshooting is obtained in either case.

5.1.2. Interest rate shock

Fig. 7 shows impulse responses to a 1 percent exogenous increase in domestic interest rates for the values of μ in Table 1. Under flexible prices, the exchange rate jumped immediately to its new long-run equilibrium after an interest rate shock with no persistence ($\mu = 0$). When prices are sticky, the shock affects real variables, which in turn have an impact on exchange rate dynamics. Higher domestic interest rates

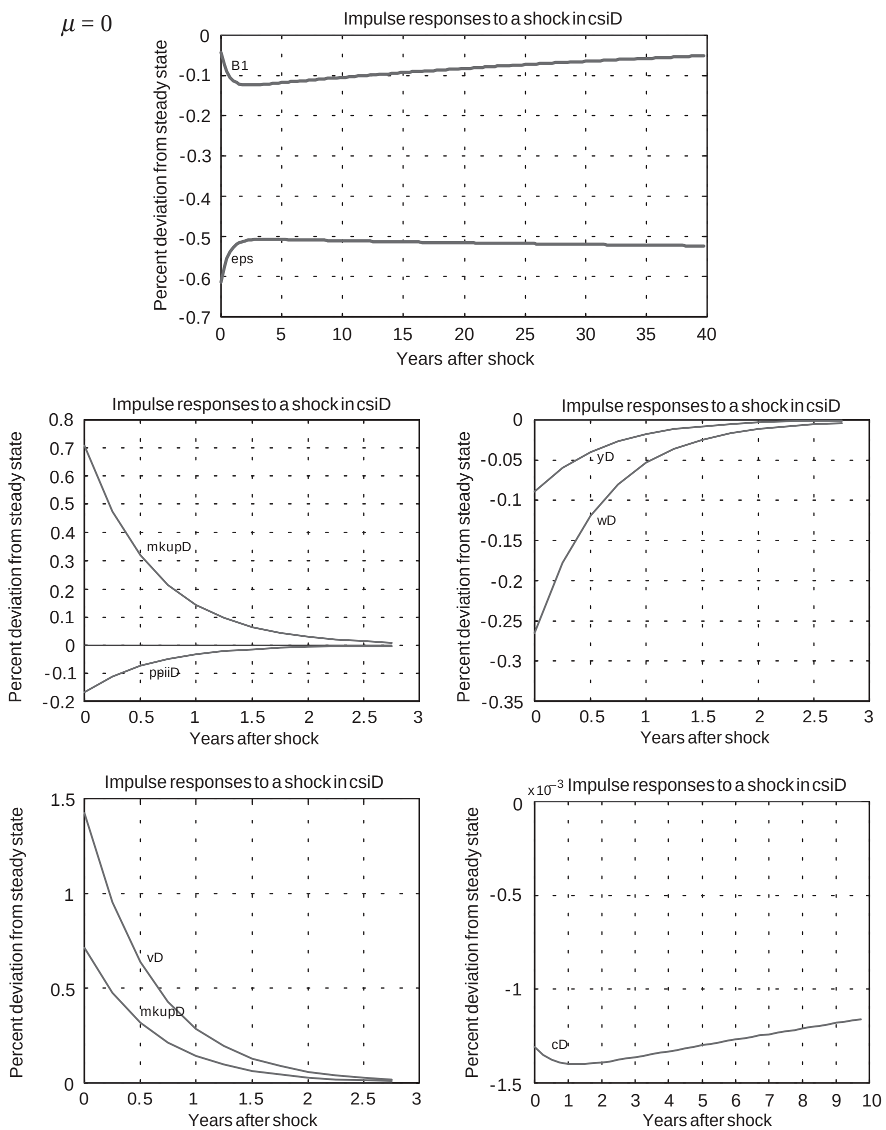

Fig. 7. Interest rate shock, sticky prices.

$\mu = 0.5$

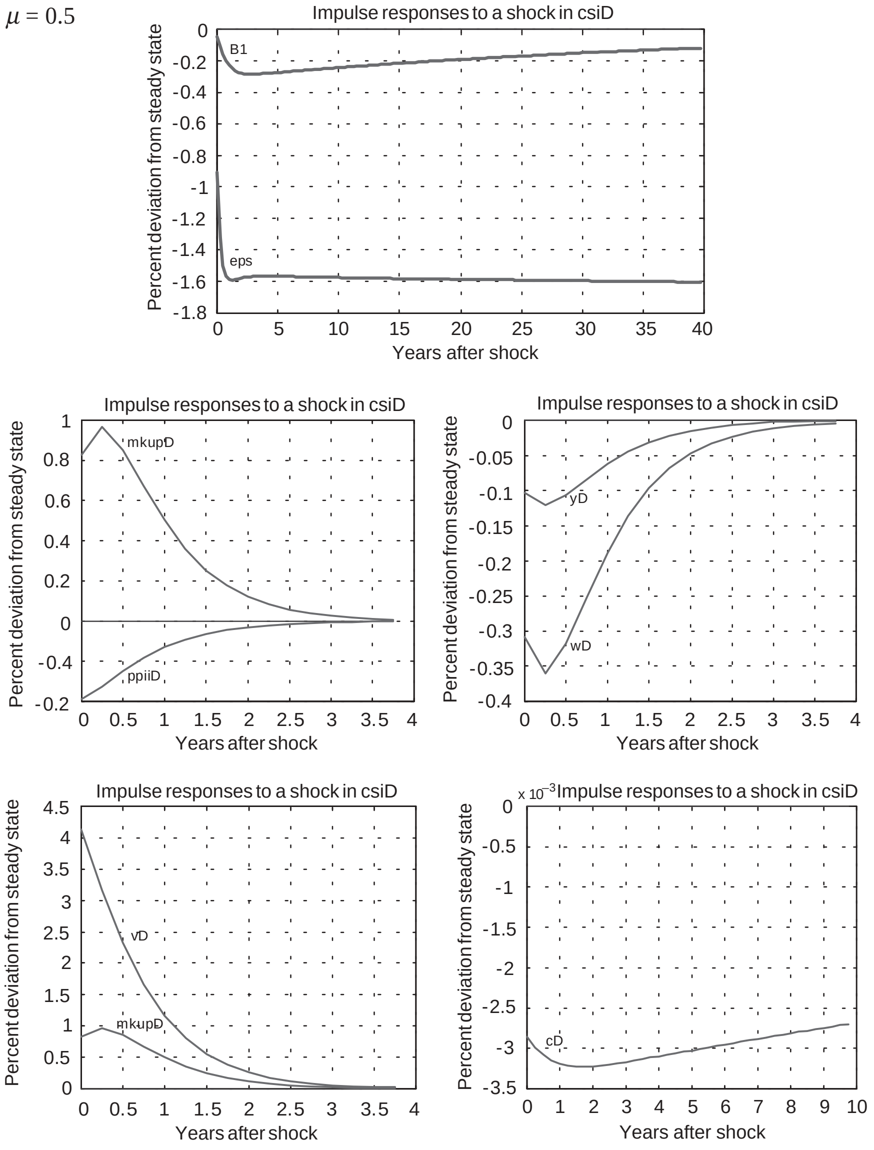

Fig. 7 (*continued*).

result in lower PPI inflation at home than abroad, though firms raise the markup component of prices to preserve profitability, and this translates into a higher relative value of home equity. Domestic labor demand falls below foreign (not shown), and so do the real wage, GDP, and (slightly) consumption. The home economy borrows from foreign to mitigate the fall in consumption. The exchange rate appreciates on impact. Its dynamics in the periods after the shock reflect those of

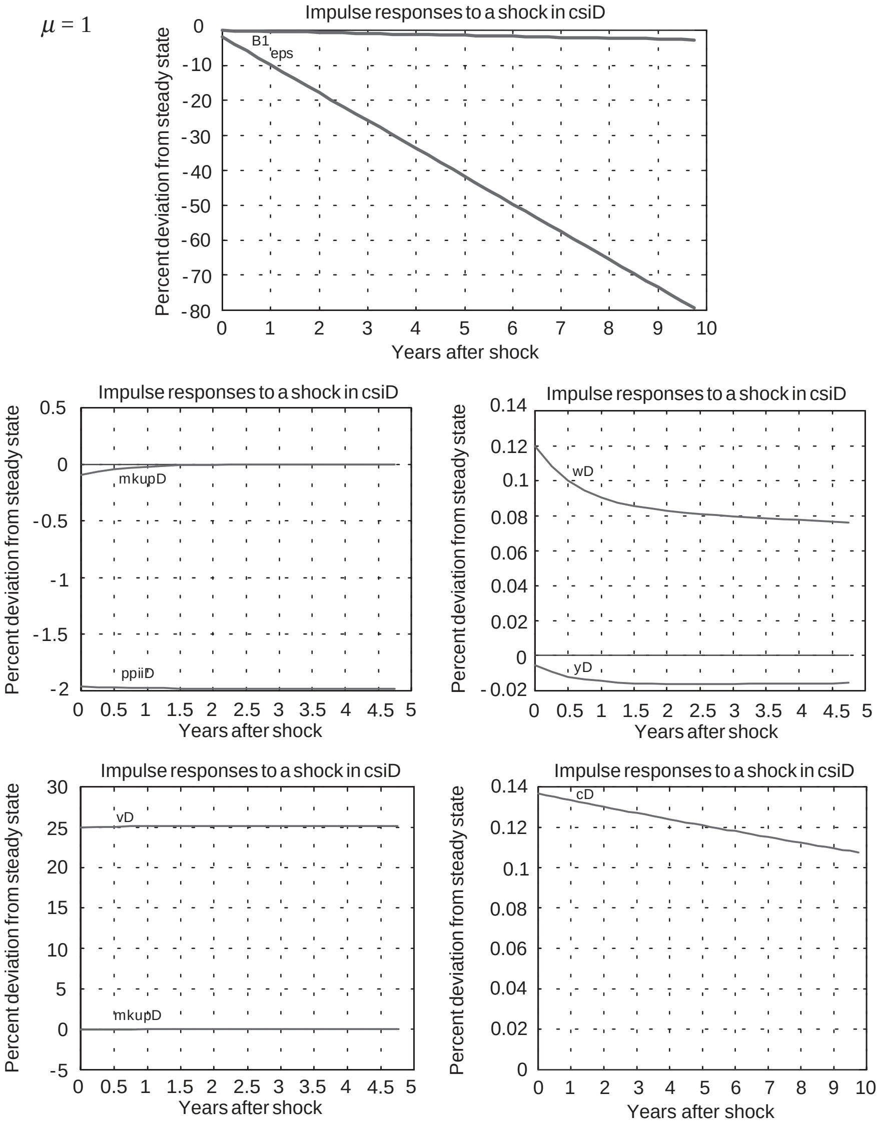

Fig. 7 (*continued*).

GDP differential and asset holdings. A GDP differential below the steady state pushes the exchange rate upwards, consistent with the lowering of the domestic interest rate in response to lower GDP. Net foreign assets below the steady state push the exchange rate downward. Because the elasticity of the exchange rate to the GDP differential ($\eta_{\varepsilon y^{\mathrm{D}}}$) is larger than that to assets ($\eta_{\varepsilon B}$) in absolute value, the former effect prevails, and the exchange rate moves upwards in the first years after the shock. However, the effect of asset dynamics on the exchange rate becomes preponderant once the GDP differential has been (almost entirely) re-absorbed. Approximately five years after the shock, the exchange rate starts moving slowly downward, mirroring the return of assets to the steady state. Eventually, the exchange rate settles on a new long-run position between the impact appreciation in period 0 and the level it had reached due to GDP dynamics. Thus, the exchange rate displays "two-ways" overshooting following a zero-persistence relative interest rate shock: it shoots below the new long-run equilibrium on impact, but it climbs above it when the GDP differential is the main driving force.

If the interest rate shock is persistent—$\mu \in (0, 1)$—the responses of markup, real wage, GDP, and consumption differential become hump-shaped, as the differential in relative prices adjusts gradually. Deviations from the steady state become more persistent. The home economy borrows from abroad to sustain consumption. Consider the case $\mu = 0.5$. The exchange rate jumps downward and continues to appreciate further until the GDP differential has reached its peak. After that, the exchange rate reverses direction along with GDP and climbs slightly. As in the case of a shock with no persistence, once the GDP differential is close to zero, asset dynamics take over, and the exchange rate moves downward to its new steady state. In the case of a persistent shock, the exchange rate undershoots its new long-run equilibrium on impact.[24]

A permanent shock ($\mu = 1$) causes domestic PPI inflation to be permanently below foreign. The domestic markup now falls on impact relative to foreign, though it eventually settles on a slightly higher value. This translates into a permanently higher domestic equity value. The initially lower markup generates higher labor demand, which mitigates the negative GDP effect of the shock. The real wage differential is above the steady state for a long time, though its new steady-state level is negative (not shown). The real GDP differential eventually returns to zero from below. Domestic consumption is above foreign for many years, but it eventually settles below, and the domestic economy is permanently in debt. The percentage deviation of the exchange rate from the steady state increases linearly in absolute value. The now usual caveat on the consequences of permanent shock for the path of the exchange rate applies. The interest rate shock has permanent real consequences by generating non-zero steady-state inflation, which imposes a permanent real cost on the economy.

As for the case of a productivity shock, the model delivers richer exchange rate dynamics than the benchmark setup in Obstfeld and Rogoff (1995). It brings a new perspective to bear on Dornbusch's (1976) results on exchange rate overshooting. In particular, our model has the potential for reconciling rational behavior and UIP

[24] Although it is hard to see this from the figure, the new steady-state level is also below the level that the exchange rate reaches with the further appreciation due to the hump-shaped response of GDP in the first years after the shock. When μ rises to 0.75, the exchange rate converges monotonically to the new steady state (see Cavallo and Ghironi, 2001). In this case, the size of the net debt accumulated by the home economy makes up for the small elasticity of the exchange rate to assets.

with the evidence of delayed overshooting in Clarida and Galí (1994) and Eichenbaum and Evans (1995).

6. Conclusions

We presented a theory of exchange rate determination that de-emphasizes the role of exogenous money supply and emphasizes the relation between the exchange rate and net foreign assets and the endogeneity of interest rate setting. Our model builds on a stationary version of Obstfeld and Rogoff (1995). We relied on the method of undetermined coefficients as in Campbell (1994) and Uhlig (1999) to obtain the solution. This technique is fully consistent with the forward-looking nature of the model. It delivers a process equation for the exchange rate rather than a solution expressed in the form of an infinite summation of future variables. This facilitates interpretation and quantitative work.

We started from a model with flexible prices and PPP. Interest rates are set to react to CPI inflation and real GDP movements. The solution for the nominal exchange rate exhibits a unit root, consistent with the empirical findings of Meese and Rogoff (1983). However, today's exchange rate depends also on the stock of real net foreign assets accumulated in the previous period. For plausible parameter values, a capital inflow (accumulation of net foreign debt) generates appreciation of the exchange rate. The quantitative relevance of net foreign assets for the latter is stronger the slower their convergence to the steady state following shocks and the higher the degree of substitutability between domestic and foreign goods in consumption.

We introduced price stickiness by assuming that it is costly to change output prices over time. We investigated the relation between asset holdings and the exchange rate quantitatively using a plausible calibration of the model. When prices are sticky, the exchange rate still exhibits a unit root. The current level of the exchange rate depends on the past GDP differential, along with net foreign assets.

The model yields a number of results on exchange rate overshooting. Under flexible prices, the exchange rate overshoots its new long-run level following a temporary (relative) productivity shock. If the shock is persistent, endogenous monetary policy and asset dynamics generate delayed overshooting. Endogenous monetary policy is responsible for exchange rate undershooting after persistent (relative) interest rate shocks. When prices are sticky, temporary shocks to relative productivity result in delayed overshooting. So do persistent shocks. Temporary relative interest rate shocks cause immediate overshooting. No overshooting may happen when interest rate shocks are persistent. Sticky-price dynamics are richer than in the Obstfeld and Rogoff (1995) model. Our model has the potential for reconciling rational behavior and UIP with the empirical results in Clarida and Galí (1994) and Eichenbaum and Evans (1995).

The flexible-price model delivers exchange rate appreciation after a favorable relative productivity shock under Taylor-type monetary policy. This is one side of the story that one would like to represent formally when trying to explain the recent behavior of the U.S. economy and the dollar exchange rate. However, the flexible-price model does not generate appreciation *cum* accumulation of net foreign debt. Because consumption smoothing is the only motive for asset accumulation, the home economy accumulates assets rather than debt. The sticky-price model generates appreciation, a net foreign debt, and a stock market expansion after a permanent favorable shock to relative productivity. But it remains to be seen whether the advent of the "new economy" has shifted U.S. productivity permanently above foreign. Adding capital accumulation as in Backus et al. (1994) is a promising way of generating the dynamics in the data for non-permanent productivity shocks. This is a direction we will pursue in future work, along with rigorous testing of the model's implications and exploring the consequences of deviations from PPP.[25] Another possible extension will be the exploration of the performance of alternative policy rules.

References

Backus, D.K., Kehoe, P.J., Kydland, F.E., 1994. Dynamics of the trade balance and the terms of trade: the J-curve? American Economic Review 84, 84–103.

Benigno, G., Benigno, P., 2001. Monetary policy rules and the exchange rate. CEPR Discussion Paper 2807.

Blanchard, O.J., 1985. Debt, deficits, and finite horizons. Journal of Political Economy 93, 223–247.

Branson, W.H., Henderson, D.W., 1985. The specification and influence of asset markets. In: Jones, R.W., Kenen, P.B. (Eds.), Handbook of International Economics, Vol. 2, North-Holland, Amsterdam, pp. 749–805.

Calvo, G.A., 1983. Staggered contracts in a utility maximizing framework. Journal of Monetary Economics 12, 983–998.

Campbell, J.Y., 1994. Inspecting the mechanism: an analytical approach to the stochastic growth model. Journal of Monetary Economics 33, 463–506.

Cavallo, M., Ghironi, F., 2001. Net foreign assets and the exchange rate: Redux revived. Working Paper 505, Department of Economics, Boston College.

Clarida, R., Galí, J., 1994. Sources of real exchange rate fluctuations: how important are nominal shocks? Carnegie-Rochester Conference Series on Public Policy, Vol. 41, pp. 1–56.

Clarida, R., Gertler, M., 1997. How the Bundesbank conducts monetary policy. In: Romer, C.D., Romer, D.H. (Eds.), Reducing Inflation: Motivation and Strategy. University of Chicago Press, Chicago, pp. 363–406.

Clarida, R., Galí, J., Gertler, M., 1998. Monetary policy rules in practice: some international evidence. European Economic Review 42, 1033–1067.

Corsetti, G., Pesenti, P., 2001. Welfare and macroeconomic interdependence. Quarterly Journal of Economics 116, 421–445.

Devereux, M.B., 2002. A tale of two currencies: the Asian crisis and the exchange rate regimes of Hong Kong and Singapore. Review of International Economics, forthcoming.

Dornbusch, R., 1976. Expectations and exchange rate dynamics. Journal of Political Economy 84, 1161–1176.

Dornbusch, R., Fischer, S., 1980. Exchange rates and the current account. American Economic Review 70, 960–971.

[25] There is a fast growing empirical literature on the relation between net foreign assets and the real exchange rate (Lane and Milesi-Ferretti, 2000, 2002) and on the issues this may pose for the U.S. (Obstfeld and Rogoff, 2001).

Eichenbaum, M., Evans, C.L., 1995. Some empirical evidence on the effects of shocks to monetary policy on exchange rates. Quarterly Journal of Economics 110, 975–1009.

Ghironi, F., 2000. Macroeconomic interdependence under incomplete markets. Working Paper 471, Department of Economics, Boston College.

Ireland, P.N., 2001. Sticky-price models of the business cycle: specification and stability. Journal of Monetary Economics 47, 3–18.

Kollmann, R., 2001. The exchange rate in a dynamic-optimizing business cycle model with nominal rigidities: a quantitative investigation. Journal of International Economics 55, 243–262.

Kraay, A., Loayza, N., Servén, L., Ventura, J., 2000. Country portfolios. NBER Working Paper, 7795.

Lane, P.R., 2001. The new open economy macroeconomics: a survey. Journal of International Economics 54, 235–266.

Lane, P.R., Milesi-Ferretti, G.M., 2000. The transfer problem revisited: net foreign assets and long-run real exchange rates. CEPR Discussion Paper 2511.

Lane, P.R., Milesi-Ferretti, G.M., 2002. External wealth, the trade balance, and the real exchange rate. European Economic Review 46, 1049–1071.

Meese, R.A., Rogoff, K., 1983. Empirical exchange rate models of the seventies: do they fit out of sample? Journal of International Economics 14, 3–24.

Obstfeld, M., Rogoff, K., 1983. Speculative hyperinflations in maximizing models: can we rule them out? Journal of Political Economy 91, 675–687.

Obstfeld, M., Rogoff, K., 1995. Exchange rate dynamics redux. Journal of Political Economy 103, 624–660.

Obstfeld, M., Rogoff, K., 2001. Perspectives on OECD economic integration: implications for U.S. current account adjustment. In: Global Economic Integration: Opportunities and Challenges, Proceedings of a Symposium Sponsored by the Federal Reserve Bank of Kansas City, pp. 169–208.

Rotemberg, J.J., 1982. Monopolistic price adjustment and aggregate output. Review of Economic Studies 49, 517–531.

Rotemberg, J.J., Woodford, M., 1992. Oligopolistic pricing and the effects of aggregate demand on economic activity. Journal of Political Economy 100, 1153–1207.

Schmitt-Grohé, S., Uribe, M., 2001. Closing small open economy models. Unpublished manuscript, Rutgers University and University of Pennsylvania.

Shiells, C.R., Stern, R.M., Deardorff, A.V., 1986. Estimates of the elasticities of substitution between imports and home goods for the United States. Weltwirtschaftliches Archiv 122, 497–519.

Smets, F., Wouters, R., 2002. Openness, imperfect exchange rate pass-through and monetary policy. Presented at the November 2002 Carnegie-Rochester Conference on Public Policy. Journal of Monetary Economics 49 (this issue).

Taylor, J.B., 1993. Discretion versus policy rules in practice. Carnegie-Rochester Conference Series on Public Policy, Vol. 39, pp. 195–214.

Uhlig, H., 1999. A toolkit for analyzing nonlinear dynamic stochastic models easily. In: Marimon, R., Scott, A. (Eds.), Computational Methods for the Study of Dynamic Economies. Oxford University Press, Oxford, pp. 30–61.

Weil, P., 1989. Overlapping families of infinitely-lived agents. Journal of Public Economics 38, 183–198.