A. Derivation of $F(x,y)$ from its Differential

- Consider a differential
  \[ M(x,y)\,dx + N(x,y)\,dy \]

- If the differential is exact then there exists a function $F(x,y)$ such that
  \[ dF = M(x,y)\,dx + N(x,y)\,dy \]

  where
  \[ M(x,y) = \left( \frac{\partial F}{\partial x} \right)_y \text{ and } N(x,y) = \left( \frac{\partial F}{\partial y} \right)_x \]

- Now from $M(x,y) = \left( \frac{\partial F}{\partial x} \right)_y$, it follows that
  \[ F(x,y) = \int M(x,y)\,dx + K(y) \]

  Note $K(y)$ is independent of $x$. To find out the form for $F(x,y)$ we have to determine $K(y)$. This is done as follows. From the second definition
  \[ N(x,y) = \left( \frac{\partial F}{\partial y} \right)_x = \frac{\partial}{\partial y} \int M(x,y)\,dx + \frac{\partial K}{\partial y} \]

  - Rearrange this equation to obtain
    \[ \frac{\partial K}{\partial y} = N(x,y) - \frac{\partial}{\partial y} \int M(x,y)\,dx \]

  - Integrate the expression
    \[ K(y) = \int \left[ N(x,y) - \frac{\partial}{\partial y} \int M(x,y)\,dx \right] dy \]

  - Substitute this into the equation for $F(x,y)$:
    \[ F(x,y) = \int M(x,y)\,dx + K(y) \]
    \[ = \int M(x,y)\,dx + \int \left[ N(x,y) - \frac{\partial}{\partial y} \int M(x,y)\,dx \right] dy \]