ECON 401

Advanced Macroeconomics

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Homework 1

Due in class on Wednesday, April 24, 2019

Problem 1

Consider the stochastic growth model we studied in class. Assume that labor supply is variable: Agents choose optimally how much labor to supply in each period. The period utility function takes the form:

$$u(C_t, N_t) = \frac{\left[C_t^{\rho}(1-N_t)^{1-\rho}\right]^{1-\gamma}}{1-\gamma}, \text{ with } \gamma > 0 \text{ and } 0 < \rho < 1.$$

Utility is no longer separable between consumption and leisure (unless we make the special assumption $\gamma = 1$). All other assumptions are unchanged.

(a) Write the first-order conditions that determine agents' optimal behavior. (Hint: The marginal utility of consumption (or leisure) depends on leisure (or consumption) here.) Explain these first-order conditions intuitively.

(b) Assume $\gamma = 1$ for the rest of this problem. This implies that the utility function becomes $\rho \log C_t + (1-\rho)\log(1-N_t)$. Solve for the balanced growth path of the model and log-linearize the model around it using the technique described in lecture. (Hint: With $\gamma = 1$, you are looking at a special case of the variable-labor model we studied in class. Rewrite the utility function as $\rho[\log C_t + (1-\rho)\log(1-N_t)/\rho]$ and note that maximizing this is the same as maximizing $\log C_t + (1-\rho)\log(1-N_t)/\rho$. Compare this to the utility function for the variable-labor model in the slides and you should see that, if you set $\theta = (1-\rho)/\rho$, you have a special case of the slides, which you can follow to solve the problem.)

(c) Use the "guess and check" procedure presented in class and the method of undetermined coefficients to solve the model.

(d) EXTRA FOR THOSE OF YOU WHO WANT TO SEE THE MODEL "AT WORK"
Use Excel to calculate the impulse responses of the percent deviations of capital, labor, output, and consumption to a one-time-only 1% positive impulse to technology for the following parameter values: $g = .005$, $r = .015$, $\delta = .025$, $\alpha = .667$, $\rho = .36$, $\overline{N} = .33$, $\sigma = 1$, and $\phi = .5$. Comment on the shape of the responses based on what you learned by studying the slides.

Problem 2

Consider again the stochastic growth model, focus on the fixed-labor case, but now allow for government spending shocks as a source of fluctuations.
The representative consumer maximizes:

$$E_t \sum_{i=0}^{+\infty} \beta^i \frac{C_{t+i}^{1-\gamma}}{1-\gamma},$$

where $0 < \beta < 1$ and $\gamma > 0$. The law of motion for capital is:

$$K_{t+1} = (1-\delta)K_t + Y_t - C_t - X_t. \qquad (*)$$

X is exogenous government consumption spending, financed through lump-sum taxation. Output is:

$$Y_t = A_t^{\alpha} K_t^{1-\alpha},\ 0 < \alpha < 1.$$

(a) Obtain the Euler equation for capital accumulation. Explain the intuition.

(b) Solve for the balanced growth path. (It is useful to treat $\overline{X}_t / \overline{Y}_t$ as an exogenous variable here. Assume $\overline{A}_{t+1} / \overline{A}_t = \overline{X}_{t+1} / \overline{X}_t = G$.)

(c) Assume log-normality (so that $\log(E_t X_{t+1}) \approx E_t(\log X_{t+1}) + (1/2)\text{var}_t(\log X_{t+1})$ for any variable X) and homoskedasticity. Log-linearize equation (*), the Euler equation, and the expression of the gross return to capital accumulation around the steady state.

(d) Assume $a_t = 0\ \forall\ t$ (there are no percentage deviations of technology from the steady state). Assume $x_t = \phi x_{t-1} + \varepsilon_t,\ E_{t-1}\varepsilon_t = 0$. Show that the model reduces to:

$$k_{t+1} = \lambda_1 k_t + \lambda_4 x_t + (1 - \lambda_1 - \lambda_2 - \lambda_4)c_t,$$

$$E_t(c_{t+1} - c_t) = -\frac{\lambda_3}{\gamma} k_{t+1},$$

$$x_t = \phi x_{t-1} + \varepsilon_t,\ E_{t-1}\varepsilon_t = 0.$$

with $\lambda_1 \equiv \frac{1+r}{1+g}$, $\lambda_2 \equiv \frac{\alpha(r+\delta)}{(1-\alpha)(1+g)}$, $\lambda_3 \equiv \frac{\alpha(r+\delta)}{1+r}$, $\lambda_4 \equiv -\frac{(r+\delta)\overline{X}_t/\overline{Y}_t}{(1-\alpha)(1+g)}$.

The solution for consumption and capital has the form:

$$c_t = \eta_{ck} k_t + \eta_{cx} x_t,$$

$$k_{t+1} = \eta_{kk} k_t + \eta_{kx} x_t.$$

What is the intuition for this solution? Briefly describe how these equations and $x_t = \phi x_{t-1} + \varepsilon_t$ can be used to trace the response of capital and consumption to a government spending shock.

(e) Briefly: Describe the possible calibration approaches to evaluate the empirical performance of models.