Optimal monetary policy with endogenous entry and product variety ☆

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ABSTRACT

Deviations from long-run price stability are optimal in the presence of endogenous entry and product variety in a sticky-price model in which price stability would be optimal otherwise Long-run inflation (deflation) is optimal when the benefit of variety to consumers falls short of (exceeds) the market incentive for creating that variety—the desired markup; Price indexation exacerbates this mechanism. Plausible preference specifications and parameter values justify positive long-run inflation rates. However, short-run price stability (around this non-zero trend) is close to optimal, even in the presence of endogenously time-varying desired markups that distort the intertemporal allocation of resources.

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1. Introduction

A recent, fast growing literature argues that changes in the range of available product varieties contribute significantly to economic dynamics and movements in prices over time spans usually associated with the length of business cycles (Bilbiie et al., 2012; Broda and Weinstein, 2010, and references therein). This paper investigates whether endogenous entry and product variety generate optimal deviations from price stability in a dynamic, stochastic, general equilibrium model with imperfect price adjustment. We study Ramsey-optimal monetary policy in a second-best environment in which lump-sum taxes are not available and inflation is the only instrument of policy. Therefore, our paper contributes to a large literature that seeks to describe optimal monetary policy in fully articulated, general-equilibrium models with nominal and real rigidities, using the tools of modern public finance (e.g. Khan et al., 2003; Adão et al., 2003).[1]

Producer entry in our model takes place that subjects to sunk costs in the expectation of future monopoly profits. On the consumer side, entry is motivated by (general homothetic) preferences that exhibit a taste for variety. Price adjustment is costly, as producers incur a quadratic adjustment cost to change their prices (Rotemberg, 1982). This generates a Phillips curve that relates the markup to producer price inflation. The central bank may try to use inflation to influence markups,

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[1] Lucas and Stokey (1983) started off the literature on Ramsey-optimal fiscal and monetary policy. Chari et al. (1991) study optimal fiscal and monetary policy under flexible prices and extend the model to include capital. Other early applications to sticky price models include King and Wolman (1999), Schmitt-Grohé and Uribe (2004), and Siu (2004).

with the goal of closing the inefficiency wedge between the marginal rate of consumption–leisure substitution and the marginal product of labor. Furthermore, when the benefit of variety to consumers and the market incentive for product creation (the markup) are not aligned, an additional distortion occurs: if the former exceeds the latter there is too little entry, and vice versa (Bilbiie et al., 2008a). The central bank will use inflation to align markups (which govern entry incentives) with the benefit of variety. When preferences are such that the elasticity of substitution between varieties depends on their number, time-variation in desired markup introduces a dynamic dimension to the distortions in labor supply and product creation by generating a misallocation of resources across time and states of nature (Bilbiie et al., 2008a). The central bank can thus in principle use inflation to smooth the intertemporal path of the markup.[2] The objective of this paper is to study how these distortions and possible objectives for the central bank shape the optimal conduct of monetary policy.

Our results are twofold, pertaining to long-run and short-run inflation. Significant deviations from long-run price stability can be optimal, and their sign and magnitude depend on the balance of market incentives for entry and welfare benefit of variety. When the flexible-price market outcome results in too much entry (the net markup is higher than the benefit of variety), the central bank uses its leverage over real activity: the optimal path of producer price inflation has a positive steady-state level, which erodes the markup and precludes suboptimal entry. Long-run deflation occurs when the market provides too little entry, for deflation boosts entry by increasing markups. Optimal long-run inflation is zero if and only if preferences are such that the incentive for product creation by firms and the benefit of variety to consumers are perfectly balanced: for instance, when the utility aggregator takes the specific constant elasticity of substitution (C.E.S.) form introduced by Dixit and Stiglitz (1977-henceforth, C.E.S.-D.S.). Importantly, optimal deviations from long-run price stability generated by departing from this knife-edge scenario can be quantitatively significant: depending on the value of the parameter that measures the benefit of variety, the optimal inflation rate ranges from an annualized 4 percent to an annualized −8 percent; the numbers are even larger under price indexation.

When preferences are such that the desired markup depends upon the scale of the economy (number of firms) and is higher than the benefit of variety, the degree of goods market regulation (which is irrelevant under C.E.S. preferences, because the scale itself is irrelevant) becomes an important determinant of the optimal long-run inflation rate. A higher entry cost reduces the steady-state number of firms, makes consumers less willing to substitute among their goods and increases desired markups; this creates more scope for using inflation in order to lower markups and discourage welfare-inefficient entry. Plausible preferences and parameter values justify the positive inflation targets adopted by central banks throughout the industrialized world (see Table 1 in Schmitt-Grohé and Uribe, 2011, for a summary).

In the short run, however, approximate price stability (around a possibly non-zero optimal trend) is a robust policy prescription. In particular, the volatility of inflation under Ramsey policy is small for all the preferences considered: the central bank uses its leverage over real activity in the long run, but not in the short run.[3] The welfare costs (in units of steady-state consumption) of perfectly stabilizing prices relative to following Ramsey-optimal policy can indeed be sizeable. Since the volatility of inflation under Ramsey policy is negligible, it can be concluded that most of the welfare cost of targeting a constant level of prices is due to the "long-run" component, i.e. to failing to adopt the Ramsey steady-state level of inflation as the central bank's target. Therefore, our conclusion is that the introduction of endogenous entry and preferences for variety more general than C.E.S.-D.S. can dramatically alter the long-run policy prescriptions obtained under fixed variety, but not the short-run implications. Lastly, we also quantify the temptation facing policymakers to renege on the optimal policies, and discuss how this is affected by the presence of endogenous entry and variety.

Our results contribute to a large and growing literature on optimal monetary policy and inflation by studying a hitherto unexplored motive for non-zero optimal inflation. To isolate the contribution of the novel feature considered here (entry and variety), our analysis abstracts from other, well understood features—e.g. government spending and monetary distortions—that have been shown to result in optimal deviations from price stability.[4] In such an environment, price stability is optimal or at least close to optimal in many models: the monetary authority does not use inflation (a distortionary tax) to try to close the constant

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[2] A different theory of endogenous desired markups with entry relies upon strategic interactions coming for instance from Cournot competition, as in Portier (1995) and Cook (2001). As discussed at length in Bilbiie et al. (2012), the "demand-based", translog-preferences model of endogenous markups used here differs from these "supply-based" explanations along two main dimensions, both of which are related to the empirical evidence pertaining to entry. First, evidence points to the fact that the vast majority of entering and exiting firms are small, which casts doubt on their ability to exert a significant influence on aggregate markups. Second, it is *product* creation and destruction by *existing* firms, rather than entry and exit by new firms, that drives the overall quantitative contribution of extensive margins to explaining aggregate fluctuations; it is therefore difficult to argue that strategic interactions drive markups down with entry, in a view of the world where entry is understood in the larger sense of product creation by already existing firms. Finally, a recent paper by Lewis and Poilly (2012) compares the empirical performance of the two frameworks by estimating the dynamic general equilibrium models on aggregate data. They find that while the translog model is a good fit of the data, in the "strategic interactions" model there is no evidence of a "competition effect" (whereby markups decrease with the number of firms); that model turns out to be statistically equivalent to the C.E.S. model.

[3] This result is consistent with standard tax smoothing arguments made in the dynamic public finance literature (see Golosov and Tsyvinski, 2006, for a review): the Ramsey planner tries to smooth inefficiency wedges over time, which results in our framework in a path for its instrument (inflation) that is close to constant. This mechanism operates in the Ramsey-optimal *fiscal* policy analysis of Chugh and Ghironi (2011), in a flexible-price model similar to ours. There, the Ramsey planner ends up choosing a smooth tax rate on dividends, for the same reason. Indeed, inflation in our framework resembles an indirect tax on dividends (see Bilbiie et al., 2008b).

[4] In particular, the presence of government spending has been shown to imply optimal deviations from price stability in a variety of economic environments. This conclusion arises in the Ramsey-optimal policy exercises of Khan et al. (2003), whose model features staggered pricing and monetary and real distortions, and Adão et al. (2003), whose model features predetermined prices. The same conclusion emerges in the linear-quadratic environment

wedge between the marginal rate of consumption–leisure substitution and the marginal product of labor implied by monopolistic competition and endogenous labor supply.[5] It is important to notice that even when optimal deviations from *short-run* price stability occur in the existing literature, the finding that price stability is the *optimal long-run* policy prescription is surprisingly robust across a wide range of economic environments.[6] Indeed, this is a common theme of all the variations of the baseline model with Calvo (1983)–Yun (1996) price rigidity considered by Woodford (2003): "The optimal long-run inflation target is zero in this model, *no matter how large* the steady-state distortions may be" (p. 462, emphasis in original). Schmitt-Grohé and Uribe (2011) comprehensively review the existing literature on optimal inflation and conclude that "the observed inflation objectives of central banks pose a puzzle for monetary theory"; optimal long-run inflation is zero or very close to zero under a wide range of economic frictions, including incomplete taxation, the zero lower bound on nominal interest rates, downward rigidity in nominal wages, and the quality bias in measured inflation.[7] Thus, endogenous entry and product variety yield a policy implication that is largely new to the literature.[8]

This paper is not the first to study optimal monetary policy under endogenous entry. Bilbiie et al. (2008b) showed that, in a model with entry and sticky prices à la Rotemberg (1982), stabilizing producer price inflation at zero in all periods is Pareto optimal in a first-best environment in which lump-sum taxes are available to finance the subsidies (or taxes) needed to correct real distortions. Bergin and Corsetti (2008) also study a model with entry and predetermined prices and show the (constrained) optimality of producer price stability in a second-best environment. Lewis (2013) studies optimal monetary policy in an economy with entry, a cash-in-advance constraint, and sticky wages. In her model, optimal inflation is used over the cycle to improve upon the flexible-wage response to shocks. In independent work, Faia (2010) uses a framework similar to ours (essentially, the Bilbiie et al., 2008b model), but with C.E.S.-D.S. preferences and oligopolistic competition as in Portier (1995) and Cook (2001), as well as government spending shocks. She finds that the Ramsey long-run prescription is "zero inflation"; however, in the short run, significant deviations from price stability are required for optimality. While apparently in stark contrast with our short-run findings, the difference can be explained by the absence of government spending from our framework since, as noted above, government spending by itself implies optimal deviations from short-run price stability. In order to isolate the potential role of entry and variety in generating deviations from price stability, our framework therefore abstracts from government spending altogether.

When considering these studies in relation to the conduct of monetary policy in reality, one may wonder whether it is appropriate for central banks to have distortions in product variety in mind when determining their inflation targets. There are two reasons for an affirmative answer. First, to the extent that variety is important for aggregate fluctuations and long-run welfare, generating the optimal amount of variety is consistent with the policy objective of a welfare-maximizing equilibrium. Second, even if one may argue that optimal variety is best implemented by regulation policy, reality shows that regulators intervene in the economy only under exceptional circumstances to affect the behavior of the largest firms (for instance, Microsoft). "Blanket" instruments that affect *all* producers at all points in time (such as inflation) are thus better suited to induce the optimal equilibrium for the aggregate economy. More generally, the exercise of this paper is one of finding the optimal monetary policy, given a certain economic environment—very much like all studies of optimal monetary policy on which this paper builds—rather than a more general public finance exercise that would try to assess which is the best policy instrument to use in order to address a certain distortion.[9]

2. The model

The model with endogenous producer entry and product variety used here builds on Bilbiie et al. (2012), augmented with price stickiness as in Bilbiie et al. (2008b). In the interest of space, this paper only includes a summary description of the economic environment and presents in some detail the ingredients that are key for the optimal monetary policy problem (the pricing and entry decisions of firms, and the nature of preferences for variety of consumers). The above mentioned papers contain a complete description of the model (whose full set of equilibrium conditions is nevertheless outlined in Table 1 for completion).

(*footnote continued*)
of Woodford (2003, Ch. 6.5), whose model features Calvo (1983) pricing and a distorted steady state (see also Benigno and Woodford, 2005). The first two papers also discuss the role of monetary distortions in shaping policy tradeoffs.

[5] Woodford (2003) and Benigno and Woodford (2005) show that, when the steady state is distorted and the distortion is large (there are no lump sum taxes available to finance the subsidies needed to restore optimality), even a constant level of government spending will lead to optimal deviations from short-run price stability. A central bank that aims at stabilizing output and inflation around their efficient levels will face a tradeoff induced by productivity shocks that act like "cost-push" shocks in the Phillips curve when steady-state government spending is not zero.

[6] Examples of environments in which long-run price stability is optimal and short-run price stability is not included i.a. models of search and matching in the labor market (Thomas, 2008; Faia, 2009) and sticky wages (Erceg et al., 2000).

[7] Two popular *theoretical* justifications for deviations from long-run price stability are the zero lower bound (see e.g. Billi, 2011) and downward nominal wage rigidity (Kim and Ruge-Murcia, 2009). Schmitt-Grohé and Uribe (2011) results imply that these distortions are not enough to generate *quantitatively significant* deviations from price stability.

[8] Other reasons complementary to ours for deviating from long-run price stability are emphasized in recent contributions: growth in firm-specific productivity (Weber, 2011); trending relative prices in a two-sector model (Wolman, 2011); and the absence of a commitment technology, i.e. discretion (Anderson et al., 2010; Van Zandweghe and Wolman, 2011).

[9] See Correia et al. (2008) for an example of how, when both consumption and labor income taxes are available, the optimal policy (and the optimal allocation itself) does not even depend on the degree of nominal rigidity.

Table 1
Model Summary.

Pricing	$\rho_t = \mu_t \dfrac{w_t}{Z_t}$
Markup	$\mu_t = \dfrac{\theta(N_t)}{[\theta(N_t)-1]\left(1-\frac{\kappa}{2}\pi_t^2\right)+\kappa\left\{(1+\pi_t)\pi_t-\beta(1-\delta)E_t\left[\frac{C_t}{C_{t+1}}\frac{N_t}{N_{t+1}}\frac{Y_{t+1}^C}{Y_t^C}(1+\pi_{t+1})\pi_{t+1}\right]\right\}}$
Variety effect	$\rho_t = \rho(N_t)$
Profits	$d_t = \left(1-\dfrac{1}{\mu_t}-\dfrac{\kappa}{2}\pi_t^2\right)\dfrac{Y_t^C}{N_t}$
Free entry	$v_t = w_t \dfrac{f_E}{Z_t}$
Number of firms	$N_t = (1-\delta)(N_{t-1}+N_{E,t-1})$
Intratemporal optimality	$h_L(L_t) = \dfrac{w_t}{C_t}$
Euler equation (shares)	$v_t = \beta(1-\delta)E_t\left[\left(\dfrac{C_{t+1}}{C_t}\right)^{-1}(v_{t+1}+d_{t+1})\right]$
Euler equation (bonds)	$(C_t)^{-1} = \beta E_t\left[\dfrac{1+i_t}{1+\pi_{t+1}^C}(C_{t+1})^{-1}\right]$
Consumption sector	$Y_t^C = \left(1-\dfrac{\kappa}{2}\pi_t^2\right)^{-1}C_t$
Aggregate accounting	$C_t + N_{E,t}v_t = w_t L_t + N_t d_t$
CPI inflation	$\dfrac{1+\pi_t}{1+\pi_t^C} = \dfrac{\rho_t}{\rho_{t-1}}$

The economy is populated by a unit mass of atomistic, identical **households**. The representative household supplies L_t hours of work in each period t in a competitive labor market for the nominal wage rate W_t and maximizes expected intertemporal utility $E_t[\sum_{s=t}^{\infty}\beta^{s-t}U(C_s,L_s)]$, where C_t is consumption and $\beta \in (0,1)$ the subjective discount factor. The period utility function takes the form $U(C_t,L_t) = \ln C_t - h(L_t)$, where $h_L(L) > 0$, $h_{LL}(L) \geq 0$, and $\varphi \equiv h_{LL}(L)L/h_L(L) \geq 0$ is the inverse Frisch elasticity of labor supply to wages (and the inverse intertemporal elasticity of substitution in labor supply).

At time t, the household consumes the basket of goods C_t, defined over a continuum of goods Ω. At any given time t, only a subset of goods $\Omega_t \subset \Omega$ is available. Let $p_t(\omega)$ denote the nominal price of a good $\omega \in \Omega_t$. For any symmetric homothetic preferences, there exists a well defined consumption index C_t and an associated welfare-based price index P_t. The demand for an individual variety, $c_t(\omega)$, is then obtained as $c_t(\omega)d\omega = C_t\partial P_t/\partial p_t(\omega)$, which uses the conventional notation for quantities with a continuum of goods as flow values. Given the demand for an individual variety, $c_t(\omega)$, the symmetric elasticity of demand $-\theta$ (where θ measures the elasticity of substitution) is in general a function of the number N_t of goods available (where N_t is the mass of Ω_t): $\theta(N_t) \equiv -\partial \ln c_t(\omega)/\partial \ln p_t(\omega)$. The *benefit of an additional product variety* is described by the relative price $\rho_t(\omega) = \rho(N_t) \equiv p_t(\omega)/P_t$ or, in elasticity form:

$$\epsilon(N_t) \equiv \frac{\rho'(N_t)}{\rho(N_t)}N_t.$$

Together, $\theta(N_t)$ and $\epsilon(N_t)$ completely characterize the effects of preferences in our model; explicit expressions can be obtained for these objects upon specifying functional forms for preferences, as will become clear in the discussion below.

There is a continuum of monopolistically competitive **firms**, each producing a different variety $\omega \in \Omega$.[10] Production requires only one factor, labor. Aggregate labor productivity is indexed by Z_t, which represents the effectiveness of one unit of labor. Productivity is exogenous and follows an $AR(1)$ process in percent deviation from its steady-state level. Output supplied by firm ω is $y_t(\omega) = Z_t l_t(\omega)$, where $l_t(\omega)$ is the firm's labor demand for productive purposes. The unit cost of production, in units of the consumption good C_t, is w_t/Z_t, where $w_t \equiv W_t/P_t$ is the real wage.

Prior to entry, firms face a *sunk entry cost* of f_E effective labor units, equal to $w_t f_E/Z_t$ units of the consumption good. All firms that enter the economy produced in every period, until they are hit with a "death" shock, which occurs with probability $\delta \in (0,1)$ in every period. In each period, there is a mass N_t of firms producing and setting prices in the economy. Firms face nominal rigidity in the form of a quadratic cost of adjusting prices over time (Rotemberg, 1982): $pac_t(\omega) \equiv \kappa[(p_t(\omega)/p_{t-1}(\omega))-1]^2(p_t(\omega)/P_t)y_t^D(\omega)/2$.[11] The demand for a firm's output, which comes from consumers and firms themselves when they change prices, has price elasticity $-\theta(N_t)$.

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[10] For convenience, we use the terms good and variety interchangeably. Note that the assumption that each firm produces a different variety implies that the number of goods available N_t is also the number of producers in period t. We refer to individual producers as firms following the standard convention in the New Keynesian literature. However, a more general—and empirically relevant—interpretation is to think of productive units as product lines at firms whose boundaries we leave unspecified. See Bilbiie et al. (2012) for more details.

[11] When a new firm sets the price of its output for the first time, we appeal to symmetry across firms and interpret p_{t-1} as the notional price that the firm would have set at time $t-1$ if it had been producing in that period. This assumption is consistent with both the original Rotemberg (1982) setup and our timing assumption that entrants in period t begin producing and setting prices at $t+1$.

2.1. Pricing and the Phillips curve

Anticipating that the *equilibrium is symmetric* the argument ω is henceforth ignored, while recalling that small-case letter pertain to firm-specific variables and capital letters refer to aggregate variables. The real profit of a firm in period t can be written as

$$d_t = \rho_t y_t^D - w_t l_t - \frac{\kappa}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 \rho_t y_t^D,$$

where $\rho_t \equiv p_t/P_t$ is the real, relative price of firm's output. The real value of the firm at time t (in units of consumption) is the expected present discounted value of future profits from $t+1$ on $v_t = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s$, where $\Lambda_{t,s} \equiv [\beta(1-\delta)]^{s-t} C_t/C_s$ is the discount factor applied by households to future dividends.

At time t, the firm chooses labor l_t and its price p_t to maximize $d_t + v_t$ subject to the demand constraint $y_t = y_t^D$, taking all aggregate variables as given. Letting λ_t denote the Lagrange multiplier on the demand constraint, the first-order condition with respect to labor yields $\lambda_t = w_t/Z_t$: the shadow value of an extra unit of output is simply the firm's marginal cost, common across all firms in the economy.

Firms set prices optimally by equating their marginal cost λ_t with marginal revenue (denoted by MR_t), which is given by the change in total revenues induced by producing one extra unit of output today:

$$MR_t = \frac{\partial}{\partial y_t}\left\{\rho_t y_t - \frac{\kappa}{2}\left(\frac{p_t}{p_{t-1}} - 1\right)^2 \rho_t y_t - \beta(1-\delta) E_t\left[\frac{C_t}{C_{t+1}}\frac{\kappa}{2}\left(\frac{p_{t+1}}{p_t} - 1\right)^2 \rho_{t+1} y_{t+1}\right]\right\}$$

$$= \rho_t\left\{\left(1 - \frac{\kappa}{2}\pi_t^2\right)\left[\frac{\theta(N_t)-1}{\theta(N_t)}\right] + \frac{\kappa\pi_t(1+\pi_t)}{\theta(N_t)} - \frac{\kappa\beta(1-\delta)}{\theta(N_t)}E_t\left[\frac{C_t}{C_{t+1}}\frac{N_t}{N_{t+1}}\frac{Y_{t+1}^C}{Y_t^C}\pi_{t+1}(1+\pi_{t+1})\right]\right\}, \tag{1}$$

where producer price inflation is defined as $\pi_t \equiv p_t/p_{t-1} - 1$ and the aggregate output of the consumption basket $Y_t^C \equiv C_t + PAC_t = N_t\rho_t y_t$. In the last expression, $PAC_t = \kappa\pi_t^2 Y_t^C/2$ is the adjustment cost aggregated across firms and $C_t = (1 - \kappa\pi_t^2/2)Y_t^C$.

The optimality condition $MR_t = \lambda_t$ delivers the non-linear Phillips curve relation of our model, reported in Table 1, where the markup is defined as the ratio of relative price ρ_t to real marginal cost λ_t. The link between markups and inflation that is at the core of the "New Keynesian" literature is also at work in our framework.[12] To start with, notice that in the absence of nominal rigidity ($\kappa = 0$), the marginal revenue is simply $MR_t = \rho_t[1 - \theta^{-1}(N_t)]$ and the markup $\mu_t = \theta(N_t)/[\theta(N_t) - 1]$.

Price adjustment costs have three effects on the pricing decision. The first is mechanical: since they are proportional to sales revenues, adjustment costs imply that inflation erodes marginal revenues (the first term inside the curly brackets) and hence increases proportionally the markup. However, because the price adjustment cost is a function of squared inflation, this effect is second-order.

Second, and most importantly, the presence of price adjustment costs leads to a positive relationship between marginal revenues and inflation, and hence an inverse relationship between markups and inflation. To understand this, focus on the case of CES preferences. Without adjustment costs, a monopolist increasing production by one unit faces a fall in price (hence, in a symmetric equilibrium, there is deflation), and lower marginal revenue. However, CES preferences imply that the markup remains constant: since price adjustment is costless, deflation has no bearing on real variables. With price adjustment costs, increasing output by one unit implies a relatively smaller fall in price, and, therefore, a smaller decline in marginal revenue (and marginal cost), and an increase in the markup relative to its flexible-price level. In a symmetric equilibrium, the fall in price relative to its previous level translates into aggregate deflation, which is therefore associated with a higher markup (and lower marginal revenue); this explains the second term in curly brackets.

Lastly, there is a third effect through expected inflation because revenues tomorrow are a function of the price today: through this channel, expected inflation tomorrow is associated with a lower marginal revenue, and a higher markup today. These last two effects vanish when goods are close substitutes (θ tends to infinity), for the increase in production implies no fall in price when the elasticity of demand is very high. The alternative preference specifications considered here do not change these basic mechanisms because of the predetermined, slow-moving nature of the number of product varieties. It is also important to notice that, in the steady state, the second effect (through realized inflation) always dominates, because the third one is discounted; namely, dropping time subscripts to refer to steady-state variables:

$$MR = \rho\left\{\left(1 - \frac{\kappa}{2}\pi^2\right)\left(1 - \frac{1}{\theta(N)}\right) + \frac{1 - \beta(1-\delta)}{\theta(N)}\kappa\pi(1+\pi)\right\} \tag{2}$$

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[12] King and Wolman (1996) and Goodfriend and King (1997) contain early analyses of the relationship between markups and inflation in the Calvo model. Schmitt-Grohé and Uribe (2008) provide a discussion of the New Keynesian Phillips curve under Rotemberg pricing, and its implications for optimal monetary policy.

Since the first direct (negative) effect is second-order, and the positive indirect net effect through the demand function is first-order, the steady-state marginal revenue is *increasing* and hence the markup is *decreasing* in inflation at low levels of inflation.[13]

2.2. Firm entry and households' intertemporal decision

In every period, there is an unbounded mass of prospective entrants. Entrants at time t only start producing at time $t+1$, which introduces a one-period time-to-build lag in the model, and all firms are subject to identical probability δ of exogenous firm destruction at the end of each period (after production and entry). A proportion δ of new entrants will therefore never produce. Prospective entrants in period t compute their expected post-entry value, v_t, given by the present discounted value of their expected stream of profits from $t+1$ on. This also represents the average value of incumbent firms *after* production has occurred (since both new entrants and incumbents then face the same probability $1-\delta$ of survival and production in the subsequent period). Entry occurs until firm value is equalized with the entry cost, resulting in the free entry condition $v_t = w_t f_E / Z_t$. This condition holds so long as the mass $N_{E,t}$ of entrants is positive. Macroeconomic shocks are assumed to be small enough for this condition to hold in every period. The timing of entry and production implies that the number of producing firms during period t is given by $N_t = (1-\delta)(N_{t-1} + N_{E,t-1})$.

Entry is financed by the households through a portfolio decision. Specifically, households maximize the present discounted value of their utility subject to a standard budget constraint; they hold two types of assets: shares in a mutual fund of firms and nominal bonds. They can buy shares in a mutual fund of $N_t + N_{E,t}$ firms ($1-\delta$ of which will exit) paying the real price v_t; and they receive the real payoff $v_t + d_t$ (selling price plus dividend) on the outstanding portfolio of shares in the mutual fund of existing N_t firms. Bonds purchased at t pay nominal interest i_t at time $t+1$. The optimal portfolio decision yields the Euler equations in Table 1, where $\pi_t^C \equiv P_t / P_{t-1} - 1$ in the Euler equation for bonds is CPI inflation (transversality conditions are omitted from Table 1). Forward iteration of the equation for share holdings and absence of speculative bubbles yield the expression for firm value used in Section 2.1 above. Finally, the first-order condition for the optimal choice of labor effort requires that the marginal disutility of labor be equal to the marginal utility from consuming the real wage received for an additional unit of labor: $h_L(L_t) = w_t / C_t$.

The equilibrium conditions of our model are summarized in Table 1. In the table, there are 12 equations and 13 endogenous variables. The model is closed by specifying monetary policy conduct, which we shall do in Section 3 below after discussing the properties of alternative preference specifications and the distortions that characterize our economy in the flexible-price case.

Before proceeding, a word of clarification on measurement that applies to any model with product variety: construction of consumer price index (CPI) data by statistical agencies does not adjust for availability of new products as in the welfare-consistent price index. An implication of this measurement issue for our model is that the "true" welfare-based CPI of our model is not observable in the data or, conversely, that the observed, measured CPI is closer to the producer price index (PPI) of our model (the average price of a product) p_t than P_t.[14] Since the PPI is the price index in the model available to a central bank trying to measure inflation, all of our normative discussion below will be cast in terms of this observable variable: consequently, "inflation" refers to "PPI inflation" below.

2.3. Preference specifications and flexible-price markups

Four alternative preference specifications are useful as special cases for illustrative purposes below. Here, we discuss the implications of these preferences for the welfare benefit of variety and the desired markups, obtained if all individual producers can adjust their prices freely in the *flexible-price equilibrium*, denoted with a star.

The first preference specification features a constant elasticity of substitution between goods as in Dixit and Stiglitz (1977). For these C.E.S. preferences, the consumption aggregator is $C_t = (\int_{\omega \in \Omega} c_t(\omega)^{\theta - 1/\theta} d\omega)^{\theta/(\theta-1)}$, where $\theta > 1$ is the symmetric elasticity of substitution across goods. The consumption-based price index is then $P_t = (\int_{\omega \in \Omega_t} p_t(\omega)^{1-\theta} d\omega)^{1/(1-\theta)}$, and the household's demand for each individual good ω is $c_t(\omega) = (p_t(\omega)/P_t)^{-\theta} C_t$. It follows that the flexible-price markup and the benefit of variety are independent of the number of goods: $\mu^*(N_t) - 1 = \epsilon^*(N_t) = \epsilon = 1/(\theta-1)$. The second specification is the C.E.S. variant with *generalized love of variety* introduced by the working paper version of Dixit and Stiglitz (1977), which disentangles monopoly power (measured by the net markup $1/(\theta-1)$) and consumer love for variety, captured by a constant parameter $\epsilon > 0$. With this specification (labeled "general C.E.S." henceforth), the consumption basket is $C_t = (N_t)^{\epsilon - 1/(\theta-1)}(\int_{\omega \in \Omega} c_t(\omega)^{\theta - 1/\theta} d\omega)^{\theta/(\theta-1)}$. The third preference specification features *exponential love-of-variety* (dubbed

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[13] To anticipate, since inflation variability entails resource costs, the welfare-relevant region for inflation will be the one where the markup is decreasing in inflation; specifically, under our baseline calibration described below the markup-minimizing level of (annualized) inflation is above 5 percent. At higher values of inflation, of course, the first negative effect dominates and the markup becomes increasing in inflation.

[14] Furthermore, adjustment for variety, when it happens, certainly neither happens at the frequency represented by periods in our model, nor using one of the specific functional forms for preferences that our model assumes. For these reasons, when investigating the properties of their model in relation to the data, Bilbiie et al. (2008b, 2012) focus on real variables deflated by a data-consistent price index: for any variable X_t in units of the consumption basket, its data-consistent counterpart is obtained as $X_{R,t} \equiv P_t X_t / p_t = X_t / \rho_t = X_t / \rho(N_t)$.

Table 2
Four preference specifications: markup, relative price and benefit of variety under flexible prices.

C.E.S.-D.S.	General C.E.S.	Exponential	Translog
$\mu^* = \dfrac{\theta}{\theta-1}$	$\mu^* = \dfrac{\theta}{\theta-1}$	$\mu^*(N_t) = \dfrac{\theta^*(N_t)}{\theta^*(N_t)-1} = 1 + \dfrac{1}{\alpha N_t}$	$\mu^*(N_t) = \dfrac{\theta^*(N_t)}{\theta^*(N_t)-1} = 1 + \dfrac{1}{\sigma N_t}$
$\rho^*(N_t) = N_t^{\frac{1}{\theta-1}}$	$\rho^*(N_t) = N_t^{e}$	$\rho^*(N_t) = e^{-\frac{1}{\alpha N_t}}$	$\rho^*(N_t) = e^{-\frac{1}{2}\frac{\tilde{N}-N_t}{\sigma N N_t}}$
$\epsilon^*(N_t) = \mu^* - 1$	$\epsilon^*(N_t) = \epsilon$	$\epsilon^*(N_t) = \dfrac{1}{\alpha N_t} = \mu^*(N_t) - 1$	$\epsilon^*(N_t) = \dfrac{1}{2\sigma N_t} = \dfrac{\mu^*(N_t)-1}{2}$

"exponential" for short) and is in some sense just the opposite of the previous: the elasticity of substitution is not constant (because of demand-side pricing complementarities), but the benefit of variety is equal to the net markup. Specifically, the elasticity of substitution is $\theta^*(N_t) = 1 + \alpha N_t$, where $\alpha > 0$ is a free parameter, and the relative price is given by $\rho^*(N_t) = e^{-1/\alpha N_t}$; hence, the benefit of variety and the markup are, respectively: $\epsilon^*(N_t) = \mu^*(N_t) - 1 = 1/\alpha N_t$. As N_t increases, goods become closer substitutes, and the elasticity of substitution increases. If goods are closer substitutes, then the markup $\mu^*(N_t)$ and the benefit of additional varieties in elasticity form ($\epsilon^*(N_t)$) must decrease[15]; for this specific functional form, the markup and benefit of variety decrease by the same amount when N_t increases. Finally, the fourth preference specification uses the *translog expenditure function* proposed by Feenstra (2003). For this specification, the symmetric price elasticity of demand is $1 + \sigma N_t$, with $\sigma > 0$; $\tilde{N}$ in the table is the measure of all possible varieties, $\tilde{N} \equiv Mass(\Omega)$. In contrast to the previous specification, the change in $\epsilon^*(N_t)$ is only *half* the change in the net markup generated by an increase in the number of producers. Table 2 summarizes the expressions for markup, relative price, and benefit of variety in elasticity form for each preference specification.

2.4. Sources of inefficiency in the flexible-price equilibrium

Bilbiie et al. (2008b) and Bergin and Corsetti (2008) show that optimal monetary policy always seeks to stabilize producer prices perfectly in a first-best environment (where lump-sum taxes/transfers are available to finance any optimal subsidies/taxes used to correct distortions that arise under flexible prices). The question asked in this paper is: would a Ramsey planner that operates in a second-best environment, having the inflation rate as her only policy instrument, choose non-zero inflation? Before delving into this question, it is useful to review the distortions that can make the *flexible-price* equilibrium inefficient in our model and hence, as will become clear shortly, constitute a reason for a positive answer[16]; there are three such distortions:

Distortion 1 affects the labor optimality condition: with elastic labor supply, and consumption goods priced at a markup over marginal cost, the household "buys" an inefficiently high amount of the cheaper good (leisure, which is not priced at a markup), and so it ends up working and consuming too little. This is true in models with endogenous variety as in models with fixed variety. The inefficiency wedge in labor optimality can be constant (as under C.E.S. preferences) or time-varying (as under exponential or translog preferences). We refer to this distortion as the "labor distortion".

Distortion 2 operates through the product creation margin: when the welfare benefit of variety $\epsilon^*(N_t)$ and the net markup $\mu^*(N_t) - 1$ (which measures the profit incentive for firms to enter the market) are not aligned within a given period, entry is inefficient from a social standpoint. When, for instance, the benefit of variety is low compared to the desired markup ($\epsilon^*(Nt) < \mu^*(N_t) - 1$), the consumer surplus of creating a new variety is lower than the profit signal received by a potential entrant; equilibrium entry is therefore too high (with the size of the distortion being governed by the difference between the two objects). The opposite holds when $\epsilon^*(Nt) > \mu^*(N_t) - 1$. We refer to this distortion as the "static entry distortion" below, to highlight that it still operates in a static model and/or in steady state, and for C.E.S. preferences for which Distortion 3 below disappears.

Distortion 3 also operates through the product creation margin: variations in desired markups over time (induced by changes in the N_t) introduce an additional discrepancy—equal to the ratio $\mu^*(N_t)/\mu^*(N_{t+1})$—between the "private" (competitive equilibrium) and "social" (Pareto optimum) return to a new variety. When there is entry, the future markup is lower than the current one, and this ratio increases, generating an additional inefficient reallocation of resources to enter in the current period. We label this the "dynamic entry distortion" below, making explicit that it operates only with preferences that allow for time-varying desired markups.

The remarkable feature of all three distortions listed above is that they depend on the markup. But with sticky prices, as explained in detail in Section 2.1, the markup is intimately related to (current and expected) inflation; the central bank can therefore attempt to correct these distortions by using inflation to affect markups. This paper studies the interaction of the

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[15] This property for the markup occurs whenever the price elasticity of residual demand decreases with quantity consumed along the residual demand curve.

[16] A more detailed discussion of the inefficiencies associated with monopolistic competition and entry in this framework is provided by Bilbiie et al. (2008a).

Table 3
Reduced model, summary.

$$N_t = (1-\delta)\left(N_{t-1} + \frac{(L_t - L_{C,t})Z_t}{f_E}\right)$$

$$\theta(N_t) - 1 + \kappa\left\{\frac{(1+\pi_t)\pi_t}{1 - \frac{\kappa}{2}\pi_t^2} - \beta(1-\delta)E_t\left[\frac{(1+\pi_{t+1})\pi_{t+1}}{1 - \frac{\kappa}{2}\pi_{t+1}^2}\frac{N_t}{N_{t+1}}\right]\right\} = \theta(N_t)L_{C,t}h_L(L_t)$$

$$h_L(L_t)\frac{f_E}{Z_t} = \beta(1-\delta)E_t\left[\left(h_L(L_{t+1})\frac{f_E}{Z_{t+1}} + \frac{1 - L_{C,t+1}h_L(L_{t+1})}{N_{t+1}}\right)\right]$$

flexible-price distortions above with the resource cost of price adjustment in shaping Ramsey-optimal monetary policy in a second-best environment.

3. Ramsey-optimal monetary policy and endogenous entry and product variety

Before discussing the solution to our policy problem, it is useful to recall the benchmark results from the plain vanilla New Keynesian model with fixed variety. In the most basic version of that framework, inefficiency is due to the sticky-price distortion and the labor distortion (elastic labor and monopolistic competition). But—in the simplest version of the model in which no "cost-push" shocks are present—the welfare costs of inflation outweigh the potential benefits obtained by (even slight) variations in inflation that would lead to markup erosion and improve the labor wedge; hence, inflation is optimally not used.[17]

The full Ramsey problem for the model with endogenous entry and product variety summarized in Table 1 is described in Appendix A. In order to build intuition for the numerical results obtained below, we seek to obtain analytical results whenever possible and simplify the problem as follows. First, since the nominal interest rate only enters the Euler equation for bonds, the problem can be regarded as one where the Ramsey planner chooses the allocation directly; once the paths of consumption and CPI inflation are known, the path of the interest rate consistent with optimality is uniquely determined by the Euler equation. Similar reasoning and repeated substitutions of all the static equilibrium conditions into the three dynamic ones (the Phillips curve, the law of motion for variety, and the Euler equation for shares) allow us to reduce the model to the three-equation system in Table 3, where $L_{C,t}$ denotes the total amount of labor used in production of existing goods $L_{C,t} \equiv N_t l_t$.[18] Therefore, the Ramsey planner chooses total labor, producer price inflation, labor allocated to the consumption sector, and the number of firms to maximize the following Lagrangian (where $\eta_{j,t}$ is the Lagrange multiplier on constraint j in Table 3):

$$\max_{L_t, \pi_t, L_{C,t}, N_{t+1}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln\left[\left(1 - \frac{\kappa}{2}\pi_t^2\right)Z_t\rho(N_t)L_{C,t}\right] - h(L_t) + \eta_{1,t}\left[N_{t+1} - (1-\delta)\left(N_t + \frac{(L_t - L_t^C)Z_t}{f_E}\right)\right] \right.$$

$$+ \eta_{2,t}\left[(1 - \theta(N_t))\left(1 - \frac{\kappa}{2}\pi_t^2\right) + \theta(N_t)\left(1 - \frac{\kappa}{2}\pi_t^2\right)L_{C,t}h_L(L_t) - \kappa\pi_t(1+\pi_t)\right.$$

$$\left. + \beta(1-\delta)\kappa\pi_{t+1}(1+\pi_{t+1})\frac{N_t}{N_{t+1}}\frac{1 - \frac{\kappa}{2}\pi_t^2}{1 - \frac{\kappa}{2}\pi_{t+1}^2}\right]$$

$$\left. + \eta_{3,t}\left[h_L(L_t)\frac{f_E}{Z_t} - \beta(1-\delta)\left(h_L(L_{t+1})\frac{f_E}{Z_{t+1}} + \frac{1 - L_{C,t+1}h_L(L_{t+1})}{N_{t+1}}\right)\right]\right\}. \tag{3}$$

The first-order conditions of this problem are outlined in Appendix A, which also contains an analytical proof of the following result.

Proposition 1. *In a model with endogenous entry, homothetic preferences for variety, elastic labor, and quadratic price adjustment costs, optimal inflation in the Ramsey equilibrium is zero in steady state if and only if preferences are such that the*

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[17] This is a standard-second best argument—that holds regardless of the value of labor supply elasticity—which can be traced back to the influential analyses of King and Wolman (1996) and Goodfriend and King (1997), using Calvo pricing. Rotemberg and Woodford (1997) showed the optimality of zero inflation when a subsidy is used to eliminate the markup distortion. Rotemberg and Woodford (1997) provide a proof of the optimality of zero steady-state inflation in the absence of a corrective subsidy, in the context of a model with two-period Taylor contracts. Benigno and Woodford (2005) prove the optimality of zero steady state inflation in the Calvo model without the corrective subsidy, and Schmitt-Grohé and Uribe (2011) generalize that result for a model with investment in physical capital. While the result holds exactly in the simplest model, it has been shown (as reviewed in the Introduction) to be robust to the introduction of other frictions.

[18] This requires using also $C_t = (1 - (\kappa/2)\pi_t^2)Z_t\rho(N_t)L_{C,t}$, which is implied by aggregate accounting. A detailed derivation is provided in an online Appendix available on the authors' webpages.

benefit of variety is equal to the desired steady-state (net) markup:

$$\epsilon(N) = \frac{1}{\theta(N) - 1} \tag{4}$$

Proof. See Appendix A. ∎

It is worth emphasizing that zero long-run inflation is optimal (as long as (4) holds) *regardless of labor supply elasticity*. In other words, the planner does *not* use her distortionary instrument (inflation) to correct the labor supply distortion in a standard second best fashion. The introduction of firm entry and endogenous product variety in an environment where (4) holds—such as, for instance, with C.E.S.-D.S. preferences—does not change the conclusions obtained in the simplest, fixed-variety model. Intuitively, since the product creation margin is efficient by virtue of the balancing of the benefit of variety with the monopoly profit incentive for entry, and the planner was already not using inflation in the fixed-variety case, endogenous entry does not give the planner any additional incentive to resort to this distortionary instrument.

When (4) does not hold, entry is distorted by the misalignment of markup and benefit of variety (the static entry distortion operates). A non-zero optimal long-run rate of inflation emerges: as discussed at length in Section 2.1, inflation affects the markup and can be used by the central bank to close the gap between the profit incentive for firm entry (the markup) and the benefit of variety for consumers (ϵ). Rewriting the price-setting optimality condition $MR = \lambda$ in steady state using the expression (2) for steady-state marginal revenue, we obtain our model's *long-run Phillips curve* (LRPC)—the relationship between steady-state markup and steady-state inflation:

$$\mu(\pi) = \frac{\theta}{(\theta - 1)\left(1 - \frac{\kappa}{2}\pi^2\right) + \kappa\frac{r + \delta}{1 + r}(1 + \pi)\pi}, \tag{5}$$

where for the sake of exposition preferences are assumed to be C.E.S.

In the knife-edge case covered by Proposition 1 ($\epsilon = 1/(\theta - 1)$) the benefit of variety is equal to the desired net markup, and so the Ramsey equilibrium results in a path of inflation with a long-run value of zero.[19]

When the benefit of variety is lower than the desired net markup ($\epsilon < 1/(\theta - 1)$), there is always *too much entry* in the monopolistically competitive equilibrium. The central bank cannot close this gap perfectly (because using inflation entails resource costs), but it can reduce it by exploiting the long-run Phillips curve (5), namely by choosing a path of inflation that lowers markups and hence *reduces entry*. Recall that, as discussed at length is Section 2.1, in an inflationary steady state, a firm doing Rotemberg pricing chooses a price level that is "too low", pushing the adjustment costs associated with increasing prices into the future; this means that markups will indeed be lower with inflation. The reverse reasoning holds when the benefit of variety is higher than the desired net markup ($\epsilon > 1/(\theta - 1)$): there is *too little entry* in the steady state with zero inflation. The central bank will generate long-run *deflation* in order to increase long-run markups and stimulate entry and variety towards their optimal level, trading this off against the resource costs of non-zero inflation.

The optimality of deviations from long-run price stability in our framework relies upon the existence of a non-vertical long-run Phillips curve. While the latter feature is also present in any model of imperfect price adjustment (with no entry) that does not necessarily imply optimal non-zero long-run inflation, it has different welfare consequences in our framework because of the intimate link between entry, variety, and the markup. Appendix B shows that our results are actually *strengthened* under an alternative assumption that weakens the long-run trade-off between inflation and real activity, namely *price indexation*.

4. Optimal inflation in the long run: quantitative results

This section quantifies the optimal long-run rate of inflation by means of a numerical example. In the simulations below, the parameterization is identical to Bilbiie et al. (2008b, 2012), namely the discount factor is $\beta = 0.99$ (implying that the steady-state interest rate is $r = 0.01$), the exogenous destruction rate $\delta = 0.025$, labor elasticity is set to 4 ($\varphi = 0.25$) and the price adjustment cost parameter $\kappa = 77$. Furthermore, the steady-state level of productivity Z is normalized to 1. The choice of preference parameters, which are specific to the functional form of preferences adopted (θ and ϵ for C.E.S., α for exponential, and σ for translog), is discussed in detail in the respective section below, together with our choice of the sunk entry cost parameter f_E.

4.1. Optimal long-run inflation under C.E.S. preferences

Under C.E.S. preferences, the sunk cost parameter f_E does not affect the steady-state of the Ramsey-optimal inflation, because it does not influence the steady-state desired markup μ^*; indeed, μ^* is pinned down exclusively by the elasticity of substitution between goods, which is set to $\theta = 3.8$, a value that is consistent with product-level data—see the discussion in

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[19] In fact, there is a second, non-zero value of inflation that solves $\mu(\pi) = (\theta - 1)^{-1}$; however, since inflation entails resource costs that are quadratic, the planner will always choose the path of inflation with smaller (in modulus) values.
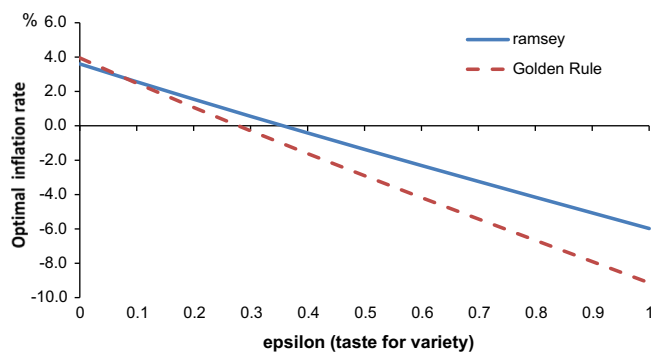
Fig. 1. Ramsey-optimal long-run (blue solid line) and "golden rule" (red dashed line) inflation rates under C.E.S. preferences as a function of benefit of variety ϵ, benchmark calibration. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Bilbiie et al. (2008b, 2012) on both the irrelevance of f_E under C.E.S. preferences and the calibration of θ. We consider different values for the parameter governing the benefit of variety, ϵ, since its value turns out to be crucial for our results.

Fig. 1 plots the steady-state value of the Ramsey-optimal inflation rate (blue solid line) under general C.E.S. as a function of the benefit of variety parameter ϵ, for an interval going from $\epsilon = 0$, which implies that there is no independent benefit to the consumer of introducing a new variety to $\epsilon = 1$, which is higher than any plausible empirical estimate of long-run average net markups. Unless the benefit of variety and monopoly power coincide (C.E.S.-D.S. preferences) and the steady state is efficient, the optimal rate of PPI inflation in the Ramsey steady state is non-zero: specifically, there is long-run inflation (deflation) when the benefit of variety is smaller (larger) than the markup. This simply illustrates our intuitive discussion following Proposition 1. The larger the difference between benefit of variety and net markup, the larger is the optimal deviation from long-run price stability. Indeed, sizable deviations from price stability occur, ranging from an annual inflation rate of almost 4 percent, when the benefit of variety is nil, to an annual deflation rate of 6 percent, when the benefit of variety is 1.[20] Appendix B studies the implications of *price indexation* for our results. A higher degree of price indexation implies even larger optimal deviations from long-run price stability. When indexation is almost full and the long-run Phillips curve is almost vertical, the rate of optimal long-run inflation (or deflation) is very large indeed. For values of the indexation parameter in line with empirical estimates (e.g. Smets and Wouters, 2007), the maximum value of long-run optimal inflation ranges from around 6 percent (when $\epsilon = 0$) to a deflation rate of 10 percent (when $\epsilon = 1$).

Fig. 1 also plots (with a red dashed line) the "*golden rule*" level of inflation denoted by π^{GR} and defined, by obvious analogy to growth theory and similarly to King and Wolman (1999), as the value of inflation that maximizes steady-state utility, subject to the steady-state version of the constraints. Naturally, this concept bears no relationship with the notion of optimality as implied by the Ramsey policy, which takes into account all dynamic trade-offs and initial conditions that are overlooked by definition by π^{GR}. For most values of the benefit of variety—except at very low values—π^{GR} is lower than the Ramsey steady-state level; noticeably, this is the case for C.E.S.-D.S. preferences, where π^{GR} is in fact negative. In other words, starting from π^{GR}, the planner has an incentive to undertake the transition to higher inflation, despite the fact that in the long run (in the Ramsey steady state) the level of utility is lower. There is, however, a welfare gain along the transition which makes it optimal for the planner to pursue it. This gain comes from the inflationary path associated with the transition, which implies falling markups, higher marginal costs and higher value of products. There is entry—in the form of creating new varieties—in the transition, and insofar as the benefit of variety is large enough, it is worth undertaking the transition.[21] Consistent with this intuition, the difference between the two inflation rates is increasing with ϵ, and vanishes when ϵ is close to zero. When ϵ is strictly zero, π^{GR} is slightly larger than the Ramsey level: just like in fixed-variety models (King and Wolman, 1999), it is then optimal to undertake a disinflationary transition.

The quantitative significance of our results on the optimal deviation from long-run price stability hinges upon one's view of a plausible value for the parameter governing the welfare benefit of variety. Decisive evidence on a direct aggregate measure of the welfare benefit of new products is not available.[22] Therefore, our exercise should be viewed as on one hand providing a novel argument for potentially significant deviations from long-run price stability, and on the other pointing to the need for more empirical investigation into the nature of preferences for variety, since this—along with markups—is the single most important determinant of optimal deviations from price stability in a framework with endogenous product variety. In that vein, a recent paper (Lewis and Poilly, 2012) estimates a number of entry models similar to ours and

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[20] As we increase the elasticity of substitution θ, the markup falls, and the blue solid line in Fig. 1 shifts downwards. For $\theta = 6$ (not pictured) long-run optimal inflation ranges from 2 percent when $\epsilon = 0$ to -7 percent when $\epsilon = 1$. Different values of the price stickiness parameter κ or the inverse labor supply elasticity φ do not change the optimal long-run inflation rate significantly (results are available upon request).

[21] A key ingredient which makes this welfare gain valuable in present-value terms is, of course, that there is discounting; hence, the long-term welfare loss from moving to a steady state with lower utility is weighed down considerably.

[22] Bils and Klenow (2001) argue that it is "probably not feasible", although they review some microeconomic estimates of the consumer surplus from introducing a new brand of a specific product.

describes the difficulties faced in identifying the love-of-variety parameter ϵ with C.E.S. preferences. However, they also find that the translog model fits the data well, partly because by restricting the benefit of variety to equal half the net desired markup it provides the additional restriction necessary to identify ϵ. Coupled with the translog specification's performance in replicating other macroeconomic stylized facts, such as the markup's correlation with the business cycle studied by Bilbiie et al. (2012), this suggests that the translog specification is an important empirical benchmark; it is therefore natural to turn our attention to its long-run properties next.

4.2. Optimal long-run inflation under translog preferences

Under translog preferences, the relevant parameter governing *both* the steady-state desired markup and the benefit of variety is σ (recall that $\mu^*(N) = 1 + (\sigma N)^{-1}$ and $\epsilon^*(N) = 1/2\sigma N$). Furthermore, because both the steady-state markup and the benefit of variety depend on the number of firms under translog, the value of the sunk entry cost f_E now matters. To understand the role of these parameters in shaping long-run incentives, consider the case of inelastic labor for illustrative purposes; in that case the steady-state number of firms under flexible prices (and with zero PPI inflation) is (see Bilbiie et al., 2012, Appendix A):

$$N^{\text{translog}} = \frac{-\delta + \sqrt{\delta^2 + 4\dfrac{\sigma}{f_E}L(r+\delta)(1-\delta)}}{2\sigma(r+\delta)}.$$

Intuitively, the steady-state number of firms is decreasing with the level of regulation, i.e. with the sunk entry cost f_E. It follows that the *elasticity of substitution* between goods is

$$1 + \sigma N^{\text{translog}} = 1 + \frac{-\delta + \sqrt{\delta^2 + 4\dfrac{\sigma}{f_E}L(r+\delta)(1-\delta)}}{2(r+\delta)} \qquad (6)$$

Evidence on the elasticity of substitution between goods can therefore only be used to pin down the ratio σ/f_E (given the values of L, r and δ), but not the individual values of σ and f_E; in other words, σ and/or f_E do individually affect the scale of the economy (the steady-state number of firms), but only their ratio affects the elasticity of substitution and the steady-state desired markup. Therefore, in the remainder of the paper, σ/f_E is treated as the relevant parameter under translog. When studying how monetary policy prescriptions are affected by different values of this parameter, one should bear in mind that larger values of σ/f_E can mean either a larger σ or a lower f_E (a more deregulated economy).[23]

Fig. 2 plots the optimal long-run inflation rate under translog preferences, as a function of σ/f_E. The optimal inflation rate is decreasing in σ/f_E, because the elasticity of substitution is increasing in that parameter. It follows that the gap between the steady-state desired net markup and the benefit of variety, which governs the relevant distortion, is decreasing in σ/f_E. Intuitively, more regulation (higher f_E) leads ceteris paribus to a lower number of firms in steady-state, to less willingness by consumers to substitute between their products, and higher desired markups. Since the benefit of variety is half the desired (net) markup, it also increases proportionally, calling for a higher optimal long-run inflation rate. Under the baseline calibration delivering an elasticity of substitution of 3.8 ($\sigma/f_E = 0.354$), the benefit of variety is 0.178 and the optimal long-run rate of inflation is 1.03 percent; price indexation, studied in detail in Appendix B, brings that value to 2 percent or higher for plausible degrees of indexation.

The figure also shows the golden rule level of inflation under translog preferences, defined as in the previous subsection, which is uniformly lower than the Ramsey steady-state level. The intuition is the same as the one provided above for C.E.S. preferences, having to do with the optimality of the inflationary path along the transition when the benefit of variety is large enough. Note that consistently with this intuition, the difference between the two inflation rates (which measures the incentive to undertake the transition) is largely invariant to σ/f_E, precisely because under translog the benefit of variety is always a fixed proportion (half) of the markup.

Evidence on entry costs (reviewed carefully i.a. by Ebell and Haefke, 2009) points to a large degree of heterogeneity across countries: while it "costs" 8.6 days or 1 percent of annual per capita GDP to start a firm in the United States (with similar numbers for Australia, the UK and Scandinavian countries), the costs are an order of magnitude higher in most continental European countries (at the extreme, a whopping 84.5 days in Spain and 48 percent of annual per capita GDP for Greece). The preference parameter σ seems unlikely to vary much across countries. Thus, our model identifies variation in the degree of entry regulation as a possible source of significant variation in the optimal inflation rate across countries.

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[23] Note also that steady-state ratios that could be conceived as allowing to calibrate the sunk cost independently are also a function of the steady-state markup, hence of the elasticity of substitution and finally only of the ratio σ/f_E. Thus, the share of labor used to pay for the sunk cost into total labor is

$$\frac{L_E}{L} = \frac{f_E N_E}{L} = \frac{\delta}{1-\delta}\frac{f_E N}{L}$$

which using the expression for N is only a function of σ/f_E; under the baseline calibration with $\sigma/f_E = 0.354$ and $L = 1/3$, we have $L_E/L = 0.325$. The output value of resources used for the sunk cost as a share of GDP is $wL_E/Y = Q/(1+Q)$, where $Q = \delta(\mu-1)/(\mu(r+\delta))$, which is again only a function of σ/f_E. Under the baseline calibration, $wL_E/Y = 0.16$.
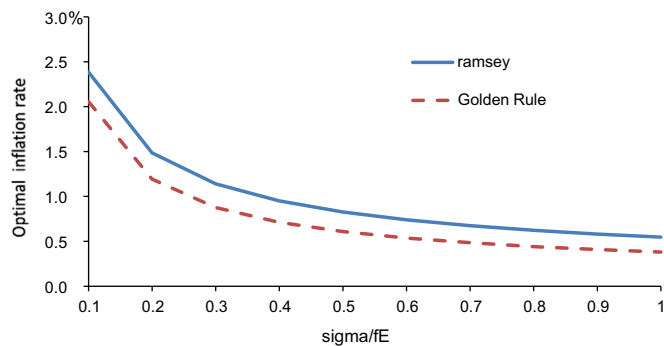
Fig. 2. Ramsey-optimal long-run (blue solid line) and "golden rule" (red dashed line) inflation rates under translog preferences as a function of σ/f_E, benchmark calibration. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

Table 4
Standard Deviations (in percentage points) under Ramsey Policy.

	C.E.S. $\epsilon = 0$	C.E.S.-D.S.	C.E.S. $\epsilon = 1$	Exponential	Translog
Inflation	0.113	0	0.193	0.019	0.036
Output	3.092	3.222	3.431	2.878	2.896

5. Optimal inflation and welfare over the business cycle

Next, our attention to two interrelated questions. First, is there a scope for the policymaker to use inflation over the business cycle in an attempt to correct the distortions described in Section 2.4 above? To answer this question, we compute the volatility of inflation under Ramsey-optimal policy; the model (consisting of the three constraints in Table 3 and the four first-order conditions in Appendix A) is solved by loglinearizing it around the steady state with optimal long-run inflation. Second, what are the welfare costs of fully stabilizing PPI inflation (around a steady-state value of zero), relative to following Ramsey-optimal policy? The method used for welfare computations consists of a second-order approximation to the equilibrium conditions and follows closely Schmitt-Grohé and Uribe (2007), which is also the source of our calibration of the productivity process—namely, an AR(1) process in logarithms with autocorrelation 0.856 and standard deviation 0.0064.

5.1. Optimal inflation volatility

In principle, inflation under Ramsey-optimal policy in this model will vary over the cycle as the planner attempts to minimize the distortions as described in Section 2.4 above. In order to quantify the inflation fluctuations in the Ramsey equilibrium, Table 4 reports the standard deviations of annualized inflation for five preference specifications[24]: C.E.S. with the benefit of variety set to $0, 1/(\theta-1)$ and 1, respectively; translog; and exponential. For the last two preference specifications we calibrate the corresponding free parameters (σ/f_E and α/f_E, respectively) following Bilbiie et al. (2012), such that the implied elasticity of substitution is the same as under C.E.S-D.S (under our baseline calibration, this implies $\sigma/f_E = \alpha/f_E = 0.354$). The standard deviation of HP-filtered output under Ramsey policy (also reported in the table) is around 3 percent for all preference specifications—a number that is empirically plausible (conditional on there being only productivity shocks).[25]

Under C.E.S.-D.S. preferences optimal policy consists of replicating the flexible-price solution: the Ramsey-equilibrium inflation volatility is zero. Although fluctuations in the flexible-price equilibrium are not optimal (because of the labor distortion), the central bank does not try to smooth these fluctuations using inflation. This conclusion is robust to changes in labor supply elasticity and the elasticity of substitution between goods (result are available upon request). When the benefit of variety and the net markup no longer coincide, optimal long-run policy does not entirely eliminate the corresponding steady-state distortion ("Distortion 2" above), and there is further scope to use inflation in the short run to correct that distortion (the intuition mirroring the one applying in steady state). Quantitatively, however, the movements in inflation are minuscule: even though the parameter values considered ($\epsilon = 0$ or 1) are chosen precisely in order to deliver a "large" distortion, the Ramsey-equilibrium inflation volatilities are, respectively, merely 0.113 and 0.193 percent.

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[24] An earlier, working paper version contains a detailed analysis of the impulse response functions under all preference specifications (Bilbiie et al., 2011).

[25] We have also calculated the standard deviations of inflation and output under a simple Taylor rule ($i_t = 1.5\pi_t$). Under that policy, the standard deviation of inflation is 0.36 percent under C.E.S. and 0.61 under translog and exponential, while that of output is again around 3 percent.

The role of optimal policy in alleviating the dynamic entry distortion (Distortion 3) is best illustrated by using exponential preferences. Recall that for this class of preferences, the conditions of Proposition 1 are met and zero long-run inflation is optimal; but desired markups vary with the number of producers (because the elasticity of substitution between goods is a decreasing function of N_t). To understand the inflation dynamics implied by these preferences, consider the New Keynesian Phillips curve loglinearized around the (optimal, under these preferences) zero-inflation steady-state:

$$\pi_t = \beta(1-\delta)E_t\pi_{t+1} - \frac{\theta(N)-1}{\kappa}\left(\hat{\mu}_t - \hat{\mu}_t^*\right), \tag{7}$$

$$\text{where } \hat{\mu}_t^* = -\frac{\theta'(N)N}{\theta(N)(\theta(N)-1)}\hat{N}_t^* = -\frac{1}{1+\alpha N}\hat{N}_t^* \tag{8}$$

is the desired, flexible-price markup (using that $\theta'(N)=\alpha$) and variables with a hat are expressed in log-deviations from steady state. Variations in desired markups are akin to the "cost-push shocks" used in the New Keynesian literature to justify short-run deviations from price stability (see e.g. Woodford, 2003); different from those shocks, however, they are entirely endogenous here. The elasticity of desired markups to the number of firms is equal to the inverse of the elasticity of substitution $1+\alpha N$ and hence will depend on α/f_E by virtue of an equation identical to (6).

A central bank who wanted to entirely undo the dynamic entry distortion would set inflation to aim for a constant path of *realized* markups, $\hat{\mu}_t = 0$; this policy is, however, not feasible because of the resource costs of price adjustment implied by inflation. Abstracting for the moment from this cost and assuming that this policy were feasible, it would imply that inflation obeys (setting $\hat{\mu}_t = 0$ in (7) and iterating forward):

$$\pi_t = \frac{\alpha N}{\kappa}E_t\sum_{i=0}^{\infty}[\beta(1-\delta)]^i\hat{\mu}_{t+i}^*. \tag{9}$$

Intuitively, the central bank needs to neutralize movements in the "natural", flexible-price markup generated by entry. Since a positive productivity shock generates an increase in entry and the number of producers and hence a fall in desired markups (by (8)), the central bank will typically need to engineer a deflationary path, in order to keep realized markups constant. However, using inflation entails resource costs, which need to be weighed against these benefits—and so inflation variations under optimal policy will typically be smaller than those implied by (9). For the baseline calibration considered in Table 4, Ramsey-equilibrium inflation volatility is only 0.019 percent. Therefore, the dynamic entry distortion does not in itself justify deviations from price stability (this conclusion is robust to using a calibration that implies a very large distortion, i.e. to a very low value of α/f_E; results are available upon request).

Finally, under translog preferences there are in principle two reasons why optimal inflation would vary over the cycle, since both the static and dynamic entry distortions operate—the former calls for *short-run inflation* (to decrease the markup level towards the benefit of variety), while the latter implies *short-run deflation* in order to smooth the path of markups. The net effect of these two forces is short-run *deflation* for standard calibrations. Yet again, the equilibrium fluctuations in Ramsey-optimal inflation are minuscule: the standard deviation is merely 0.036 percent under the baseline calibration, suggesting that short-run price stability (around a non-zero long-run trend) is close to optimal when both entry distortions apply.[26]

To sum up, our results concerning inflation volatility under Ramsey policy are in line with those obtained by a large literature (some of which are reviewed in the introduction) that has found that short-run price stability is a robust policy prescription in models featuring a variety of distortions. The inflation volatility numbers reported in Table 4 are of the same order of magnitude as those found by Schmitt-Grohé and Uribe (2004, 2007) in different models featuring a number of other distortions.

5.2. The welfare cost of price stability

How much would be lost by pursuing a policy of full price stability (zero inflation at all times) rather than the Ramsey-optimal policy studied in this paper?[27] Such welfare comparisons face two main challenges in our model. First, the model has an endogenous state variable (the number of firms); any meaningful welfare comparison should therefore ensure that the initial value for the endogenous state variable is the same across the two policy scenarios and take into account the full transition. A side implication of this is that, of course, welfare comparisons based merely on the steady state are potentially spurious. Second, the Ramsey equilibrium features two additional, non-fundamental state variables: the lagged Lagrange multipliers on the dynamic constraints of the planner. These variables are naturally absent from the equilibrium under the alternative policy, which raises the issue of choosing their *initial values* under Ramsey policy. The literature, starting with the

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[26] This result is robust to considering different values of σ/f_E (lower values imply a higher elasticity of desired markups to number of producers, and hence a stronger dynamic entry distortion—since under translog an equation like 8 also holds, with σ instead of α) and different values for the other parameters; the results of these robustness exercises are available upon request.

[27] Welfare losses are defined, in the Lucas (1987) tradition, as the units of steady-state consumption that we would need to give the household in order to make it indifferent (in terms of expected present discounted utility) between a certain policy (zero inflation in and out of steady state) and the benchmark Ramsey-optimal equilibrium. Our computation method follows closely the method of Schmitt-Grohé and Uribe (2007)—details are available upon request.
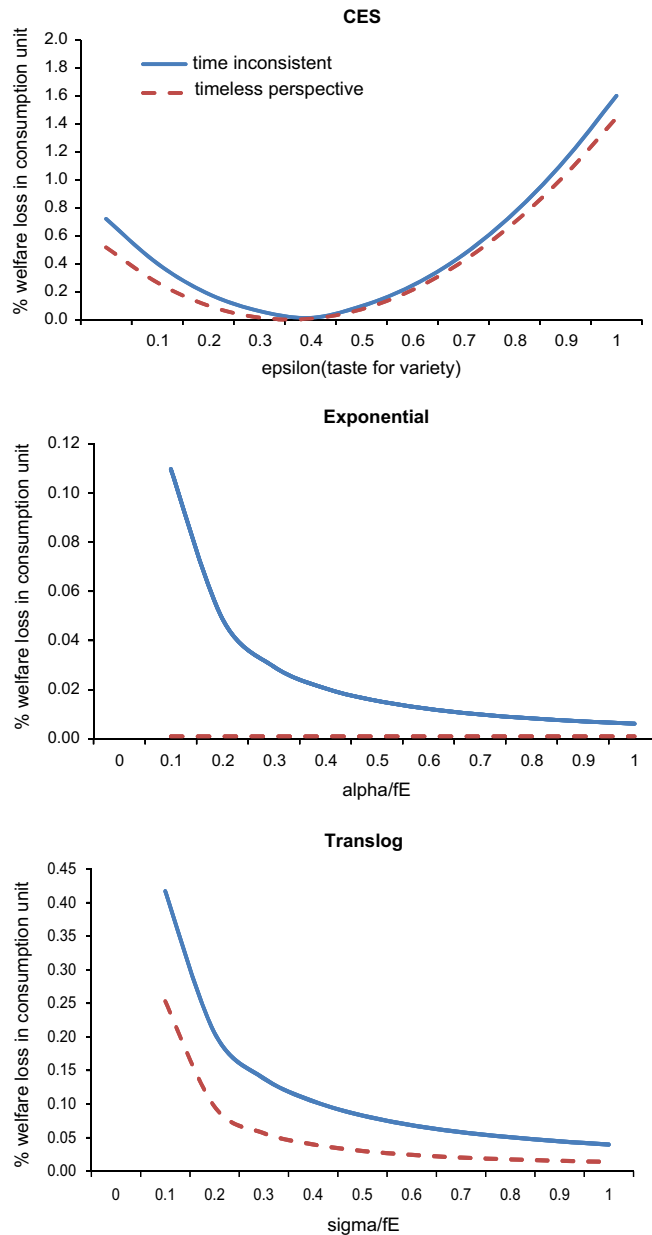
Fig. 3. Welfare losses of fully stabilizing inflation relative to timeless-optimal (red dashed) and time-inconsistent (blue solid) Ramsey policy. Units are percentage points of steady-state consumption. (For interpretation of the references to color in this figure caption, the reader is referred to the web version of this paper.)

influential paper of King and Wolman (1999), suggests two possibilities: zero, or the Ramsey steady-state values. Under the former choice, policy is *not timeless-optimal*: the inflation rate chosen in the initial period t_0 has no consequence for expectations prior to t_0; therefore, policy chosen in any period after t_0 is not a continuation of policy chosen in t_0. The planner has an incentive, under this scenario, to exploit initial conditions because there is no value to fulfilling the constraint at t_0. When the initial values of the co-state variables are set to their Ramsey equilibrium steady-state values, policy is *timeless-optimal*: the value of relaxing the initial-period constraint is no longer zero (as in the previous scenario), but equal to the value that is optimal from a long-run perspective, as in King and Wolman (1999) and Khan et al. (2003).[28]

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[28] Woodford (2003, Ch. 7) uses a different definition of timeless-optimal policy, whereby initial values depend in a precise way on the state of the world.

Fig. 3 plots the welfare losses of price stability under each of our three preference classes (C.E.S., translog and exponential); for each preference specification, the welfare loss is computed relative to the two Ramsey policy scenarios described above, labeled "time-inconsistent" (blue solid line) and "timeless-optimal" (red dashed line).

For any preference specification, the welfare loss is potentially due to three factors: (i) long-run (having the wrong long-run inflation target); (ii) short-run (being unable to use inflation over the cycle) and (iii) incentives to exploit initial conditions. Given our results in the previous section on the magnitude of inflation volatility under Ramsey policy, it can be conjectured that the bulk of the welfare difference between the zero-inflation policy and the timeless-optimal Ramsey policy is in fact due to factor (i): having the wrong long-run inflation target. Consistent with this intuition, the welfare loss relative to timeless-optimal policy for exponential preferences (for which the optimal long-run inflation target is indeed zero) is nil: there is no loss associated to not using inflation to correct the dynamic entry distortion. On the other hand, the loss from having the wrong inflation target and not being able to correct the *static entry distortion* can be quite large under C. E.S. (0.6 percent of steady-state consumption with $\epsilon = 0$, and about 1.5 percent with $\epsilon = 1$). Under translog, its value depends on σ/f_E: at very low values of the parameter the welfare loss becomes sizeable, but for values close to our benchmark calibration ($\sigma/f_E = 0.354$), which deliver a reasonable elasticity of substitution, the loss is only about 0.05 percent of steady-state consumption.

Given a preference specification, the welfare loss relative to non-timeless-optimal Ramsey policy is uniformly higher than the one relative to timeless-optimal policy: the difference between the two is indeed a proxy for the policymaker's incentive to renege on the previously chosen policy.[29] The clearest illustration of this is obtained for exponential preferences: unlike in the timeless-optimal scenario studied above, there is a loss associated to fully stabilizing prices, which comes exclusively from the inability of the policymaker to exploit initial conditions.

The policymaker's temptations to renege previous commitments come in our framework from two sources. The first one is associated to the multiplier on the price setting constraint and is the standard one operating in the fixed-variety sticky-price model (King and Wolman, 1999, provide for an early version of that analysis); in our framework, the incentive to exploit this channel is strengthened because unexpected inflation affects markups, which determine the entry decision. The second source is novel, and is related to the multiplier on the dynamic constraint governing product creation (the Euler equation for shares).

This can be best understood under exponential preferences, because the second channel is the only one operating in that framework (the steady-state value of the Lagrange multiplier on the Phillips curve is zero, as already shown analytically in the proof of Proposition 1). Remember that, as discussed above when deriving (9), the central bank's objective under exponential preferences is to minimize markup variability. In response to a positive productivity shock, the planner faces a path of desired markups that is decreasing, following an inverted hump-shape, in response to the hump-shaped increase in the number of products. Under non-timeless-optimal policy, the central bank chooses a higher initial inflation rate initially (relative to timeless-optimal policy) in order to bring markups down already from the initial period and hence smooth the intertemporal path of markups. It can do so, precisely because the value of the firm (which, through the free entry condition, is linked to marginal cost and hence to the markup) need not fulfill the Euler equation for shares constraint in the initial period. The magnitude of this temptation is, however, very small: the welfare gain from reneging on the timeless-optimal plan is, for our baseline calibration $\alpha/f_E = 0.354$, merely 0.02 percent of steady-state consumption.

The temptation to ignore initial constraints is somewhat larger when both initial conditions can be exploited: the welfare gain from doing so is about 0.2 percent under C.E.S. (with $\epsilon = 0$) and almost 0.1 percent under translog preferences (at $\sigma/f_E = 0.354$).

6. Conclusions

A large literature studies optimal monetary policy in the presence of imperfect price adjustment and other real or monetary distortions. A general conclusion of this literature is that the optimal long-run rate of inflation is zero or very close to zero. Moreover, in an environment in which productivity shocks are the only source of uncertainty, perfectly stabilizing prices over the business cycle (and hence replicating the flexible-price allocation) is optimal, or nearly so. This paper argues that the optimal *long-run* rate of inflation can be significantly different from zero in an environment with endogenous entry and product variety, but price stability (around this long-run trend) is close to optimal over the cycle. The sign and magnitude of the optimal long-run inflation rate depend on the balance between the market incentives for entry (measured by the desired markup) and the welfare benefit of product variety to consumers. When the market outcome results in too much entry relative to what the consumer values (when the markup is higher than the benefit of variety), positive long-run inflation is optimal because it erodes long-run markups and profit margins, and it reduces entry. In the opposite case, long-run deflation is optimal, because it increases steady-state markups and hence provides more incentives for entry. The long-run rate of inflation is zero only in the knife-edge case of C.E.S.-D.S. preferences, for which the net markup is equal to the benefit of variety. With translog preferences, an important determinant of the optimal long-run rate of inflation is the sunk

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[29] The only exception is, trivially, when both welfare losses are zero: under C.E.S.-D.S. preferences or when either α/f_E or σ/f_E become very large so that goods are closely substitutable and markups are low.

entry cost: less regulation implies more firms, more closely substitutable goods, and lower desired markups. This instead means that there is less scope for using inflation in order to reduce markups.

The welfare loss from having a flat price level can be sizeable, depending on the benefit of variety parameter and, under translog, on the degree of entry regulation. Since the volatility of inflation under Ramsey policy is negligible for all preference specification considered, it follows that the bulk of the welfare loss of pursuing a constant price level is due not recognizing the non-zero long-run target for inflation implied by Ramsey policy. Finally, we quantify the policymaker's temptations to renege on previously-chosen policies, for which our framework provides a new source: achieving higher utility through the product creation margin.

Our analysis provides a hitherto unexplored argument for potentially significant deviations from long-run price stability, with deviations of potential magnitudes not encountered in other economic environments no matter the type of underlying distortions (Schmitt-Grohé and Uribe, 2011, review exhaustively the robustness of the "zero optimal inflation" prescription). While our conclusions for optimal monetary policy are derived using the Rotemberg (1982) model, they should in principle carry to other forms of nominal rigidity, such as the widely used Calvo (1983)–Yun (1996) model. Three ingredients are essential for our results to hold, and they are all likely to be robust to the specific type of price stickiness. The first consists of the distortions outlined in Section 2.4, which all depend on the markup; this is orthogonal to the type of price rigidity. The second is the inverse relationship between average markups and inflation outlined in Section 2.1; in the Calvo–Yun model too, a similar negative relationship between inflation and average markups holds, as explained originally in King and Wolman (1996) and Goodfriend and King (1997). The third is a direct welfare cost of inflation, such as the quadratic resource cost in the Rotemberg framework; but this is also true of the Calvo–Yun model through dispersion in relative prices, which can also be expressed as a quadratic function of inflation (see Rotemberg and Woodford, 1997).

Since the single most important determinant of optimal long-run inflation is the balance of markups and benefit of variety, our findings point to the need for continued study of the determinants of markups and serious empirical investigation of the nature of preferences for variety.[30] The one preference specification (translog) that is not subject to identification problems related to the benefit of variety (Lewis and Poilly, 2012) and has several merits in fitting business cycle facts pertaining to entry, markups and profits (Bilbiie et al., 2012), implies that the optimal long-run rate of inflation is at least 1 percent under reasonable parameter values but with no indexation, and is increasing with the degree of entry regulation and with price indexation.

Acknowledgments

Appendix A. Ramsey-optimal policy and variety

In the model with endogenous entry and product variety summarized in Table 1, the Ramsey problem is solved by assuming that the central bank chooses the optimal paths of all 13 endogenous variables that maximize the present discounted value of household utility, taking as constraints the 12 private agents' decision rules. Adding one Lagrange multiplier on each constraint, one obtains a system of 25 equations (12) private agents' decision rules and 13 first-order conditions and 25 unknowns (13 original variables and 12 Lagrange multipliers). The resulting system can be solved numerically by standard perturbation methods.

The problem is simplified as described in text, to obtain the reduced Ramsey (3) whose solution is outlined here. The four first-order conditions with respect to labor, inflation, labor in the consumption sector, and the number of product varieties

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[30] While research on markups has already been extensive (see, for instance, Rotemberg and Woodford, 1999, and, more recently, Nekarda and Ramey, 2010), there is little empirical evidence on the benefit of variety to consumers. We elaborate further on this point below.

next period are, respectively, at any time t:

$$-h_L(L_t)-(1-\delta)\eta_{1,t}\frac{Z_t}{f_E}+\theta(N_t)\eta_{2,t}L_{C,t}h_{LL}(L_t)\left(1-\frac{\kappa}{2}\pi_t^2\right)$$
$$+\eta_{3,t}h_{LL}(L_t)\frac{f_E}{Z_t}-\eta_{3,t-1}(1-\delta)\left(h_{LL}(L_t)\frac{f_E}{Z_t}-\frac{L_{C,t}h_{LL}(L_t)}{N_t}\right)=0,$$

$$-\frac{\pi_t}{1-\frac{\kappa}{2}\pi_t^2}-\eta_{2,t}\left[\begin{array}{c}\theta(N_t)\pi_tL_{C,t}h_L(L_t)+(1-\theta(N_t))\pi_t\\+(1+2\pi_t)+\beta(1-\delta)\pi_{t+1}(1+\pi_{t+1})\frac{N_t}{N_{t+1}}\frac{\kappa\pi_t}{1-\frac{\kappa}{2}(\pi_{t+1})^2}\end{array}\right]+\eta_{2,t-1}(1-\delta)\frac{N_{t-1}}{N_t}\left(1-\frac{\kappa}{2}\pi_{t-1}^2\right)\frac{1+\frac{\kappa}{2}\pi_t^2+2\pi_t}{\left(1-\frac{\kappa}{2}\pi_t^2\right)^2}=0,$$

$$\frac{1}{L_{C,t}}+(1-\delta)\eta_{1,t}\frac{Z_t}{f_E}+\theta(N_t)\eta_{2,t}h_L(L_t)\left(1-\frac{\kappa}{2}\pi_t^2\right)+\eta_{3,t-1}(1-\delta)\frac{h_L(L_t)}{N_t}=0,$$

$$\beta\frac{\epsilon(N_{t+1})}{N_{t+1}}+\eta_{1,t}-\eta_{1,t}\beta(1-\delta)-\beta\eta_{2,t+1}\theta'(N_{t+1})\left(1-\frac{\kappa}{2}\pi_{t+1}^2\right)\left[1-L_{C,t+1}h_L(L_{t+1})\right]-\eta_{2,t}\beta(1-\delta)\kappa\pi_{t+1}(1+\pi_{t+1})\frac{N_t}{N_{t+1}^2}\frac{1-\frac{\kappa}{2}\pi_t^2}{1-\frac{\kappa}{2}\pi_{t+1}^2}$$
$$+\beta^2(1-\delta)\eta_{2,t+1}\kappa\pi_{t+2}(1+\pi_{t+2})\frac{1-\frac{\kappa}{2}\pi_{t+1}^2}{1-\frac{\kappa}{2}\pi_{t+2}^2}\frac{1}{N_{t+2}}+\beta(1-\delta)\eta_{3,t}\frac{1-L_{C,t+1}h_L(L_{t+1})}{N_t^2}=0,$$

where the last equation uses $\epsilon(N_t)\equiv\rho'(N_t)/\rho(N_t)N_t$; furthermore, initial conditions (at time $t=-1$) are necessary for the two Lagrange multipliers corresponding to forward-looking constraints, $\eta_{2,-1}$; $\eta_{3,-1}$ as in i.a. King and Wolman (1999). The choice of these initial values is discussed further in text.

The non-stochastic steady state is such that:

$$-h_L(L)-(1-\delta)\eta_1+\theta(N)\eta_2L_Ch_{LL}(L)\left(1-\frac{\kappa}{2}\pi^2\right)$$
$$+\eta_3h_{LL}(L)-\eta_3(1-\delta)h_{LL}(L)\left(1-\frac{L_C}{N}\right)=0,$$

$$-\frac{\pi}{1-\frac{\kappa}{2}\pi^2}-\eta_2\left[\begin{array}{c}\theta(N)\pi L_Ch_L(L)+[1-\theta(N)]\pi\\+(1+2\pi)+\beta(1-\delta)\pi(1+\pi)\frac{\kappa\pi}{1-\frac{\kappa}{2}(\pi)^2}\end{array}\right]+\eta_2(1-\delta)\frac{1+\frac{\kappa}{2}\pi^2+2\pi}{1-\frac{\kappa}{2}\pi^2}=0,$$

$$\frac{1}{L_C}+(1-\delta)\eta_1+\theta(N)\eta_2h_L(L)\left(1-\frac{\kappa}{2}\pi^2\right)+\eta_3(1-\delta)\frac{h_L(L)}{N}=0,$$

$$\beta\frac{\epsilon(N)}{N}+\eta_1[1-\beta(1-\delta)]-\beta\eta_2\theta'(N)\left[1-\frac{\kappa}{2}\pi^2\right]\left[1-L_Ch_L(L)\right]-(1-\beta)\beta(1-\delta)\kappa\frac{\eta_2\pi(1+\pi)}{N}+\beta(1-\delta)\eta_3\frac{1-L_Ch_L(L)}{N^2}=0,$$

$$[\theta(N)-1]\left[1-\frac{\kappa}{2}\pi^2\right]+\kappa[1-\beta(1-\delta)](1+\pi)\pi-\theta(N)\left(1-\frac{\kappa}{2}\pi^2\right)L_Ch_L(L)=0,$$

$$\delta N-(1-\delta)\left(L-L^C\right)=0,$$

$$[1-\beta(1-\delta)]h_L(L)-\beta(1-\delta)\frac{1-L_Ch_L(L)}{N}=0,$$

where the first four equations are the steady-state versions of the first-order conditions outlined above, and the last three correspond to the constraints of the reduced Ramsey problem in Table 3.

Since the problem outlined in (3) is a concave optimization problem, it will have a unique steady-state solution. The proof of Proposition 1 starts by proving the "only if" statement (i.e. that if the steady-state of the solution features zero inflation, then preferences satisfy (4)). Conjecture therefore that $\pi=0$ in steady state. The first-order condition for the choice of inflation (the second equation above) then implies that the Lagrange multiplier on the Phillips curve is zero:

$$\eta_2=0.$$

Naturally, the constraint associated to imperfect price adjustment is not binding in steady state when there is zero inflation. The other conditions evaluated at this equilibrium imply (using the notation $\beta^{-1}=1+r$, so r is the discount rate):

$$1+(1-\delta)\frac{\eta_1}{h_L(L)}-\frac{\eta_3}{L}\varphi[\delta+[\theta(N)-1](r+\delta)]=0,$$

$$1+(1-\delta)\eta_1L_C+(1-\delta)\frac{\theta(N)-1}{\theta(N)}\frac{\eta_3}{N}=0,$$

$$\epsilon(N)+(r+\delta)\eta_1N+(1-\delta)\frac{1}{\theta(N)}\frac{\eta_3}{N}=0,$$

$$\frac{\theta(N)-1}{\theta(N)}=L_Ch_L(L),$$

$$\delta N = (1-\delta)\left(L - L^C\right),$$

$$h_L(L)N = \frac{1-\delta}{r+\delta}\frac{1}{\theta(N)}.$$

The second and third equations imply

$$1 + (1-\delta)\eta_1 L_C - \epsilon(N)[\theta(N)-1] - [\theta(N)-1](r+\delta)\eta_1 N = 0. \tag{10}$$

But the fourth and sixth equations imply:

$$[\theta(N)-1](r+\delta)N = (1-\delta)L_C,$$

which substituted into (10) yields

$$\epsilon(N) = \frac{1}{\theta(N)-1}.$$

This proves the "only if" part of Proposition 1. But since the steady state is unique, the "if" statement follows immediately: since there exists one steady state for preferences satisfying (4) and the steady state is unique by virtue of the concavity of the Ramsey problem, it follows that if preferences satisfy (4), then steady-state inflation is necessarily zero.

Appendix B. Price indexation

This appendix outlines some of the implications of price indexation for Ramsey-optimal monetary policy. Assume that firms index to *past inflation* and pay an adjustment cost given by[31]:

$$pac_t(\omega) \equiv \frac{\kappa}{2}\left[\frac{p_t(\omega)}{p_{t-1}(\omega)}\left(\frac{p_{t-1}(\omega)}{p_{t-2}(\omega)}\right)^{-\gamma} - 1\right]^2 \frac{p_t(\omega)}{P_t} y_t^D(\omega), \tag{11}$$

where $\gamma \in [0,1]$ is the *indexation parameter*. Under this indexation scheme, it can be easily shown that the long-run Phillips curve becomes

$$\mu(\pi) - 1 = \frac{\theta}{(\theta-1)\left\{1 - \frac{\kappa}{2}[(1+\pi)^{1-\gamma}-1]^2\right\} + \kappa\frac{r+\delta}{1+r}(1+\pi)^{1-\gamma}\left[(1+\pi)^{1-\gamma}-1\right]} - 1. \tag{12}$$

Note that this nests the no-indexation case when $\gamma = 0$ and the full-indexation case when $\gamma = 1$. Under full indexation, however, the steady-state inflation rate will be *indeterminate*: there is no long-run cost of using inflation ((11) evaluated at the steady-state implies $pac=0$) and no benefit of inflation (the long-run Phillips curve (12) is vertical $\mu = \theta/(\theta-1)$). For values of γ in the open interval $(0,1)$, our long-run results change as follows. The optimal rate of inflation (deflation) is increasing (in absolute value) with the indexation parameter γ. When indexation is almost full (γ is close to 1) the optimal rate of long-run inflation (deflation) is indeed very large.

The reasons why indexation implies larger deviations from long-run price stability are twofold: first, indexation lowers the welfare cost associated with a given long-run inflation rate (the steady-state adjustment cost $PAC = \kappa/2((1+\pi)^{1-\gamma}-1)^2 Y^C$ is decreasing in γ). Second, indexation causes the long-run Phillips curve to steepen, and hence implies that larger inflation rates are required to achieve a certain change in long-run markup that is necessary in order to bring it closer to the benefit of variety.

Fig. B.1 illustrates these results quantitatively, plotting the optimal long-run rate of inflation for C.E.S. preferences as a function of the indexation parameter γ, for the two extreme value of the benefit of variety: $\epsilon = 0$ and $\epsilon = 1$, respectively.[32] For empirically plausible degrees of indexation (estimated for instance by Smets and Wouters, 2007, in the range between 0.25 and 0.5), optimal long-run inflation rates range from around 6 percent inflation (for $\epsilon = 0$) to around 10 percent deflation (for $\epsilon = 1$). A similar picture occurs under translog preferences (not plotted), where the optimal long-run rate of inflation, given $\sigma/f_E = 0.353$, ranges from 1.03 percent under no indexation to approximately 10 percent when $\gamma = 0.9$ (results are available upon request).

The working paper version (Bilbiie et al., 2011) also illustrates the implications of price indexation for short-run optimal policy, showing that indexation does not significantly affect the conclusions of the analysis for the no-indexation case, for all the preferences considered. Most importantly, the paths of real variables (consumption, hours, and number of firms) are invariant to the indexation parameter: indexation makes inflation less costly, but it also makes the Phillips curve more vertical, meaning that a larger inflation rate has a smaller effect on real variables. To conclude, indexation affects significantly the optimal monetary policy prescriptions in the long run, but not in the short run.

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[31] A simple indexation scheme whereby firms index to a constant inflation rate $\tilde{\pi}$, rather than past inflation, would merely imply that the optimal long-run inflation rate is uniformly increased by the constant $\tilde{\pi}$, without affecting any of the other results.

[32] The domain of γ is restricted to values lower than 0.9 because, for larger values, the optimal rate of long-run inflation (deflation) becomes extremely large (close to 600 percent inflation and 300 percent deflation, respectively, for $\gamma = 0.99$).
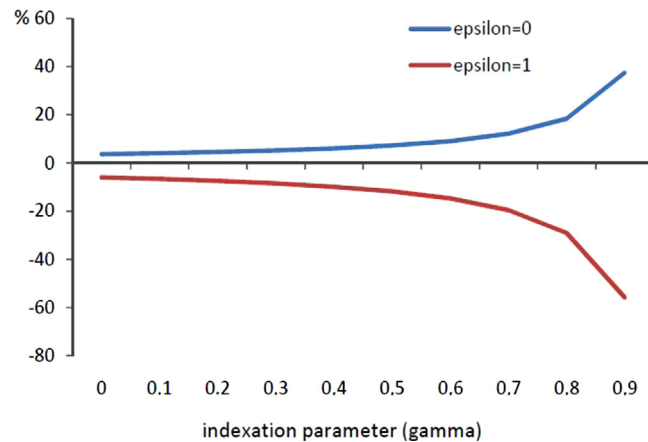
Fig. B.1. The optimal long-run inflation rate as a function of the indexation parameter γ.

Appendix C. Supplementary data

Supplementary data associated with this article can be found in the online version at http://dx.doi.org/10.1016/j.jmoneco.2014.02.006.

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