Interest rate rules for fixed exchange rate regimes

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Abstract

This paper shows that properly designed interest rate rules can be consistent with maintaining exchange rate stability. It sheds light on the relation between interest rate rules, exchange rate regimes, and determinacy of the rational expectation equilibrium in a modern macroeconomic framework.
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1. Introduction

The performance of alternative rules for interest rate setting by central banks of open economies has been the subject of increasing attention in the literature.[1] Interest rate rules can be used to achieve a variety of policy goals. Among these, properly designed interest rate rules can be consistent with maintaining exchange rate stability.

This paper sheds light on the relation between interest rate rules, exchange rate regimes, and determinacy of the rational expectation equilibrium of the economy.

As discussed in Obstfeld and Rogoff (1996),[2] although a fixed exchange rate implies equality between the domestic and foreign interest rates, pegging the domestic interest rate to the foreign one is not sufficient to fix the exchange rate in all periods. Indeed, simple interest rate pegging by the follower country in the exchange rate arrangement results in indeterminacy of the exchange rate and (plausibly) of the real economy. The solution proposed by Obstfeld and Rogoff combines interest rate pegging with the specification of at least one point in the money supply path, so that the level of the nominal exchange rate is determined. This is the device used by Taylor (1994) and Wieland (1996).

Our contribution consists of showing how it is possible to implement a fixed exchange rate arrangement in a framework in which policy in the follower country does not need to specify any point in the path of money supply. We propose a class of interest rate rules for the follower country that determine a unique equilibrium with a fixed exchange rate when combined with the credible threat to suspend currency convertibility if the exchange rate settles on an explosive path. The rules we consider produce equality between the domestic and foreign interest rate *endogenously* in all periods as a feature of the rational expectations equilibrium. We show that there is a multiplicity of rules consistent with a fixed exchange rate regime for the same exchange rate parity. Multiple equilibria can arise only when the commitment to the rule that yields exchange rate stability and determinacy (or to the threat of suspending convertibility) is not perfectly credible.

Determinacy of the fixed exchange rate does not necessarily imply determinacy of other domestic (or foreign) variables. Assuming that also the leader country is following an interest rule, for the world economy to be determinate, it is necessary that the interest setting rules of *both* countries be consistent with determinacy.[3] The rule followed by the leader country determines the *nature* of the fixed exchange rate regime because it sets the course of monetary policy for the world economy. In this sense, there is a multiplicity of fixed exchange rate regimes for the same exchange rate parity: changes in the rule of the leader country face the follower with changes in the global monetary environment and in the welfare

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[1] Several contributions are collected in the web page on 'Monetary Policy Rules in Open Economies,' http://www.geocities.com/monetaryrules/mpoe.htm.

[2] P. 556, footnote 44.

[3] Woodford (2003, Chapter 2) discusses the importance of ensuring equilibrium determinacy in monetary models. See also Carlstrom and Fuerst (2001).

level implied by sticking to the fixed exchange rate commitment. However, for given policy of the leader country, the rule through which the exchange rate commitment is implemented by the follower is welfare-neutral in a determinate world economy.

The structure of the paper is as follows. Section 2 revisits the problem discussed in Obstfeld and Rogoff (1996) by showing that interest rate pegging does not yield a fixed exchange rate and generates indeterminacy, holding the monetary rule of the leader country exogenous. Section 3 shows how to design interest rate rules that are consistent with a fixed exchange rate commitment. Section 4 discusses the importance of the rule followed by the leader country for determinacy of the world equilibrium and the nature of the global monetary regime. Section 5 illustrates the arguments of the previous sections by means of simple log-linear examples. Section 6 concludes.

2. The mirage of fixing the exchange rate through interest rate pegging

We begin by revisiting the pitfalls of interest rate pegging discussed by Obstfeld and Rogoff (1996). We assume that the world economy consists of two countries: home and foreign. Home is the follower country in the exchange rate regime – the country that is trying to peg the exchange rate; foreign is the leader. We keep the formal apparatus at a minimum and do not present a full-fledged microfounded model of the two economies. Initially, we hold the leader's policy rule (if any) as given and focus on the follower country. We only restrict the foreign country's policymaking to be consistent with a 'leadership' position in a fixed exchange rate regime by assuming that the foreign country never targets a level of the exchange rate that differs from that chosen by home.[4]

Agents in this country maximize a utility function that depends on consumption and, possibly, other arguments, such as leisure or money balances. We assume that the period utility function is additively separable in the various arguments and well-behaved in each of them.[5] Among the financial assets that agents can hold, there are bonds denominated in units of the domestic currency and bonds denominated in units of the foreign currency. The time t interest rate on home (foreign) currency bonds is i_t (i_t^*). Agents receive interest payments at time $t+1$. We denote consumption with C, the Consumer Price Index (CPI) with P, and the agents' discount factor with β. E_t is the rational expectation operator, conditional on information available at time t.

The Euler equation for holdings of domestic bonds is

$$\beta(1 + i_t)E_t\left(\frac{U'(C_{t+1})}{P_{t+1}}\right) = \frac{U'(C_t)}{p_t}. \tag{1}$$

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[4]We relax this assumption below.

[5]The assumption of additive separability simplifies the notation in what follows. It does not affect the nature of our results in any significant way.

Letting S denote the exchange rate (in units of the domestic currency per unit of the foreign one), holdings of foreign currency bonds are determined by

$$\beta(1 + i_t^*)E_t\left(S_{t+1}\frac{U'(C_{t+1})}{P_{t+1}}\right) = S_t\frac{U'(C_t)}{P_t}. \tag{2}$$

Eqs. (1) and (2) imply

$$(1 + i_t)E_t\left(\frac{U'(C_{t+1})}{P_{t+1}}\right) = (1 + i_t^*)E_t\left(\frac{S_{t+1}}{S_t}\frac{U'(C_{t+1})}{P_{t+1}}\right). \tag{3}$$

This equation ensures that the consumer is indifferent at the margin between domestic and foreign bonds. In a perfect foresight framework, it reduces to

$$1 + i_t = (1 + i_t^*)\frac{S_{t+1}}{S_t}, \tag{4}$$

the familiar uncovered interest parity condition (UIP).

Let us focus temporarily on Eq. (4). Setting $i_t = i_t^*$ at all dates t implies $S_{t+1} = S_t$ in all periods in which UIP holds. For this reason, one may think that the interest rate rule $i_t = i_t^*$ is consistent with a fixed exchange rate regime. But the UIP condition is violated ex post in all periods in which unexpected shocks happen.[6] In these circumstances, $i_t = i_t^*$ will ensure zero depreciation between t and $t + 1$. Yet, if an unexpected shock happens at the beginning of period t, an instantaneous movement of the level of the exchange rate will be observed at time t, as UIP does not hold ex post between periods $t - 1$ and t. Thus, in a perfect foresight setting, $i_t = i_t^*$ fails to yield stability of the exchange rate.

The same problem exists in a stochastic model under rational expectations. If the domestic interest rate is pegged to the foreign rate, condition (3) boils down to

$$E_t\left[\frac{U'(C_{t+1})}{P_{t+1}}\left(1 - \frac{S_{t+1}}{S_t}\right)\right] = 0. \tag{5}$$

$S_{t+1} = S_t$ in all periods solves this equation.[7] However, Eq. (5) must hold at all dates only in expected value. Unanticipated deviations of the exchange rate from the constant path – generated by unexpected disturbances to the economy – will still be consistent with the arbitrage condition and rule $i_t = i_t^*$.

Failure to stabilize the level of the exchange rate is not the only problem of interest rate pegging. Even if setting $i_t = i_t^*$ were sufficient to implement a fixed exchange rate equilibrium, a fundamental problem of determinacy of the latter would still exist, as there can be an infinite number of fixed exchange rate equilibria under the rule $i_t = i_t^*$. To see this, suppose the domestic central bank targets the level of the exchange rate S^*. $S_t = S^*$ at all dates t is a solution to Eq. (5). But so is $S_t = S^* + \mu$ at all dates t, where μ can assume any value. There are infinitely many possible fixed exchange rate equilibria under rational expectations.

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[6]With unexpected shocks here we refer to the surprises that are allowed for in perfect foresight models at the beginning of the period t in which the consumer is solving her/his optimization problem.

[7]The marginal utility of consumption and the price level are always positive.

We can relate the indeterminacy of the exchange rate under interest rate pegging to Kareken and Wallace's (1981) indeterminacy result.[8] There, agents in the domestic and foreign economies are free to hold the currencies of both countries and are indifferent between the two currencies as long as their rates of return are equal. Purchasing power parity (PPP) holds. Governments print money to finance spending. In equilibrium, absence of unexploited arbitrage opportunities between the two currencies implies that there cannot be anticipated changes in the exchange rate. However, any constant value of the exchange rate $S \in (0, \infty)$ can be an equilibrium, because combining the two governments' budget constraints yields one equation in two variables: the exchange rate and the price level. As under the $i_t = i_t^*$ policy in our example, monetary policy does not pin down the exchange rate in the Kareken-Wallace model once arbitrage opportunities have been removed by equalization of returns.

To summarize: (i) Interest rate pegging fails to deliver a fixed exchange rate. For example, there are always equilibria in which the exchange rate is not fixed and jumps unexpectedly at each point in time. (ii) In addition, there exists a multiplicity of fixed exchange rate equilibria. Results (i) and (ii) together yield a simple 'second generation' explanation of currency crises that focuses on sudden 'sunspot'-driven shifts from one path of the exchange rate to another.[9] These shifts would be entirely consistent with the rule followed by the central bank and with agents' rational optimizing behavior. Given the link between asset prices and the real economy through the Euler Eqs. (1) and (2), indeterminacy and instability of the exchange rate would translate into indeterminacy and instability of the real economy.[10] Results (i) and (ii) imply that analyses of the properties of fixed exchange rates should not focus on interest rate pegging as the operational rule that implements the regime. Taking $i_t = i_t^*$ as the rule consistent with a fixed exchange rate fails to consider points (i) and (ii).

Having established this, we turn to the design of interest rules for exchange rate stability (and determinacy).

3. Designing interest rate rules for exchange rate stability

We begin this section by presenting our proposed rule for exchange rate stability. We next show that, given a credible commitment to the rule we propose, the exchange rate is either uniquely fixed at the central bank's target level, or it explodes to infinite or implodes to zero with positive probability. We then prove that we can rule out these divergent exchange rate paths by supplementing the interest rate rule

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[8]See also Ljungqvist and Sargent (2004, pp. 593–594).

[9]'First generation' explanations focus on fundamental determinants of currency crises (Krugman, 1979). Obstfeld (1994) is an example of 'second generation' model.

[10]We are implicitly assuming that the economy is characterized by some degree of nominal rigidity. In a flexible-price model with (separable) money in the utility function, domestic real balances will be determinate under the rule $i_t = i_t^*$, even if the exchange rate is not, if foreign policy ensures a unique path for i_t^*.

with an additional commitment to suspend market convertibility along these paths. We conclude the section by discussing some implications of our proposed rule for regime credibility and tying our results to earlier literature on exchange rate determinacy and determinacy in monetary models.

3.1. The rule

Suppose the home central bank can *credibly* commit to the rule

$$1 + i_t = (1 + i_t^*)\phi\left(\frac{S_t}{S^*}\right), \tag{6}$$

in which $\phi(.)$ is a *function* with characteristics that we will discuss later.

Two features of rule (6) are worth immediate remarks. First, all elements of rule (6) are known at time t, as the rule involves a direct reaction only to variables that are part of the information set at time t. Second, the zero bound on the nominal interest rate implies a lower bound on the function $\phi(.)$:

$$\phi\left(\frac{S_t}{S^*}\right) \geqslant \frac{1}{1 + i_t^*} \geqslant 0, \tag{7}$$

where the second inequality holds strictly as long as i_t^* is finite.

Substituting rule (6) into the arbitrage condition (3), we obtain

$$E_t\left\{\frac{U'(C_{t+1})}{P_{t+1}}\left[S_{t+1} - S_t\phi\left(\frac{S_t}{S^*}\right)\right]\right\} = 0,$$

which can be rewritten as

$$\sum_{z_{t+1} \in Z_{t+1}} \pi(z_{t+1})\left\{\frac{U'(C_{t+1}(z_{t+1}))}{P_{t+1}(z_{t+1})}\left[S_{t+1}(z_{t+1}) - S_t\phi\left(\frac{S_t}{S^*}\right)\right]\right\} = 0, \tag{8}$$

where Z_{t+1} is the set of all the finite number of states of nature at time $t + 1$ and z_{t+1} is a particular state at date $t + 1$.

We require that the function $\phi(.)$ satisfy certain properties in addition to $\phi(.)$ being a function and the lower bound (7):

- $\phi(1) = 1$;
- $\phi(.)$ continuous;
- $\phi(.)$ monotone non-decreasing;
- $\phi(.)$ differentiable;
- $\phi(.)$ strictly increasing in a neighborhood of $S_t = S^*$.

3.2. Exchange rate dynamics under rule (6)

First, we show that given the above properties of the function $\phi(.)$ in rule (6), there exists only one possible path in which the exchange rate remains always fixed. Along this path, $S_t = S^* \; \forall t$. Second, we show that if $S_t \neq S^*$ at any time t, there exists a

positive probability that the exchange rate will go either to infinite or to zero as time tends to infinite.

As for the first step, observe that, given the properties of the function $\phi(.)$ and the fact that $U'(C_{t+1}(z_{t+1}))/P_{t+1}(z_{t+1})$ is positive, a necessary and sufficient condition for Eq. (8) to be satisfied under a fixed exchange rate is that the term in square brackets be zero in all states z_{t+1} at time $t+1$, i.e.,

$$S_{t+1}(z_{t+1}) = S_t \phi\left(\frac{S_t}{S^*}\right) \tag{9}$$

in all states z_{t+1}. This is a set of non-linear difference equations – one for each state of nature at time $t+1$ – that can be solved graphically, as we do in Fig. 1. (S_t is on the x-axis, $S_{t+1}(z_{t+1})$ is on the y-axis. Of course, there will be a graph for each state of nature.)

$S_t = 0$ at all dates t is always a solution to Eq. (9). However, as long as there is positive demand and finite supply of the foreign currency in the world economy, this solution will be ruled out by agents' optimal behavior. Demand for money can be motivated in several ways in a microfounded model, including the familiar money-in-the-utility-function approach or a cash-in-advance constraint. Money demand will be strictly positive if the opportunity cost of holding currency is finite. A positive demand for the foreign currency, combined with the foreign central bank's commitment to a finite quantity of currency, will ensure that its value in terms of the domestic one is strictly positive, i.e., $S_t > 0$.

$S_t = S^*$ at all dates t is another solution to Eq. (9) under the assumption that $\phi(1) = 1$, which implies $S_t = S^*$ at all dates and in all states of nature. Given the restrictions on the function $\phi(.)$, there is a unique rational expectations, fixed exchange rate solution to Eq. (9).
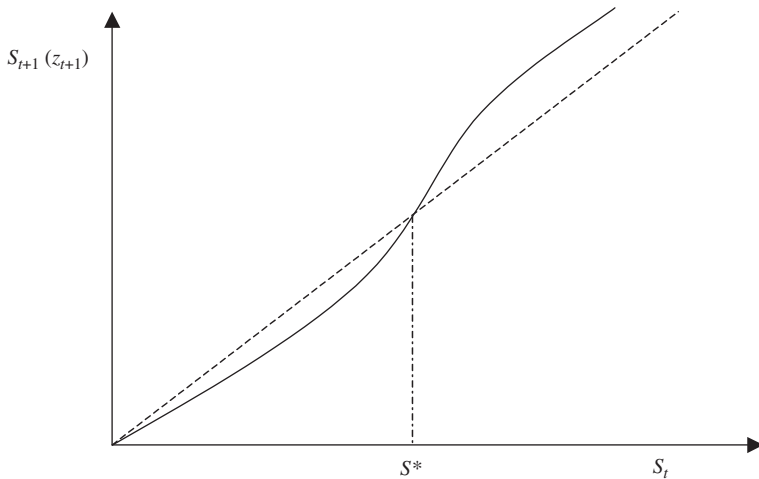

Fig. 1.

Now consider the second step. We show that there are other possible paths for the exchange rate, but if $S_t \neq S^*$ at any time t, there exists a positive probability that the exchange rate will go either to infinite or to zero as time tends to infinite.

Suppose that $S_t > S^*$. Eq. (8) implies that either all the terms in square brackets are zeros or some of the terms are positive and others are negative. In the first case, following Fig. 1, the exchange rate at time $t + 1$, in all states of nature, will be such that $S_{t+1} > S_t$, given the properties on $\phi(.)$. In the second case, if some square-bracket terms are negative, it must be the case that at least one of the terms is positive, in order for the equality in (8) to be satisfied. This implies that there will be at least a state z'_{t+1} in which $S_{t+1}(z'_{t+1}) > S_t \phi(S_t/S^*) > S_t$. Thus, under both situations there exists a state z'_{t+1} with positive probability of occurrence in which $S_{t+1}(z'_{t+1}) > S_t > S^*$. We can then repeat the same argument at time $t + 2$, and find a state z'_{t+2} in which $S_{t+2}(z'_{t+2}) > S_{t+1}(z'_{t+1}) > S_t > S^*$. By iterating the argument, we obtain that, if $S_t > S^*$, with a positive probability there is a monotone increasing sequence $S_t < S_{t+1}(z'_{t+1}) < S_{t+2}(z'_{t+2}) < \cdots < S_{t+n}(z'_{t+n}) < \cdots$.

We now show that this sequence is unbounded, so that the exchange rate goes to infinite as time tends to infinite. By contradiction, let us assume that there is an upper bound $\bar{S}$. It follows that a monotone non-decreasing bounded sequence converges to a finite limit. A convergent sequence is also a Cauchy sequence, i.e., $\forall \varepsilon > 0$ there exists an n such that for integers m and k with $m, k > n$ it is

$$|S_{t+k}(z'_{t+k}) - S_{t+m}(z'_{t+m})| < \varepsilon. \tag{10}$$

Considering without losing generality that $k > m$, our construction of the sequence under analysis implies

$$\begin{aligned}
&S_{t+k}(z'_{t+k}) - S_{t+m}(z'_{t+m}) \\
&> \left[\phi\left(\frac{S_{t+k-1}(z'_{t+k-1})}{S^*}\right) \phi\left(\frac{S_{t+k-2}(z'_{t+k-2})}{S^*}\right) \cdots \phi\left(\frac{S_{t+m+1}(z'_{t+m+1})}{S^*}\right) S_{t+m}(z'_{t+m}) \right. \\
&\left. - S_{t+m}(z'_{t+m}) \right].
\end{aligned}$$

Given the properties of the function $\phi(.)$, we observe that for any k and m with $k, m > n$, and $k > m$ without loss of generality,

$$\phi\left(\frac{S_{t+k-1}(z'_{t+k-1})}{S^*}\right) \phi\left(\frac{S_{t+k-2}(z'_{t+k-2})}{S^*}\right) \cdots \phi\left(\frac{S_{t+m+1}(z'_{t+m+1})}{S^*}\right) \geqslant 1 + c$$

for c that depends on k and m but is strictly positive for any possible choice of k and m. Moreover $S_{t+m}(z'_{t+m}) > S^*$. It then follows that

$$S_{t+k}(z'_{t+k}) - S_{t+m}(z'_{t+m}) > S^* c > 0,$$

which restricts the values of ε for which (10) holds. This contradicts the arbitrariness of ε. It follows that the sequence is unbounded.

A similar argument applies to the case $S_t < S^*$. In this case, there is a positive probability that the exchange rate decreases monotonically as time tends to infinite, and the proof that the monotone decreasing sequence can only converge to zero as

time tends to infinite is similar to that for divergence to infinite in the monotone increasing case.[11]

To summarize, we have shown that either the exchange rate is fixed or there is a positive probability that the exchange rate will explode or implode in an infinite time.[12] How can we rule out these explosive equilibria and determine a unique rational expectations equilibrium with a fixed exchange rate under rule (6)?

3.3. A commitment to suspend convertibility along an explosive path

We proved that, if the exchange rate is not fixed at the target level S^*, there is a positive probability that it will follow a monotone increasing sequence of the form $S_t < S_{t+1}(z'_{t+1}) < S_{t+2}(z'_{t+2}) < \cdots < S_{t+n}(z'_{t+n}) < S_{t+n+1}(z'_{t+n+1}) \cdots$. Suppose that, after the exchange rate reaches the level $S_\tau(z'_\tau)$, the home policymaker suspends convertibility of the currency at the market exchange rate at time $\tau + 1$ and forces conversion at an exchange rate level determined as follows.

At time τ, in state of nature z'_τ, equilibrium in asset trading requires that

$$\sum_{z_{\tau+1} \in Z_{\tau+1}} \pi(z_{\tau+1}) \left\{ \frac{U'(C_{\tau+1}(z_{\tau+1}))}{P_{\tau+1}(z_{\tau+1})} \left[S_{\tau+1}(z_{\tau+1}) - S_\tau(z'_\tau) \phi \left(\frac{S_\tau(z'_\tau)}{S^*} \right) \right] \right\} = 0, \quad (11)$$

as the interest rate rule (6) is still effective. Since

$$S_\tau(z'_\tau) \phi \left(\frac{S_\tau(z'_\tau)}{S^*} \right) > S_\tau(z'_\tau),$$

there exists at least one level of the exchange rate $\tilde{S}_{\tau+1}(z_{\tau+1})$ that satisfies the inequalities

$$S_\tau(z'_\tau) \phi \left(\frac{S_\tau(z'_\tau)}{S^*} \right) > \tilde{S}_{\tau+1}(z_{\tau+1}) > S_\tau(z'_\tau)$$

for each state of nature $z_{\tau+1} \in Z_{\tau+1}$. Suppose that the policymaker forces conversion of the currency at the value $\tilde{S}_{\tau+1}(z_{\tau+1})$ in each state of nature at time $\tau + 1$. When this happens, the equilibrium condition (11) is violated, and investors suffer capital losses almost surely. To avoid this, individuals with holdings of foreign currency will convert them into domestic currency at time τ. Therefore, $S_\tau(z'_\tau)$ cannot be an equilibrium value for the exchange rate at time τ in state of nature z'_τ. The combination of the interest rate rule (6) with forced conversion at the rate $\tilde{S}_{\tau+1}(z_{\tau+1})$ causes the explosive path to unravel. A similar argument holds for the case in which the exchange rate is imploding toward zero (or $1/S_t$ is exploding toward infinity). Commitment by the foreign government to an analogous forced conversion mechanism, combined with the rule followed by the home policymaker, will cause

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[11]The fact that the exchange rate goes to zero only asymptotically implies that this case is not ruled out by a requirement of positive money demand and finite supply.

[12]The existence of explosive paths for the nominal exchange rate is not specific to the formulation of monetary policy in terms of interest rate feedback rules, but it would arise also in the case of monetary targeting, as noted in Woodford (2003, p. 129).

this path to unravel.[13] In turn, causing the divergent paths to unravel is sufficient to remove all equilibria but $S_t = S^*$ in all periods because, as we demonstrated, divergent paths with strictly positive probability are *necessarily* part of any rational expectation equilibrium if the exchange rate ever deviates from the target level S^*.

3.4. Discussion

Credible commitment to rule (6) (and suspension of convertibility along an explosive path), combined with the proper choice of the function $\phi(.)$ and agents' rational optimizing behavior, yields exchange rate stability and determinacy. Because $S_t = S^*$ in the unique equilibrium of the economy, the rule yields $i_t = i_t^*$ endogenously at each date t as a feature of the rational expectation equilibrium. In other words, given credible commitment to – say – raise the domestic (gross) interest rate above the foreign one by the amount dictated by the function $\phi(.)$ each time S_t tends to move above S^*, the central bank will never actually need to exercise its threat in equilibrium, and interest rate equalization will follow.

Since the requirements that the function $\phi(.)$ should satisfy in order to be consistent with a fixed exchange rate are weak, it follows that an infinite number of rules are consistent with the implementation of a fixed exchange rate regime by the home central bank.[14]

What happens if the home central bank's commitment to a rule of the form (6) satisfying the requirements above is not perfectly credible?

Suppose that there exists a finite $S^{**} > S^*$ such that $\phi(S^{**}/S^*) = 1$, because it is perceived that the central bank will not actually raise the interest rate if the exchange rate rises well above a certain threshold (but is still below the threshold at which convertibility is suspended). In this case, there exists another equilibrium that can be associated with an infinite number of stochastic equilibrium trajectories converging to it (see Fig. 2).[15] A sudden change in expectations away from the solution S^* – possibly triggered by a change in the color of sunspots – will cause the exchange rate to move from S^* to S^{**} and yield a 'second generation' exchange rate crisis.[16]

The relatively weak requirements we impose on the function $\phi(.)$ are indeed designed to ensure that credibility of the commitment to the rule that determines a

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[13]Suspension of market convertibility works in a fashion akin to Obstfeld and Rogoff's (1983) fractional backing mechanism in ruling out explosive trajectories.

[14]We should also note that the class of rules in (6) does not exhaust the possible policies that would implement a determinate, fixed exchange rate – including interest rate policies. For instance, in a model with money in the utility function, logarithmic utility (with a weight $\chi > 0$ for utility from real money balances, M_t/P_t), and PPP, setting the interest rate so that

$$\frac{1 + i_t}{i_t} = \frac{1}{\chi C_t} \frac{M_t}{S^* P_t^*}$$

will result in $S_t = S^*$ (and $i_t = i_t^*$) in all periods. But rule (6) is preferable since it is more transparent and does not require knowledge of the money demand equation.

[15]The specific trajectory will depend on the realizations of the states of nature during the transition. In Fig. 2, the home interest rate is lower than i_t^* to the right of S^{**}. Of course, other cases are possible.
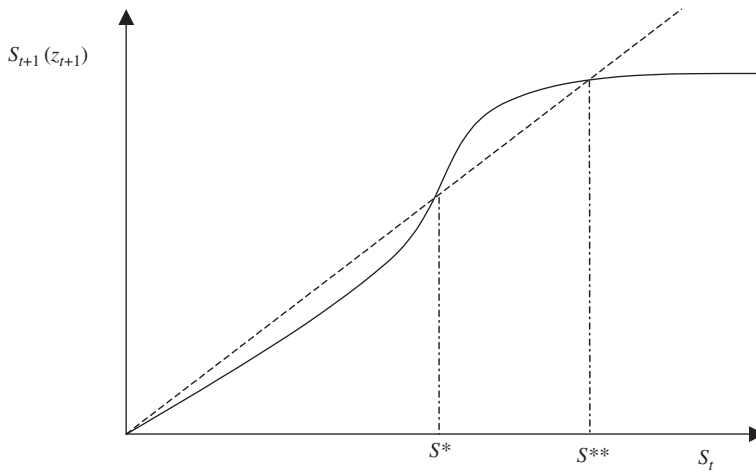
[16]The experience of Sweden in 1992 is a good example.

Fig. 2.

unique fixed exchange rate equilibrium is as strong as possible. For example, requiring $\phi(.)$ to be strictly convex – along with the other conditions – would have been consistent with the purpose of determining a unique equilibrium. However, a strictly convex function $\phi(.)$ would force the home central bank to raise its interest rate at a very steep pace in case of deviations of the exchange rate above S^*. In a more general model, this may indeed weaken the commitment to the rule and set the scope for situations of the type we just discussed.[17]

We conclude this section by observing that we can relate the indeterminacy result under interest rate pegging and our proposed solution to Sargent and Wallace's (1975) criticism of interest rate targeting and the discussion in Woodford (2003, Chapter 2). The rule $i_t = i_t^*$ specifies the path of the domestic interest rate in terms of a path (that of the foreign interest rate) that is 'exogenous,' in the sense that it does not make the domestic interest rate directly a function of the endogenous variable that the rule would like to pin down – the exchange rate. (If the home economy is not a small open economy, i_t^* may be affected by domestic economic developments depending on the design of foreign policy. Yet, the key observation is that the rule $i_t = i_t^*$ does not specify the path of i_t as a function of S_t or of other variables that are related to S_t in a way that would pin it down.) As such, $i_t = i_t^*$ is an example of the interest rate targeting policies criticized by Sargent and Wallace for causing indeterminacy. Making the path of i_t a function of S_t in the proper way yields exchange rate determinacy (and generates $i_t = i_t^*$ as an endogenous outcome), much as Woodford's Wicksellian reaction of the interest rate to the price level in a closed economy yields price level determinacy.

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[17]Of course, multiple equilibria exist also if the commitment to (6) is perfectly credible but the threat of suspending convertibility along an explosive path is not. However, we would argue that the requirement that policymakers are committed to implementing actions that would terminate an *explosive* path is a plausible restriction on policy behavior.

4. World determinacy: the role of the leader country

So far, we have focused on interest rate setting by the follower country in the exchange rate arrangement. We now turn briefly to the role of the leader country, considering the traditional $n - 1$ country argument from the perspective of equilibrium determinacy.

A properly designed rule for the home economy, combined with the commitment to suspend convertibility along an explosive path, yields a determinate, fixed exchange rate. However, this is not sufficient to ensure determinacy of other domestic (or foreign) variables. For example, if PPP holds, a fixed exchange rate implies that the domestic price level is tied to the foreign one. If the latter is subject to indeterminacy because foreign monetary policy does not ensure its determinacy, the domestic price level will be indeterminate too. Similarly, domestic real variables may be subject to indeterminacy caused by monetary policymaking in the leader country. The rule followed by this country is crucial to ensure determinacy of the world economy. Because home imports foreign monetary policy in equilibrium, interest rate setting in the foreign country determines the characteristics of the world economy equilibrium – including uniqueness.

We do not discuss the issue in detail here, rather we focus on a simple example to make our point. Suppose the home economy is following rule (6), with a proper choice of $\phi(.)$ and the commitment to suspend convertibility along an explosive path. The foreign economy follows a rule of the type

$$1 + i_t^* = \phi^*(Y_t^*, \pi_t^*), \tag{12}$$

where Y_t^* and π_t^* are foreign GDP and CPI inflation, respectively.

Rule (12) is a generalized Taylor-type rule for interest rate setting. The choice of the function $\phi^*(.,.)$ determines its characteristics. If $\phi^*(.,.)$ is not chosen appropriately, instability and indeterminacy of the world economy may result.

A multiplicity of choices of $\phi^*(.,.)$ will ensure stability and determinacy. The specific function chosen in this subset will set the characteristics of the determinate equilibrium of the world economy. In particular, different choices of $\phi^*(.,.)$ will yield different welfare levels in the foreign and home economies. The foreign rule determines the nature of the fixed exchange rate regime. So, we may say that there is a multiplicity of fixed exchange rate regimes depending on the rule followed by the leader, each of them characterized by a different level of welfare. Each regime can be then implemented by a set of welfare-neutral interest setting rules in the follower country: given the rule of the leader country, the follower's interest setting rules consistent with the exchange rate being fixed at S^* all yield the same level of welfare in a determinate world economy, since they all yield $i_t = i_t^*$ in equilibrium.

5. Some log-linear examples

To substantiate the arguments we made in the previous sections, we consider some simple examples using a log-linear approximation to the equilibrium conditions

around a deterministic steady state. We compare alternative rules for interest rate setting by the follower country in terms of their ability to generate a locally determinate equilibrium with the exchange rate at its steady-state, target level in all periods. Consistent with our focus on fixed exchange rates, we assume zero steady-state depreciation. We note that if the exchange rate is not determinate (and at its target level) in a local analysis, it cannot be determinate (and at its target level) in a global analysis. The examples illustrate the role of foreign interest rate setting and the follower's reaction to it. Additionally, they make it possible to link our paper to the earlier work on rational expectation, open economy models of the 1970s and 1980s that simply posited log-linear relations of the type below without starting from explicit microfoundations. The examples show that our results would hold also in these earlier models under the assumption that monetary policy is conducted by setting interest rates.

We use hats below to denote percent deviations from steady-state levels (of gross rates in the cases of interest and inflation rates). Log-linearizing the Euler equations for home and foreign bonds yields the UIP condition

$$\hat{\imath}_t - \hat{\imath}_t^* = \mathrm{E}_t \hat{S}_{t+1} - \hat{S}_t.$$

From this, we see immediately that the rule $\hat{\imath}_t = \hat{\imath}_t^*$ implies $\mathrm{E}_t \hat{S}_{t+1} = \hat{S}_t$ – and thus results in exchange rate indeterminacy – regardless of the policy followed by the foreign country (and any other feature of the economy).[18] Instead, the rule $\hat{\imath}_t = \hat{\imath}_t^* + \phi_S \hat{S}_t$, $\phi_S > 0$ – a log-linear example of the general rule (6) we studied above – implies that the exchange rate is determined by

$$(1 + \phi_S)\hat{S}_t = \mathrm{E}_t \hat{S}_{t+1},$$

which has the unique fixed exchange rate solution $\hat{S}_t = 0$ for all t.[19]

As an alternative, suppose now that the home country reacts to exchange rate depreciation rather than the level of the exchange rate (in addition to reacting to $\hat{\imath}_t^*$): $\hat{\imath}_t = \hat{\imath}_t^* + \phi_S \Delta \hat{S}_t$, where Δ denotes first difference ($\Delta \hat{S}_t \equiv \hat{S}_t - \hat{S}_{t-1}$). Then, UIP and this policy rule imply

$$\phi_S \Delta \hat{S}_t = \mathrm{E}_t \Delta \hat{S}_{t+1},$$

which has unique solution $\Delta \hat{S}_t = 0$ for all t as long as $\phi_S > 1$. This policy stabilizes the rate of depreciation uniquely at its steady-state level. Given zero steady-state depreciation, also this policy results in a fixed exchange rate.[20]

A reaction to the level (or the rate of change) of the exchange rate is thus crucial for home to accomplish a desired exchange rate target. However, for home to accomplish its goal, it is important not only that its rule incorporate a reaction to the exchange rate, but also that policy react to the foreign interest rate $\hat{\imath}_t^*$. To see this,

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[18]In particular, the rule $\hat{\imath}_t = \hat{\imath}_t^*$ results in exchange rate indeterminacy even if the foreign country follows a rule in which there is a price level target and/or an exchange rate target ($\hat{\imath}_t^* = \phi_P^* \hat{P}_t^* - \phi_S^* \hat{S}_t$).

[19]Note that the rule $\hat{\imath}_t = \hat{\imath}_t^* + \phi_S \hat{S}_t$, $\phi_S > 0$ achieves the determinate fixed exchange rate $\hat{S}_t = 0$ even when foreign policy includes a feedback to the exchange rate as in the rule $\hat{\imath}_t^* = \phi_P^* \hat{P}_t^* - \phi_S^* \hat{S}_t$.

[20]If steady-state depreciation were not zero, the rules $\hat{\imath}_t = \hat{\imath}_t^* + \phi_S \hat{S}_t$, $\phi_S > 0$, and $\hat{\imath}_t = \hat{\imath}_t^* + \phi_S \Delta \hat{S}_t$, $\phi_S > 1$, would yield a crawling peg rather than a fixed exchange rate.

suppose the home central bank is only reacting to the level of the exchange rate, so that $\hat{\imath}_t = \phi_S \hat{S}_t$, $\phi_S > 0$. Then, UIP implies

$$(1 + \phi_S)\hat{S}_t = \mathrm{E}_t \hat{S}_{t+1} + \hat{\imath}_t^*.$$

Given a unique, bounded path of the foreign interest rate, there is a unique solution for the exchange rate, but it is not guaranteed that this is fixed.[21]

To highlight this point further, consider the following example. Suppose that prices are flexible, PPP holds, so that $\hat{P}_t = \hat{S}_t + \hat{P}_t^*$, and international asset markets are complete, so that $\hat{r}_t^{*n} = \hat{r}_t^n$, where $\hat{r}_t^n$ and $\hat{r}_t^{*n}$ are the natural real interest rates at home and abroad, assumed bounded. We may then write the log-linear Euler equations as

$$\hat{\imath}_t = \mathrm{E}_t \hat{\pi}_{t+1} + \hat{r}_t^n, \tag{13}$$

$$\hat{\imath}_t^* = \mathrm{E}_t \hat{\pi}_{t+1}^* + \hat{r}_t^{*n}, \tag{14}$$

with

$$\hat{\pi}_t = \hat{\pi}_t^* + \Delta \hat{S}_t,$$

and where $\hat{\pi}_t \equiv \hat{P}_t - \hat{P}_{t-1}$ and $\hat{\pi}_t^* \equiv \hat{P}_t^* - \hat{P}_{t-1}^*$. Assume that the home central bank follows the rule $\hat{\imath}_t = \phi_S \hat{S}_t$, $\phi_S > 0$, and the foreign central bank targets the price level: $\hat{\imath}_t^* = \phi_P^* \hat{P}_t^*$, $\phi_P^* > 0$. Eq. (14) then implies

$$\phi_P^* \hat{P}_t^* = \mathrm{E}_t(\hat{P}_{t+1}^* - \hat{P}_t^*) + \hat{r}_t^{*n}, \tag{15}$$

from which it is immediate to conclude that there is a bounded, unique solution for the foreign price level $\hat{P}_t^*$ and, therefore, the foreign interest rate $\hat{\imath}_t^*$. Eq. (13) and PPP then imply

$$\phi_S \hat{S}_t = \mathrm{E}_t(\hat{S}_{t+1} - \hat{S}_t) + \mathrm{E}_t(\hat{P}_{t+1}^* - \hat{P}_t^*) + \hat{r}_t^n,$$

or, using (15),

$$\phi_S \hat{S}_t = \mathrm{E}_t(\hat{S}_{t+1} - \hat{S}_t) + \phi_P^* \hat{P}_t^*.$$

Since there is a unique, bounded path of the foreign price level, $\phi_S > 0$ implies that there is a unique, bounded path of the exchange rate, though the exchange rate is not fixed. In fact, the solution for the exchange rate at time t is

$$\hat{S}_t = \phi_P^* \sum_{j=0}^{\infty} (1 + \phi_S)^{-(j+1)} \mathrm{E}_t \hat{P}_{t+j}^*,$$

and, under this policy, $\hat{S}_t = 0$ only in the special case in which $\hat{P}_{t+j}^* = 0$ in all periods. Including the reaction to foreign policy in domestic interest rate setting, so that $\hat{\imath}_t = \hat{\imath}_t^* + \phi_S \hat{S}_t$, $\phi_S > 0$, is thus necessary to accomplish the desired exchange rate target.

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[21]Similarly, for the case $\hat{\imath}_t = \phi_S \Delta \hat{S}_t$, there is a unique solution for $\Delta \hat{S}_t$ with $\phi_S > 1$, but it is not guaranteed that $\Delta \hat{S}_t = 0$ in all periods.

5.1. Different exchange rate targets

As the examples above clarify, the response of the domestic interest rate to the foreign one, in addition to the reaction to the exchange rate (level or depreciation), ensures that home will accomplish its exchange rate target even if the foreign central bank follows an interest rate rule with feedback to the exchange rate.[22] The same mechanism implies that home achieves its exchange rate target even if the foreign central bank is responding to deviations of the exchange rate from a *different* target level in its policymaking. This is seen most transparently in a perfect foresight example. Under perfect foresight, UIP holds as

$$\hat{\imath}_t = \hat{\imath}_t^* + \hat{S}_{t+1} - \hat{S}_t,$$

where a hat now denotes log (rather than log-deviation from steady state) and, as before, we take logs of gross interest rates. Suppose the foreign central bank is following the rule

$$\hat{\imath}_t^* = \phi_P^*(\hat{P}_t^* - \hat{P}^*) - \phi_S^*(\hat{S}_t - \hat{S}'),$$

where $\hat{S}'$ is the log-exchange rate target level of the foreign central bank. The home central bank follows the rule

$$\hat{\imath}_t = \hat{\imath}_t^* + \phi_S(\hat{S}_t - \hat{S}^*), \quad \phi_S > 0.$$

Combining this rule with UIP yields a difference equation that has unique solution $\hat{S}_t = \hat{S}^*$ for all t, regardless of the feedback to the deviation from a different target in the foreign rule. As long as foreign policy is not committed to *fixing* $\hat{S}_t$ to $\hat{S}'$ – a scenario that we consider implausible since fixed exchange rates usually involve either a leader country that pays little attention to the exchange rate or a cooperative arrangement in which the exchange rate target is shared across central banks – internalization of foreign policy into home implies that home accomplishes its exchange rate target.

6. Conclusions

As discussed by Obstfeld and Rogoff (1996), pegging the domestic interest rate to that of a foreign, leader country does not yield a fixed exchange rate. It results in instability and indeterminacy. We propose a solution to this problem in terms of interest rate feedback rules for the follower country so that we ensure the determinacy of the fixed exchange rate equilibrium in a rational expectations setting under relatively weak conditions. A multiplicity of welfare-neutral rules will do. The class of rules we propose differs from other approaches in the literature (such as Taylor, 1994; Wieland, 1996) because it does not require the specification of any point in the money supply path. The rule of the leader country is important to have

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[22]This was arguably the behavior of the Bundesbank under the European Monetary System.

determinacy of the world equilibrium and will set the other macroeconomic features of the fixed exchange rate regime.

Acknowledgements

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