Relative Price Dynamics and the Aggregate Economy[*]

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Abstract

We explore the role of relative prices in the dynamics of a two-sector, New Keynesian model with heterogeneous nominal rigidity, immobile labor, and endogenous interest rate setting. We show that labor immobility is necessary to generate endogenously persistent dynamics in our model, but it is not sufficient. When labor is immobile, aggregate inflation and output depend on the relative price across sectors – an endogenous state variable – if nominal rigidity differs across sectors and/or the central bank responds to sectoral inflation rates asymmetrically. We analyze the determinants of this endogenous persistence and study the extent to which relative prices impart persistence to aggregates by means of numerical examples. Persistence through relative prices significantly prolongs the responses of aggregates to transitory shocks. We show that heterogeneity in nominal rigidity (combined with labor immobility) is more important for this than the overall degree of stickiness in the economy. Relative price dynamics result in hump-shaped aggregate responses following productivity shocks in the relatively sticky sector. Depending on parameter values, they generate humps in aggregate output – but not in inflation – following monetary policy shocks if these are sufficiently persistent.

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1. Introduction

What is the role of relative prices in the determination of aggregate output and inflation dynamics? We explore this question in a two-sector, New Keynesian model with heterogeneous nominal rigidity and immobile labor across sectors.

The literature on nominal rigidity and aggregate dynamics between the late 1970s and early 1990s attributed a central role to relative price or wage movements in the determination of aggregate inflation and output. For instance, relative wages play a central role in Taylor (1980). Ball and Mankiw (1995) attribute a similarly central role to relative prices, and Gordon (1985) estimates significant effects of key relative prices on the aggregate U.S. Phillips curve.

The New Keynesian model that has become the benchmark for monetary business cycle and policy analysis since the end of the 1990s puts relative price distortions at the heart of its *normative* implications. Maintaining price stability to mimic the flexible price equilibrium is optimal because it removes the misallocation of resources implied by the fact that not all firms are adjusting prices at the same time in the Calvo (1983)-Yun (1996) model.[1]

Yet, the benchmark, log-linear model (as described, for instance, in Clarida, Galí, and Gertler, 1999) attributes no role at all to relative price changes in explaining aggregate dynamics. To see this point, it is sufficient to observe that the New Keynesian Phillips curve implied by the (log-linear) Calvo-Yun model is identical to that generated by the equally standard Rotemberg (1982) model of nominal rigidity, in which there is no relative price dispersion at all, and the cost of inflation comes in the form of the resource cost implied by a firm-level, convex cost of price adjustment (Roberts, 1995).

The reason for this result is that the Calvo-Yun or Rotemberg models do not feature structural heterogeneity across firms that can impart an explicit role to relative price dynamics in the log-linear economy. All firms are identical and choose the same price in all periods in the Rotemberg setup. Crucially, all firms face the *same* probability of price adjustment in the Calvo-Yun world. The implication of the latter hypothesis is that relative price dynamics are "approximated away" when the aggregate Calvo-Yun model is log-linearized.[2]

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[1] We are implicitly assuming that, when policy is conducted under discretion, the flexible price equilibrium is efficient owing to the presence of appropriate taxes and subsidies.

[2] To be accurate, from a positive perspective, relative prices matter in the benchmark model to the extent that strategic complementarity in firm pricing under monopolistic competition affects the output gap coefficient in the New Keynesian Phillips curve (see Woodford, 2003). This differs from the role for relative prices that we have in

This paper develops a sticky price, Calvo-Yun-Rotemberg model that preserves a central role for relative price dynamics in the transmission of macroeconomic shocks. We do so by assuming that the economy consists of two sectors that can be characterized by different degrees of nominal rigidity.[3] Households derive utility from consumption of a non-durable basket that aggregates goods produced in the two sectors in a C.E.S. fashion.[4] Firms *within* each sector face the same probability of adjusting the price level in each period or the same convex cost of price adjustment. Yet, if labor is immobile across sectors, the assumption that nominal rigidity differs *across* sectors generates a first-order role for the cross-sectoral relative price in the aggregate economy that is not lost in log-linearization, as we demonstrate by obtaining a generalized New Keynesian Phillips curve for aggregate inflation that features a relative price term.[5] The intuition for this term is simple. *Ceteris paribus*, if sector 1 prices are relatively more flexible than sector 2's, an increase in the relative price of sector 1 output that shifts aggregate demand toward sector 2 causes aggregate inflation to fall, as a larger portion of aggregate demand falls on the relatively sticky sector.

We assume that monetary policy is conducted by setting the nominal interest rate endogenously in response to movements in aggregate output and an average of sectoral inflation rates.[6] Aggregate inflation and output respond to changes in the relative price between the two sectors when nominal rigidity differs across them, but also when nominal rigidity is the same and the central bank responds to sectoral inflation rates asymmetrically. Asymmetric policy responses cause aggregate output to respond to relative prices in the intertemporal IS equation

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mind, whereby the log-linear model features relative prices as variables whose dynamics can affect those of aggregates.

[3] Bils and Klenow (2004) document overwhelming evidence of differences in the degree of nominal rigidity across sectors of the U.S. economy. Similar evidence has been documented for a number of European countries. See, for instance, Álvarez and Hernando (2004), Fabiani, Gattulli, and Sabbatini (2004), Loupias and Ricart (2004), Stahl (2004), and Veronese, Fabiani, Gattulli, and Sabbatini (2005). These studies also address the extent to which price rigidity is time dependent versus state dependent. We focus on heterogeneous time dependent stickiness. A growing literature explores the consequences of state dependent rigidity. Among others, see Dotsey, King, and Wolman (1999), Golosov and Lucas (2003), Hernández (2003), and Klenow and Kryvtsov (2005).

[4] We abstract from the role of durable goods, explored by Barsky, House, and Kimball (2004) and Erceg and Levin (2002).

[5] Related Phillips curve results for models with heterogeneous nominal rigidity are in Benigno (2004), Hernández (2003), and Woodford (2003, pp. 200-204). We discuss the relation between our work and these contributions at various points below. Independently of this paper and without reciprocal knowledge, Carvalho (2006) obtained related results. See also Dixon and Kara (2005) for a Taylor model with heterogeneous rigidity.

[6] Depending on the weights, the average can coincide with the CPI or reduce to inflation in one sector only. As shown in Carlstrom, Fuerst, and Ghironi (2006), responding to any average of sectoral inflation rates (including

(the Fisher equation), introducing a channel through which relative prices affect aggregate inflation (even if there is no direct effect through the aggregate Phillips curve).[7] With immobile labor, the cross-sectoral relative price is an endogenous state variable in the model, which introduces persistence in dynamics. We study the determinants of relative price persistence and how this affects aggregates analytically, focusing on the effect of changes in the parameters of the policy rule.

The assumption that labor does not move across sectors plays an important role in our model. A contribution of this paper is to show that heterogeneous nominal rigidity and/or asymmetric policy responses to sectoral inflation rates are necessary for endogenous aggregate persistence, but they are not sufficient. If labor is mobile, real wages are equalized across sectors, and relative prices are simply proportional to the cross-sectoral productivity differential. In this case, there is no added persistence from relative price effects *regardless* of heterogeneous nominal rigidity or the characteristics of interest rate setting.[8]

Equipped with a set of analytical results and intuitions, we then analyze the extent to which relative price movements impart persistence in the dynamics of aggregate output and inflation under labor immobility by means of numerical examples.[9]

We find that relative price effects result in hump-shaped responses to productivity shocks if these affect the sector in which prices are relatively sticky, but no hump is observed following shocks to the flexible sector.[10] The model does not generate hump-shaped responses of aggregate inflation and output to monetary policy shocks (which affect aggregate demand, and

reacting only to one sector) ensures local equilibrium determinacy so long as the response to this measure of inflation is sufficiently aggressive.

[7] Our interest in asymmetric policy reactions is entirely positive rather than normative, as we further discuss below. It is motivated by the observation that central banks often respond to a subset of prices in the economy rather than the entire CPI, allowing for the fact that this asymmetry in responses may originate in something other than differences in nominal rigidity.

[8] Endogenous persistence via relative prices disappears also if the goods produced in the two sectors are perfect substitutes, regardless of labor immobility and differences in nominal rigidity. In this case, the ability of consumers to arbitrage across the outputs of the two sectors ensures that their relative price is always equal to 1. But the distinction between sectors loses much of its interest in this case, and so does the plausibility of different degrees of nominal rigidity.

[9] Dependence of aggregate inflation on an inertial relative price arises also in the one-sector model when price stickiness is combined with wage stickiness. In that case, the relative price of labor in terms of consumption – the real wage – is an endogenous state variable featured in both Phillips curves for price and wage inflation. Erceg, Henderson, and Levin (2000) explore the normative properties of this setup. We extend our model to allow for potentially heterogeneous nominal wage rigidity across sectors in an appendix.

[10] Aggregate productivity shocks (*i.e.*, equal shocks across sectors) do not yield hump-shaped responses of aggregates in the scenarios and for the parameter values we consider.

thus shift the demand for both sectors in the same direction) if the shocks are transitory – though relative price movements significantly prolong the response. Depending on parameter values, sufficiently persistent shocks result in hump-shaped aggregate output responses, but no hump in inflation.

We conclude from our exercise that accounting for relative price movements with immobile labor is not entirely sufficient to shield the *basic*, fully forward-looking Calvo-Yun-Rotemberg model from the criticism that it is unable to reproduce the hump-shaped patterns observed in the data in response to monetary policy shocks, especially transitory ones. Other features of the economy – which we omit to focus on the role of relative prices in an analytically tractable model – must be responsible for such richer dynamics.[11] However, relative price effects can impart significant, empirically plausible, extra persistence to the economy.[12]

The rest of the paper is organized as follows. Section 2 relates our modeling strategy and results to recent literature on endogenous persistence in monetary business cycle models. Section 3 presents our model. Section 4 explores the properties of the solution under different assumptions about nominal rigidity and interest rate setting. Section 5 analyzes the consequences of labor mobility. Section 6 presents impulse responses to productivity and monetary policy shocks, focusing on the case of immobile labor. Section 7 concludes.

2. Endogenous Persistence and Our Modeling Approach

Chari, Kehoe, and McGrattan (2000) provided the starting point for much of the recent literature on the endogenous persistence properties of New Keynesian models. They demonstrate that a microfounded version of the Taylor (1980) model does not generate a "contract multiplier" in line with Taylor's results: Once the parameter restrictions from explicit microfoundations are imposed, the Taylor model does not produce the persistent output responses to permanent money shocks highlighted in Taylor's article.[13] Our results echo Chari, Kehoe, and McGrattan's in that relative price effects result in hump-shaped output responses

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[11] Some such features are explored in Christiano, Eichenbaum, and Evans (2005).
[12] The extended version of the model allowing for heterogeneous nominal wage rigidity across sectors confirms the main conclusions of our exercise.
[13] In fact, also a version of the Taylor model in which two groups of firms set prices in staggered fashion does not produce hump-shaped responses of aggregate inflation and output, even if the solution of the model depends on past group-level inflation.

only for persistent interest rate shocks. (In addition, the responses of cross-sectoral relative prices and quantities to monetary policy shocks display the counterfactual implications highlighted by Bils, Klenow, and Kryvtsov, 2003.) On the other hand, we find that – even when they do not contribute humps – relative price changes do induce significant extra persistence in aggregate output dynamics. In the absence of any relative price effect, a zero-persistence interest rate shock causes aggregate output to deviate from the steady state only during the quarter of the shock. For plausible parameter values, relative price effects can prolong the response by over two years. We show that heterogeneous nominal rigidity is more important for this result than the overall degree of price stickiness in the economy.

By highlighting the role of cross-sectoral labor immobility, our paper is related to a branch of the literature that focuses on the role of factor specificity in enhancing the persistence properties of monetary business cycle models. As in Huang (2006) and references therein, factor specificity plays a central role for endogenous aggregate persistence, as sectoral specificity of labor is necessary for endogenously persistent aggregate dynamics. However, in contrast to Huang's model with firm-specific factors and Taylor-type staggered prices, sector-level factor specificity is not sufficient in our model with Calvo-Yun-Rotemberg pricing, as there is no endogenous persistence if nominal rigidity is identical across sectors and the central bank responds to the CPI.

A different, related literature focuses on the role of durable goods. Barsky, House, and Kimball (2004) and Erceg and Levin (2002) develop multi-sector, sticky-price models in which households consume durable and non-durable goods. In particular, Barsky, House, and Kimball focus on the consequences of heterogeneous nominal rigidity across durables and non-durables and show that flexible prices in the durable sector can result in (approximate) money neutrality even if non-durable prices are sticky.

Ohanian, Stockman, and Kilian (1995) provide the first precursor of our study of which we are aware. They study the transmission properties of a two-sector model where prices are sticky in one sector and flexible in the other. Their model features labor mobility and capital is the relevant endogenous state variable. They show that price stickiness in a sector can cause slower equilibrium adjustment of prices also in the flexible sector (relative to the situation in which both sectors have flexible prices).

Our model – with immobile labor, no capital, and no durable goods – builds on results and intuitions that are familiar in the open economy literature on monetary interdependence across countries, which often relies on similar assumptions. Benigno (2004) highlights the role of the terms of trade as a state variable in a two-country model of a monetary union in which nominal rigidity can differ across countries. He focuses on normative issues and the choice of the optimal target of inflation for the central bank of the monetary union.[14] Our model is closely related to Benigno's, in that it can be reinterpreted as a model of a monetary union, with different nominal rigidity, immobile labor, and perfect consumption insurance across countries. The cross-sectoral relative price in our model (which we refer to as the terms of trade between the two sectors, since it is the rate at which consumers can trade sectoral goods for one another) plays the same role as the terms of trade across countries in imparting persistence to the world economy in open economy models.[15] A contribution of this paper is thus to cast intuitions that are familiar in international monetary economics in a closed-economy setting.[16]

By focusing on the endogenous persistence properties of multi-sector economies, our work is also related to a literature that explores the implications of production chains in which a sector produces goods that are used as intermediate inputs in a different, consumption-producing sector. Huang and Liu (2001, 2004) show that such production chains can impart considerable endogenous persistence to aggregate dynamics in response to monetary shocks. Bouakez, Cardia, and Ruge-Murcia (2005) develop and estimate a sticky-price model of monetary transmission that shares features with ours and with the production-chain models of Huang and Liu. An additional contribution of our paper is to shed light on mechanisms that are at work in the richer, quantitative model of Bouakez, Cardia, and Ruge-Murcia.

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[14] Benigno and López-Salido (2006) use Benigno's model to study the persistence of inflation rates in the euro area, but they focus on country-level inflation rates – the equivalent of sectoral inflation rates in our model. Aoki (2001) develops a two-sector, closed-economy model in which prices are flexible in one of the two sectors. Like Benigno (2004), he focuses on the normative implications of his setup.

[15] As one would expect, if nominal rigidity is identical across countries in those models, the dynamics of the aggregate world economy are insulated from the terms of trade.

[16] Woodford (2003, pp. 200-204) develops a similar model, also building on Benigno (2004), but he does not focus on its positive properties. Hernández (2003) develops a model with time- and state-dependent pricing, in which firms can choose where to allocate themselves in terms of their frequency of price adjustment (low versus high). He shows that this implies a generalized New Keynesian Phillips curve that depends on the relative mass of firms in the high- versus low-frequency-of-price-change mode along with the terms of trade between the two groups. He too does not focus on persistence.

3. The Model

Our model is a two-sector version of the standard New Keynesian setup used in the recent literature on monetary policy. We first describe the behavior of households and firms, respectively, and then turn to the linearized system that will be the focus of our analysis.

3.A. The Representative Household

The economy is populated by a continuum of households between 0 and 1. The representative household consists of two agents. One of these supplies labor to firms in sector 1, the other supplies labor to firms in sector 2. These agents jointly maximize an intertemporal utility function that depends on the household's consumption of a basket of goods C_t, on the household's holdings of real money balances M_{t+1}/P_t (where M_{t+1} is nominal money holdings and P_t is the CPI), and on the disutility of the two agents from supplying labor in sectors 1 and 2, L_t^1 and L_t^2, respectively:

$$E_0 \sum_{t=0}^{\infty} \beta^t U\left(C_t,\ M_{t+1}/P_t,\ L_t^1,\ L_t^2\right), \quad 1 > \beta > 0, \tag{1}$$

where E_0 denotes the expectation operator conditional on information available at $t = 0$. For simplicity, we will assume:

$$U\left(C_t,\ M_{t+1}/P_t,\ L_t^1,\ L_t^2\right) = \log C_t + V\left(\frac{M_{t+1}}{P_t}\right) - \left(L_t^1\right)^2/2 - \left(L_t^2\right)^2/2, \tag{2}$$

where the function $V(\cdot)$ has the usual properties. The consumption basket C_t is a CES aggregate of sub-baskets of individual goods produced in sectors 1 and 2:

$$C_t = \left[b^{\frac{1}{\omega}} \left(C_t^1\right)^{\frac{\omega-1}{\omega}} + (1-b)^{\frac{1}{\omega}} \left(C_t^2\right)^{\frac{\omega-1}{\omega}} \right]^{\frac{\omega}{\omega-1}}, \qquad \omega > 0,\ 1 > b > 0. \tag{3}$$

Sectors 1 and 2 are populated by monopolistically competitive firms, which produce differentiated brands of the sectors' goods. Sector 1 consists of firms in the interval between 0 and b; sector 2 consists of firms between b and 1. The sectoral consumption sub-baskets are:

$$C_t^1 = \left[\left(\frac{1}{b}\right)^{\frac{1}{\theta}} \int_0^b \left(C_t^1(z)\right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}},\ C_t^2 = \left[\left(\frac{1}{1-b}\right)^{\frac{1}{\theta}} \int_b^1 \left(C_t^2(z)\right)^{\frac{\theta-1}{\theta}} dz \right]^{\frac{\theta}{\theta-1}},\ \theta > 1. \tag{4}$$

Given the consumption index in (3), the CPI equals:

$$P_t = \left[b\left(P_t^1\right)^{1-\omega} + (1-b)\left(P_t^2\right)^{1-\omega}\right]^{\frac{1}{1-\omega}}, \tag{5}$$

where P_t^1 and P_t^2 are the price sub-indexes for sectors 1 and 2, respectively:

$$P_t^1 = \left[\frac{1}{b}\int_0^b \left(P_t^1(z)\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}, \quad P_t^2 = \left[\frac{1}{1-b}\int_b^1 \left(P_t^2(z)\right)^{1-\theta} dz\right]^{\frac{1}{1-\theta}}, \tag{6}$$

and $P_t^j(z)$ denotes the price of individual brand *z* produced in sector *j*, *j* = 1, 2.

Given these price indexes, the household allocates its consumption to individual brands of the goods in each sector according to the demand schedule:

$$C_t^j(z) = \left(\frac{P_t^j(z)}{P_t^j}\right)^{-\theta}\left(\frac{P_t^j}{P_t}\right)^{-\omega} C_t, \qquad j = 1, 2. \tag{7}$$

Since our focus is on symmetric equilibria within each sector we henceforth drop the firm-specific index *z*, and instead consider a representative firm in each sector *j* = 1, 2.

The representative household enters the period with M_t cash balances and B_{t-1} holdings of nominal bonds. At the beginning of the period, the household visits the financial market, where it carries out bond trading and receives a monetary transfer X_t from the monetary authority. The two agents then split and offer labor in sectors 1 and 2. They meet on the way home from work and go shopping for consumption goods. Before entering the goods market, the household has cash holdings $M_t + X_t + R_{t-1}B_{t-1} - B_t$, where R_{t-1} is the gross nominal interest rate between $t-1$ and t. Agents receive their nominal wage bills ($W_t^1 L_t^1$ and $W_t^2 L_t^2$) and lump-sum profit rebates from firms (Π_t^1 and Π_t^2) at the end of the period. Thus, the household ends the period with cash balances given by the budget constraint:

$$M_{t+1} = M_t + X_t + R_{t-1}B_{t-1} + W_t^1 L_t^1 + W_t^2 L_t^2 + \Pi_t^1 + \Pi_t^2 - B_t - P_t C_t. \tag{8}$$

We assume that the money balances that enter the utility function (those that matter for time-*t* transaction services) are those with which the household *leaves* the time-*t* goods market, *i.e.*, cash held after goods market trading. In the terminology of Carlstrom and Fuerst (2001), this is the "cash-when-I'm-done" (CWID) timing of money-in-the-utility-function models.[17] Bond-pricing and money demand equations are given by:

$$\frac{U_C(t)}{P_t} = R_t \beta E_t\left(\frac{U_C(t+1)}{P_{t+1}}\right), \tag{9}$$

$$\frac{U_m(t)}{U_C(t)} = \frac{R_t - 1}{R_t}, \tag{10}$$

where $U_C(t)$ denotes the marginal utility of consumption at time t and $U_m(t)$ is the marginal utility of time-t real money balances.[18] Labor supplies are determined by:

$$-\frac{U_{L^1}(t)}{U_C(t)} = \frac{W_t^1}{P_t}, \tag{11}$$

$$-\frac{U_{L^2}(t)}{U_C(t)} = \frac{W_t^2}{P_t}, \tag{12}$$

where $-U_{L^1}(t)$ $(-U_{L^2}(t))$ is the marginal disutility of supplying labor to sector 1 (2) firms. We allow for the possibility that real wages in sectors 1 and 2 may differ because of labor immobility.

3.B. Firms

Sectors 1 and 2 are populated by monopolistically competitive firms that produce differentiated varieties of the goods in each sector. Price setting in sectors 1 and 2 is subject to Calvo-Yun-Rotemberg type nominal rigidity. Given the standard nature of the environment we only sketch a description of firm behavior. Recall that since our focus is on symmetric equilibria we will consider the behavior of a representative firm in each sector.

Firms in each sector produce output according to the linear technology:

$$Y_t^j = \Phi_t^j L_t^j, \qquad j = 1, 2, \tag{13}$$

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[17] We utilize the traditional CWID timing to be consistent with the majority of the literature. The alternative to CWID timing is "cash-in-advance" (CIA) timing, where the money that aids utility (or reduces transactions costs) is the money the household has when entering the goods market, $M_t + X_t + R_{t-1}B_{t-1} - B_t$.

[18] As shown by Carlstrom and Fuerst (2001), the Fisher equation with CIA timing is $\frac{U_C(t)}{P_t} = \beta R_{t+1}\frac{U_C(t+1)}{P_{t+1}}$ (under perfect foresight for simplicity). Therefore, it is easy to verify that a current-looking interest rate rule with CWID timing is equivalent to a backward-looking rule with CIA timing except for one feature: With CIA timing, exogenous shocks to the interest rate at time t affect the economy only if they persist beyond time t.

where Y_t^j and L_t^j are the typical firm's output and labor demand in sector *j*, and Φ_t^j is sector *j*'s productivity. Firms in each sector *j* = 1, 2, face the downward-sloping demand schedule (7).

Firms choose the amount of labor to be employed and the price of their output to maximize profits in a familiar fashion. Pricing is subject to nominal rigidity. The optimal price in sector *j* satisfies:

$$\frac{P_t^j}{P_t} = \frac{1}{Z_t^j}\frac{W_t^j}{P_t}\frac{1}{\Phi_t^j}, \quad j = 1, 2, \tag{14}$$

where Z_t^j is marginal cost in sector *j*, so that $\left(1/Z_t^j\right)$ is the markup of price over marginal cost, identical across firms in each sector. Equation (14) follows from either a Calvo-Yun type setup for price stickiness[19] or a quadratic cost of price adjustment as in Rotemberg (1982). Yun (1996) provides the details that link the behavior of marginal cost in each sector to price growth in each sector. For simplicity we omit these details, but simply state the log-linearized version below (equation (22)).

3.C. The Log-Linearized System and Equilibrium

We assume that policy is such that there is a unique steady state to this model. As customary, we log-linearize the equilibrium conditions around this steady state. Lower-case letters denote percentage deviations from steady-state levels (w_t^j is the log deviation of the *real* wage W_t^j/P_t). When interest and inflation rates are concerned, we consider percentage deviations of *gross* rates from the respective steady-state levels. (Net inflation is equal to zero in steady state.)

Household behavior is defined by the labor supply equations (11)-(12), the Fisher equation (9), and the demand curves (7).[20] Using the equilibrium condition $\varphi_t^j + l_t^j = c_t^j$, where φ_t^j is the percentage deviation of Φ_t^j from the steady state, these optimality conditions can be expressed as:

$$w_t^j = c_t + c_t^j - \varphi_t^j, \quad j = 1, 2. \tag{15}$$

$$E_t c_{t+1} - c_t = r_t - E_t \pi_{t+1}, \tag{16}$$

[19] Calvo (1983), Yun (1996).

[20] Money is determined residually by the money demand equation (10) under our assumptions on monetary policy.

$$c_t^j = -\omega\left(p_t^j - p_t\right) + c_t, \;\; j = 1, 2. \tag{17}$$

From (5), the CPI is linked to the sectoral prices via

$$p_t = bp_t^1 + (1-b)p_t^2, \tag{18}$$

and prices and inflation are linked by

$$\pi_t = \Delta p_t, \text{ and } \pi_t^j = \Delta p_t^j, \;\; j = 1, 2, \tag{19}$$

where Δ denotes first differences ($\Delta x_t \equiv x_t - x_{t-1}$ for any variable *x*).

Turning to firm behavior, the pricing equation (14) has the form

$$z_t^j = w_t^j + p_t - p_t^j - \varphi_t^j, \;\; j = 1, 2. \tag{20}$$

We assume that the productivity shocks φ_t^j follow autoregressive processes of the form:

$$\varphi_t^j = \rho_j \varphi_{t-1}^j + \varepsilon_t^j, \; j = 1, 2, \tag{21}$$

where ε_t^j is a zero-mean, Normally distributed innovation to productivity in sector *j*. We allow sectoral productivity innovations to have different variances and be correlated across sectors. Sectoral inflation rates are determined by the familiar New Keynesian Phillips curve:[21]

$$\pi_t^j = \lambda_j z_t^j + \beta E_t \pi_{t+1}^j, \;\; j = 1, 2, \tag{22}$$

where $\lambda_j > 0$ measures the degree of nominal rigidity in sector *j*. We allow sectors to differ in the extent to which prices are sticky.[22]

To close the model we need to define monetary policy. We specify monetary policy as a Taylor rule in which the nominal interest rate is a function of inflation in the two sectors and aggregate output, which equals consumption in equilibrium:

$$r_t = \tau_1 b\pi_t^1 + \tau_2(1-b)\pi_t^2 + \tau_C c_t + \varphi_t^r, \tag{23}$$

where $\tau_1, \tau_2, \tau_C \geq 0$, and the exogenous monetary policy shock φ_t^r follows the AR(1) process

$$\varphi_t^r = \rho_r \varphi_{t-1}^r + \varepsilon_t^r, \tag{24}$$

with $1 \geq \rho_r \geq 0$ and ε_t^r a zero-mean, i.i.d., Normal innovation. If $\tau_1 = \tau_2$, the central bank reacts to CPI inflation. In general, the central bank is free to target different measures of inflation by letting τ_1 differ from τ_2. Focusing on the case $\tau_C = 0$ and writing the policy rule as

[21] See Yun (1996) for the Calvo-Yun setup, or Roberts (1995) for the quadratic cost adjustment scenario.

$r_t = \tau\left[(1-\eta)\pi_t^1 + \eta\pi_t^2\right]$ with $1 \geq \eta \geq 0$, Carlstrom, Fuerst, and Ghironi (2006, CFG below) show that $\tau > 1$ is necessary and sufficient for local determinacy for any value of η, including when the central bank is reacting to only one sector of the economy and the weight of this sector in consumption is arbitrarily small (and including also the case in which prices are fully flexible in one sector). We assume that local determinacy holds below.[23]

To summarize, the equilibrium of the model consists of the ten sectoral variables $z_t^j, w_t^j, \pi_t^j, c_t^j, p_t^j$, for j = 1, 2, and the four aggregate variables p_t, π_t, r_t, and c_t, that satisfy the fourteen restrictions in (15)-(20), (22), and (23) given the exogenous paths of sectoral productivities φ_t^j and the monetary policy shock φ_t^r implied by (21) and (24).

4. Price Rigidity, Policy, and the Role of Terms of Trade Dynamics

Analyzing the properties of our two-sector economy is easier if we define sectoral relative prices (the prices of the goods produced in the two sectors in units of the consumption basket) and the relative price between the two sectors (the price of the sector 1 good in units of sector 2 good). We refer to this relative price as the terms of trade below, as it is the rate at which households can trade the sector 1 good for the sector 2 one. We focus on the case in which labor is immobile across sectors in this section and explore the consequences of labor mobility in the next section.

Equations (15), (17), and (20) can be combined to yield:

$$z_t^j = 2c_t - (1+\omega)rp_t^j - 2\varphi_t^j, \; j = 1, 2, \tag{25}$$

where we have defined the relative price of sector j as:

$$rp_t^j \equiv p_t^j - p_t, \; j = 1, 2. \tag{26}$$

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[22] In the Calvo-Yun setup, $\lambda_j = (1-\delta_j)(1-\delta_j\beta)/\delta_j$, where $1-\delta_j$ is the probability of price adjustment in each period for firms in sector j.

[23] It is easy to recover results for the CFG rule by setting $\tau_1 = \tau(1-\eta)/b$ and $\tau_2 = \tau\eta/(1-b)$ in equation (23) and below. In this case, the central bank reacts to CPI inflation when $\eta = 1 - b$. We use the specification in (23) as it allows more flexibility in evaluating the consequences of changes in policy parameters. With the CFG rule, more aggressiveness in the reaction to sector 1 inflation (a lower value of η) is necessarily associated to less aggressiveness in the reaction to sector 2 inflation. The specification in (23) allows us to vary the reaction coefficient to a sectoral inflation rate holding the reaction to inflation in the other sector constant.

In equilibrium, the marginal cost of sector *j* depends negatively on that sector's relative price. An increase in rp_t^1 shifts demand toward sector 2. It causes labor demand in sector 1 to fall and a lower marginal cost in that sector.

Define the terms of trade across sectors as:

$$T_t \equiv rp_t^1 - rp_t^2 = p_t^1 - p_t^2 . \tag{27}$$

The definitions of relative prices, terms of trade, and the CPI index imply that sectoral relative prices are proportional to the terms of trade:

$$rp_t^1 = (1-b)T_t , \tag{28}$$

$$rp_t^2 = -bT_t . \tag{29}$$

An improvement in sector 1's terms of trade (say, due to an increase in p_t^1 for given p_t^2) causes rp_t^1 to increase by a factor $1-b$ because of the impact of p_t^1 on the aggregate price level p_t.

Substituting (25) into (22) yields sectoral inflation equations:[24]

$$\pi_t^j = \lambda_j \left[2c_t - (1+\omega) rp_t^j - 2\varphi_t^j \right] + \beta E_t\left(\pi_{t+1}^j\right), \; j = 1, 2. \tag{30}$$

Combining the Fisher equation (16) with the policy rule (23) yields:

$$E_t\left(c_{t+1}\right) - c_t = \tau_1 b \pi_t^1 + \tau_2 (1-b) \pi_t^2 + \tau_C c_t + \varphi_t^r - E_t\left(\pi_{t+1}\right). \tag{31}$$

Equations (18), (19), (26), (27), (30), and (31) constitute a system of ten equations in ten unknowns (the sectoral inflation rates, price levels, and relative prices; the CPI level and inflation rate; consumption, and the terms of trade). The presence of lagged price levels in the system implies a solution in which current variables are functions of past sectoral price levels. The same symmetry properties of the CPI index that ensure that sectoral relative prices are proportional to the terms of trade also imply that we can write the solutions for today's inflation rates and consumption as functions of the past *differential* across sectoral prices, *i.e.*, the past terms of trade. We can guess that the solution for sectoral inflation rates and consumption has the form:

$$\pi_t^1 = \alpha_1 T_{t-1} + \gamma_1 \varphi_t^1 + \gamma_2 \varphi_t^2 + \gamma_{r,1} \varphi_t^r , \tag{32}$$

$$\pi_t^2 = \alpha_2 T_{t-1} + \gamma_3 \varphi_t^1 + \gamma_4 \varphi_t^2 + \gamma_{r,2} \varphi_t^r , \tag{33}$$

$$c_t = \alpha_3 T_{t-1} + \gamma_5 \varphi_t^1 + \gamma_6 \varphi_t^2 + \gamma_{r,3} \varphi_t^r . \tag{34}$$

Equations (32) and (33) and the definition of the sectoral inflation rates immediately return the solutions for sectoral price levels. Also, (32) and (33) can be used in conjunction with the definition of CPI inflation to obtain the solution for the latter as:

$$\pi_t = \alpha_4 T_{t-1} + \gamma_7 \varphi_t^1 + \gamma_8 \varphi_t^2 + \gamma_{r,4} \varphi_t^r, \tag{35}$$

where $\alpha_4 \equiv b\alpha_1 + (1-b)\alpha_2$, $\gamma_7 \equiv b\gamma_1 + (1-b)\gamma_3$, $\gamma_8 \equiv b\gamma_2 + (1-b)\gamma_4$, and $\gamma_{r,4} \equiv b\gamma_{r,1} + (1-b)\gamma_{r,2}$.

Using the solutions for the sectoral price levels makes it possible to verify that the terms of trade obey:

$$T_t = \alpha_5 T_{t-1} + (\gamma_1 - \gamma_3)\varphi_t^1 + (\gamma_2 - \gamma_4)\varphi_t^2 + (\gamma_{r,1} - \gamma_{r,2})\varphi_t^r, \tag{36}$$

where $\alpha_5 \equiv 1 + \alpha_1 - \alpha_2$. The autoregressive root α_5 (which is inside the unit circle) is responsible for persistence in terms of trade dynamics beyond the persistence of the sectoral productivity shocks φ_t^1 and φ_t^2, and the monetary policy shock φ_t^r. Persistent terms of trade dynamics imply persistent movements in the sectoral relative prices rp_t^1 and rp_t^2.

If the elasticities of aggregate output and CPI inflation to the past terms of trade (α_3 and α_4, respectively) are zero, aggregate output and CPI inflation display no endogenous persistence regardless of the terms of trade adjusting over time as a state variable. If α_3 and α_4 are zero (as in the benchmark one-sector model), the responses of aggregate output and inflation to shocks are only as persistent as the shocks themselves (*i.e.*, the only persistence is exogenous) and the responses to non-permanent shocks display immediate peaks followed by monotonic decay toward the steady state. But if the elasticities of aggregate output and CPI inflation to the past terms of trade are different from zero, endogenous terms of trade persistence translates into endogenous persistence in aggregate output and CPI inflation beyond the persistence of exogenous shocks.[25] This endogenous aggregate persistence happens when nominal rigidity differs across sectors ($\lambda_1 \neq \lambda_2$) and/or when the central bank is reacting to sectoral inflation rates differently ($\tau_1 \neq \tau_2$, or $\eta \neq 1-b$ in CFG). In these cases, the autoregressive parameter in the

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[24] These equations are the analog to (2.27) in Woodford (2003, p. 203).

[25] Sectoral output levels are given by $c_t^1 = -\omega(1-b)T_t + c_t$ and $c_t^2 = \omega b T_t + c_t$, respectively. Given the solutions for the terms of trade and aggregate output, it is then easy to recover solutions for output levels in the two sectors.

terms of trade equation (36) measures the speed at which aggregate output and CPI inflation return to the steady state beginning in the period *after* that in which exogenous shocks have returned to zero.[26]

Solutions for the elasticities α_i, $i = 1, 2, 3$; γ_h, $h = 1, ..., 6$, and $\gamma_{r,s}$, $s = 1, \ldots, 3$ can be recovered with the method of undetermined coefficients as described in Appendix A. Given solutions for these elasticities, the values of α_4, α_5, γ_7, γ_8, and $\gamma_{r,4}$ can then be recovered using the definitions above. (We shall see, however, that it is easier to solve for α_5 first when nominal rigidity is the same across sectors.)

The role of different degrees of nominal rigidity and different sectoral reactions in the Taylor rule in generating endogenous persistence can be understood better by considering first the special cases in which nominal rigidity is identical across sectors and the central bank reacts to CPI inflation and in which nominal rigidity is identical across sectors but the central bank attaches more weight to one of the two sectors in its reaction function.

4.A. Identical Nominal Rigidity, Reaction to CPI Inflation

Suppose that sectors 1 and 2 are characterized by identical degrees of nominal rigidity ($\lambda_1 = \lambda_2 = \lambda$) and that the central bank is reacting to CPI inflation ($\tau_1 = \tau_2$). Equation (30) and the definition of CPI inflation imply the aggregate Phillips curve:

$$\pi_t = 2\lambda c_t + \beta E_t(\pi_{t+1}) - 2\lambda\left[b\varphi_t^1 + (1-b)\varphi_t^2\right]. \tag{37}$$

Equation (31) becomes:

$$E_t(c_{t+1}) - c_t = \tau\pi_t + \tau_C c_t + \varphi_t^r - E_t(\pi_{t+1}). \tag{38}$$

If nominal rigidity is the same across sectors and the central bank is reacting to CPI inflation, sectoral relative price movements have no impact on CPI inflation and aggregate consumption dynamics. This is the same result obtained by Benigno (2004).

When $\lambda_1 = \lambda_2$, equation (30) and the definitions of sectoral inflation rates imply:

[26] For example, following a zero-persistence shock at time 0, if the elasticity to the past terms of trade is zero, CPI inflation returns to the steady state in period 1. If the elasticity to the past terms of trade differs from zero, it determines the size of the CPI inflation movement in period 1, and α_5 then measures the speed of adjustment of CPI inflation to the steady state from period 2 on. Note that the elasticities of aggregate and sectoral endogenous variables to the terms of trade do not depend on the persistence of exogenous shocks.

$$\beta E_t T_{t+1} - [1+\beta+\lambda(1+\omega)]T_t + T_{t-1} = 2\lambda(\varphi_t^1 - \varphi_t^2). \tag{39}$$

This equation has one eigenvalue outside the unit circle and one inside. The terms of trade remain endogenously persistent, *i.e.*, $\alpha_5 \neq 0$ in (36), where α_5 is now the stable eigenvalue of (39) (this can also be seen by subtracting equation (A.7) in Appendix A from (A.6), manipulating the resulting equation, and using the definition of α_5). The solution is:

$$\alpha_5 = e = \frac{1+\beta+\lambda(1+\omega) - \sqrt{[1+\beta+\lambda(1+\omega)]^2 - 4\beta}}{2\beta}, \tag{40}$$

where $1 > \alpha_5 > 0$.[27] Calvo-Yun-Rotemberg nominal rigidity implies endogenous terms of trade persistence.[28] However, as pointed out by Benigno (2004), even if prices are sticky, the terms of trade are completely insulated from monetary policy when $\lambda_1 = \lambda_2$.[29] This holds regardless of whether the central bank is reacting to CPI inflation or τ_1 differs from τ_2 (or $\eta \neq 1-b$ in CFG). In addition, if $\tau_1 = \tau_2$, aggregate output and inflation dynamics are shielded from terms of trade persistence: $\alpha_3 = \alpha_4 = 0$ in equations (34) and (35). (CPI inflation and aggregate output are both fully forward-looking variables and the standard results on absence of endogenous persistence in a one-sector Calvo-Yun-Rotemberg model hold.) Instead, sectoral inflation rates and outputs remain persistent: $\alpha_4 \equiv b\alpha_1 + (1-b)\alpha_2 = 0$ and $\alpha_5 \equiv 1 + \alpha_1 - \alpha_2 = e$ imply $\alpha_1 = -(1-b)(1-e)$ and $\alpha_2 = b(1-e)$. It is easy to verify that the elasticities of c_t^1 and c_t^2 to T_{t-1} are $-\omega(1-b)e$ and ωbe, respectively.

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[27] The root α_5 is real for all plausible parameter values. It tends to 0 if $\lambda \to \infty$ (see footnote 28) and/or $\omega \to \infty$. If goods are perfect substitutes ($\omega \to \infty$), the ability of consumers to arbitrage across the outputs of the two sectors ensures that their relative price is always 1, *i.e.*, $T_t = 0$ regardless of relative sectoral productivity. (To see this, divide both sides of (39) by $1+\beta+\lambda(1+\omega)$ and take the limit for $\omega \to \infty$.)

[28] Dividing both sides of (39) by $1+\beta+\lambda(1+\omega)$ and taking the limit for $\lambda \to \infty$ (the case of price flexibility) yields $T_t = -[2/(1+\omega)](\varphi_t^1 - \varphi_t^2)$, *i.e.*, there is no endogenous terms of trade persistence when prices are flexible. Given $T_t = T_{t-1} + \pi_t^1 - \pi_t^2$, under flexible prices, sectoral inflation rates adjust so as to remove the effect of the past terms of trade on the current level in equilibrium.

[29] Note that it is also $\gamma_{r,1} = \gamma_{r,2}$ in equation (36) when $\lambda_1 = \lambda_2$.

4.B. Identical Nominal Rigidity, Different Sectoral Inflation Reactions

Suppose $\lambda_1 = \lambda_2 = \lambda$, but $\tau_1 \neq \tau_2$ ($\eta \neq 1-b$ in CFG). Equation (37) still describes CPI inflation dynamics. However, in this case, the sectoral inflation rates in (31) cannot be aggregated into CPI inflation, exposing aggregate consumption to endogenous persistence via terms of trade effects. To see this, let $\tau \equiv b\tau_1 + (1-b)\tau_2$, $\tau^D \equiv \tau_1 - \tau_2$, and $x_t^D \equiv x_t^1 - x_t^2$ for any variable x_t^j, $j = 1, 2$. The interest rate rule (23) can be rewritten as:

$$r_t = \tau\pi_t + \tau^D b(1-b)\pi_t^D + \tau_C c_t + \varphi_t^r . \tag{41}$$

Asymmetric policy responses to sectoral inflation rates induce a response of the interest rate to the sectoral inflation differential (which equals the change in the terms of trade: $\pi_t^D = T_t - T_{t-1}$). The Fisher equation (31) becomes:

$$E_t c_{t+1} - c_t = \tau\pi_t + \tau^D b(1-b)\pi_t^D + \tau_C c_t + \varphi_t^r - E_t\pi_{t+1} . \tag{42}$$

This equation and the sectoral Phillips curves in (30) imply that the solution for aggregate consumption depends on the past terms of trade (and thus displays endogenous persistence) even if nominal rigidity is equal across sectors. When $\lambda_1 = \lambda_2 = \lambda$, the Phillips curves in (30) yield:

$$\pi_t^D = -\lambda\left[(1+\omega)T_t + 2\varphi_t^D\right] + \beta E_t\pi_{t+1}^D . \tag{43}$$

The sectoral inflation differential is a function of the terms of trade, and thus inherits the persistence of the latter. Combined with (42), this implies that aggregate consumption is no longer insulated from terms of trade movements. Put differently, when $\tau_1 \neq \tau_2$, c_t becomes a function of relative prices (as one can also see from direct inspection of (31) and the sectoral Phillips curves in (30)). Since, in general, sectoral inflation rates are endogenously persistent regardless of whether sectors are characterized by the same degree of nominal rigidity (α_1 and α_2 differ from zero regardless of λ_1 versus λ_2), it follows that consumption is exposed to endogenous persistence via terms of trade effects when $\lambda_1 = \lambda_2 = \lambda$ but $\tau_1 \neq \tau_2$.[30] Equation (37) then implies that CPI inflation depends on relative prices too – and thus the past terms of trade.

[30] We analyze special cases in which *one* sector's inflation rate displays no endogenous persistence below. Note that the presence of T_{t-1} in (42) (since $\pi_t^D = T_t - T_{t-1}$) is not sufficient for endogenous persistence of aggregate inflation and output. This requires endogenous persistence of relative prices and the terms of trade. We show

The results above show that the argument in Benigno (2004) and Woodford (2003, pp. 200-204) that aggregate inflation and output are insulated from terms of trade dynamics when nominal rigidity is identical across sectors is conditional on the nature of monetary policy. Benigno and Woodford focus on the choice of the optimal inflation target for the central bank's welfare-based loss function and do not analyze the consequences of policy implementation through endogenous interest rate setting. Our exercise makes it transparent that if the central bank is not reacting to CPI inflation in setting the interest rate, the result that aggregate output and inflation are insulated from terms of trade dynamics if nominal rigidity is equal across sectors no longer holds, and α_3 and α_4 differ from zero in equations (34) and (35). Of course, this actually matters for shock transmission to aggregates to the extent that the terms of trade move in response to shocks. It is straightforward to verify that relative prices do not play any role in the transmission of interest rate shocks, because they do not move at all, if nominal rigidity is equal across sectors, regardless of what measure of inflation the central bank is responding to (*i.e.*, in equations (A.4) and (A.5), $\gamma_{r,1} = \gamma_{r,2}$ if $\lambda_1 = \lambda_2$, regardless of τ_1 versus τ_2). The reason is that interest rate shocks are aggregate and, if $\lambda_1 = \lambda_2$, they affect sectoral prices in the same way even if $\tau_1 \neq \tau_2$. Instead, even if $\lambda_1 = \lambda_2$, relative prices move in response to sector-specific productivity shocks and matter for their transmission to aggregates when $\tau_1 \neq \tau_2$.[31, 32]

One can solve for the elasticities to the past terms of trade in this scenario by starting from the observation that the AR(1) root for the terms of trade is still $\alpha_5 = e$ as in equation

below that, when labor is mobile across sectors, equation (42) still holds, but there is no endogenous persistence because relative prices and the terms of trade become simply proportional to the sectoral productivity differential.

[31] Given $\lambda_1 = \lambda_2$, and regardless of τ_1 versus τ_2, the only situation in which relative prices do not move in response to productivity shocks is when these are purely aggregate and such that $\varphi_t^1 = \varphi_t^2$ in all periods (thus requiring perfectly correlated innovations and $\rho_1 = \rho_2$). In this case, it is possible to verify that $\gamma_1 - \gamma_3 + \gamma_2 - \gamma_4 = 0$, implying no movement in the terms of trade.

[32] We should also note that the situation in which nominal rigidity is the same across sectors but the central bank does not respond to CPI inflation is not justified on normative grounds. As Benigno (2004) shows, it is optimal to target CPI inflation when $\lambda_1 = \lambda_2$. The reason for studying the case in this subsection is thus entirely positive, as in reality central banks may be responding to inflation in a subset of the economy even if nominal rigidity is identical across sectors.

(40).[33] Hence, the elasticities of sectoral inflation rates π_t^1 and π_t^2 to the past terms of trade – α_1 and α_2, respectively – are such that:[34]

$$\alpha_1 = \frac{\lambda\left[2\alpha_3 - (1+\omega)(1-b)e\right]}{1-\beta e}, \tag{44}$$

$$\alpha_2 = \frac{\lambda\left[2\alpha_3 + (1+\omega)be\right]}{1-\beta e}. \tag{45}$$

It follows that the elasticity of CPI inflation to the past terms of trade is:

$$\alpha_4 \equiv b\alpha_1 + (1-b)\alpha_2 = \frac{2\alpha_3\lambda}{1-\beta e}. \tag{46}$$

Equations (44)-(46) and $1+\alpha_1-\alpha_2 = e$ can be substituted into equation (A.8) in Appendix A to obtain an equation that can be solved for α_3 (the elasticity of aggregate output to the past terms of trade) as:

$$\alpha_3 = \frac{\lambda b(1-b)(1+\omega)(\tau_1-\tau_2)e}{(1-e+\tau_C)(1-\beta e)+2\lambda\left[b\tau_1+(1-b)\tau_2-e\right]}. \tag{47}$$

In turn, using this equation in conjunction with (44)-(46) yields solutions for α_1, α_2, and α_4.

Equations (44)-(47) make it possible to draw some conclusions on the effect of changes in the policy parameters τ_1, τ_2, and τ_C on the extent to which aggregate variables respond to the past terms of trade in the model, *i.e.*, on the extent to which aggregates inherit endogenous terms of trade persistence.

Equation (47) shows that the elasticity of aggregate output to the past terms of trade decreases if the central bank reacts more aggressively to output, *i.e.*, if τ_C rises. Because both α_1 and α_2 are increasing functions of α_3 (equations (44) and (45)), a more aggressive reaction to output dampens the effect of the past terms of trade on current sectoral inflation rates. (Changes in the extent to which aggregate output is subject to endogenous persistence affect the sensitivity of sectoral inflation to the past terms of trade through the sectoral inflation equations in (30).) In turn, less sensitivity of sectoral inflation rates to the past terms of trade implies that aggregate inflation is less responsive to T_{t-1} (equation (46)). Other things given, consistent with standard intuition, a central bank that reacts aggressively to an expansion in aggregate output is more

[33] Note that this implies that there is no endogenous persistence also in this case if $\lambda \to \infty$ and/or $\omega \to \infty$.
[34] This follows from equations (A.6) and (A.7) in Appendix A and the definition of α_5.

successful at driving the latter and inflation more quickly back to the steady state for any given value of the inflation reaction parameters τ_1 and τ_2. (It is important to remember though that changes in τ_C will affect also the elasticities of aggregate output and CPI inflation to the productivity and monetary policy shocks – the γ's in equations (34) and (35).)

Analyzing the consequences of changes in τ_1 and τ_2 is more complicated. Differentiating the solution for α_3 with respect to τ_1 or τ_2 yields complicated expressions that cannot be signed without the aid of numerical values for the parameters of the model. Figures 1 and 2 report the results of a numerical example. For illustrative purposes, we set the parameter values at $\beta = .99$, $\lambda = .086$ (as implied by $\beta = .99$ and the commonly made assumption that prices are sticky on average for four quarters, *i.e.*, a Calvo-Yun probability of price adjustment equal to .25), $\omega = 2$, $b = .5$ (sectors have equal size), $\tau_C = .5$ (as in Taylor, 1993). Figure 1 plots α_3 as a function of τ_1 and τ_2 over the ranges [1, 20] for both reaction coefficients. (The diagram looks similar for higher or lower values of λ; for the case of a Cobb-Douglas consumption basket, $\omega = 1$; or for the [1, 100] range for τ_1 and τ_2.) The elasticity of output to the past terms of trade increases with τ_1 for given τ_2 and decreases with τ_2 for given τ_1. The top two portions of Figure 2 plot α_3 as a function of τ_1 over the range [1, 20] for $\tau_2 = 1.5$ and α_3 as a function of τ_2 for $\tau_1 = 1.5$, respectively. Given the same reference value for a "slice" of Figure 1, the increase in α_3 caused by higher τ_1 is identical (in absolute value) to the decrease caused by larger τ_2. This is a consequence of cross-sectoral symmetry caused by the assumption $b = .5$.[35] Higher τ_1 (τ_2) for given τ_2 (τ_1) causes both α_1 and α_2 to increase (decrease) by the same amount. The effects are exactly symmetric if the initial reference values of τ_2 and τ_1 are identical.

The intuition for the results in Figure 2 is as follows. Suppose the initial values of τ_1 and τ_2 are identical. We know from the discussion above that α_3 is zero in this case. If both τ_1 and τ_2 are raised by the same amount, it must be that case that α_3 equals zero also with the higher values of the inflation reaction parameters. Hence the symmetric, opposite effects on α_3. Now suppose that τ_1 is raised, holding τ_2 constant at, say, 1.5. Suppose τ_1 is increased to 2.5. α_3 becomes positive. The reason is that expected output growth in equation (31) becomes a

[35] If b were different from .5, the effects would be rescaled so that equal increases in reaction coefficients starting from identical values leave α_3 at zero.

function of sector 1 inflation as well as CPI inflation. Given $\tau_1 = \tau_2 + \tau_1^D$, where $\tau_1^D > 0$ is the extra reaction to sector 1 inflation, equation (31) can be rearranged as:

$$c_t = \frac{E_t(c_{t+1}) - \left[\tau_1^D b \pi_t^1 + \tau_2 \pi_t + \varphi_t^r - E_t(\pi_{t+1})\right]}{1 + \tau_C}. \tag{48}$$

Equations (28) and (30) imply that sector 1 inflation is a decreasing function of the terms of trade (since an increase in the latter shifts demand away from sector 1). It follows that, for any given level of expected future aggregate output and CPI inflation, today's aggregate output is an increasing function of today's terms of trade. Given that the latter are a state variable, today's aggregate output becomes an increasing function of yesterday's terms of trade.

The logic for the case of an increase in τ_2 is similar. From equations (29) and (30), sector 2 inflation is an increasing function of the terms of trade (since an increase in the latter shifts demand in favor of sector 2). Thus, the Fisher equation implies that today's aggregate output becomes a decreasing function of the terms of trade.[36]

For completeness of illustration, Figure 2.c. displays the slice of Figure 1 taken at τ_1 = 2.5. Suppose that, given τ_1 now equal to 2.5, the central bank raises τ_2 too. For the reasons discussed above, α_3 decreases. If τ_2 is raised above 2.5, the negative effect of the terms of trade on consumption via the Fisher equation prevails, and aggregate output becomes a decreasing function of the terms of trade (footnote 36).

A special case is also informative on the properties of the model. Consider the situation in which $\omega = 1$, $\tau_C = 0$, $\tau_1 = 0$. Consumers have Cobb-Douglas preferences over the goods produced in the two sectors, as in Benigno (2004). The central bank reacts only to inflation in sector 2. In this case, $\alpha_3 = -be$, $\alpha_1 = -2\lambda e/(1-\beta e)$, and $\alpha_2 = 0$. In addition, it is possible to verify that the elasticities of c_t^1 and c_t^2 to T_{t-1} are $-e$ and 0, respectively. If the central bank reacts only to sector 2 inflation and $\omega = 1$, there is no endogenous persistence in that sector's

[36] Given $\tau_2 = \tau_1 + \tau_2^D$, where $\tau_2^D > 0$ is the extra reaction to sector 2 inflation, the Fisher equation (31) can be rewritten as:

$$c_t = \frac{E_t(c_{t+1}) - \left[\tau_2^D (1-b) \pi_t^2 + \tau_1 \pi_t + \varphi_t^r - E_t(\pi_{t+1})\right]}{1 + \tau_C}.$$

inflation rate and output. The elasticity of CPI inflation to the past terms of trade is $-2\lambda be/(1-\beta e)$, or $-2\lambda be/(1-e) = -b\left[\sqrt{\lambda(\lambda+2)}-\lambda\right]$ if $\beta \to 1$.[37]

4.C. Different Nominal Rigidity, Reaction to CPI Inflation

If nominal rigidity differs across sectors ($\lambda_1 \neq \lambda_2$), equation (30) and the definitions of sectoral and aggregate inflation rates can be manipulated to obtain:

$$\beta E_t rp_{t+1}^j - \left[1+\beta+\lambda_j(1+\omega)\right]rp_t^j + rp_{t-1}^j = \pi_t - 2\lambda_j\left(c_t - \varphi_t^j\right) - \beta E_t \pi_{t+1}, \; j = 1, 2. \tag{49}$$

Because sectoral relative prices are proportional to the terms of trade (equations (28) and (29)), they inherit the persistence of the latter. Equation (49) shows that inflation becomes endogenously persistent if nominal rigidity differs between sectors 1 and 2 regardless of τ_1 versus τ_2 (or η versus $1-b$ in CFG). Averaging equation (49) across sectors and using the definition of the consumer price index yields:

$$\begin{aligned}\pi_t &= \beta E_t \pi_{t+1} + 2\left[b\lambda_1 + (1-b)\lambda_2\right]c_t - (1+\omega)\left[b\lambda_1 rp_t^1 + (1-b)\lambda_2 rp_t^2\right] \\ &-2\left[b\lambda_1\varphi_t^1 + (1-b)\lambda_2\varphi_t^2\right].\end{aligned} \tag{50}$$

Hence, if $\lambda_1 \neq \lambda_2$, inflation inherits the persistence of the terms of trade, and so does consumption through equation (31).[38] The intuition is simple: Changes in relative prices redistribute demand across sectors in the economy. When nominal rigidity differs in the two sectors, the sectoral relative prices move at different speeds. This introduces persistence in the aggregate economy, as demand redistribution across heterogeneous sectors affects aggregate output and prices. Aggregate adjustment must then continue until all relative and aggregate prices and quantities have reached the steady state.

Subtracting equation (49) for $j = 2$ from the equation for $j = 1$ and using equations (28) and (29) yields the difference equation for terms of trade dynamics with different degrees of nominal rigidity across sectors:

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[37] In the symmetric case in which $\omega = 1$, $\tau_C = 0$, and $\tau_2 = 0$ (the central bank reacts only to inflation in sector 1), it is $\alpha_3 = (1-b)e$, $\alpha_1 = 0$, and $\alpha_2 = 2\lambda e/(1-\beta e)$. The elasticities of c_t^1 and c_t^2 to T_{t-1} are 0 and e, respectively. If the central bank reacts only to sector 1 inflation and $\omega = 1$, the elasticities of sector 1 inflation and output to T_{t-1} are zero. The elasticity of CPI inflation to the past terms of trade is $2\lambda(1-b)e/(1-\beta e)$, or $2\lambda(1-b)e/(1-e) = (1-b)\left[\sqrt{\lambda(\lambda+2)}-\lambda\right]$ if $\beta \to 1$.

[38] Equation (50) also follows from aggregating (30) across sectors.

$$\beta E_t T_{t+1} - \{1 + \beta + (1+\omega)[(1-b)\lambda_1 + b\lambda_2]\}T_t + T_{t-1} = -2(\lambda_1 - \lambda_2)c_t + 2(\lambda_1\varphi_t^1 - \lambda_2\varphi_t^2). \tag{51}$$

It is no longer possible to analyze terms of trade dynamics separately from those of aggregate output and inflation. Substituting (28) and (29) into (50) yields:

$$\pi_t = \beta E_t \pi_{t+1} + 2[b\lambda_1 + (1-b)\lambda_2]c_t - (1+\omega)b(1-b)(\lambda_1 - \lambda_2)T_t - 2[b\lambda_1\varphi_t^1 + (1-b)\lambda_2\varphi_t^2]. \tag{52}$$

Finally, if $\tau_1 = \tau_2 = \tau$, consumption growth is determined by (38).

Equations (38), (51), and (52) constitute a system of three difference equations in three unknowns – the terms of trade, CPI inflation, and aggregate output. In particular, equation (52) is a generalized New Keynesian Phillips curve for an economy with heterogeneous nominal rigidity. Suppose that $\lambda_1 > \lambda_2$ (sector 1 has relatively flexible prices). *Ceteris paribus*, an increase in the terms of trade that shifts demand from sector 1 to sector 2 causes aggregate inflation to decrease because a larger portion of aggregate demand is now allocated to the relatively sticky sector.

The presence of aggregate consumption in equation (51) implies that, when $\lambda_1 \neq \lambda_2$, terms of trade dynamics are no longer insulated from monetary policy. In particular, the stable AR(1) root α_5 in the terms of trade solution (36) becomes a function of τ and τ_C.[39] We show in Appendix B that α_5 now solves the equation:

$$\begin{aligned} J(\alpha_5) \equiv\; & 2(1+\omega)b(1-b)(\lambda_1-\lambda_2)^2(\tau-\alpha_5)\alpha_5 \\ & - \left\{ \begin{matrix} (1+\omega)[(1-b)\lambda_1 + b\lambda_2]\alpha_5 \\ -(1-\alpha_5)(1-\beta\alpha_5) \end{matrix} \right\} \left\{ \begin{matrix} (1-\alpha_5+\tau_C)(1-\beta\alpha_5) \\ +2[b\lambda_1 + (1-b)\lambda_2](\tau-\alpha_5) \end{matrix} \right\} = 0. \end{aligned} \tag{53}$$

One can verify that setting $\lambda_1 = \lambda_2$ in this equation yields a quartic equation for α_5, the only stable solution of which when the condition for local determinacy holds is e as in equation (40).[40] In the general case $\lambda_1 \neq \lambda_2$, the condition for local determinacy being satisfied by policy ensures that the quartic equation (53) has only one root inside the unit circle.[41] As when $\lambda_1 = \lambda_2$, the root α_5 tends to 0 if the sectoral goods are perfect substitutes ($\omega \to \infty$). Regardless of heterogeneous nominal rigidity, arbitrage across sectors by consumers ensures that relative prices are equal to 1 ($T_t = 0$) if goods are perfect substitutes, eliminating endogenous persistence

[39] CFG demonstrates that, if $\tau_C = 0$, $\tau > 1$ is necessary and sufficient for local determinacy of the equilibrium in this scenario. (If it were $\tau < 1$, there would be multiple stable solutions for α_5.)

[40] As mentioned above, the condition for local determinacy is simply $\tau > 1$ if $\tau_C = 0$.

[41] For simplicity, we assume that parameters are such that the relevant root is always real.

via relative price movements.[42] But the distinction between sectors becomes uninteresting in this case and heterogeneity in nominal rigidity implausible. In the general case in which substitutability is finite, the root α_5 is a function of the policy parameters τ and τ_C.[43] We are interested in how changes in these policy parameters affect α_5, the endogenous persistence of the terms of trade, and, in turn, the elasticity of aggregate output, α_3, and other variables to the past terms of trade.

It is possible to verify that, if $\lambda_1 > \lambda_2$, α_3 is an increasing function of α_5. In turn, changes in α_5 have opposite direct effects on α_1 and α_2. Since these effects are proportional to λ_1 and λ_2, if $b = .5$ (sectors have equal size), the effect of a change in α_5 on α_4 is dictated by the sector with the larger λ, *i.e.*, the sector in which prices are more flexible.[44]

Consider the following parameter values for illustrative purposes: $\beta = .99$, $\lambda_1 = .505$ (sector 1 prices are sticky for two quarters on average), $\lambda_2 = .086$, $\omega = 2$, $b = .5$. Holding τ constant at 1.5, raising τ_C from an initial value of zero to .1 and then .5 causes positive values of α_3 and α_5 to decrease and the absolute value of a negative elasticity α_4 to increase. (Since λ_1 is bigger than λ_2, the increasing absolute value of a negative α_1 more than offsets a decreasing α_2.) The intuition is as follows. Equation (51) ties terms of trade and aggregate output dynamics. If the central bank reacts more aggressively to output deviations from the steady state, both output and the terms of trade become less persistent (smaller α_3 and α_5). However, this comes at the cost of increased sensitivity of CPI inflation to the past terms of trade (larger α_4 in absolute value). This differs from what happened in the previous subsection, where higher τ_C caused both

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[42] To see this, divide both sides of (51) by $1+\beta+(1+\omega)[(1-b)\lambda_1 + b\lambda_2]$ and take the limit for $\omega \to \infty$.

[43] This is true also if one of the λ's tends to infinite, *i.e.*, if prices are flexible in one sector. To see this, observe that equations (51) and (52) reduce to $0 = 2c_t - (1+\omega)(1-b)T_t - 2\varphi_t^1$ (after appropriate division of both sides and rearranging) if $\lambda_1 \to \infty$. (This is the same equation that one obtains from (25) by setting $z_t^1 = 0$ as implied by price flexibility in sector 1 and recalling (28).) Using this result in conjunction with the Fisher equation (38) and the definitions of CPI and sectoral inflation rates yields an equation that relates current and expected sector 2 inflation to expected, current, and past terms of trade, and the exogenous shocks. Equation (30) for $j = 2$, equation (29), and the restriction $0 = 2c_t - (1+\omega)(1-b)T_t - 2\varphi_t^1$ then provide a second equation that relates sector 2 inflation to the terms of trade. The solution of this system features dependence of current sector 2 inflation and terms of trade on the past terms of trade, thus imparting endogenous persistence to aggregate output and inflation. This is of course no longer the case if both λ's tends to infinite, in which case we have again $T_t = -[2/(1+\omega)](\varphi_t^1 - \varphi_t^2)$.

[44] The results above can be verified by using equations (B.5), (B.1), (B.2), and (B.3) in the appendix, respectively.

α_3 and α_4 to become smaller in value. There, changes in τ_C had no effect on terms of trade dynamics (which are insulated from monetary policy when $\lambda_1 = \lambda_2$) and, given the AR(1) root of the terms of trade, e, α_3 and α_4 were proportional to each other (equation (46)). A change in τ_C affected the elasticity of CPI inflation to the past terms of trade, α_4, only through its effect on the elasticity of aggregate output, α_3. When $\lambda_1 \neq \lambda_2$, two important changes take place. First, the AR(1) root of the terms of trade solution in equation (36), α_5, becomes a function of policy. Comparing equations (47) and (B.4), this implies that policy affects α_3 directly *and* indirectly, via its effect on α_5. Second, policy no longer affects the elasticity of aggregate inflation to the terms of trade, α_4, only through its effect on the elasticity of aggregate output, α_3, but there is an additional effect due to the impact of the different speed of price adjustment across sectors on the Phillips curve for aggregate inflation. This manifests itself at the denominator of (B.3), where α_5 is now endogenous to policy, and at the numerator, where the effect of policy through α_5 is proportional to the negative of the difference between λ_1 and λ_2. The latter term can cause α_4 to be negative when $\lambda_1 > \lambda_2$, even if α_3 is positive. In particular, for α_4 to be positive, it must be:

$$\alpha_3 > \frac{(\lambda_1 - \lambda_2)b(1-b)(1+\omega)\alpha_5}{2[b\lambda_1 + (1-b)\lambda_2]}. \tag{54}$$

If $b = .5$, the threshold for α_4 to be positive becomes $[(\lambda_1 - \lambda_2)(1+\omega)\alpha_5]/[4(\lambda_1 + \lambda_2)]$. Given $\alpha_5 > 0$ and $\lambda_1 > \lambda_2$, plausible parameter values imply that α_3 and α_4 have opposite sign. A higher τ_C can then cause the absolute value of α_4 to increase if it induces output persistence (α_3) to decline sufficiently faster than terms of trade persistence (α_5).[45]

If τ is raised from 1.5 to 2 while holding τ_C at zero, the elasticity of CPI inflation to the past terms of trade (α_4) falls in absolute value, whereas the terms of trade become more persistent (α_5 rises) and aggregate output becomes more sensitive to T_{t-1} (α_3 rises). Intuitively, a more aggressive reaction to CPI inflation reduces the persistence of the latter. However, it does so at the cost of more endogenous persistence in output movements.[46]

[45] As expected, if we swap sectors 1 and 2, making sector 2 the relatively flexible sector for the same parameter values as in the numerical example above, α_5 is unaffected, and α_3 and α_4 change sign, with the same absolute values as before. Higher τ_C causes the absolute value of α_3 to decrease and α_4 to increase.

4.D. Different Nominal Rigidity, Different Sectoral Inflation Reactions

Suppose $\lambda_1 \neq \lambda_2$ and $\tau_1 \neq \tau_2$. Nominal rigidity differs across sectors and the central bank reacts to sectoral inflation rates asymmetrically. Aggregate output, terms of trade, and CPI inflation dynamics are determined by equations (42), (51), and (52). Equations (B.1)-(B.3) and (B.5) still hold. Hence, once one has determined the properties of the elasticity of T_t to T_{t-1} (α_5) in this case, it is easy to recover those of the elasticities of aggregate output (α_3) and CPI inflation (α_4) to T_{t-1} as in Section 4.C.[47] We thus focus on α_5, noting that the case of this subsection also provides information on the consequences of changing the policy response to a sectoral inflation rate, holding the response to the other sector unchanged, and starting from a situation in which the central bank was responding to CPI inflation.

We show in Appendix B that α_5 now solves the equation:

$$J^G(\alpha_5) \equiv 2(1+\omega)b(1-b)(\lambda_1-\lambda_2)[\lambda_1(\tau_1-\alpha_5)-\lambda_2(\tau_2-\alpha_5)]\alpha_5$$
$$-\begin{Bmatrix} (1+\omega)[(1-b)\lambda_1+b\lambda_2]\alpha_5 \\ -(1-\alpha_5)(1-\beta\alpha_5) \end{Bmatrix}\begin{Bmatrix} (1-\alpha_5+\tau_C)(1-\beta\alpha_5) \\ +2[b\lambda_1(\tau_1-\alpha_5)+(1-b)\lambda_2(\tau_2-\alpha_5)] \end{Bmatrix}=0. \qquad (55)$$

Assume $\tau^D \equiv \tau_1 - \tau_2 < 0$. It is possible to verify that $J^G(\alpha_5) - J(\alpha_5)$ (where $J(\alpha_5)$ is defined in (53)) is a convex parabola that crosses the horizontal axis at $\alpha_5^* \in (0,1)$ and $\alpha_5^{**} > 1$, where:

$$\alpha_5^* = \frac{1+\beta+\lambda_1(1+\omega)-\sqrt{[1+\beta+\lambda_1(1+\omega)]^2-4\beta}}{2\beta},$$

i.e., α_5^* has the same expression as the solution for α_5 in the case of equal nominal rigidity across sectors ($\lambda_1 = \lambda_2$) in equation (40), with λ_1 replacing λ, and it is independent of τ^D. Assume further that $\lambda_1 > \lambda_2$, *i.e.*, prices are relatively stickier in sector 2. Our claim is that if α_5^* is smaller than the stable root of $J(\alpha_5)$, then the stable root of $J^G(\alpha_5)$ must be smaller than the stable root of $J(\alpha_5)$. The reason is that since $J^G(\alpha_5)$ crosses $J(\alpha_5)$ only once between 0 and 1, if it crosses to the left of the stable root of $J(\alpha_5)$, it must cross the horizontal axis between α_5^* and the

[46] The numerical results are unchanged if we use a calibration that matches Bils and Klenow's (2004) results: β = .99, λ_1 = 10, λ_2 = .052, ω = 1, b = .9 (see below).

[47] Perfect substitutability across goods produced in the two sectors would eliminate endogenous persistence via relative prices also in this general case.

stable root of $J(\alpha_5)$.[48] This implies that terms of trade persistence when sector 2 prices are relatively stickier ($\lambda_1 > \lambda_2$) and the central bank is reacting more aggressively to sector 2 inflation ($\tau^D < 0$) is lower than persistence when the policy response is identical across sectors.

Now recall that the stable root of $J(\alpha_5)$ is the autoregressive coefficient of T_t when $\lambda_1 \neq \lambda_2$ and $\tau_1 = \tau_2$. Hence, our proof boils down to arguing that the solution for α_5 when nominal rigidity is such that $\lambda_1 > \lambda_2$ and the central bank responds to CPI inflation is larger than the solution for α_5 if nominal rigidity is identical across sectors and equal to λ_1, *i.e.*, T_t is more persistent when $\lambda_1 > \lambda_2$ than when the extent of price flexibility is kept equal across sectors at the higher (λ_1) level. This is intuitive: If we hold λ_1 constant and decrease λ_2 below it, the sectoral price p_t^2 is adjusted at lower speed than before, while p_t^1 is adjusted at the same speed, resulting in slower movement of $T_t \equiv p_t^1 - p_t^2$. Although we could not verify the point formally, it held true for all the combinations of parameter values we tried. It is worth noting that, if $\lambda_1 > \lambda_2$, the assumption $\tau^D < 0$ ($\tau_2 > \tau_1$) is consistent with the normative prescription of Benigno (2004) that policy should react more aggressively to inflation in the relatively sticky sector. It is intuitive that this would reduce terms of trade persistence relative to a response to the CPI as the central bank is reacting more aggressively to the relatively stickier sector.[49] Holding $\tau^D < 0$ constant, a more aggressive reaction to output (an increase in τ_C) causes the persistence parameter α_5 to fall for plausible parameter values for the same reason as in the case $\tau_1 = \tau_2$.

4.E. Summary

To summarize, when labor is immobile (and sectoral goods are imperfect substitutes), endogenous aggregate persistence arises in our two-sector model economy if nominal rigidity differs across sectors and/or the central bank responds asymmetrically to (size-weighted) sectoral

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[48] $J^G(\alpha_5)$ crosses $J(\alpha_5)$ from above at α_5^* since $J^G(\alpha_5) - J(\alpha_5)$ is a convex parabola. Again, we focus on the case in which the relevant root is real for simplicity.

[49] Note that, letting $\tau_1 = \tau$, we can rewrite the policy rule (23) as $r_t = \tau b \pi_t^1 + \left(\tau - \tau^D\right)\!(1-b)\pi_t^2 + \tau_C c_t + \varphi_t^r$. It follows that sufficiently large values of $\tau^D > 0$ for given τ could lead to indeterminacy, by causing the interest rate to decrease in response to sector 2 inflation. This possibility is a consequence of the different specification of the policy rule relative to CFG, and a violation of the determinacy condition stated there. For conventional parameter values, given $\lambda_1 > \lambda_2$, small $\tau^D > 0$ would not cause indeterminacy, but it would increase terms of trade persistence because of the weaker reaction to the relatively stickier sector.

inflation rates in interest rate setting. In both cases, relative price adjustment is the key for endogenous aggregate persistence. Relative price movements shift aggregate demand across sectors. When nominal rigidity differs, relative prices rp_t^1 and rp_t^2 adjust at different speeds, *i.e.*, sectoral demands adjust at different speeds. If sector 1 is relatively more flexible than sector 2, an increase in the cross-sectoral relative price that shifts aggregate demand toward sector 2 has a direct negative effect on current aggregate inflation as a larger share of aggregate demand falls on the relatively stickier sector. When nominal rigidity is identical across sectors but the central bank is not reacting to CPI inflation, interest rate setting induces aggregate output to respond to the dynamics of relative prices. In turn, this results in endogenous persistence of aggregate CPI inflation as well.

We remark that the endogenous persistence effect of terms of trade dynamics on CPI inflation and aggregate output does not depend on the assumption that monetary policy is conducted through endogenous interest rate setting. Although the values of the elasticities to the past terms of trade and the exogenous shocks depend on the reaction coefficients in the Taylor rule (23), endogenous aggregate persistence through terms of trade movements arises also if monetary policy is conducted by selecting an exogenous path for money supply as long as nominal rigidity differs across sectors.[50] As long as this is the case, the demand-redistribution effect of relative price changes will affect aggregate inflation dynamics through the impact of relative prices on equilibrium sectoral marginal costs. The persistence effect via dependence of aggregate inflation on the past sectoral price differential would continue to operate through this channel.

5. Labor Mobility

How do the results of the previous section change if we remove the assumption of labor immobility? We demonstrate in this section that labor mobility would remove endogenous persistence *regardless* of heterogeneity in nominal rigidity and asymmetry in policy reactions to sectoral inflation rates.[51] When labor is mobile, real wages are equalized across sectors (marginal costs differ only to the extent that there are differences in productivity), and the terms

[50] This is the case studied by Carvalho (2006).
[51] As we mentioned above, endogenous persistence via relative prices disappears (regardless of heterogeneous nominal rigidity, labor immobility, and policy) also if goods produced in the two sectors are perfect substitutes.

of trade are simply proportional to the cross-sectoral productivity differential. There is no endogenous persistence in relative prices and, therefore, in aggregates.

To see this observe that, when labor is mobile (and $w_t^1 = w_t^2$ for all t), the sectoral wage equations in (15) imply $c_t^D = \varphi_t^D$. The sectoral demand equations in (17) and the definition of the terms of trade yield $c_t^D = -\omega T_t$. Combining this with $c_t^D = \varphi_t^D$ shows that, if labor is mobile, the terms of trade are simply proportional to the productivity differential across sectors: $T_t = -\frac{1}{\omega}\varphi_t^D$, regardless of nominal rigidity and policy. It follows that there is no endogenous persistence in terms of trade dynamics and the sectoral relative prices rp_t^1 and rp_t^2, determined by $rp_t^1 = -\frac{1-b}{\omega}\varphi_t^D$ and $rp_t^2 = \frac{b}{\omega}\varphi_t^D$.

One can verify that the sectoral inflation equations in (30) hold also when labor is not sector-specific and that aggregate inflation in the presence of heterogeneous nominal rigidity is determined by equation (52). Therefore:

$$\pi_t = \beta E_t \pi_{t+1} + 2\left[b\lambda_1 + (1-b)\lambda_2\right]c_t + \frac{(1+\omega)b(1-b)(\lambda_1 - \lambda_2)}{\omega}\varphi_t^D - 2\left[b\lambda_1\varphi_t^1 + (1-b)\lambda_2\varphi_t^2\right]. \quad (56)$$

Combining this equation with the Fisher equation (42) (for the general case $\tau_1 \neq \tau_2$), the sectoral Phillips curves in (30), and the results above on rp_t^1 and rp_t^2, we obtain a system of four forward-looking difference equations for aggregate CPI inflation, aggregate output, and the sectoral inflation rates. Therefore, assuming that policy is such that the equilibrium is locally determinate, the solutions for these variables can be recovered uniquely as functions of the sectoral productivity shocks and the monetary policy shock. When labor is mobile, there is no endogenous persistence in the model, regardless of sectoral nominal rigidity and policy.

When labor is free to flow across sectors, marginal costs are equalized except for productivity differentials (it is easy to verify that $z_t^D = -\frac{\omega-1}{\omega}\varphi_t^D$). The same property extends to the relative prices rp_t^1 and rp_t^2, and the sectoral prices p_t^1 and p_t^2. In turn, absence of endogenous persistence in relative prices implies absence of endogenous persistence in sectoral quantities and in aggregates.

An extensive literature has explored the role of factor specificity in enhancing the persistence properties of monetary business cycle models. Huang (2006) illustrates the mechanism in detail with respect to firm-specific factors: Factor specificity leads to relative price movements that strengthen strategic complementarity in firm pricing and thus contribute to the propagation of shocks over time.[52] In our model, labor immobility is an assumption of sector-specificity of labor – rather than firm-level specificity within a given sector – but its role is akin to that in Huang, as labor immobility can generate persistence via cross-sectoral relative price effects. There is one important difference, though. In Huang's model with firm-specific factors and Taylor-type staggered prices, factor specificity is sufficient to generate endogenous persistence. In our model with sector-specific labor and Calvo-Yun-Rotemberg pricing, this is not the case. Combined with the results of Section 4, the results of this section show that labor immobility is *necessary* for endogenous persistence, but it is *not sufficient*. Even when labor is completely immobile, there is no endogenous persistence in aggregate inflation and output if nominal rigidity is identical across sectors and policy is reacting to CPI inflation.[53]

6. Relative Prices and Shock Transmission: Examples

The previous sections studied the determinants of relative price effects in our two-sector model economy analytically. To substantiate intuitions and investigate the importance of relative price effects in shock transmission, this section presents the results of numerical examples computing impulse responses of sectoral and aggregate variables to productivity and interest rate shocks with immobile labor. We focus especially on the extent to which endogenous persistence via terms of trade dynamics results in noticeable endogenous persistence in aggregate output and inflation and whether this persistence can result in hump-shaped responses to shocks.

There is pervasive empirical evidence that the responses of aggregate output and inflation to monetary policy shocks display hump-shaped dynamics. Fuhrer and Moore (1995) argued

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[52] Huang (2006) contains an extensive list of references on the role of factor specificity. Most recently, Altig, Christiano, Eichenbaum, and Linde (2005) find that the assumption of firm-specific capital is important for the persistence properties of their model.

[53] An obvious extension of our analysis, which we leave for future work, would be the modeling of intermediate degrees of labor mobility and the evaluation of the consequences of intermediate mobility for shock propagation and persistence.

that this evidence highlights a crucial shortcoming of the benchmark New Keynesian model described in Clarida, Galí, and Gertler (1999): Since there is no endogenous state variable in the benchmark model, responses to non-permanent, exogenous shocks decay monotonically after the initial jump and are only as persistent as the shocks themselves. Several solutions have been proposed for this problem, starting from the inclusion of backward-looking price setters by Fuhrer and Moore.[54] Our interest in this section is thus also in whether relative price effects can explain hump-shaped responses through the dependence of aggregate output and inflation on the past terms of trade in the fully forward-looking model of this paper. We shall see that the answer is partially positive only when monetary policy shocks are sufficiently persistent – and even in this case the hump-shaped pattern is limited to output: Even if relative price effects do contribute significantly to extra persistence, they cannot explain humps following interest rate shocks for reasonable choices of parameter values if the shocks are transitory.

6.A. Calibration

Clearly, a central issue in evaluating the quantitative relevance of the mechanisms we explored in Section 4 is the calibration of the nominal rigidity parameters λ_1 and λ_2, and the sectoral size parameter b. For illustrative purposes, in Section 4 we used parameter values that are common in the New Keynesian literature. However, according to Bils and Klenow (2004), 90 percent of firms in the U.S. economy have essentially flexible prices within the quarter. Other firms preset prices for roughly five quarters. Continuing to interpret sector 1 in our model as the relatively flexible sector, the Bils-Klenow evidence corresponds to $b = .9$, $\lambda_1 = 10$, and $\lambda_2 = .052$. We choose these values as benchmark calibration in the exercise below. Note that the calibration of the sectoral size and nominal rigidity parameters is quite unfavorable to the nominal rigidity hypothesis, as a very small portion of the economy is subject to any noticeable rigidity beyond the one quarter horizon. However, there is a significant discrepancy across flexible and sticky sectors of the economy, setting the scope for a potentially significant effect of relative prices on dynamics. We keep $\beta = .99$ as in Section 4 and, as in examples in Bils and

[54] Galí (2003) argues that the baseline New Keynesian Phillips curve is a reasonable description of aggregate U.S. and European data. See Fuhrer (2005) for a counterargument centered on the absence of endogenously persistent terms from the baseline Phillips curve. Our model can be seen as providing a microfoundation for such terms under fully forward-looking price setting. Other models of persistence are based on imperfect information, such as Mankiw and Reis (2002) and Nimark (2005).

Klenow, we set the elasticity of substitution across sectors $\omega = 1$. In our benchmark calibration, we assume that the central bank is responding to CPI inflation and set the policy rule parameters to the values in Taylor (1993): $\tau_1 = \tau_2 = 1.5$ and $\tau_C = .5$.

6.B. Impulse Responses

6.B.1. Productivity Shock in the Flexible Sector

Figure 3 presents impulse responses to a one-percent increase in sector 1 productivity with persistence $\rho_1 = .95$. Percent deviations of endogenous variables from the steady state are on the vertical axis, and the number of years after the shock is on the horizontal axis.[55] To evaluate the role of heterogeneity in nominal rigidity, we compare the responses for the benchmark parameterization above (round markers) to those for the case in which $\lambda_1 = \lambda_2 = .052$ (cross markers). This alternative scenario is more favorable to the nominal rigidity hypothesis than the Bils-Klenow (BK) case as all firms in the economy have sticky prices for an average of five quarters.

Heterogeneous nominal rigidity results in amplified, more persistent responses. When sector 1 prices are as sticky as sector 2's, the shock causes sector 1's marginal cost (markup) to decrease (increase), as sector 1 firms do not lower their relative price as much as they would under flexible prices. When sector 1 prices are essentially flexible, the sectoral markup remains constant, and the relative price falls by more. This results in an amplified, more persistent downward movement of the terms of trade – and in a more significant expansion of sector 1 output and aggregate GDP – in the BK case. Sector 2 output is initially boosted by larger aggregate demand, but is then driven below the steady state by the strengthening of sector 2's terms of trade (since T_t falls, the price of sector 2's output relative to sector 1's rises), which shifts demand toward sector 1. Note that sector 2 inflation displays a hump-shaped response, with the peak decrease happening three quarters after the shock. But the small size of sector 2 implies that the response of aggregate inflation shows no hump.

[55] In this figure and the following ones, the first row shows the responses of aggregate inflation and output, the interest rate, and the terms of trade; the second row shows sectoral inflation rates and relative prices; the third row

6.B.2. Productivity Shock in the Sticky Sector

Figure 4 shows the responses to a one-percent productivity increase in sector 2 with persistence $\rho_2 = .95$. As before, round markers denote the BK case and cross markers denote the scenarios with equal, more significant price stickiness. Heterogeneous nominal rigidity results in hump-shaped responses of aggregate inflation and output to a productivity shock in the relatively sticky sector.

When sector 1 prices are flexible (and $\omega = 1$), the relative price of this sector's output increases just enough to shield sector 1's employment and output from a productivity shock in sector 2. Therefore, the dynamics of aggregate output are driven by the response of sector 2 output in the BK scenario with essentially flexible sector 1 prices. Since sector 2 output peaks only about seven quarters after the shock, so does aggregate output. In turn, terms of trade dynamics are responsible for the delayed peak response of sector 2 output.[56] Instead, when nominal rigidity is identical across sectors, the shock causes sector 1's markup to fall slightly on impact, boosting sectoral output temporarily before relative price appreciation shifts demand to sector 2. Consistent with the analytical results above, aggregate output is not affected by relative prices, and its response declines monotonically after the initial expansion.

Interestingly, the hump-shaped decline in aggregate inflation that follows a sector 2 shock with heterogeneous nominal rigidity is driven by an amplified, hump-shaped response of inflation in the flexible sector. A productivity shock in the sticky sector has a very small direct effect on inflation in sector 1, which is thus mainly driven by the terms of trade. Given relative sector size, this generates a hump-shaped path of aggregate inflation. When nominal rigidity is identical across sectors, aggregate inflation is insulated from relative prices, even if the response of sector 1 inflation remains hump-shaped.

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shows sectoral outputs and real wages; and the forth row shows sectoral employments and marginal costs (the negatives of markups).

[56] It is possible to verify that $\partial c_t^1 / \partial \varphi_t^2 = -\omega(1-b)\partial T_t / \partial \varphi_t^2 + \partial c_t / \partial \varphi_t^2 = -\omega(1-b)(\gamma_2 - \gamma_4) + \gamma_6 = 0$ and $\partial c_t^1 / \partial T_{t-1} = -\omega(1-b)\partial T_t / \partial T_{t-1} + \partial c_t / \partial T_{t-1} = -\omega(1-b)\alpha_5 + \alpha_3 = 0$ if $\omega = 1$ and $\lambda_1 \to \infty$. It follows that $c_t = (1-b)c_t^2$ in this case. Hence, given $c_t^2 = bT_t + c_t$ with $\omega = 1$, both sectoral and aggregate output inherit the persistence of the terms of trade under sticky sector 2 prices, since $c_t^2 = T_t$ and $c_t = (1-b)T_t$. The assumption $\omega = 1$ is important here because it ensures that terms of trade changes redistribute demand across sectors in directly proportional fashion ($c_t^1 - c_t^2 = -T_t$). When sector 1 prices are flexible, it is then optimal for sector 1

6.B.3. Monetary Policy Shock

Figure 5 shows the responses to a 1 percent decrease in the interest rate with zero persistence. This is an aggregate demand shock that increases demand for both sectors. A tradeoff emerges: The BK calibration (round markers), with much less overall rigidity but much more heterogeneity, delivers more persistence at the cost of a much smaller aggregate output response than the case of identical rigidity (cross markers). To further illustrate the tradeoff, Figure 5 includes also the responses for an alternative scenario (square markers), more in line with the New Keynesian literature than the BK calibration, in which we assume that prices are essentially flexible (λ_1 = 10) in half of the economy (b = .5) and they are sticky for an average of four quarters (λ_2 = .086) in the other half. We label this case HNK (for "Half" New Keynesian) below.

When nominal rigidity is pervasive, the impact response of aggregate output is significantly amplified. But aggregate output returns to the steady state in the quarter after the shock. In the BK and HNK scenarios, prices adjust more slowly in sector 2 than in sector 1, which results in an increase in the terms of trade. Different degrees of nominal rigidity imply that marginal cost rises (the markup decreases) by more in sector 2 than in sector 1, where nearly flexible prices imply an almost constant markup. Output expands initially in both sectors, owing to larger demand for both. When nominal rigidity is heterogeneous, aggregate inflation falls slightly below the steady state after the impact period. The reason is that terms of trade appreciation reduces sector 1 demand and marginal cost very slightly below the steady state after the initial expansion. As a consequence, sector 1 inflation falls, and so aggregate inflation, which returns to the steady state in approximately six quarters. The effect of terms of trade dynamics prolongs the response of aggregate output by approximately two years in the BK case and somewhat less in the HNK scenario (where the terms of trade are less persistent). Thus, heterogeneity is more important for output persistence in response to monetary policy shocks than the extent to which nominal rigidity is pervasive in the aggregate economy, but much heterogeneity with little aggregate stickiness results in very small real effects of shocks.[57]

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firms and workers to absorb a sector 2 productivity shock completely through price and wage changes, keeping sectoral output and effort unchanged.

[57] In addition, the responses for the BK scenario display counterfactual implications highlighted by Bils, Klenow, and Kryvtsov (2003), who find evidence that the relative price of the flexible sector falls in the first eight months

Figure 6 repeats the exercise of Figure 5 for a shock with persistence .9. The dynamics share many qualitative features with Figure 5 with two main differences: First, aggregate inflation does not fall below the steady state. When the policy shock is persistent, aggregate demand expansion prevails on relative price effects, and sector 1 output and marginal cost do not drop below the steady state after the initial period, preventing sector 1 inflation from falling. Second, and more important, even if the model still generates no hump in aggregate inflation, the response of aggregate output displays a hump in the BK and HNK cases, with the peak happening approximately a year after the initial impulse in the BK calibration. This is a consequence of relative price dynamics, as the slower adjustment of sector 2 prices induces a hump-shaped response of the terms of trade and sector 2's output to the shock. No such hump-shaped response is observed in the standard one-sector model with sticky prices and flexible wages (replicated by the scenario $\lambda_1 = \lambda_2 = .052$), regardless of shock persistence.

6.C. The Role of Policy

The examples above focus on the consequences of heterogeneous nominal rigidity for given monetary policy. Here, we provide examples of the implications of changes in the parameters of the interest setting rule for given heterogeneity in nominal rigidity. We take the BK calibration with Taylor's (1993) rule as benchmark and compare shock transmission in this scenario to the following alternatives: (i) $\tau_1 = \tau_2 = 1.5$, $\tau_C = 0$ (a rule in which the central bank responds only to CPI inflation with coefficient 1.5); (ii) $\tau_1 = 1.5$, $\tau_2 = 5$, $\tau_C = .5$ (an asymmetric Taylor rule with a stronger response to inflation in the relatively sticky sector); and (iii) $\tau_1 = 0$, $\tau_2 \to \infty$ (approximated by $\tau_2 = 1{,}000{,}000$), $\tau_C = 0$ (given near price flexibility in sector 1, this rule approximates the overall flexible-price equilibrium by stabilizing sector 2 inflation at zero – it is the policy that would be recommended as approximately optimal under our assumptions by Aoki, 2001, and Benigno, 2004, if monetary frictions were negligible).[58] In the next three

after a monetary policy shock and increases only in the following months, with little noticeable effect on relative sectoral quantities. This issue is associated also to the identification of shocks in empirical work, and we leave it for exploration in richer, quantitative models.

[58] One can design monetary policies that stabilize sector 2 inflation at zero without assuming an arbitrarily large response to sector 2 inflation, which would quite impractical in a real world in which policy mistakes are possible. Given determinacy, all rules that fully stabilize sector 2 inflation deliver identical dynamics in response to shocks. Since our interest is on shock transmission under the policy that approximates flexible prices, rather than implementation of the latter, we use the interest rate rule in (iii) as a convenient shortcut rather than a policy prescription.

figures, round markers denote the benchmark scenario, cross markers denote alternative (i), square markers denote alternative (ii), and star markers denote alternative (iii). For brevity, we focus our discussion below on the differences in key results that are the focus of our analysis.

Figure 7 presents responses to a one-percent increase in sector 1 productivity. Alternatives (i) and (ii) dampen fluctuations in relative prices and inflation. Importantly, alternative (iii) removes any effect of the shock on sector 2 employment and output. This mirrors a result we obtained above for the case of a sector 2 shock, which had no effect on sector 1 output due to near price flexibility in that sector. In the scenario of Figure 7, policy (iii) mimics perfectly flexible prices in sector 2. Thus, in conjunction with the assumption $\omega = 1$, it implies that sector 2's relative price adjusts so that sectoral employment and output are insulated from a productivity shock in the other sector.

Note that the path of the terms of trade displays only a minimal hump under policy (iii). This is consistent with analytical results above. As we showed, the terms of trade display no endogenous persistence if prices are flexible: They jump with the shock and return to the steady state monotonically in the following periods, reaching the steady state when the shock dies out. A policy that approximates flexible-prices reproduces these dynamics, front-loading the response of the terms of trade and relative prices to shocks.[59] This is especially important in Figure 8, which presents the responses to a one-percent increase in sector 2 productivity. In this case, the policy (iii) that approximates the flexible-price equilibrium removes the hump in aggregate output dynamics that is instead present under all other rules considered. Since policy (iii) front-loads the response of the terms of trade to shocks, it removes the hump in dynamics that was caused by a hump-shaped terms of trade path. The responses of sectoral and aggregate outputs are correspondingly front-loaded.

Figure 9 presents the responses to a one-percent interest rate decrease with zero persistence. As expected, this shock has no noticeable real effect under the policy that approximates flexible prices. Dynamics under alternatives (i) and (ii) – and the extent of persistence in responses – are quite similar to the benchmark. Consistent with the results in

[59] The hump in relative price dynamics does not disappear completely in Figure 7 because sector 1 prices are not *perfectly* flexible.

Section 4, lowering the policy reaction to aggregate output increases output and terms of trade persistence, albeit slightly, while increasing the reaction to sector 2 inflation reduces it.[60]

6.D. Sticky Wages

How would wage stickiness affect results? Appendix C develops a version of the model that builds on Erceg, Henderson, and Levin (2000) and allows for potentially heterogeneous stickiness in nominal wages. Sectoral real wages become additional endogenous state variables in the solution of the model with sticky wages. We focus on the transmission of monetary policy shocks under the benchmark Taylor (1993) rule with $\tau_1 = \tau_2 = 1.5$ and $\tau_C = .5$. Figure 10 presents impulse responses to the zero persistence interest rate shock in the extended model, comparing them to the model with flexible wages. Cross markers denote the flexible-wage model (BK calibration). As for price stickiness, we tilt the benchmark calibration against nominal rigidity in the extension of the BK scenario with sticky prices and wages (round markers). We assume that wages are essentially flexible within the quarter in sector 1, setting the corresponding rigidity parameter to 10, and they are sticky for approximately a year in sector 2, setting the relevant parameter to .086. Finally, square markers denote the sticky-wage-and-price version of the HNK case, in which both prices and wages are sticky for approximately a year in half of the economy (sector 2). As Figure 10 shows, adding wage stickiness does not alter the main qualitative features of most impulse responses (except real wage and marginal cost in the sector with relatively sticky wages). Importantly, wage stickiness contributes to the amplitude of the aggregate output response to the monetary expansion.[61] As without wage stickiness, if the interest rate shock is persistent, there is no hump in inflation, but a hump is observed in the response of aggregate output, which is amplified by wage rigidity.

[60] The hump-shaped aggregate output response to a persistent interest rate shock discussed above is present also under alternatives (i) and (ii). Another policy change that one may consider is the introduction of interest rate smoothing. Reproducing the exercise of Figure 9 with a .75 smoothing coefficient in interest rate setting yields very similar conclusions. Omitted figures are available on request.

[61] Similar conclusions to those of the sticky-price-only model hold if we assume that all wages are sticky for one year: The amplitude of impact responses is substantially increased, but persistence drops.

7. Conclusions

We developed a New Keynesian, two-sector model with heterogeneous nominal rigidity and immobile labor that restores a role for relative prices in the determination of aggregate output and inflation. Our interest was in exploring the determinants of these relative price effects analytically – to shed light on mechanisms that are at work in richer, quantitative models such as that of Bouakez, Cardia, and Ruge-Murcia (2005) – and to explore the extent to which relative price dynamics contribute to observed persistence in aggregate output and inflation dynamics. The model generates humps in the responses of aggregate output and inflation to productivity shocks in the relatively sticky sector of the economy. However, it does not result in such humps following monetary policy shocks unless these are persistent. Other sources – such as those explored in Christiano, Eichenbaum, and Evans (2005) – must be responsible for delayed peak responses of aggregates to transitory monetary policy shocks.

Notwithstanding the mixed conclusions on the humps front, we find that relative price effects implied by empirically plausible heterogeneity in nominal rigidity do contribute significantly to persistence, and that heterogeneity in nominal rigidity (combined with labor immobility) is more important for persistence than the overall degree of stickiness in the economy. Thus, the results of our exercise support the development of multi-sector monetary models as a promising avenue for future research and suggest that empirical estimation of Phillips curve equations should take into account the potential presence of relative price terms.

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Appendix A. The Undetermined Coefficients Solution

Equations (32) and (33) and the definitions of sectoral inflation rates imply:

$$p_t^1 = p_{t-1}^1 + \alpha_1 T_{t-1} + \gamma_1 \varphi_t^1 + \gamma_2 \varphi_t^2 + \gamma_{r,1}\varphi_t^r, \tag{A.1}$$

$$p_t^2 = p_{t-1}^2 + \alpha_2 T_{t-1} + \gamma_3 \varphi_t^1 + \gamma_4 \varphi_t^2 + \gamma_{r,2}\varphi_t^r, \tag{A.2}$$

Subtracting (A.2) from (A.1) yields the solution for the terms of trade, equation (36).

Equations (32) and (33) and the definition of CPI inflation yield the solution for the latter, equation (35). It follows that the aggregate price level obeys:

$$\begin{aligned} p_t = p_{t-1} &+ [b\alpha_1 + (1-b)\alpha_2]T_{t-1} + [b\gamma_1 + (1-b)\gamma_3]\varphi_t^1 \\ &+ [b\gamma_2 + (1-b)\gamma_4]\varphi_t^2 + [b\gamma_{r,1} + (1-b)\gamma_{r,2}]\varphi_t^r \end{aligned}, \tag{A.3}$$

(Equations (A.1)-(A.3) show that price levels are characterized by unit roots regardless of λ_1 versus λ_2 and τ_1 versus τ_2 in our model.)

Equations (28), (29), and (36) yield the solutions for relative prices:

$$rp_t^1 = (1-b)\left[(1+\alpha_1-\alpha_2)T_{t-1} + (\gamma_1-\gamma_3)\varphi_t^1 + (\gamma_2-\gamma_4)\varphi_t^2 + (\gamma_{r,1}-\gamma_{r,2})\varphi_t^r\right], \tag{A.4}$$

$$rp_t^2 = -b\left[(1+\alpha_1-\alpha_2)T_{t-1} + (\gamma_1-\gamma_3)\varphi_t^1 + (\gamma_2-\gamma_4)\varphi_t^2 + (\gamma_{r,1}-\gamma_{r,2})\varphi_t^r\right]. \tag{A.5}$$

Substituting the solutions for sectoral inflation rates, relative prices, and consumption into equations (30) and (31), and equating coefficients on T_{t-1} in each of the resulting equations yields the following system:

$$\alpha_1 = \lambda_1[2\alpha_3 - (1+\omega)(1-b)(1+\alpha_1-\alpha_2)] + \beta\alpha_1(1+\alpha_1-\alpha_2), \tag{A.6}$$

$$\alpha_2 = \lambda_2[2\alpha_3 + (1+\omega)b(1+\alpha_1-\alpha_2)] + \beta\alpha_2(1+\alpha_1-\alpha_2), \tag{A.7}$$

$$\alpha_3(1+\alpha_1-\alpha_2) - \alpha_3 = \tau_1 b\alpha_1 + \tau_2(1-b)\alpha_2 + \tau_C\alpha_3 - [b\alpha_1 + (1-b)\alpha_2](1+\alpha_1-\alpha_2). \tag{A.8}$$

Equations (A.6)-(A.8) can be solved for the elasticities α_i, i = 1, 2, 3.

Equating coefficients on φ_t^1 yields:

$$\gamma_1 = \lambda_1[2\gamma_5 - (1+\omega)(1-b)(\gamma_1-\gamma_3) - 2] + \beta[\gamma_1\rho_1 + \alpha_1(\gamma_1-\gamma_3)], \tag{A.9}$$

$$\gamma_3 = \lambda_2[2\gamma_5 + (1+\omega)b(\gamma_1-\gamma_3)] + \beta[\gamma_3\rho_1 + \alpha_2(\gamma_1-\gamma_3)], \tag{A.10}$$

$$\begin{aligned} \gamma_5\rho_1 - \gamma_5 + \alpha_3(\gamma_1-\gamma_3) &= \tau_1 b\gamma_1 + \tau_2(1-b)\gamma_3 + \tau_C\gamma_5 \\ &- \{[b\gamma_1 + (1-b)\gamma_3]\rho_1 + [b\alpha_1 + (1-b)\alpha_2](\gamma_1-\gamma_3)\}. \end{aligned} \tag{A.11}$$

Given solutions for α_i, i = 1, 2, 3, this system can be solved for the elasticities γ_h, h = 1, 3, 5.

Similarly, equating coefficients on φ_t^2 yields a system that can be solved for γ_h, h = 2, 4, 6, given solutions for α_i, i = 1, 2, 3:

$$\gamma_2 = \lambda_1\left[2\gamma_6 - (1+\omega)(1-b)(\gamma_2 - \gamma_4)\right] + \beta\left[\gamma_2\rho_2 + \alpha_1(\gamma_2 - \gamma_4)\right], \tag{A.12}$$

$$\gamma_4 = \lambda_2\left[2\gamma_6 + (1+\omega)b(\gamma_2 - \gamma_4) - 2\right] + \beta\left[\gamma_4\rho_2 + \alpha_2(\gamma_2 - \gamma_4)\right], \tag{A.13}$$

$$\begin{aligned}&\gamma_6\rho_2 - \gamma_6 + \alpha_3(\gamma_2 - \gamma_4) = \tau_1 b\gamma_2 + \tau_2(1-b)\gamma_4 + \tau_C\gamma_6 \\ &-\left\{\left[b\gamma_2 + (1-b)\gamma_4\right]\rho_2 + \left[b\alpha_1 + (1-b)\alpha_2\right](\gamma_2 - \gamma_4)\right\}.\end{aligned} \tag{A.14}$$

Finally, equating coefficients on φ_t^r yields a system for $\gamma_{r,h}$, h = 1, 2, 3, given the solutions for α_i, i = 1, 2, 3:

$$\gamma_{r,1} = \lambda_1\left[2\gamma_{r,3} - (1+\omega)(1-b)(\gamma_{r,1} - \gamma_{r,2})\right] + \beta\left[\gamma_{r,1}\rho_r + \alpha_1(\gamma_{r,1} - \gamma_{r,2})\right], \tag{A.15}$$

$$\gamma_{r,2} = \lambda_2\left[2\gamma_{r,3} + (1+\omega)b(\gamma_{r,1} - \gamma_{r,2})\right] + \beta\left[\gamma_{r,2}\rho_r + \alpha_2(\gamma_{r,1} - \gamma_{r,2})\right], \tag{A.16}$$

$$\begin{aligned}&\gamma_{r,3}\rho_r - \gamma_{r,3} + \alpha_3(\gamma_{r,1} - \gamma_{r,2}) = 1 + \tau_1 b\gamma_{r,1} + \tau_2(1-b)\gamma_{r,2} + \tau_C\gamma_{r,3} \\ &-\left\{\left[b\gamma_{r,1} + (1-b)\gamma_{r,2}\right]\rho_r + \left[b\alpha_1 + (1-b)\alpha_2\right](\gamma_{r,1} - \gamma_{r,2})\right\}.\end{aligned} \tag{A.17}$$

Appendix B. Solving for Terms of Trade Persistence with Heterogeneous Nominal Rigidity

B.1. Reaction to CPI Inflation

It is possible to verify that equations (44) and (45) for the elasticities of sectoral inflation rates π_t^1 and π_t^2 to the past terms of trade change to:

$$\alpha_1 = \frac{\lambda_1\left[2\alpha_3 - (1+\omega)(1-b)\alpha_5\right]}{1-\beta\alpha_5}, \tag{B.1}$$

$$\alpha_2 = \frac{\lambda_2\left[2\alpha_3 + (1+\omega)b\alpha_5\right]}{1-\beta\alpha_5}, \tag{B.2}$$

when nominal rigidity differs across sectors. Hence, the elasticity of aggregate inflation to the past terms of trade is now given by:

$$\alpha_4 \equiv b\alpha_1 + (1-b)\alpha_2 = \frac{2\alpha_3\left[b\lambda_1 + (1-b)\lambda_2\right] - (\lambda_1 - \lambda_2)b(1-b)(1+\omega)\alpha_5}{1-\beta\alpha_5}. \tag{B.3}$$

Substituting (B.1)-(B.3) into (A.8) and rearranging yields the following expression for α_3 as a function of α_5 and structural parameters:

$$\alpha_3 = \frac{b(1-b)(1+\omega)(\lambda_1 - \lambda_2)(\tau - \alpha_5)\alpha_5}{(1-\alpha_5+\tau_C)(1-\beta\alpha_5) + 2[b\lambda_1 + (1-b)\lambda_2](\tau - \alpha_5)}. \tag{B.4}$$

Now, subtracting (B.2) from (B.1), adding 1 to both sides of the resulting equation, and recalling the definition of α_5 yields a second equation that can be solved for α_3 as a function of α_5 and structural parameters:

$$\alpha_3 = \frac{(1+\omega)[(1-b)\lambda_1 + b\lambda_2]\alpha_5 - (1-\alpha_5)(1-\beta\alpha_5)}{2(\lambda_1 - \lambda_2)}. \tag{B.5}$$

Finally, equating (B.4) and (B.5) and rearranging returns equation (53).

B.2. Different Sectoral Inflation Reactions

Substituting (B.1)-(B.3) into (A.8) with $\tau_1 \neq \tau_2$ and rearranging yields the analog to (B.4):

$$\alpha_3 = \frac{(1+\omega)b(1-b)\left[\lambda_1(\tau_1 - \alpha_5) - \lambda_2(\tau_2 - \alpha_5)\right]\alpha_5}{(1-\alpha_5+\tau_C)(1-\beta\alpha_5) + 2\left[b\lambda_1(\tau_1-\alpha_5) + (1-b)\lambda_2(\tau_2-\alpha_5)\right]}. \tag{B.6}$$

Equating the expressions for α_3 (the elasticity of output to the past terms of trade) in (B.5) and (B.6) and rearranging yields equation (55).

Appendix C. The Sticky Wage Model

In this appendix we describe an extension of the model that incorporates potentially heterogeneous nominal wage rigidity. Building on Erceg, Henderson, and Levin (2000), we assume that there is a continuum of households that supply two types of labor. Different from the main text, household members now have market power over their labor supply. Household members set nominal wages subject to Calvo-Yun-Rotemberg nominal rigidity. To avoid the heterogeneity in household consumption that this may induce, we assume that there is a complete array of state-contingent bonds for consumption insurance across households, but not for leisure insurance.

The period utility function of household h is

$$U^h(C_t(h), M_{t+1}(h)/P_t, L_t^1(h), L_t^2(h)) = \log C_t(h) + V(M_{t+1}(h)/P_t) - (L_t^1(h))^2/2 - (L_t^2(h))^2/2, \tag{C.1}$$

where $C_t(h)$ is the consumption basket of household *h*, described by equations (3) and (4), $M_{t+1}(h)/P_t$ denotes the real money balances held by the household, and $L_t^j(h)$ is labor supplied by household *h* in sector *j*.

We assume that there is an employment agency that buys labor of type *j* = 1, 2 from all households to produce a labor index L_t^j, sold then to producers in sector *j* in a competitive market. Firms in sector *j* have the production function (13), where L_t^j is now the labor index produced by the employment agency for sector *j* (a constant-elasticity aggregator of labor services supplied by members of the households in sector *j*):

$$L_t^j = \left[\int_0^1 \left(L_t^j(h)\right)^{\frac{\theta_w - 1}{\theta_w}} dh\right]^{\frac{\theta_w}{\theta_w - 1}}, \quad j = 1, 2, \tag{C.2}$$

The employment agency's demand for labor of type *j* from household *h* is:

$$L_t^j(h) = \left(\frac{W_t^j(h)}{W_t^j}\right)^{-\theta_w} L_t^j, \quad \text{where} \quad W_t^j = \left[\int_0^1 \left(W_t^j(h)\right)^{1-\theta_w} dh\right]^{\frac{1}{1-\theta_w}}, \quad j = 1, 2. \tag{C.3}$$

Households choose $C_t(h)$, $M_{t+1}(h)/P_t$, and bonds in every period, and they choose $W_t^1(h)$ and $W_t^2(h)$ subject to Calvo-Yun-Rotemberg rigidity, which we allow to differ across sectors 1 and 2, to maximize

$$E_0\left[\sum_{t=0}^{\infty} \beta^t U^h(C_t(h), M_{t+1}(h)/P_t, L_t^1(h), L_t^2(h))\right], \tag{C.4}$$

subject to the budget constraint and the labor demand equations in (C.3).[62]

The first-order conditions include the bond-pricing equation (9), where R_t is the risk-free nominal interest rate, and the money demand equation (10).[63] The first-order conditions (11) and (12) no longer hold. It is useful to define the marginal rates of substitution:

[62] As in Erceg, Henderson, and Levin (2000), agents have access to a complete set of state-contingent bonds, ensuring equalization of equilibrium labor effort across workers in each sector. We omit the modified budget constraint for this case.

$$MRS_t^j(h) \equiv -\frac{U_{L^j}^h(t)}{U_C^h(t)}, \quad j = 1, 2. \tag{C.5}$$

One can then show that the log-linearized first-order conditions with respect to the wage rates $W_t^j(h)$ yield the sectoral wage inflation equations:

$$\pi_t^{W^j} = \beta E_t \pi_{t+1}^{W^j} + \lambda_j^W \left[mrs_t^j - w_t^j \right], \quad j = 1, 2, \tag{C.6}$$

where $\pi_t^{W^j}$ is the percent deviation of gross nominal wage inflation in sector *j* from the steady state, w_t^j is the percent deviation of the real wage in sector *j*, $\lambda_j^W > 0$ measures the degree of wage rigidity in sector *j*, and

$$mrs_t^j = c_t + l_t^j \tag{C.7}$$

is the average marginal rate of substitution across households.

From the definition of real wages:

$$w_t^j = w_{t-1}^j + \pi_t^{W^j} - \pi_t. \tag{C.8}$$

This equation implies that sectoral real wages are additional endogenous state variables in the model with nominal wage rigidity – similarly to the real wage being an endogenous state variable in Erceg, Henderson, and Levin (2000).

Log-linearizing (C.2) and imposing equilibrium, we have:

$$c_t^j = l_t^j + \varphi_t^j \ . \tag{C.9}$$

The equilibrium of the economy can be described by the same system of equations as in the main text, but replacing the equations that determine w_t^j (equations (15)) with (C.8), and including (C.6), (C.7), and (C.9) to obtain solutions for $\pi_t^{W^j}$, l_t^j and mrs_t^j.

[63] Note that we do not need to rewrite the condition (10) in terms of household-specific consumption because access to complete state-contingent claims to consumption implies $C_t(h) = C_t$.

Figure 1. Aggregate Output Elasticity to the Terms of Trade as a Function of Policy

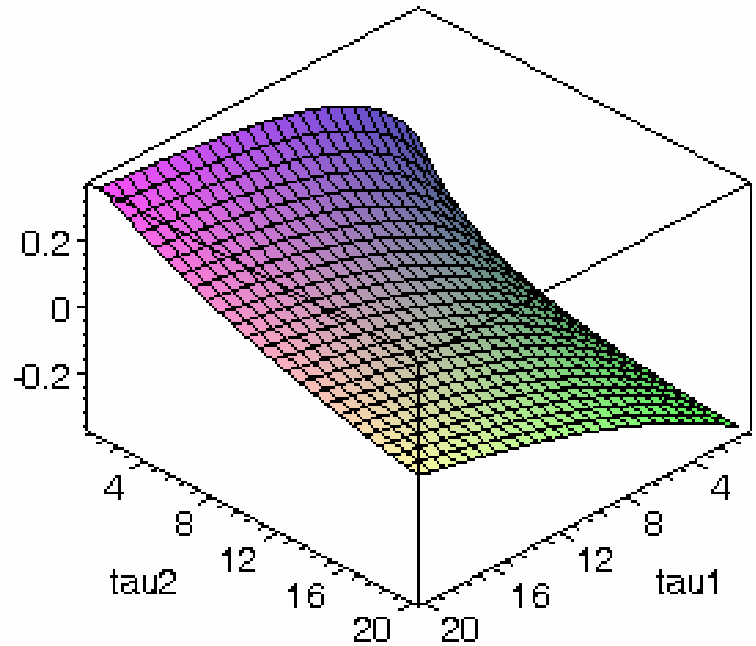
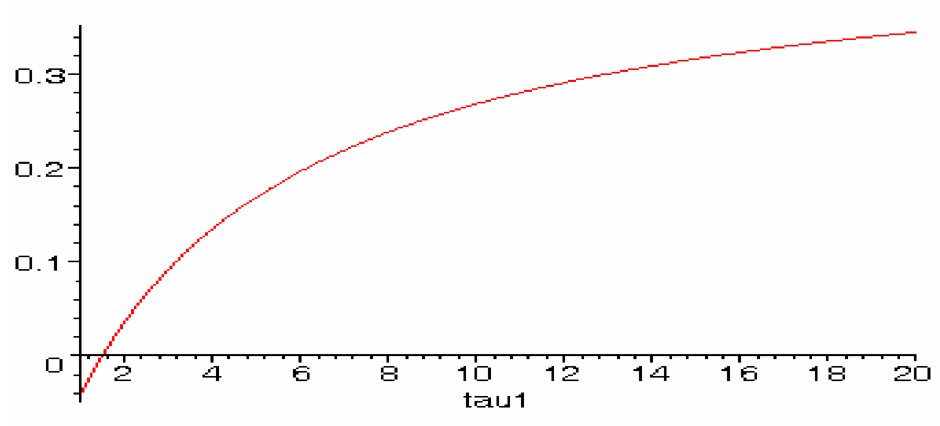
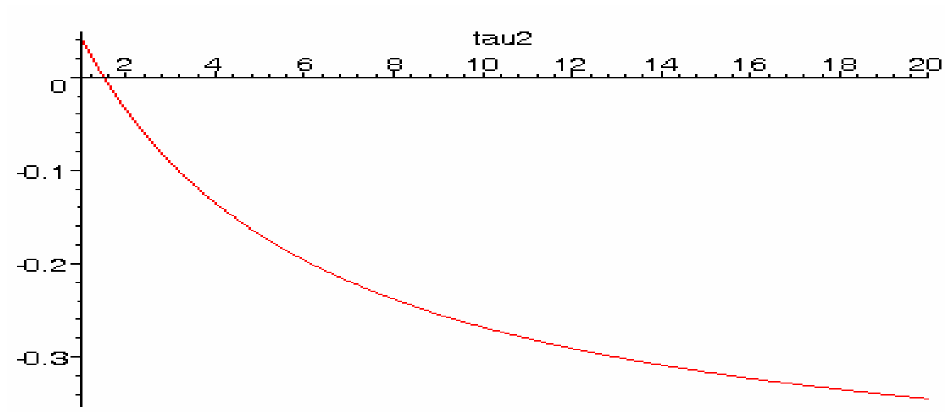


Figure 2. Aggregate Output Elasticity to the Terms of Trade as a Function of Policy, Slices

2.a.



2.b.



2.c.

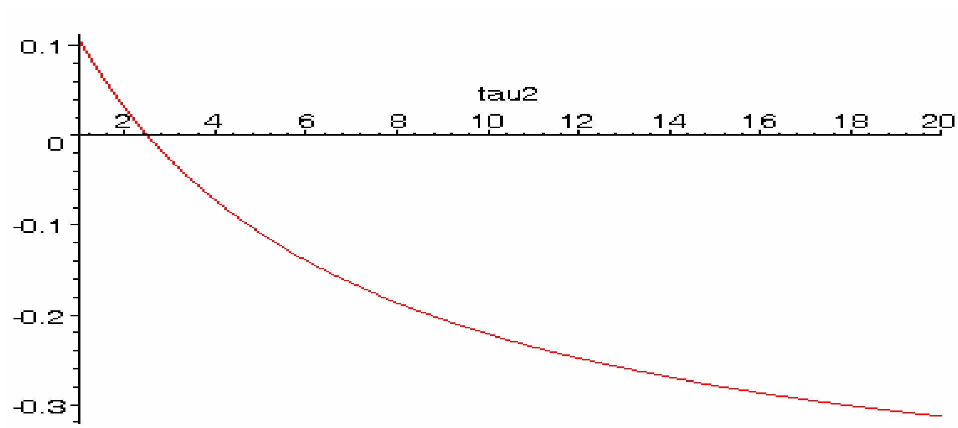


Figure 3. Impulse Responses, Productivity Shock in the Flexible Sector

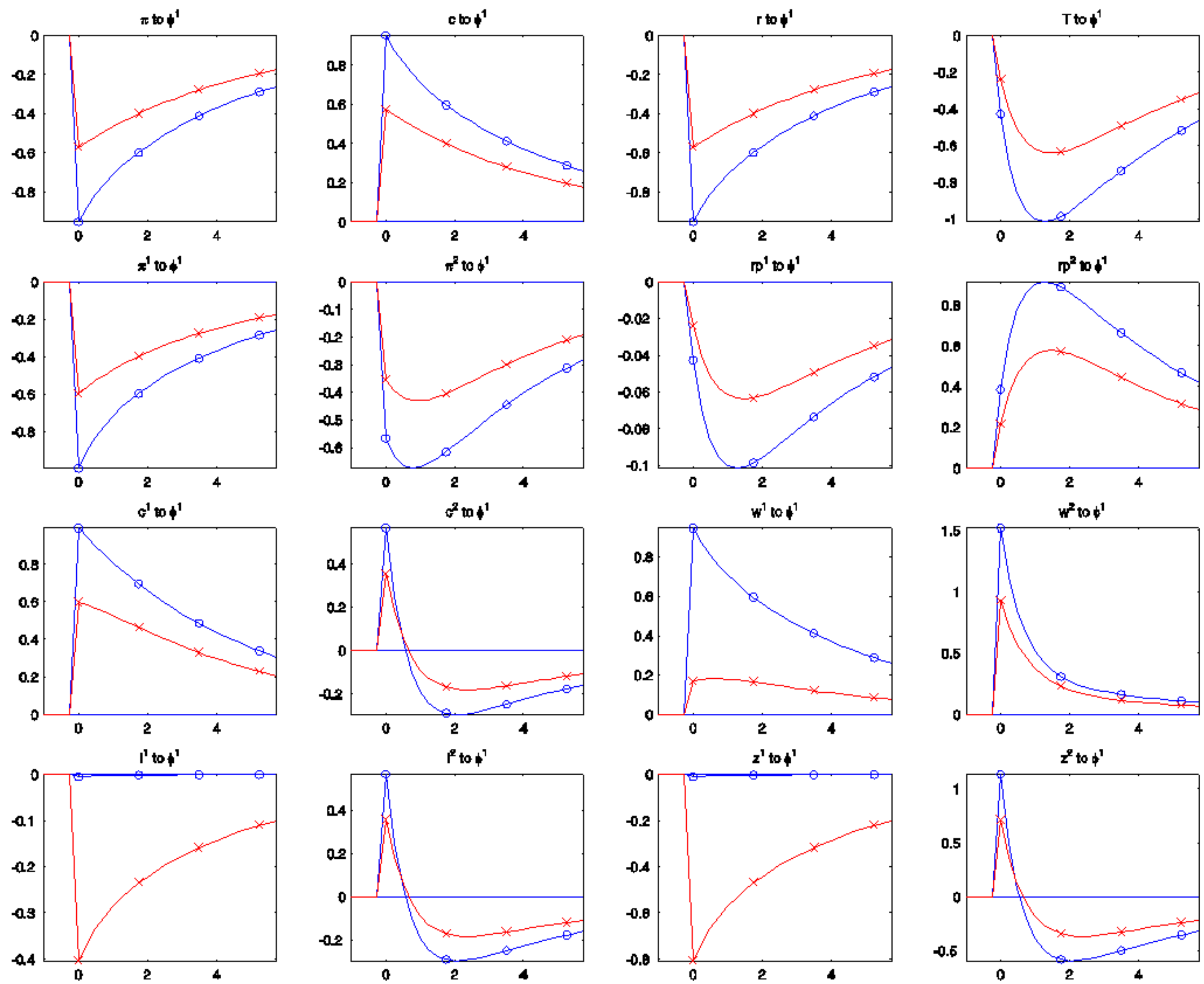


Figure 4. Impulse Responses, Productivity Shock in the Sticky Sector

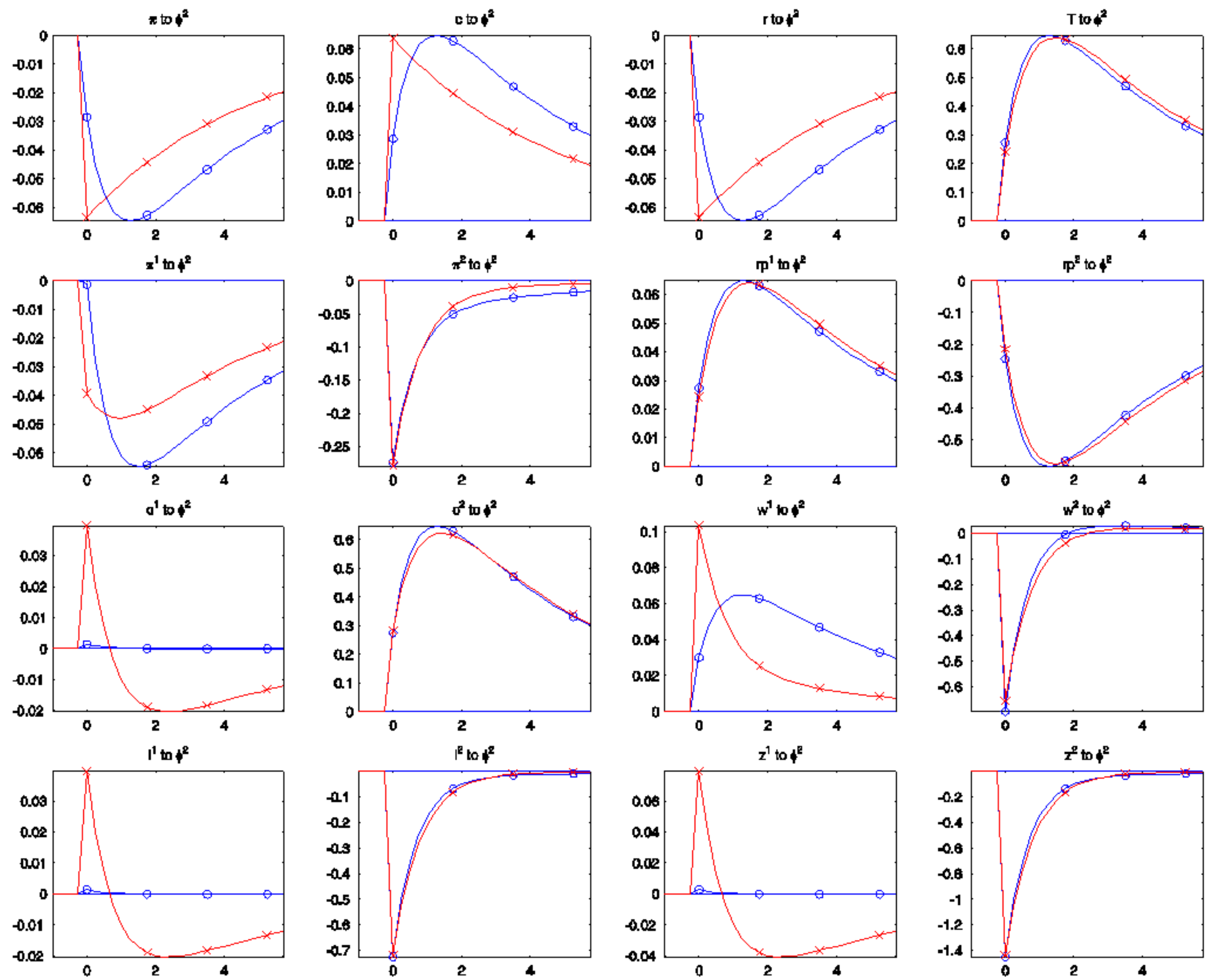


Figure 5. Impulse Responses, Zero-Persistence Interest Rate Shock

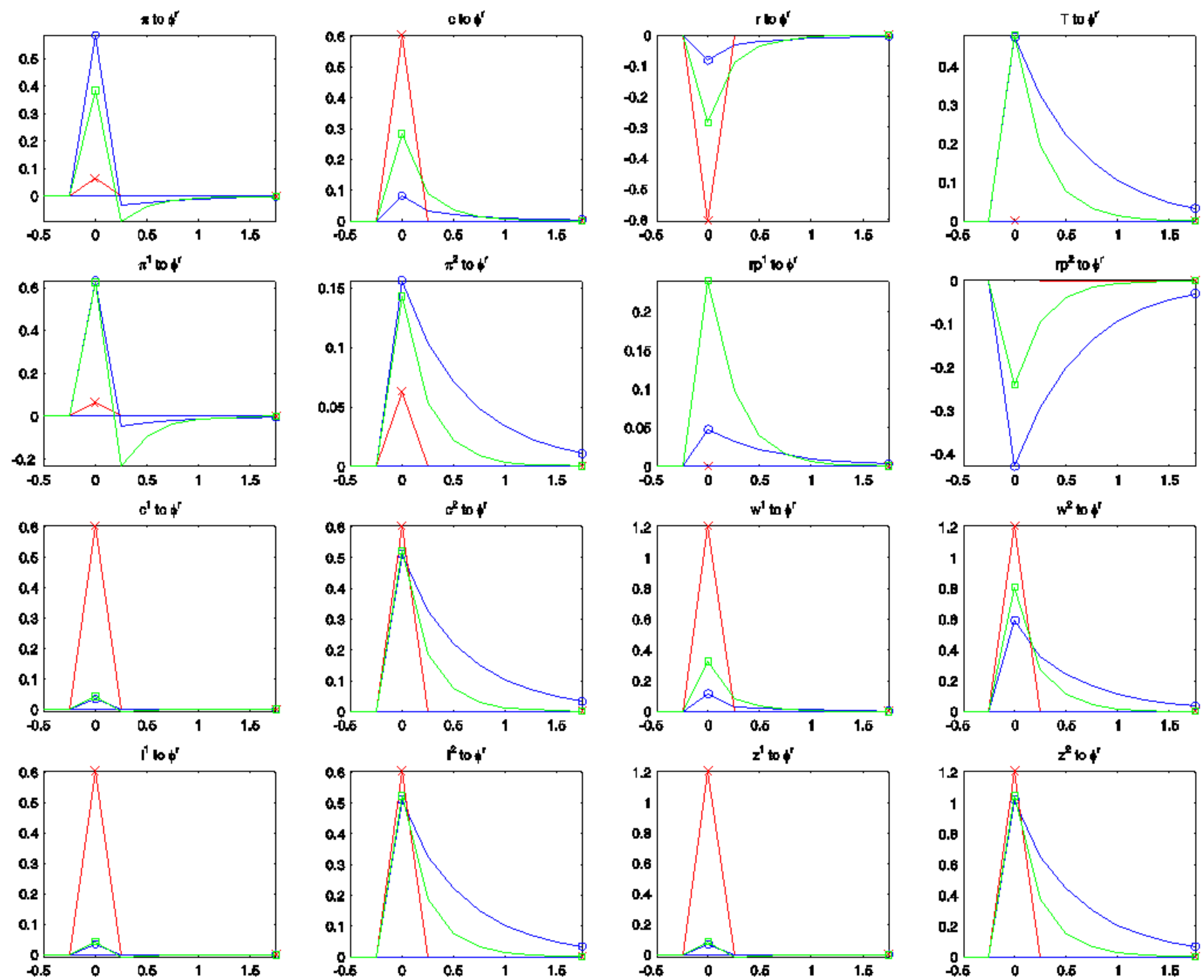


Figure 6. Impulse Responses, Persistent Interest Rate Shock

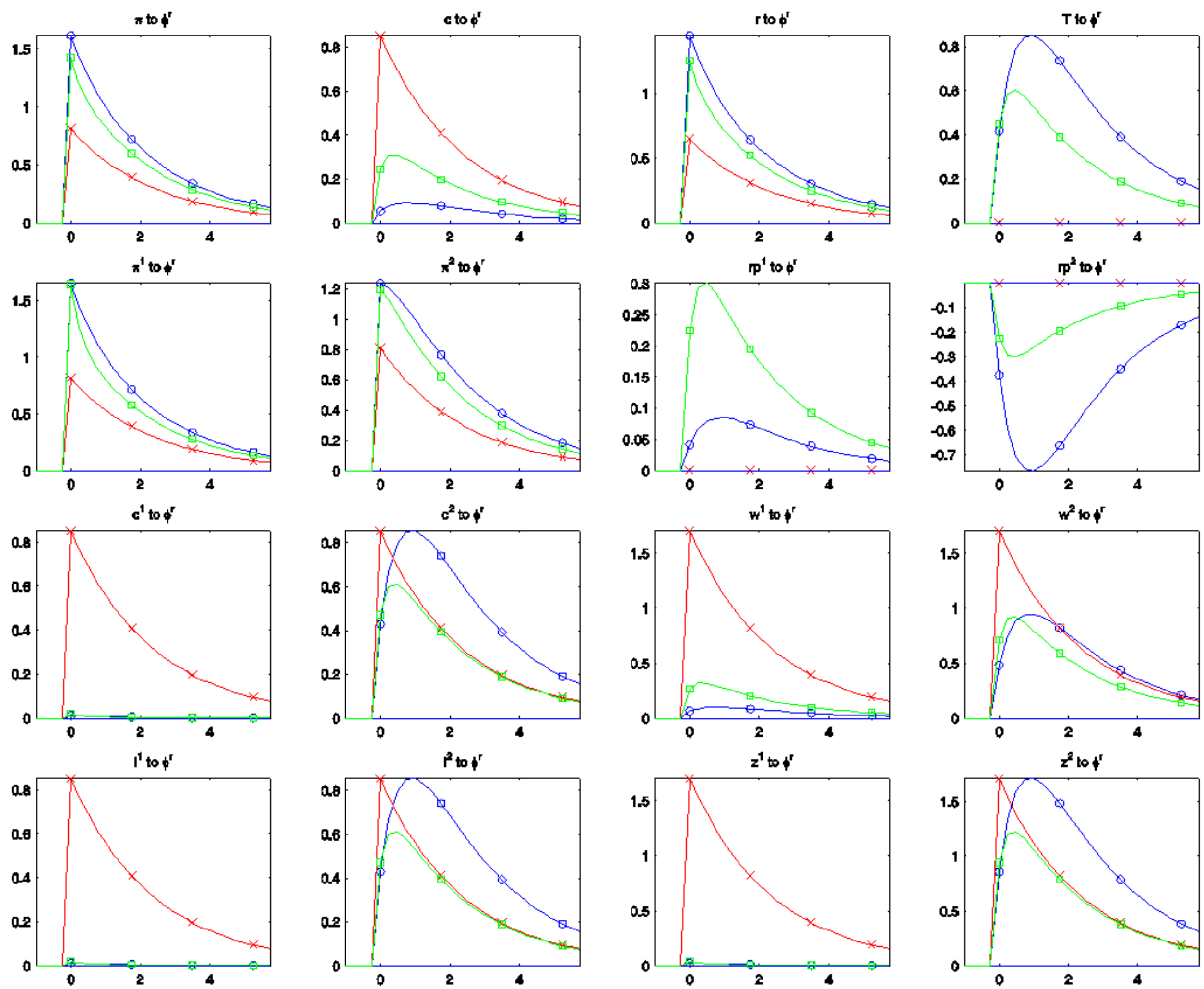


Figure 7. The Role of Policy: Impulse Responses, Productivity Shock in the Flexible Sector

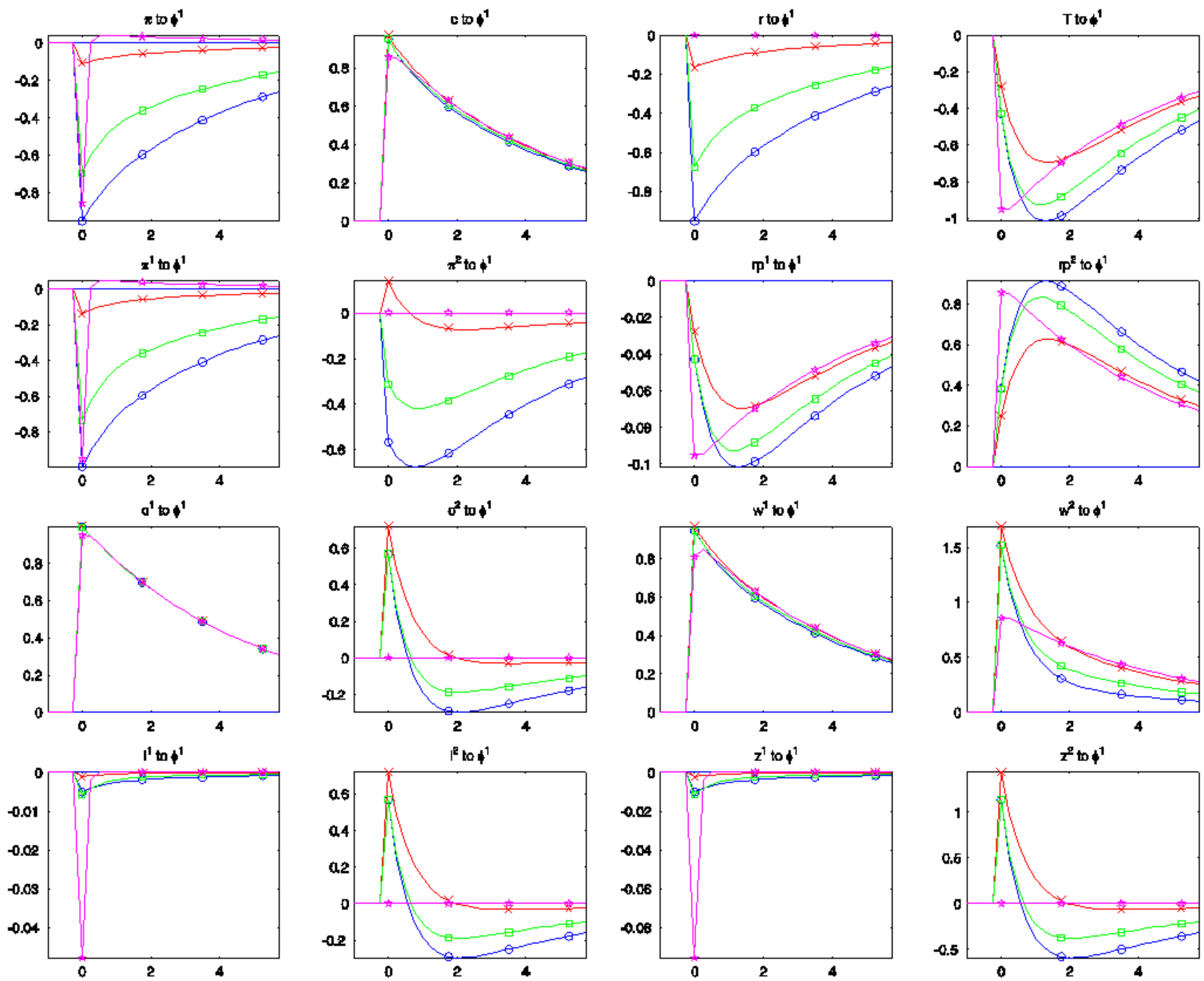


Figure 8. The Role of Policy: Impulse Responses, Productivity Shock in the Sticky Sector

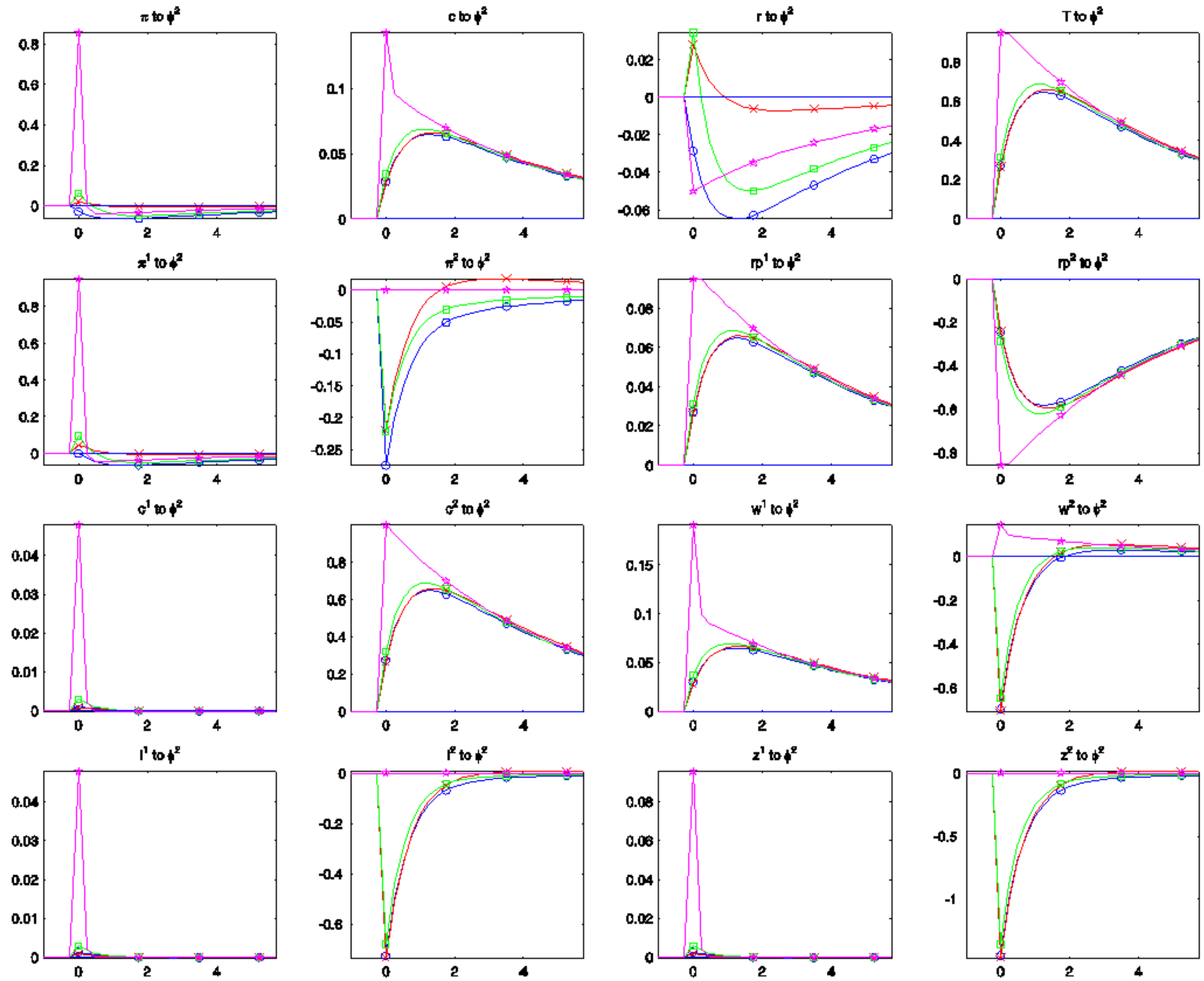


Figure 9. The Role of Policy: Impulse Responses, Zero-Persistence Interest Rate Shock

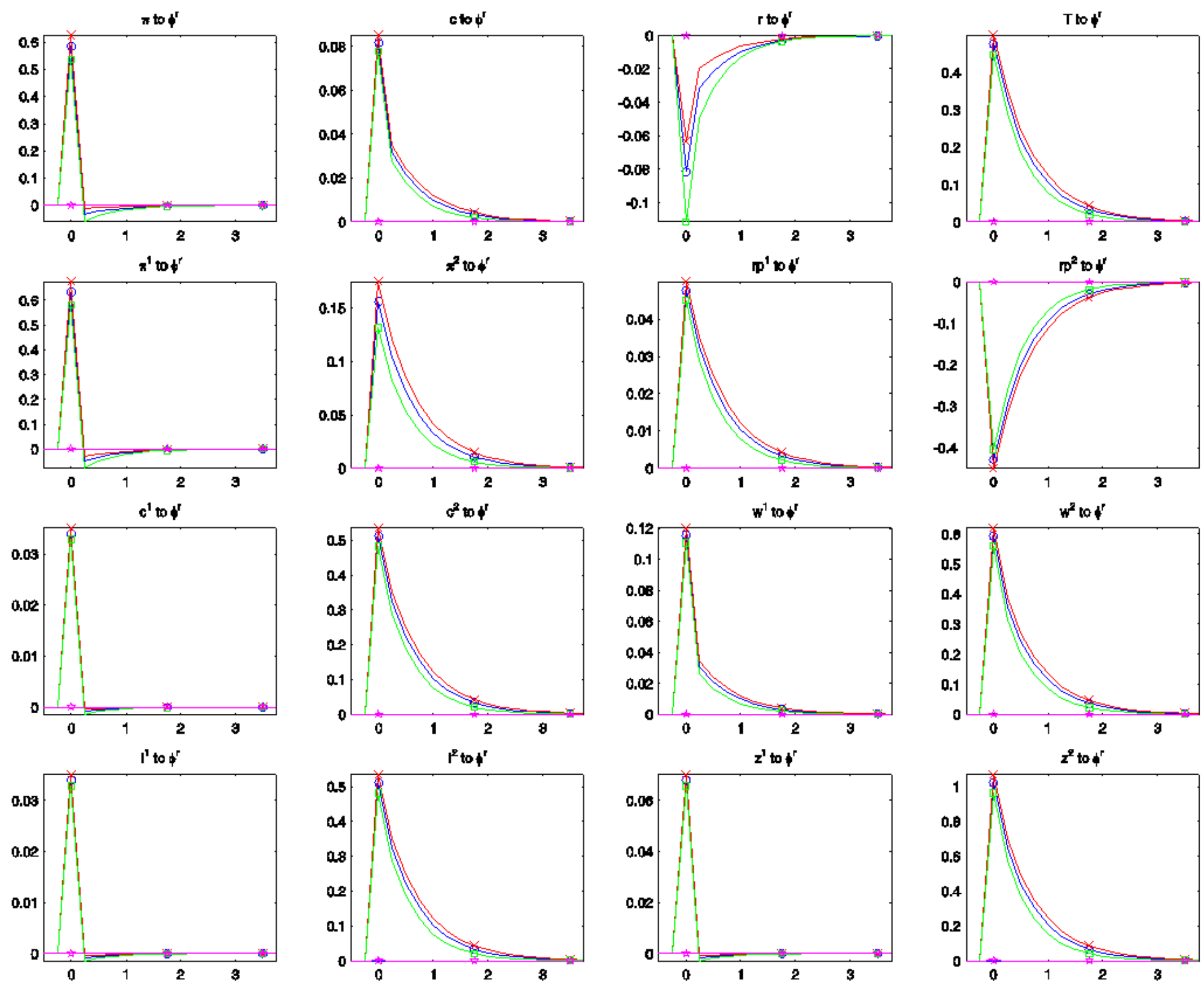


Figure 10. Sticky Prices and Wages: Impulse Responses, Zero-Persistence Interest Rate Shock