Appendix to: Optimal Monetary Policy with Endogenous Entry and Product Variety

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Abstract

Proof that the reduced equilibrium conditions in Table 3 are equivalent to the equilibrium conditions in Table 1

The equilibrium conditions outlined in Table 1 define the contraints of the Ramsey problem. The problem can be simplified by using the more compact but equivalent representation of the constraints outlined in Table 3, where we eliminate all the intratemporal optimality conditions. This Appendix describes the steps necessary to obtain that equivalent representation.

We start from the equilibrium conditions outlined in Table 1 (we refer to each equation using its label in that Table). In deriving the reduced model we also use the "production function" for consumption:

$$C_t = \left(1 - \frac{\kappa}{2}\pi_t^2\right) Z_t \rho\left(N_t\right) L_{C,t} = \left(1 - \frac{\kappa}{2}\pi_t^2\right) Y_t^C$$

which can be obtained from the aggregate accounting equation using the other equilibrium conditions. We substitute out all the intratemporal equilibrium conditions as follows. First, rearrange the markup equation (Phillips curve), combined with the pricing condition and the production function for consumption to get

$$\begin{aligned} & \left[\theta\left(N_t\right) - 1\right] + \kappa \left\{ \frac{\left(1+\pi_t\right)\pi_t}{\left(1 - \frac{\kappa}{2}\pi_t^2\right)} - \beta\left(1-\delta\right) E_t \left[\frac{N_t}{N_{t+1}} \frac{\left(1+\pi_{t+1}\right)\pi_{t+1}}{\left(1 - \frac{\kappa}{2}\pi_{t+1}^2\right)} \right] \right\} \\ = \quad & \theta\left(N_t\right) \frac{w_t}{\rho_t Z_t \left(1 - \frac{\kappa}{2}\pi_t^2\right)} = \theta\left(N_t\right) h_L\left(L_t\right) L_{C,t} \end{aligned}$$

where the second equality uses again the production function for consumption and the intratemporal optimality condition.

The number of entrants can be expressed, using the aggregate accounting equation and the definition of profits, as

$$N_{E,t}v_t = w_t L_t + \left(1 - \frac{1}{\mu_t} - \frac{\kappa}{2}\pi_t^2\right) Y_t^C - C_t$$

Replacing the production function for consumption, the free entry condition and the pricing contidion we obtain

$$N_{E,t} = \frac{Z_t}{f_E} L_t - \frac{1}{f_E}\frac{1}{\rho_t} Y_t^C = \frac{Z_t\left(L_t - L_{C,t}\right)}{f_E},$$

where the last equality used the production function for the consumption good. Replacing this in the state equation for the number of firms we obtain the second equation in Table 3.

Finally, replacing the free entry condition, the intratemporal optimality condition, the definition of profits and the production function for consumption into the Euler equation for shares we have

$$h_L\left(L_t\right)\frac{f_E}{Z_t} = \beta\left(1-\delta\right)E_t\left[h_L\left(L_{t+1}\right)\frac{f_E}{Z_{t+1}} + \left(1 - \frac{1}{\mu_{t+1}\left(1 - \frac{\kappa}{2}\pi_{t+1}^2\right)}\right)\frac{1}{N_{t+1}}\right]$$

Using the free entry condition again to substitute for the markup and the production function for the consumption good, respectively, we have:

$$\begin{aligned} h_L\left(L_t\right)\frac{f_E}{Z_t} &= \beta\left(1-\delta\right)E_t\left[h_L\left(L_{t+1}\right)\frac{f_E}{Z_{t+1}} + \left(1 - \frac{w_{t+1}}{Z_{t+1}\rho\left(N_{t+1}\right)\left(1 - \frac{\kappa}{2}\pi_{t+1}^2\right)}\right)\frac{1}{N_{t+1}}\right] \\ &= \beta\left(1-\delta\right)E_t\left[h_L\left(L_{t+1}\right)\frac{f_E}{Z_{t+1}} + \left(1 - \frac{w_{t+1}L_{C,t+1}}{C_{t+1}}\right)\frac{1}{N_{t+1}}\right] \end{aligned}$$

Finally, using the intratemporal optimality condition we obtain the third equation in Table 3.