Monetary Policy and Business Cycles with Endogenous Entry and Product Variety: Appendix

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A The Reduced Benchmark Model and the Efficient Equilibrium

The system of equilibrium conditions in Table 1 can be further reduced by substituting all the equations that hold *within* a given period. The reduced system is shown in Table A.1 for the case $\varphi = 0$.

Table A.1. Reduced Benchmark Model ($\varphi = 0$)

Markup	$\mu_t = \frac{\theta}{(\theta-1)\left[1-\frac{\kappa}{2}\pi_t^2\right]+\kappa\left\{(1+\pi_t)\pi_t-\beta(1-\delta)E_t\left[\frac{1-\frac{\kappa}{2}\pi_t^2}{1-\frac{\kappa}{2}\pi_{t+1}^2}\frac{N_t}{N_{t+1}}(1+\pi_{t+1})\pi_{t+1}\right]\right\}}$
Number of firms	$N_{t+1} = (1-\delta)\left(N_t + \frac{1}{f_{E,t}}\left[Z_t - Y_t^C N_t^{\frac{1}{1-\theta}}\right]\right)$
Euler equation (shares)	$\frac{f_{E,t}}{\mu_t} N_t^{\frac{1}{\theta-1}} = \beta\left(1-\delta\right)E_t\left[\left(\frac{C_{t+1}}{C_t}\right)^{-1}\left(\frac{f_{E,t+1}}{\mu_{t+1}}N_{t+1}^{\frac{1}{\theta-1}} + \left(1-\frac{1}{\mu_{t+1}}\right)\frac{\left[1-\frac{\kappa}{2}\pi_{t+1}^2\right]C_{t+1}}{N_{t+1}}\right)\right]$
Euler equation (bonds)	$(C_t)^{-1} = \beta E_t\left[\frac{1+i_t}{1+\pi_{t+1}}\left(\frac{N_{t+1}}{N_t}\right)^{\frac{1}{\theta-1}}(C_{t+1})^{-1}\right]$

As for the system in Table 1, the reduced system is closed by specifying a rule for nominal interest rate setting, the setting of the labor subsidy τ_t^L, and processes for the exogenous entry cost $f_{E,t}$ and productivity Z_t.

Based on the first-best policy exercise in the text, the efficient equilibrium of this economy is obtained when policy mimics the flexible-price equilibrium through producer price stability ($\pi_t = 0$

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$\forall t$). Denoting variables in this equilibrium with a star, the solution for the efficient equilibrium is obtained from the system in Table A.2.

Table A.2. Benchmark Model ($\varphi = 0$): The Efficient Equilibrium

Markup	$\mu_t^* = \frac{\theta}{\theta-1}$
Number of firms	$N_{t+1}^* = (1-\delta)\left(N_t^* + \frac{1}{f_{E,t}}\left[Z_t - C_t^*\left(N_t^*\right)^{\frac{1}{1-\theta}}\right]\right)$
Euler equation (shares)	$\frac{f_{E,t}}{\mu}\left(N_t^*\right)^{\frac{1}{\theta-1}} = \beta(1-\delta)E_t\left[\left(\frac{C_{t+1}^*}{C_t^*}\right)^{-1}\left(\frac{f_{E,t+1}}{\mu}\left(N_{t+1}^*\right)^{\frac{1}{\theta-1}} + \left(1-\frac{1}{\mu}\right)\frac{C_{t+1}^*}{N_{t+1}^*}\right)\right]$
Euler equation (bonds)	$\left(C_t^*\right)^{-1} = \beta E_t\left[\left(1+i_t^*\right)\left(\frac{N_{t+1}^*}{N_t^*}\right)^{\frac{1}{\theta-1}}\left(C_{t+1}^*\right)^{-1}\right]$

Log-linearizing under assumptions of log-normality and homoskedasticity, the last three equations in this table are:

$$\mathsf{N}_{t+1}^* = (1+r)\,\mathsf{N}_t^* - (r+\delta)(\theta-1)\,\mathsf{C}_t^* + \left[(r+\delta)(\theta-1)+\delta\right]\mathsf{Z}_t - \delta\mathsf{f}_{E,t}, \tag{1}$$

$$\mathsf{C}_t^* = \frac{1-\delta}{1+r}E_t\mathsf{C}_{t+1}^* - \left(\frac{1-\delta}{1+r}\frac{1}{\theta-1} - \frac{r+\delta}{1+r}\right)\mathsf{N}_{t+1}^* + \frac{1}{\theta-1}\mathsf{N}_t^* - \frac{1-\delta}{1+r}E_t\mathsf{f}_{E,t+1} + \mathsf{f}_{E,t}, \tag{2}$$

$$E_t\mathsf{C}_{t+1}^* = \mathsf{C}_t^* + \mathsf{i}_t^* + \frac{1}{\theta-1}\mathsf{N}_{t+1}^* - \frac{1}{\theta-1}\mathsf{N}_t^*. \tag{3}$$

Equations (1)-(2) fully determine consumption and the number of producers as a function of exogenous shocks (as in BGM). Equation (3) can then be used to obtain the Wicksellian interest rate i_t^*. Subtracting this equation from the log-linear version of the Euler equation for bond holdings in the sticky-price equilibrium and using $\tilde{\mathsf{Y}}_{R,t}^C = \hat{\mathsf{C}}_{R,t} \equiv \mathsf{C}_t - [1/(\theta-1)]\,\mathsf{N}_t - \{\mathsf{C}_t^* - [1/(\theta-1)]\,\mathsf{N}_t^*\} = \mathsf{C}_{R,t} - \mathsf{C}_{R,t}^*$ and $\hat{\imath}_t \equiv \mathsf{i}_t - \mathsf{i}_t^*$ yields $E_t\tilde{\mathsf{Y}}_{R,t+1}^C - \tilde{\mathsf{Y}}_{R,t}^C = \hat{\imath}_t - E_t\pi_{t+1}$, which we use in the derivation of the interest rate rule that supports the efficient, flexible-price allocation.

B Proof of Proposition 1

We study a hypothetical scenario in which a benevolent planner maximizes lifetime utility of the representative household by choosing quantities directly. The "production function" for aggregate consumption output is $Y_t^C = Z_t N_t^{\frac{1}{\theta-1}} L_t^C$, implying that consumption is given by $C_t =$

$\left[1-\frac{\kappa}{2}\left(\pi_t\right)^2\right] Z_t N_t^{\frac{1}{\theta-1}} L_t^C$. Hence, the problem solved by the planner can be written as:

$$\max_{\{L_s^C,\pi_s\}_{s=t}^{\infty}} E_t\left[\sum_{s=t}^{\infty}\beta^{s-t}U\left(\left[1-\frac{\kappa}{2}\left(\pi_s\right)^2\right] Z_s N_s^{\frac{1}{\theta-1}} L_s^C\right)\right],$$
$$s.t.\ N_{t+1}=\left(1-\delta\right)N_t+\left(1-\delta\right)\frac{\left(L-L_t^C\right)Z_t}{f_{E,t}}, \tag{4}$$

or, substituting the constraint into the utility function and treating next period's state as the choice variable:

$$\max_{\{N_{s+1},\pi_s\}_{s=t}^{\infty}} E_t\left\{\sum_{s=t}^{\infty}\beta^{s-t}U\left[\left[1-\frac{\kappa}{2}\left(\pi_s\right)^2\right] Z_s N_s^{\frac{1}{\theta-1}}\left(L-\frac{1}{\left(1-\delta\right)}\frac{f_{E,s}}{Z_s}N_{s+1}+\frac{f_{E,s}}{Z_s}N_s\right)\right]\right\}.$$

The first-order condition with respect to inflation is simply:

$$\pi_t=0\ \forall t, \tag{5}$$

whereas the first order condition with respect to N_{t+1} for a given level of inflation is:

$$\begin{aligned}&U'\left(C_t\right)\left[1-\frac{\kappa}{2}\left(\pi_t\right)^2\right] Z_t N_t^{\frac{1}{\theta-1}}\frac{1}{1-\delta}\frac{f_{E,t}}{Z_t}\\ &=\beta E_t\left\{\begin{array}{c} U'\left(C_{t+1}\right)\left[1-\frac{\kappa}{2}\left(\pi_{t+1}\right)^2\right] Z_{t+1}\frac{1}{\theta-1}N_{t+1}^{\frac{1}{\theta-1}-1} \\ \cdot\left[L-\frac{1}{(1-\delta)}\frac{f_{E,t+1}}{Z_{t+1}}N_{t+2}+\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}+\left(\theta-1\right)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}\right]\end{array}\right\}.\end{aligned}$$

The term in square brackets in the right-hand side of this equation is:

$$L-\frac{1}{\left(1-\delta\right)}\frac{f_{E,t+1}}{Z_{t+1}}N_{t+2}+\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}+\left(\theta-1\right)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}=L_{t+1}^C+\left(\theta-1\right)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}.$$

Hence, the first-order condition becomes:

$$U'\left(C_t\right)\left[1-\frac{\kappa}{2}\left(\pi_t\right)^2\right] N_t^{\frac{1}{\theta-1}} f_{E,t}=\beta\left(1-\delta\right)E_t\left\{\begin{array}{c} U'\left(C_{t+1}\right)\left[1-\frac{\kappa}{2}\left(\pi_{t+1}\right)^2\right] Z_{t+1}\frac{1}{\theta-1}N_{t+1}^{\frac{1}{\theta-1}-1} \\ \cdot\left[L_{t+1}^C+\left(\theta-1\right)\frac{f_{E,t+1}}{Z_{t+1}}N_{t+1}\right]\end{array}\right\},$$

leading to

$$U'\left(C_t\right)\left[1-\frac{\kappa}{2}\left(\pi_t\right)^2\right] N_t^{\frac{1}{\theta-1}} f_{E,t}=\beta\left(1-\delta\right)E_t\left\{U'\left(C_{t+1}\right)\left[\frac{1}{\theta-1}\frac{C_{t+1}}{N_{t+1}}+\left[1-\frac{\kappa}{2}\left(\pi_{t+1}\right)^2\right] N_{t+1}^{\frac{1}{\theta-1}} f_{E,t+1}\right]\right\}. \tag{6}$$

Combining (6) and (5) we obtain the equation that, together with the dynamic constraint (4) fully determines the planner equilibrium:

$$U'(C_t)\, N_t^{\frac{1}{\theta-1}} f_{E,t} = \beta (1-\delta) E_t \left\{ U'(C_{t+1}) \left[\frac{1}{\theta-1} \frac{C_{t+1}}{N_{t+1}} + N_{t+1}^{\frac{1}{\theta-1}} f_{E,t+1} \right] \right\}. \tag{7}$$

General Homothetic Preferences

The proof is identical to that of Proposition 1, with $\rho(N_t)$ replacing $N_t^{\frac{1}{\theta-1}}$ and $\epsilon(N_{t+1})$ replacing $1/(\theta-1)$ in (7). Implementation of this first-best optimum in the decentralized economy is ensured by the interest rate rule in equation (15) of the main text, together with one of the optimal subsidies studied in Bilbiie, Ghironi, and Melitz (2006)[1] to induce markup equalization across states and over time and balance the benefit from variety with the profit incentives for entry. Finally, in order to ensure optimality of the steady-state with zero inflation when we log-linearize around such a steady-state in the translog case, we only need to impose a subsidy in steady state. We impose an entry subsidy/tax that, as show in Bilbiie, Ghironi, and Melitz (2006), is equivalent to doubling the entry cost in the steady state of the translog model, i.e., entrants need to pay $2f_{E,t}$ rather than $f_{E,t}$. None of the steady state ratios (and hence none of the log-linearized equations) is modified by this, since the entry cost is irrelevant for all of them.

[1] Bilbiie, F. O., F. Ghironi, and M. J. Melitz (2006): "Monopoly Power and Endogenous Variety in Dynamic Stochastic General Equilibrium: Distortions and Remedies," manuscript, University of Oxford, Boston College, and Princeton University.

C The Log-Linear Model

Table C.1 Log-Linear, Benchmark Model, Summary

Pricing	$\rho_t = \mu_t + \mathsf{w}_t - \mathsf{Z}_t$
Markup	$\pi_t = \beta(1-\delta) E_t \pi_{t+1} - \frac{\theta-1}{\kappa}\mu_t$
Variety effect	$\rho_t = \frac{1}{\theta-1}\mathsf{N}_t$
Profits	$\mathsf{d}_t = \mathsf{Y}_{C,t} - \mathsf{N}_t + (\theta-1)\mu_t$
Free entry	$\mathsf{v}_t = \mathsf{f}_{E,t} + \mathsf{w}_t - \mathsf{Z}_t$
Number of firms	$\mathsf{N}_{t+1} = (1-\delta)\mathsf{N}_t + \delta \mathsf{N}_{E,t}$
Intratemporal optimality	$\mathsf{L}_t = \varphi(\mathsf{w}_t - \mathsf{C}_t)$
Euler equation (shares)	$\mathsf{E}_t\mathsf{C}_{t+1} = \mathsf{C}_t + \frac{1-\delta}{1+r}\mathsf{E}_t\mathsf{v}_{t+1} - \mathsf{v}_t + \frac{r+\delta}{1+r}\mathsf{E}_t\mathsf{d}_{t+1}$
Euler equation (bonds)	$\mathsf{E}_t\mathsf{C}_{t+1} = \mathsf{C}_t + \mathsf{i}_t - \mathsf{E}_t\pi^C_{t+1}$
Output of consumption sector	$\mathsf{Y}_{C,t} = \mathsf{C}_t$
Aggregate accounting	$\mathsf{Y}_{C,t} + \frac{vN_E}{Y_C}\mathsf{v}_t + \frac{vN_E}{Y_C}\mathsf{N}_{E,t} = \frac{wL}{Y_C}\mathsf{w}_t + \frac{wL}{Y_C}\mathsf{L}_t + \frac{dN}{Y_C}\mathsf{d}_t + \frac{dN}{Y_C}\mathsf{N}_t$
CPI inflation	$\pi_t - \pi^C_t = \rho_t - \rho_{t-1}$
Nominal interest rate	$\mathsf{i}_t =$ interest rate rule from main text

Since we log-linearize around a steady state with zero inflation, the steady-state ratios needed above are as in BGM:

$$\frac{vN_E}{Y_C} = \frac{\delta}{r+\delta}\frac{1}{\theta}, \quad \frac{dN}{Y_C} = \frac{1}{\theta}, \quad \frac{wL}{Y_C} = \frac{\theta-1}{\theta} + \frac{\delta}{r+\delta}\frac{1}{\theta}.$$

For the case of translog preferences, the only equations that change in the Table above are the second and third. The markup equation is replaced by (17) in the main text, and the variety effect is $\rho_t = [2(\theta-1)]^{-1}\mathsf{N}_t$.

D Proof of Proposition 2

Focus on the rule $i_t = \tau E_t \pi_{t+1}$ in the inelastic labor case. The proof of determinacy is similar to that in Carlstrom, Fuerst, and Ghironi (2006).[2]

Recall that the steady state of the model is such that $1 + d/v = (1+r)/(1-\delta) = 1/[\beta(1-\delta)]$. Define $\gamma \equiv d/v$. Note that, for plausible parameter values (β close to 1, δ small), γ is very small

[2] Carlstrom, C. T., T. S. Fuerst, and F. Ghironi (2006): "Does It Matter (for Equilibrium Determinacy) What Price Index the Central Bank Targets?" *Journal of Economic Theory* 128: 214-231.

(0.036001 when $\beta = 0.99$ and $\delta = 0.025$). In particular, assume that β, δ, and θ are such that $1 - \gamma(\theta - 1) > 0$. This condition is satisfied by all plausible values of β, δ, and θ.

Omitting shocks and focusing on perfect foresight for the purposes of analyzing determinacy, we can rewrite the system as

$$A_1 \begin{bmatrix} \pi_{t+1} \\ \mathsf{C}_{t+1} \\ \mu_{t+1} \\ \mathsf{N}_{t+1} \end{bmatrix} = A_0 \begin{bmatrix} \pi_t \\ \mathsf{C}_t \\ \mu_t \\ \mathsf{N}_t \end{bmatrix},$$

where:

$$A_1 \equiv \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ -(\tau - 1) & 1 & 0 & -\frac{1}{\theta-1} \\ 0 & 1 & 1-\gamma(\theta-1) & -\frac{1-\gamma(\theta-1)}{\theta-1} \end{bmatrix},$$

$$A_0 \equiv \begin{bmatrix} 1+\gamma & 0 & \frac{(\theta-1)(1+\gamma)}{\kappa} & 0 \\ 0 & -\frac{\gamma(\theta-1)}{\beta(1+\gamma)} & 0 & \frac{1}{\beta} \\ 0 & 1 & 0 & -\frac{1}{\theta-1} \\ 0 & 1+\gamma & 1+\gamma & -\frac{1+\gamma}{\theta-1} \end{bmatrix}.$$

Our interest is in the eigenvalues of the matrix $A \equiv A_1^{-1} A_0$.

The characteristic equation has the form:

$$J(e) = J_4 e^4 + J_3 e^3 + J_2 e^2 + J_1 e + J_0 = 0,$$

where:

$$J_4 \equiv -\beta\kappa(1+\gamma)\left[1 - \gamma(\theta - 1)\right],$$

$$J_3 \equiv \beta(1+\gamma)^2\left[\kappa - (\theta-1)(\tau-1)\right] + \kappa\left[1-\gamma(\theta-1)\right]\left[1+\beta(1+\gamma)(2+\gamma)\right],$$

$$J_2 \equiv (\theta-1)(\tau-1)\left[\gamma^2(1+\gamma)(\theta-1) + (1+\gamma)(1+3\beta\gamma) + \beta\left(1+\gamma^3\right)\right]$$
$$- \kappa\left\{3(1+\beta) - \beta\gamma^3(\theta-2) - \gamma^2\left[(\theta-1)(1+2\beta) - 5\beta\right] - \gamma\left[2(\theta-2)(1+\beta) - \beta(\theta+4)\right]\right\}$$

$$J_1 \equiv (1+\gamma)\left\{(1+\gamma)\left[\kappa - (\theta-1)(\tau-1)\right] + \kappa\left[2 + \beta(1+\gamma(2+\gamma)) - \gamma(\theta-1)\right]\right\}$$

$$J_0 \equiv -\kappa(1+\gamma)^2.$$

For determinacy (and stability), three roots of J must be outside the unit circle and one root must be within the unit circle. The condition $1-\gamma(\theta-1)>0$ ensures $J_4<0$. Plausible values of the price stickiness coefficient κ, the policy parameter τ, and substitutability θ also imply $\kappa > (\theta-1)(\tau-1)$, hence $J_3>0$.[3] The same plausible parameter values (and small γ) imply $J_2<0$ and $J_1>0$. Finally, $J_0<0$. Since $J(0)<0$, $J'(0)>0$, $J''(0)<0$, and $J'''(0)>0$, all the roots of J have positive real parts. The product of the four roots is equal to $J_0/J_4>1$. Furthermore,

$$J(1)=\gamma(1+\gamma)(\theta-1)(\tau-1)\left[\beta(1+\gamma)+\gamma(\theta-1)-1\right].$$

Since it is always $\theta>(1+\gamma)(1-\beta)/\gamma$, $J(1)$ has the sign of $\tau-1$. Therefore, if $\tau<1$, there are either 0 or 2 roots in $(0,1)$, so that we can never have determinacy. Hence, $\tau>1$ is necessary for determinacy.

We now turn to sufficiency.

Since $J(0)<0$ and $J(1)>0$ for $\tau>1$, we know that J has (at least) two real roots, one in the unit circle and one outside. Let us refer to these two real roots as $e_1<1$ and $e_2>1$. Our task is to examine the remaining two roots of J and demonstrate that they are outside the unit circle if $\tau>1$. The strategy is to examine these two roots in the neighborhood of $\tau=1$ defined by $\tau=1+\varepsilon$, with $\varepsilon>0$ and arbitrarily small. We can show that we have determinacy in this neighborhood whether the remaining roots are real or complex. In addition, we will show that as τ increases, these roots cannot pass back into the unit circle.

Focus on the case in which the two remaining roots are real

We first demonstrate that, in this case, J must have three roots outside the unit circle.

Define the function $h(x)\equiv J(e)$ where $x\equiv e-1$. The function h is also a quartic with coefficients h_0, h_1, h_2, h_3, and h_4. Note that $h_0=J(1)$, $h1=J'(1)$, $h_2=J''(1)/2$, $h_3=J'''(1)/3!$, etc. Inspection of the J function implies that $h_0>0$ and $h_4<0$. Also,

$$h_3\equiv 6\left\{\begin{array}{c}\kappa\left[\beta(1+\gamma)^2+(1+\beta\gamma(1+\gamma))(1-\gamma(\theta-1))\right]\\ -\beta(1+\gamma)\left[(1+\gamma)(\theta-1)(\tau-1)+2\kappa(1-\gamma(\theta-1))\right]\end{array}\right\}.$$

If $\tau\to 1$ and $\beta\to 1$, $h_3>0$. The same holds for any plausible value of τ and β. Hence, Descartes' Rule of Signs implies that there is indeterminacy if and only if $h_1>0$ and $h_2>0$. In the neighborhood of $\tau=1$, both $J'(1)$ and $J''(1)$, however, cannot be greater than or equal to zero

[3] For all plausible estimates, κ is much larger than θ and τ.

since

$$(1-\gamma) J'(1|\tau=1)+\gamma J''(1|\tau=1) = -\kappa\gamma^2 \left[\beta\gamma^2 (3\theta-2) + \gamma (2\beta-1)(\theta-2) + (1-\beta)(\theta+2)\right] < 0,$$

under the weak, sufficient conditions $\beta > 1/2$ and $\theta > 2$. This then implies that $h(x)$ $(J(e))$ has three roots greater than zero (unity). Hence, we have determinacy (and stability) for τ just slightly greater than unity for any plausible parametrization.

As long as these two roots remain real, they must remain outside the unit circle for larger values of τ. This is true because $J(0) < 0$ and $J(1) > 0$ for all $\tau > 1$, so that the only way for there to be indeterminacy is to have three roots within the unit circle. This can never be the case without the roots first becoming complex. Therefore, as we increase τ out of the neighborhood $1+\varepsilon$, we must continue to have exactly one root in the unit circle.

The proof for the case of complex roots is similar to that in Carlstrom, Fuerst, and Ghironi (2006). We omit it to save space, but it is available on request.

E Endogenous Aggregate Stickiness and Producer Entry: The Model

This appendix develops the model in which new entrants do not pay a cost of price adjustment relative to a previous period's price.

The Price Index

Recall that a new entrant in period t starts producing (and setting prices) in period $t+1$. We can write the price index at time t as:

$$P_t = \left\{ \left[N_t - (1-\delta) N_{E,t-1}\right] (\tilde{p}_t)^{1-\theta} + (1-\delta) N_{E,t-1} \left(p_t^{t-1}\right)^{1-\theta} \right\}^{\frac{1}{1-\theta}}, \tag{8}$$

where p_t^{t-1} is the price chosen for period t by firms that entered in period $t-1$ and $\tilde{p}_t$ is an average price for firms that entered in periods $t-2$, $t-3$, and beyond, defined by:

$$\tilde{p}_t \equiv \left[\frac{1}{N_{t-1}} \sum_{s=2}^{\infty} (1-\delta)^{s-1} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

Assume $\lim_{T\to\infty} (1-\delta)^{T-1} N_{t-T} = 0$. Then, using the law of motion $N_t = (1-\delta)(N_{t-1} + N_{E,t-1})$, it is possible to rewrite the price index as:

$$P_t = \left[\sum_{s=1}^{\infty} (1-\delta)^s N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} . \tag{9}$$

This is a compact expression for P_t as weighted average of the prices chosen for that period by firms that entered in all past periods, weighted by their probability of survival. Equation (9) involves an infinite number of state variables (all the lags of the number of entrants). However, we show below that it can be reduced to a finite, small number of state variables in log-linear form.

Firms

A firm ω that entered in period $\upsilon \leq t-1$ produces output according to $y_t^{\upsilon,S}(\omega) = Z_t l_t^{\upsilon}(\omega)$ and faces demand

$$y_t^{\upsilon,D}(\omega) = \left(\frac{p_t^{\upsilon}(\omega)}{P_t} \right)^{-\theta} (C_t + PAC_t),$$

where:

$$PAC_t \equiv \sum_{s=2}^{\infty} (1-\delta)^s N_{E,t-s} pac_t^{t-s}(\omega).$$

Note that the aggregate cost of price adjustment at time t aggregates only the costs of price adjustment borne by firms that entered in periods $t-2$, $t-3$, and beyond, since firms that entered in period $t-1$ pay no cost of price adjustment in period t. The firm-level cost of price adjustment takes the same form as in the benchmark model:

$$pac_t^{\upsilon}(\omega) \equiv \frac{\kappa}{2} \left(\frac{p_t^{\upsilon}(\omega)}{p_{t-1}^{\upsilon}(\omega)} - 1 \right)^2 \rho_t^{\upsilon}(\omega) y_t^{\upsilon,D}(\omega), \quad \upsilon \leq t-2,$$

where $\rho_t^{\upsilon}(\omega) \equiv p_t^{\upsilon}(\omega)/P_t$.

The value of a firm that entered in any period $\upsilon \leq t$ is given by the expected present discounted value of the stream of profits it will generate from period $t+1$ on:

$$v_t^{\upsilon}(\omega) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s^{\upsilon}(\omega), \quad \upsilon \leq t.$$

In particular, the value of a new entrant is:

$$v_t^{t}(\omega) = E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s^{t}(\omega),$$

and the free entry condition will now require equalization of this value to the sunk entry cost: $v_t^t(\omega) = w_t f_{E,t}/Z_t = v_t^t$ (identity of marginal cost across firms ensures symmetry of the equilibrium within cohorts of firms).

A new price setter in period t now maximizes

$$d_t^{t-1}(\omega) + E_t \sum_{s=t+1}^{\infty} \Lambda_{t,s} d_s^{t-1}(\omega),$$

where

$$d_t^{t-1}(\omega) = \rho_t^{t-1}(\omega)\, y_t^{t-1,D}(\omega) - w_t l_t^{t-1}(\omega),$$
$$d_s^{t-1}(\omega) = \rho_s^{t-1}(\omega)\, y_s^{t-1,D}(\omega) - w_s l_s^{t-1}(\omega) - \frac{\kappa}{2}\left(\frac{p_s^{t-1}(\omega)}{p_{s-1}^{t-1}(\omega)} - 1\right)^2 \rho_s^{t-1}(\omega)\, y_s^{t-1,D}(\omega), \quad s \geq t+1.$$

The initial price setting decision yields:

$$p_t^{t-1}(\omega) = \mu_t^{t-1}(\omega)\, P_t \frac{w_t}{Z_t}, \tag{10}$$

with

$$\mu_t^{t-1}(\omega) = \frac{\theta}{(\theta - 1) - \kappa E_t\left[\Lambda_{t,t+1} \frac{N_t}{N_{t+1}} \frac{Y_{t+1}^C}{Y_t^C}\left(1 + \pi_{t+1}^{t-1}(\omega)\right) \pi_{t+1}^{t-1}(\omega)\right]}. \tag{11}$$

Because the firm knows that it will face a cost of price adjustment in the future, its first price setting decision is not just the flexible-price markup $\theta/(\theta-1)$ over marginal cost, but it incorporates the incentive to smooth price changes between the initial choice and the following one.[4]

Price setting decisions in the following periods – and the implied markup equation – and price setting decisions by firms that entered prior to $t-1$ take the same form as in the benchmark model.

The Household's Budget Constraint and Portfolio Decision

We assume that the household now decides at time t how many shares to hold in each cohort of firms that entered the economy up to and including period t. In real terms, exploit symmetry

[4] The version of the model in which new entrants charge a constant markup over marginal cost is obtained by setting the scaling parameter for the cost of price adjustment (κ) to zero in the markup equation (11) for new price setters.

within cohorts, the budget constraint is now:

$$
\begin{aligned}
& B_{t+1}+\sum_{s=0}^{\infty}(1-\delta)^{s} v_{t}^{t-s} N_{E,t-s} x_{t+1}^{t-s}+C_{t} \\
& =\left(1+r_{t}\right) B_{t}+\sum_{s=0}^{\infty}(1-\delta)^{s+1}\left(d_{t}^{t-s-1}+v_{t}^{t-s-1}\right) N_{E,t-s-1} x_{t}^{t-s-1}+\left(1+\tau_{t}^{L}\right) w_{t} L_{t}+t_{t}^{L} .
\end{aligned}
$$

In period t, the household receives dividends from its holdings of shares in firms that entered in periods $t-1$, $t-2$, and beyond, and the value of selling its share holdings. It then buys holdings of shares to be carried into $t+1$ in all producing firms at time t plus the new entrants in period t (as in the benchmark model). The Euler equation for share holdings is:

$$
v_{t}^{t-s}=\beta(1-\delta) E_{t}\left[\frac{C_{t}}{C_{t+1}}\left(d_{t+1}^{t-s}+v_{t+1}^{t-s}\right)\right], \quad s \geq 0 .
$$

As in the benchmark model, iteration of this equation yields the value of the firm. In particular, the Euler equation for investment in new firms is:

$$
v_{t}^{t}=\beta(1-\delta) E_{t}\left[\frac{C_{t}}{C_{t+1}}\left(d_{t+1}^{t}+v_{t+1}^{t}\right)\right] . \tag{12}
$$

Aggregate Accounting

Imposing the equilibrium conditions $B_{t+1}=B_{t}=0$ and $x_{t+1}^{t-s}=x_{t}^{t-s-1}=1$, and the government budget constraint $t_{t}^{L}=-\tau_{t}^{L} w_{t} L_{t}$, we have the aggregate accounting relation:

$$
v_{t}^{t} N_{E,t}+C_{t}=w_{t} L_{t}+\sum_{s=0}^{\infty}(1-\delta)^{s+1} d_{t}^{t-s-1} N_{E,t-s-1} . \tag{13}
$$

The sum of investment in new firms and consumption must be equal to total income (labor income and dividend income paid by all cohorts of firms that produce in period t). As for the price index equation, the aggregate accounting equation (13) involves an infinite number of state variables, but it can be reduced to a small number of states in log-linear form. Note that the value of new entrants, determined by the Euler equation (12), is now the asset price that determines the allocation of resources to consumption or investment in new firms, obeying the free-entry condition $v_{t}^{t}=w_{t} f_{E,t} / Z_{t}$.

Some Log-Linear Relations

We log-linearize the model around the same steady state with zero inflation in all nominal prices as the benchmark model to facilitate comparison of the two setups in terms of their implications for the propagation of shocks. This sub-section shows how log-linearization makes it possible to reduce the number of state variables in solving the model and reports the log-linear versions of equations that feature an infinite number of state variables in level form.

Pricing and Price Index Dynamics

In log-linear terms , the price and markup equations (10) and (11) yield:

$$\mathsf{p}_t^{t-1} = \mu_t^{t-1} + \mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t, \tag{14}$$

$$\mu_t^{t-1} = \frac{\kappa\beta(1-\delta)}{\theta-1} E_t \pi_{t+1}^{t-1}, \tag{15}$$

where we used symmetry of the equilibrium across firms in the same cohort, and μ_t^{t-1} and π_{t+1}^{t-1} now denote percent deviations from steady state (of gross inflation in the case of π_{t+1}^{t-1}).

Observe that price setting for any firm after its first price setting choice is such that:

$$\mathsf{p}_t^{\upsilon} = \mu_t^{\upsilon} + \mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t, \tag{16}$$

$$\mu_t^{\upsilon} = -\frac{\kappa}{\theta-1}\left[\pi_t^{\upsilon} - \beta(1-\delta) E_t \pi_{t+1}^{\upsilon}\right], \quad \upsilon < t-1 \tag{17}$$

where υ is the date of entry. Therefore, considering any two cohorts υ and $\upsilon - 1$,

$$\mathsf{p}_t^{\upsilon} - \mathsf{p}_t^{\upsilon-1} = \mu_t^{\upsilon} - \mu_t^{\upsilon-1}.$$

Combining this with the Phillips curves for the two cohorts and using the definitions of cohort-specific inflation rates implies that the price differential between cohorts obeys the difference equation:

$$\kappa\beta(1-\delta) E_t\left(\mathsf{p}_{t+1}^{\upsilon} - \mathsf{p}_{t+1}^{\upsilon-1}\right) - \{\theta - 1 + \kappa[1+\beta(1-\delta)]\}\left(\mathsf{p}_t^{\upsilon} - \mathsf{p}_t^{\upsilon-1}\right) + \kappa\left(\mathsf{p}_{t-1}^{\upsilon} - \mathsf{p}_{t-1}^{\upsilon-1}\right) = 0.$$

The characteristic polynomial for this equation is a convex parabola with one root inside the unit circle and one outside. Therefore, the equation has unique solution $\mathsf{p}_t^{\upsilon} - \mathsf{p}_t^{\upsilon-1} = 0$, or $\mathsf{p}_t^{\upsilon} = \mathsf{p}_t^{\upsilon-1}$. To a first-order approximation, firms that are in their second (or higher) period of price setting choose the same price, and thus the same producer price inflation rate: Given the initial condition $\mathsf{p}_{-1}^{\upsilon} = \mathsf{p}_{-1}^{\upsilon-1}$ if a shock happens at time 0, this implies $\pi_t^{\upsilon} = \pi_t^{\upsilon-1}$ and, in turn, $\mu_t^{\upsilon} = \mu_t^{\upsilon-1}$ for any cohorts υ and $\upsilon - 1$ that are not in the first period of price setting.

By exploiting this property of log-linearized price setting, it is possible to verify that log-linearized, welfare-consistent consumer price inflation depends negatively on variety growth and positively on a weighted average of inflation in the first pricing of new entrants at $t-1$ relative to new entrants at $t-2$ and inflation in the pricing of the "representative" cohort υ that entered in period $t-3$ or further in the past:

$$\pi_t^C = -\frac{1}{\theta - 1}\left(\mathsf{N}_t - \mathsf{N}_{t-1}\right) + \delta\left(\mathsf{p}_t^{t-1} - \mathsf{p}_{t-1}^{t-2}\right) + (1-\delta)\,\pi_t^{\upsilon}. \tag{18}$$

The equations above make it possible to fully determine the dynamics of all prices and price indexes of interest. The inflation rate π_t^{υ} is determined by the generic cohort υ's pricing and Phillip's curve. The time t price chosen by firms that entered at $t-1$ is determined by the price and markup equations (14) and (15). Note that these two equations together imply:

$$\mathsf{p}_t^{t-1} = \frac{\kappa\beta(1-\delta)}{\theta - 1 + \kappa\beta(1-\delta)} E_t \mathsf{p}_{t+1}^{t-1} + \frac{\theta - 1}{\theta - 1 + \kappa\beta(1-\delta)}\left(\mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t\right). \tag{19}$$

From the perspective of period t, a firm that entered at $t-1$ and is setting the price for $t+1$ is no longer in its first period of price setting. Hence, the result obtained above applies and we may rewrite (19) as:

$$\mathsf{p}_t^{t-1} = \frac{\kappa\beta(1-\delta)}{\theta - 1 + \kappa\beta(1-\delta)} E_t \mathsf{p}_{t+1}^{\upsilon} + \frac{\theta - 1}{\theta - 1 + \kappa\beta(1-\delta)}\left(\mathsf{P}_t + \mathsf{w}_t - \mathsf{Z}_t\right), \quad \upsilon \leq t-1. \tag{20}$$

Finally, the time $t-1$ price chosen by entrants at $t-2$ is a state variable. At the time of a shock $(t = 0)$, p_{t-1}^{t-2} is zero, and consumer price inflation is simply:

$$\pi_0^C = \mathsf{P}_0 = \delta \mathsf{p}_0^{-1} + (1-\delta)\,\mathsf{p}_0^{\upsilon},$$

where

$$\mathsf{p}_0^{-1} = \frac{\kappa\beta(1-\delta)}{\theta - 1 + \kappa\beta(1-\delta)} E_t \mathsf{p}_1^{\upsilon} + \frac{\theta - 1}{\theta - 1 + \kappa\beta(1-\delta)}\left(\mathsf{P}_0 + \mathsf{w}_0 - \mathsf{Z}_0\right),$$

and υ is the representative cohort that entered prior to period 0. In period 1,

$$\pi_1^C = -\frac{1}{\theta - 1}\mathsf{N}_1 + \delta\left(\mathsf{p}_1^0 - \mathsf{p}_0^{-1}\right) + (1-\delta)\,\pi_1^{\upsilon},$$

where p_0^{-1} was determined above and p_1^0 is determined by (20). And so on.

Aggregate Accounting, GDP, and the Labor Market

Exploiting symmetry of (log-linearized) behavior across cohorts of firms that are not in their first period of price setting, the log-linear version of the aggregate accounting identity (13) is:

$$\frac{vN_E}{C}\left(\mathsf{v}_t^t + \mathsf{N}_{E,t}\right) + \mathsf{C}_t = \frac{wL}{C}\left(\mathsf{w}_t + \mathsf{L}_t\right) + \frac{1}{\theta}\left[\mathsf{N}_t + \delta\mathsf{d}_t^{t-1} + (1-\delta)\,\mathsf{d}_t^{\upsilon}\right],$$

with

$$\begin{aligned}\mathsf{d}_t^{t-1} &= \mathsf{y}_t^{t-1} + \theta\mu_t^{t-1} + \mathsf{w}_t - \mathsf{Z}_t, \quad \mathsf{y}_t^{t-1} = -\theta\rho_t^{t-1} + \mathsf{C}_t,\\ \mathsf{d}_t^{\upsilon} &= \mathsf{y}_t^{\upsilon} + \theta\mu_t^{\upsilon} + \mathsf{w}_t - \mathsf{Z}_t, \quad \mathsf{y}_t^{\upsilon} = -\theta\rho_t^{\upsilon} + \mathsf{C}_t.\end{aligned}$$

GDP is:

$$\mathsf{Y}_t = \frac{wL}{Y}\left(\mathsf{w}_t + \mathsf{L}_t\right) + \frac{1}{\theta}\frac{C}{Y}\left[\mathsf{N}_t + \delta\mathsf{d}_t^{t-1} + (1-\delta)\,\mathsf{d}_t^{\upsilon}\right],$$

Finally, labor market equilibrium requires:

$$\mathsf{L}_t = N_E\left(\mathsf{N}_{E,t} + \mathsf{f}_{E,t}\right) + (1-N_E)\left[\mathsf{N}_t + \delta\mathsf{y}_t^{t-1} + (1-\delta)\,\mathsf{y}_t^{\upsilon}\right] - \mathsf{Z}_t,$$

with sectoral labor allocation:

$$\begin{aligned}\mathsf{L}_t^E &= \mathsf{N}_{E,t} + \mathsf{f}_{E,t} - \mathsf{Z}_t,\\ \mathsf{L}_t^C &= \mathsf{N}_t + \delta\mathsf{y}_t^{t-1} + (1-\delta)\,\mathsf{y}_t^{\upsilon} - \mathsf{Z}_t.\end{aligned}$$

Analytical Details

Derivation of Equation (9)

Observe that

$$N_t - (1-\delta)\,N_{E,t-1} = (1-\delta)\,N_{t-1},$$

so that

$$P_t = (1-\delta)^{\frac{1}{1-\theta}} \left[N_{t-1} \left(\tilde{p}_t\right)^{1-\theta} + N_{E,t-1} \left(p_t^{t-1}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}. \tag{21}$$

Using the law of motion for N_t and the assumption $\lim_{T\to\infty} (1-\delta)^{T-1} N_{t-T} = 0$,

$$N_{t-1} = \sum_{s=2}^{\infty} (1-\delta)^{s-1} N_{E,t-s}. \tag{22}$$

From the definition of $\tilde{p}_t$, we have:

$$\begin{aligned}
\tilde{p}_t &\equiv \left[\frac{(1-\delta) N_{E,t-2}}{N_{t-1}} \left(p_t^{t-2}\right)^{1-\theta} + \frac{(1-\delta)^2 N_{E,t-3}}{N_{t-1}} \left(p_t^{t-3}\right)^{1-\theta} + \frac{(1-\delta)^3 N_{E,t-4}}{N_{t-1}} \left(p_t^{t-4}\right)^{1-\theta} + \ldots \right]^{\frac{1}{1-\theta}} \\
&= \left(N_{t-1}\right)^{-\frac{1}{1-\theta}} \left[\sum_{s=2}^{\infty} (1-\delta)^{s-1} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\
&= (1-\delta)^{-\frac{1}{1-\theta}} \left(N_{t-1}\right)^{-\frac{1}{1-\theta}} \left[\sum_{s=2}^{\infty} (1-\delta)^{s} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.
\end{aligned}$$

Finally, substituting into (21) yields (9):

$$\begin{aligned}
P_t &= (1-\delta)^{\frac{1}{1-\theta}} \left[\sum_{s=2}^{\infty} (1-\delta)^{s-1} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} + N_{E,t-1} \left(p_t^{t-1}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\
&= (1-\delta)^{\frac{1}{1-\theta}} \left[\sum_{s=1}^{\infty} (1-\delta)^{s-1} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}} \\
&= \left[\sum_{s=1}^{\infty} (1-\delta)^{s} N_{E,t-s} \left(p_t^{t-s}\right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.
\end{aligned}$$

Derivation of Equation (18)

First, observe that log-linearizing (22) yields:

$$\mathsf{N}_{t-1} = \frac{\delta}{1-\delta} \sum_{s=2}^{\infty} (1-\delta)^{s-1} \mathsf{N}_{E,t-s}. \tag{23}$$

Log-linearizing (9) yields:

$$
\begin{aligned}
\mathsf{P}_t &= \frac{\delta}{1-\theta}\left[\sum_{s=1}^{\infty}(1-\delta)^{s-1}\mathsf{N}_{E,t-s}+(1-\theta)\sum_{s=1}^{\infty}(1-\delta)^{s-1}\mathsf{p}_t^{t-s}\right] \\
&= \frac{\delta}{1-\theta}\left[\mathsf{N}_{E,t-1}+\sum_{s=2}^{\infty}(1-\delta)^{s-1}\mathsf{N}_{E,t-s}+(1-\theta)\sum_{s=1}^{\infty}(1-\delta)^{s-1}\mathsf{p}_t^{t-s}\right] \\
&= \frac{\delta}{1-\theta}\left[\mathsf{N}_{E,t-1}+\frac{1-\delta}{\delta}\mathsf{N}_{t-1}+(1-\theta)\sum_{s=1}^{\infty}(1-\delta)^{s-1}\mathsf{p}_t^{t-s}\right] \\
&= \frac{1}{1-\theta}\mathsf{N}_t+\delta\left[\mathsf{p}_t^{t-1}+\sum_{s=2}^{\infty}(1-\delta)^{s-1}\mathsf{p}_t^{t-s}\right], \qquad (24)
\end{aligned}
$$

where the third line used (23) and the fourth line used $\mathsf{N}_t=(1-\delta)\,\mathsf{N}_{t-1}+\delta\mathsf{N}_{E,t-1}$ (the log-linear law of motion for N_t). Importantly, (24) features only one state variable (N_t), by exploiting the log-linear solution of the law of motion for the number of firms (23).

Consider now the first difference of (24):

$$
\begin{aligned}
\pi_t^C &\equiv \mathsf{P}_t-\mathsf{P}_{t-1} \\
&= \frac{1}{1-\theta}\left(\mathsf{N}_t-\mathsf{N}_{t-1}\right)+\delta\left[\mathsf{p}_t^{t-1}-\mathsf{p}_{t-1}^{t-2}+\sum_{s=2}^{\infty}(1-\delta)^{s-1}\left(\mathsf{p}_t^{t-s}-\mathsf{p}_{t-1}^{t-s-1}\right)\right]. \qquad (25)
\end{aligned}
$$

From the result we obtained above that $\mathsf{p}_t^{\upsilon}-\mathsf{p}_t^{\upsilon-1}=0$ for all firms that entered prior to period $t-1$, it follows that $\mathsf{p}_t^{t-s}-\mathsf{p}_{t-1}^{t-s-1}$ in (25) can be written independently of s as $\mathsf{p}_t^{\upsilon}-\mathsf{p}_{t-1}^{\upsilon}$, where υ now simply denotes a representative cohort of firms that entered before $t-2$. Hence,

$$
\begin{aligned}
\pi_t^C &= -\frac{1}{\theta-1}\left(\mathsf{N}_t-\mathsf{N}_{t-1}\right)+\delta\left[\mathsf{p}_t^{t-1}-\mathsf{p}_{t-1}^{t-2}+\left(\mathsf{p}_t^{\upsilon}-\mathsf{p}_{t-1}^{\upsilon}\right)\sum_{s=2}^{\infty}(1-\delta)^{s-1}\right] \\
&= -\frac{1}{\theta-1}\left(\mathsf{N}_t-\mathsf{N}_{t-1}\right)+\delta\left(\mathsf{p}_t^{t-1}-\mathsf{p}_{t-1}^{t-2}+\frac{1-\delta}{\delta}\pi_t^{\upsilon}\right) \\
&= -\frac{1}{\theta-1}\left(\mathsf{N}_t-\mathsf{N}_{t-1}\right)+\delta\left(\mathsf{p}_t^{t-1}-\mathsf{p}_{t-1}^{t-2}\right)+(1-\delta)\,\pi_t^{\upsilon}.
\end{aligned}
$$

F The New Keynesian Phillips Curve with Non-C.E.S. Preferences

Log-linearizing the markup equation around a zero-inflation steady state under the usual assumptions of lognormality and homoskedasticity yields:

$$
\pi_t=\beta\,(1-\delta)\,E_t\pi_{t+1}-\frac{\theta\,(N)-1}{\kappa}\mu_t-\frac{\theta'\,(N)\,N}{\theta\,(N)\,\kappa}\mathsf{N}_t. \qquad (26)
$$

For the special case of translog preferences with $\theta(N) = 1 + \sigma N$, $\sigma > 0$, we obtain:

$$\pi_t = \beta(1-\delta) E_t \pi_{t+1} - \frac{\sigma N}{\kappa} \mu_t - \frac{\sigma N}{1+\sigma N} \frac{1}{\kappa} \mathsf{N}_t.$$

Finally, we impose the calibration scheme $\theta(N) = 1 + \sigma N = \theta$, where the latter is the elasticity of substitution in the C.E.S. case.[5] Then, the Phillips curve becomes:

$$\pi_t = \beta(1-\delta) E_t \pi_{t+1} - \frac{\theta - 1}{\kappa} \mu_t - \frac{\theta - 1}{\theta \kappa} \mathsf{N}_t. \tag{27}$$

Regarded as a markup equation, (27) implies that markup variation comes from two sources: changes in the inflation rate and product variety. Regarded as an equation for inflation dynamics, (27) implies an extra degree of persistence compared to the C.E.S. case, coming from the presence of the state variable N_t via its impact on the elasticity of substitution across products. Since the benefit of product variety in log-linear terms is now $\rho_t = \epsilon(N) \mathsf{N}_t$, the markup is related to marginal cost by $\mu_t = \epsilon(N) \mathsf{N}_t - (\mathsf{w}_t - \mathsf{Z}_t)$, which for translog preferences (under the calibration scheme above) yields:

$$\mu_t = \frac{1}{2(\theta - 1)} \mathsf{N}_t - (\mathsf{w}_t - \mathsf{Z}_t).$$

Substituting this into (27) yields:

$$\pi_t = \beta(1-\delta) E_t \pi_{t+1} + \frac{\theta - 1}{\kappa} (\mathsf{w}_t - \mathsf{Z}_t) - \left(\frac{1}{2\kappa} + \frac{\theta - 1}{\theta \kappa} \right) \mathsf{N}_t.$$

[5] This is achieved, in the translog case, by finding the implicit unique value $\sigma^* = (\theta - 1)/N^{CES}$, where N^{CES} is the steady-state value of N under C.E.S. preferences.