ECON 401

Advanced Macroeconomics

Midterm Exam

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Wait for my signal before starting to read the exam.

This is a closed-books, closed-notes exam.
You have 110 minutes to work on it.
You should answer five questions for a total of 100 points.
Read the entire exam and think about the questions before starting to work.
Do not panic if you do not make it to complete all five answers.
Any form of cheating will result in a zero score for the exam.

Question 1: 18 Points

Suppose the economy's representative household maximizes the expected intertemporal utility function

$$E_t \sum_{s=t}^{\infty} \beta^{s-t} u\left(C_s\right)$$

subject to the constraints given by the law of motion for capital:

$$K_{t+1} = \left(1 - \delta\right) K_t + I_t$$

and the budget constraint:

$$C_t + I_t = \tilde{r}_t K_t + w_t,$$

where E_t denotes the expectation operator conditional on information available at time t, $\beta \in (0,1)$ is the discount factor, C_t is consumption in period t, K_{t+1} is capital at the beginning of period $t+1$, $\delta \in (0,1)$ is the rate of capital depreciation, I_t is investment in period t, $\tilde{r}_t$ is the rental rate on capital, w_t is the real wage, and labor supply has been normalized to 1. Households rent capital to firms in a perfectly competitive market

The production function is $Y_t = A_t^{\alpha} K_t^{1-\alpha}$, where $\alpha \in (0,1)$ and A_t is an exogenous technology shock.

- Denote with R_{t+1} the return to capital accumulation at $t+1$. What is the expression for R_{t+1}? Explain.
- Explain how to obtain the Euler equation for capital accumulation intuitively.

Question 2: 18 Points

In the same setup as in Question 1, assume that the period utility function takes the form

$$u(C_t) = \frac{C_t^{1-\gamma}}{1-\gamma}, \quad \text{with } \gamma > 0.$$

Assume log-normality and homoskedasticity. Log-normality implies that, given variable X_t,

$$\log\left[E_t\left(X_{t+1}\right)\right] = E_t\left[\log\left(X_{t+1}\right)\right] + \frac{1}{2} var_t\left[\log\left(X_{t+1}\right)\right].$$

- Show that log-normality and homoskedasticity imply that the log-linear version of the Euler equation you obtained in Question 1 has the form:

$$E_t\left(c_{t+1} = c_t\right) = \sigma E_t r_{t+1},$$

  where $c_t \equiv d\log C_t$, $r_t \equiv d\log R_t$, and $\sigma \equiv 1/\gamma$.

- What is σ? What does it measure?

Question 3: 18 Points

Let the solution of the real business cycle model be described by the relations:

$$\begin{aligned} k_{t+1} &= \eta_{kk}k_t + \eta_{ka}a_t, \\ c_t &= \eta_{ck}k_t + \eta_{ca}a_t, \\ y_t &= \eta_{yk}k_t + \eta_{ya}a_t, \end{aligned}$$

with

$$a_t = \phi a_{t-1} + \varepsilon_t,$$

where lower-case letters denote percentage deviations of the corresponding variables from their steady-state levels, the η's are elasticities obtained with the method of undetermined coefficients, and $\phi \in [0, 1]$.

- What is the intuition for this solution? Put differently, why do we guess that the solution for endogenous variables depends on k_t and a_t?

- Why can we guess that this is the unique solution of the log-linearized model?

- Suppose there is a technology innovation $\varepsilon_0 = 1$ at time $t = 0$, followed by no other innovation in subsequent periods. Use the relations above to compute the responses of capital, consumption, and output to the innovation in periods 0, 1, and 2.

- What is the response of technology to the innovation if $\phi = 1$? Do capital, consumption, and output return to the initial steady state in this case? Why?

Question 4: 22 Points

Suppose a final consumption producer uses intermediate inputs produced by monopolistically competitive firms according to the production function

$$Y_t = \left[\int_0^1 y_{i,t}^{\frac{\theta-1}{\theta}} di\right]^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$. The final consumption producer operates in a perfectly competitive market and takes the price of consumption, P_t, as given. The producer chooses how much to demand of each individual input $y_{i,t}$ to maximize

$$P_t Y_t - \int_0^1 p_{i,t} y_{i,t} di,$$

subject to the production function above, where $p_{i,t}$ is the price the final producer pays for intermediate input i.

- Show that optimal input demand by the final good producer is determined by:

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t}\right)^{-\theta} Y_t.$$

- Explain this demand function.

Intermediate good producers produce goods with the production function

$$y_{i,t} = Z_t N_{i,t},$$

where Z_t is exogenous productivity and $N_{i,t}$ is labor employed by producer i. This production function implies that the marginal cost of production for the intermediate producer is w_t/Z_t, where w_t is the real wage (in units of the final consumption good). The producer sets the price $p_{i,t}$ to maximize

$$p_{i,t} y_{i,t} - P_t \frac{w_t}{Z_t} y_{i,t}$$

subject to the demand function you obtained above.

- Show that optimal price setting implies

$$\frac{p_{i,t}}{P_t} = \frac{\theta}{\theta - 1}\frac{w_t}{Z_t}.$$

- Can you think of a scenario (other than $\theta \to \infty$) in which the fact that intermediate goods are priced at a markup over marginal cost does not imply that market-determined output in this economy is inefficiently low?

Question 5: 24 Points

The first-order condition for optimal labor supply in the real business cycle model implies:

$$U_{1-N_t}(C_t, 1-N_t) = w_t U_{C_t}(C_t, 1-N_t),$$

where C_t is consumption in period t, $1-N_t$ is leisure, and w_t is the real wage.

- Explain this condition intuitively.

- Suppose the production function is $Y_t = N_t$ and the labor market is perfectly competitive. What is the value of the real wage? Why?

Now suppose the representative household is subject to a cash-in-advance constraint for its consumption. The first-order condition for optimal labor supply becomes:

$$U_{1-N_t}(C_t, 1-N_t) = w_t \left(1 + \frac{i_t}{1+i_t}\right)^{-1} U_{C_t}(C_t, 1-N_t).$$

where i_t is the nominal interest rate.

- Suppose production and the labor market assumptions are as above, and that a benevolent central banker is choosing monetary policy to maximize welfare. What is the optimal interest rate setting that the central banker would choose? What is the intuition for this result?