6.1)  a) The easiest way to solve this is to remember that for one mole of an ideal gas molecules:

\[ E = \frac{N_A m c^2}{2} = \frac{3RT}{2} \Rightarrow c^2 = \frac{3RT}{N_A m} \Rightarrow c_{\text{rms}} = \sqrt{c^2} = \sqrt{\frac{3RT}{N_A m}} = \sqrt{\frac{3RT}{MW}} \]

\[ c_{\text{rms}} = \frac{\sqrt{c^2}}{M_W} = \frac{(3)(8.31 J/\text{mole} \cdot K)(273K)}{0.002 \text{ kg/mole}} = 0.00184 \text{ m/s} \]

b) \[ E = \frac{3RT}{2} = \frac{(3)(8.31 J/\text{mole} \cdot K)(273K)}{2} = 3.40 \text{ kJ/mole} \]

c) \[ V = \frac{RT}{P} = \frac{(1\text{ mole})(0.0821 L \cdot \text{atm}/\text{mole} \cdot K)(273K)}{1\text{ atm}} = 22.4 L/\text{mole} \]

\[ \left(\frac{0.001 L}{22.4 L/\text{mole}}\right)(6.02 \times 10^{23} \text{ molecules/mole}) = 2.69 \times 10^{19} \text{ molecules} \]

d) \[ \ell = \frac{1}{\sqrt{2\pi (N/V) \sigma^2}} = \frac{1}{(1.41)(3.14)(6.02 \times 10^{23} / 22.4 L)(0.001 L/cm^3)(2.5 \times 10^{-8} \text{ cm})^2} = 1.34 \times 10^{-5} \text{ cm} \]

e) \[ z = 4\sqrt{\frac{RT}{MW}} \sigma^2 \left(\frac{RT}{MW}\right)^{1/2} = (4)(3.14)(2.689 \times 10^{19} \text{ molecules/cm}^3) \times \]

\[ (2.5 \times 10^{-8} \text{ cm})^3 \left(\frac{8.31 \times 10^7 \text{ ergs/mole} \cdot K)(273K)}{0.002 \text{ kg/mole}}\right)^{1/2} = 1.264 \times 10^{10} \text{ s}^{-1} \]

f) \[ Z = \left(\frac{N}{V}\right) \left(\frac{z}{2}\right) = (2.689 \times 10^{19} \text{ cm}^3) \left(\frac{1.264 \times 10^{10} \text{ s}^{-1}}{2}\right) = 1.699 \times 10^{29} \text{ cm}^3 \text{ s}^{-1} \]

6.2)  a) \( n_2 = 0 \) occurs when \( T = 0 K \).
b) \( n_1 = n_2 \) occurs when \( T = \infty \)

c) \( \frac{n_2}{n_1} = 1.000015 = \exp \left( -\frac{h \nu}{kT} \right) \Rightarrow \ln(1.00015) = 1.5 \times 10^{-5} = -\frac{h \nu}{kT} = -\frac{(6.626 \times 10^{-34} \text{ J} \cdot s)(-10^8 \text{ s}^{-1})}{(1.38 \times 10^{-23} \text{ J} / \text{ K})T} \)

Solving... \( T = 320 \text{ K} \)

6.3)

a) \( D = \frac{kT}{f} \Rightarrow f = \frac{kT}{D} = \frac{(1.38 \times 10^{-23} \text{ J} / \text{ K})(293 \text{ K})}{4.0 \times 10^{-11} \text{ m}^2 / \text{s}} \approx 10^{-10} \text{ kg} / \text{s} \)

b) \( V = \frac{V_2}{N_A} = \frac{4}{3} \pi R_0^3 \Rightarrow R_0 = \left( \frac{V_2}{N_A} \frac{3}{4 \pi} \right)^{1/3} = \left( \frac{(3)(0.739 \text{ cm}^3 / \text{ g}) 156,000 \text{ g} / \text{ mole}}{6.02 \times 10^{23} / \text{ mole}} \frac{(4 \pi)}{156,000 \text{ g} / \text{ mole}} \right)^{1/3} = 6.74 \times 10^{-8} \text{ g} / \text{s} \)

Then

\( f_0 = 6 \pi \eta R_0 = (6)(3.14)(0.01 \text{ g} / \text{ cm} \cdot \text{s}) \left( \frac{(3)(0.739 \text{ cm}^3 / \text{ g}) 156,000 \text{ g} / \text{ mole}}{6.02 \times 10^{23} / \text{ mole}} \frac{(4 \pi)}{156,000 \text{ g} / \text{ mole}} \right)^{1/3} = 6.74 \times 10^{-8} \text{ g} / \text{s} \)

c) If IgC is spherical then

\( f = 6 \pi \eta R \) and \( f_0 = 6 \pi \eta R_0 \Rightarrow \frac{f}{f_0} = \frac{10^{-7} \text{ g} / \text{s}}{6.74 \times 10^{-8} \text{ g} / \text{s}} = 1.6 = \frac{R}{R_0} \)

But for a hydrated, spherical molecule \( R = \left( \frac{3m(\bar{v}_2 + \delta_1 \bar{v}_1)}{4 \pi} \right)^{1/3} \)

Hence \( \frac{R}{R_0} = \left( \frac{\bar{v}_2 + \delta_1 \bar{v}_1}{\bar{v}_2} \right)^{1/3} = 1.6 = \left( \frac{(0.739 \text{ cm}^3 / \text{ g} + \delta_1 1.00 \text{ cm}^3 / \text{ g})}{0.739 \text{ cm}^3 / \text{ g}} \right)^{1/3} \)

Solving... \( \delta_1 = 2.22 \text{ g water/g IgC} \)

c) The volume per molecule of unhydrated IgC is

\( V_{\text{molecule}} = M \frac{\bar{V}_2}{N_A} = \frac{(156,000 \text{ g} / \text{ mole})(0.739 \text{ cm}^3 / \text{ g})}{6.02 \times 10^{23} \text{ molecules} / \text{ mole}} = 1.91 \times 10^{-19} \text{ cm}^3 / \text{ molecule} \)

d) The volume of a prolate ellipsoid is \( \frac{4}{3} \pi ab^2 = 1.91 \times 10^{-19} \text{ cm}^3 \). But to proceed further, we have to eliminate one of the unknowns a or b. This can be done in two ways. First, consult Figure 6.8 and notice that for a prolate ellipsoid with frictional ratio \( \frac{f}{f_0} = 1.5 \Rightarrow \frac{a}{b} \approx 10. \) Then \( a = 10b. \) Alternatively, one could use the equation for the frictional coefficient ratio, given in Additional Problem 2 below, as a function of \( a/b \)...make a calibration chart, and similarly derive the fact that \( a = 10b. \) Any way you do it...eliminate a from the volume equation...

\( \frac{4}{3} \pi ab^2 = \frac{4}{3} \pi (10b)b^2 = 1.91 \times 10^{-19} \text{ cm}^3 \Rightarrow b = 1.7 \times 10^{-7} \text{ cm} \Rightarrow a = 10b = 170 \times 10^{-7} \text{ cm} \)
1) The differential equation for one dimensional diffusion has the form
\[ \frac{dC_2}{dt} = \frac{k_g T}{f} \frac{d^2 C_2}{dx^2} = D_2 \frac{d^2 C_2}{dx^2} \]
where \( C_2(x,t) \) is the solute concentration, \( D_2 \) is the diffusion coefficient of the solute and \( f \) is the coefficient of friction of a solute particle.

a) Prove that \( C_2(x,t) = \frac{C_0}{\sqrt{4\pi D_2 t}} e^{-x^2/4D_2t} \) is a solution for this equation. Hint:

This means that you must show that if the expression for \( C_2 \) is differentiated once with respect to \( t \), the result equals the second derivative of \( C_2 \) with respect to \( x \) times \( D_2 \).

Solution:
\[ C(x,t) = \frac{C_0}{\sqrt{4\pi D_2 t}} e^{-x^2/4D_2t} \]
is a solution to the equation if its partial derivative wrt time \( t \) equals its second partial derivative wrt \( x \) times the diffusion coefficient \( D \). So we have to calculate \( \frac{\partial C}{\partial t} \) and \( \frac{\partial^2 C}{\partial x^2} \).

\[ \frac{\partial C}{\partial t} = \frac{\partial}{\partial t} \left( \frac{C_0}{\sqrt{4\pi D_2 t}} e^{-x^2/4D_2t} \right) = \frac{C_0}{\sqrt{4\pi D_2 t}} \frac{\partial}{\partial t} \left( t^{-1/2} e^{-x^2/4D_2t} \right) = \frac{1}{2t} \frac{C_0}{\sqrt{4\pi D_2 t}} \left( \frac{x^2}{2D_2} - 1 \right) e^{-x^2/4D_2t} \]

\[ \frac{\partial C}{\partial x} = \frac{\partial}{\partial x} \left( \frac{C_0}{\sqrt{4\pi D_2 t}} e^{-x^2/4D_2t} \right) = -\frac{C_0 e^{-x^2/4D_2t}}{\sqrt{4\pi D_2 t}} \frac{\partial}{\partial x} \left( \frac{x^2}{4D_2} \right) = -\frac{C_0 e^{-x^2/4D_2t}}{2D_2 \sqrt{4\pi D_2 t}} x \]

\[ \frac{\partial^2 C}{\partial x^2} = \frac{\partial}{\partial x} \left( \frac{C_0 e^{-x^2/4D_2t}}{2D_2 \sqrt{4\pi D_2 t}} \right) = -\frac{C_0 e^{-x^2/4D_2t}}{2D_2 \sqrt{4\pi D_2 t}} \left( \exp\left( -\frac{x^2}{4D_2t} \right) - \frac{x^2}{2D_2} \exp\left( -\frac{x^2}{4D_2t} \right) \right) \]

\[ = \frac{1}{2D_2 \sqrt{4\pi D_2 t}} \frac{C_0}{\sqrt{4\pi D_2 t}} \left( \frac{x^2}{2D_2} - 1 \right) e^{-x^2/4D_2t} = \frac{1}{D} \frac{\partial C}{\partial t} \]

b) Make graphs of the functions \( xC_2(x,t) \), and \( x^2 C_2(x,t) \) versus \( x \). Assume for simplicity at \( C_0=1 \text{cm} \) and \( D_2 t=1 \text{cm}^2 \). Based on these graphs, will the mean displacement \( \bar{x} \) and the root mean square (rms) displacement \( x_{rms} = \sqrt{\bar{x}^2} \) be equal? Explain. From your graphs…estimate the mean displacement and the root-mean-square displacement.
Answer: I used Microsoft Excel to generate the graphs required. Note the function \( xC(x) \) is an odd function of \( x \) and hence the area under the curve \( xC(x) \) is zero between \(-A\) and \(+A\). Analytically this means

\[
\bar{x} = \int_{-\infty}^{+\infty} xC(x)dx = 0 \quad \text{and so the average displacement is zero. On the other hand} \quad x^2C(x) \text{ is an even function of } x \text{ and its integral is clearly not zero and}
\]

\[
\bar{x^2} = \int_{-\infty}^{+\infty} x^2C(x)dx = 2Dt = 2 \Rightarrow x_{rms} = \sqrt{\bar{x^2}} = \sqrt{2} . \text{ You could derive this result by estimating the area under the graph using Simpson’s rule.}
\]

c) Graph \( C_2(x,t) \) for \( D_2t = 1 \text{cm}^2, 4 \text{cm}^2, \text{ and } 16 \text{cm}^2 \). Calculate the rms displacements from each graph.
Again I did this with Microsoft Excel. The rms displacements for $Dt=1$, 4, and 16 are $\sqrt{2}, 2\sqrt{2}, 4\sqrt{2} \cdots$ (from $x_{rms} = \sqrt{2Dt}$).

2) Consider a flea that is constrained to jump in either the $-x$ or the $+x$ direction. Initially the flea is at $x=0$. Suppose the flea executes $N$ jumps, each of length $l$.

Assume the number of jumps in the $+x$ direction is $N_+ = \frac{N+m}{2}$ and the number of jumps in the $-x$ direction is $N_- = \frac{N-m}{2}$.

a) The number of ways that a flea can be displaced $m$ jumps in the $+x$ direction after $N$ total jumps is $W = \frac{N!}{N_+!N_-!}$. If $N$ is large show that

$$\ln W \approx \text{const} \tan t - \frac{(N+m)}{2} \ln \left(1 + \frac{m}{N}\right) - \frac{(N-m)}{2} \ln \left(1 - \frac{m}{N}\right)$$

(Hint: use Stirling’s Approximation)
\[ W = \frac{N!}{N_+!N_-!} \Rightarrow \ln W = \ln (N!) - \ln (N_+) - \ln (N_-) \]

Let \[ N_+ = \frac{N + m}{2} \ldots \text{and} \ldots \ln (N_+) = N_+ \ln N_+ - N_+ \]

\[ \ln W = \ln (N!) - \frac{N + m}{2} \ln \left(\frac{N + m}{2}\right) + \frac{N + m}{2} - \frac{N - m}{2} \ln \left(\frac{N - m}{2}\right) + \frac{N - m}{2} \]

\[ = \ln (N!) + N - \frac{N + m}{2} \ln \left(\frac{N + m}{2}\right) - \frac{N - m}{2} \ln \left(\frac{N - m}{2}\right) \]

\[ = \text{cons} \tan t \cdot \frac{N + m}{2} \ln \left(\frac{N + m}{2}\right) - \frac{N - m}{2} \ln \left(\frac{N - m}{2}\right) \]

b) A function \( f(x) \) can be approximated near \( x=0 \) using the expansion

\[ f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots \]

This is called McLaurin’s Expansion. Derive the first four terms in the McLaurin Expansions for \( \ln(1+x) \) and \( \ln(1-x) \).

\[ f(x) = \ln (1 \pm x) \Rightarrow f(0) = \ln (1 \pm 0) = 0 \]

\[ f'(x) = \pm (1 \pm x)^{-1} \Rightarrow f'(0) = \pm 1 \]

\[ f''(x) = -(1 \pm x)^{-2} \Rightarrow f''(0) = -1 \]

\[ f'''(x) = \pm 2(1 \pm x)^{-3} \Rightarrow f'''(0) = \pm 2 \]

\[ \therefore f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \ldots \]

\[ = \pm x - \frac{x^2}{2} \pm \frac{x^3}{3} + \ldots \]

c) Apply the McLaurin expansion to the logarithm terms in the expression for \( \ln W \) in part a and show that \( W(m) = \text{cons} \tan t * e^{-m^2/2N} \).
\[
\ln W = \text{cons} \tan t - \frac{N + m}{2} \ln \left( \frac{N + m}{2} \right) - \frac{N - m}{2} \ln \left( \frac{N - m}{2} \right)
\]

\[
= \text{cons} \tan t - \frac{N + m}{2} \ln \left( \frac{N}{2} \left( 1 + \frac{m}{N} \right) \right) - \frac{N - m}{2} \ln \left( \frac{N}{2} \left( 1 - \frac{m}{N} \right) \right)
\]

\[
= \text{cons} \tan t - \frac{N + m}{2} \left\{ \ln \left( \frac{N}{2} \right) + \ln \left( 1 + \frac{m}{N} \right) \right\} - \frac{N - m}{2} \left\{ \ln \left( \frac{N}{2} \right) + \ln \left( 1 - \frac{m}{N} \right) \right\}
\]

\[
= \text{cons} \tan t - N \ln \left( \frac{N}{2} \right) - \frac{N + m}{2} \ln \left( 1 + \frac{m}{N} \right) - \frac{N - m}{2} \ln \left( 1 - \frac{m}{N} \right)
\]

\[
= \text{cons} \tan t - \frac{N + m}{2} \left\{ \frac{m}{N} - \frac{1}{2} \left( \frac{m}{N} \right)^2 \right\} - \frac{N - m}{2} \left\{ \frac{1}{2} \left( \frac{m}{N} \right)^2 \right\}
\]

\[
= \text{cons} \tan t - \frac{N + m}{2} \left\{ \frac{m}{N} - \frac{1}{2} \left( \frac{m}{N} \right)^2 \right\} - \frac{N - m}{2} \left\{ \frac{1}{2} \left( \frac{m}{N} \right)^2 \right\}
\]

\[
= \text{cons} \tan t - \frac{m^2}{N} + \frac{m^2}{2N} = \text{cons} \tan t - \frac{m^2}{2N}
\]

d) If \( m \) is the number of excess jumps that the flea executes in the +x direction, and if each jump has a length \( l \), the total distance that the flea travels from the origin \( x=0 \) after \( N \) jumps is \( x=lm \). Show that the probability that the flea will be at \( x=lm \) after \( N \) jumps is

\[
P(x) = \frac{1}{\sqrt{2\pi Nl^2}} e^{-x^2/2Nl^2} = \frac{1}{\sqrt{2\pi Nl^2 t}} e^{-x^2/2Nl^2 t}
\]

where the rate at which the flea jumps is \( N' = \frac{dN}{dt} \).

Note...\( x = l \cdot m \Rightarrow \ln W = \text{cons} \tan t - \frac{x^2}{2Nl^2} \)

\[\therefore W(x) \approx \text{cons} \tan t \times \exp \left\{ - \frac{x^2}{2Nl^2} \right\} \]

\[
P(x) = \frac{W(x)}{\int_{-\infty}^{\infty} W(x)dx} = \frac{\exp \left\{ - \frac{x^2}{2Nl^2} \right\}}{\int_{-\infty}^{\infty} e^{x^2/2Nl^2} dx} = \frac{\exp \left\{ - \frac{x^2}{2Nl^2} \right\}}{2 \int_{0}^{\infty} e^{x^2/2Nl^2} dx}
\]

\[
= \frac{1}{\sqrt{2\pi Nl^2}} \exp \left\{ - \frac{x^2}{2Nl^2} \right\} = \frac{1}{\sqrt{2\pi Nl^2 t}} \exp \left\{ - \frac{x^2}{2Nl^2 t} \right\}
\]

e) Prove that the function \( P(x) \) in part d satisfies the one dimensional diffusion equation (see additional problem 1) if \( 2D_2 = N'l^2 \)
\[
P(x) = \frac{1}{\sqrt{2\pi N}} \exp \left( -\frac{x^2}{2N\ell^2} \right) = \frac{1}{\sqrt{4\pi D t}} \exp \left( -\frac{x^2}{4Dt} \right) \]

…which is proportional to \(C(x,t)\), the solution of the one-dimensional diffusion equation (see problem 1).

3) The enzyme urease (jack bean) has a molecular weight 482,700 gm/mole, a diffusion coefficient \(D = 3.46 \times 10^{-11} \text{m}^2/\text{s}\) (in water at 293K), and a specific volume \(V_2 = 0.73 \text{mL/gm}\).

a) Calculate the frictional coefficient \(f\) of urease in water at \(T = 293K\). Also, calculate the hydrodynamic radius of urease at \(T = 293K\).

\[
f = \frac{k_B T}{D} = \frac{1.38 \times 10^{-23} \text{J/K}}{3.46 \times 10^{-11} \text{m}^2/\text{s}} = 1.17 \times 10^{-10} \text{kg/s}
\]

\[
f' = 6\pi \eta R \Rightarrow R = \frac{f'}{6\pi \eta} = \frac{1.17 \times 10^{-10} \text{kg/s}}{(6\pi)(0.001005 \text{kg/m/s})} = 6.21 \times 10^{-9} \text{m}
\]

b) Assuming urease is an unhydrated sphere, calculate its radius and its frictional coefficient.

\[
\frac{4}{3} \pi R_0^3 = \frac{MW \cdot V_2}{N_A}
\]

\[
R_0^3 = \frac{3}{4\pi} \left( \frac{MW \cdot V_2}{N_A} \right) = \left( \frac{3}{4\pi} \right) \left( \frac{(482.7 \text{kg/mole})(0.73 \text{mL/gm}) \left( \frac{1 \text{m}^3}{10^6 \text{mL}} \right) \left( \frac{1000 \text{gm}}{1 \text{kg}} \right) }{6.02 \times 10^{23}} \right)
\]

\[
= 140 \times 10^{-27} \text{m}^3 \Rightarrow R_0 = 5.19 \times 10^{-9} \text{m}
\]

\[
\therefore f_0 = 6\pi \eta R_0 = (6\pi)(0.001005 \text{kg/m/s})(5.19 \times 10^{-9} \text{m}) = 0.98 \times 10^{-10} \text{kg/s}
\]

c) Calculate the number of waters of hydration associated with a single urease molecule at \(T = 293K\).

\[
\frac{f}{f_0} = \frac{1.17 \times 10^{-10} \text{kg/s}}{0.98 \times 10^{-10} \text{kg/s}} = 1.19
\]

\[
\text{Then} \left( \frac{f}{f_0} \right)^3 = \left( \frac{R}{R_0} \right)^3 = \frac{V_2}{V_1} + \delta \frac{V_1}{V_2} = 1.69
\]

\[
\delta = 0.69 \frac{V_2}{V_1} = 0.69 \left( \frac{0.73 \text{mL/gm protein}}{1 \text{mL/gm water}} \right) = 0.504 \text{gm water per gm protein}
\]

\[
\therefore \delta \left( \frac{1 \text{mole}}{18 \text{gm water}} \right) \left( \frac{482,700 \text{gm protein}}{1 \text{mole}} \right) = 13,515 \text{moles water per mole protein}
\]
4) Viscosity of a gas revisited. Consider $N$ He atoms in a container of volume $V$ at a pressure of $P=1$ atm and a temperature of $T=298\text{K}$.

a) For a pure, one component gas the viscosity is $\eta = \frac{N\bar{c}ml}{2V}$ where $l$ is the mean free path, $m$ is the mass of a single gas particle, and $\bar{c}$ is the average gas particle speed. Calculate the viscosity of helium gas at $T=298\text{K}$. Assume helium behaves ideally under these conditions.

$$\frac{PV}{nRT} = \frac{N}{N_A} \Rightarrow \frac{N}{V} = \frac{P \cdot N_A}{R \cdot T} = \frac{(1\text{atm})(6.02 \times 10^{23}\text{mole}^{-1})}{(0.0821\text{L} \cdot \text{atm} \cdot \text{mole}^{-1} \cdot \text{K}^{-1})(298\text{K})}$$

$$\frac{N}{V} = (2.46 \times 10^{22}\text{molecules} / \text{L})\left(\frac{10^3\text{L}}{\text{m}^3}\right) = 2.46 \times 10^{25}\text{molecules} / \text{m}^3$$

$$l = \frac{1}{\sqrt{2\pi d^2\left(\frac{N}{V}\right)}} = \frac{1}{\sqrt{2\pi\left(2.14 \times 10^{-10}\text{m}\right)^2\left(2.46 \times 10^{25}\text{m}^{-3}\right)}}$$

$$l = \frac{1}{\sqrt{2\pi\left(4.58 \times 10^{-20}\text{m}^2\right)\left(2.46 \times 10^{25}\text{m}^{-3}\right)}} = \frac{1}{50.03 \times 10^5\text{m}} = 2 \times 10^{-7}\text{m}$$

$$\bar{c} = \sqrt{\frac{8k_B T}{\pi m}} = \sqrt{\frac{8RT}{\pi MW}} = \sqrt{\frac{8\left(8.31\text{JK}^{-1}\text{mole}^{-1}\right)(298\text{K})}{(3.14)(0.004\text{kg} / \text{mole})}} = \sqrt{1.58 \times 10^6\text{m} / \text{s}} = 1270\text{m} / \text{s}$$

$$\eta = \frac{N\bar{c}ml}{2V} = (0.5)\left(\frac{2.46 \times 10^{25}\text{m}^{-3}}{6.02 \times 10^{23}\text{mole}^{-1}}\right)(2 \times 10^{-7}\text{m})\left(1.27 \times 10^3\text{m} / \text{s}\right)(0.004\text{kg} / \text{mole})$$

$$\eta = 2.08 \times 10^{-5}\text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1}$$

b) If a temperature gradient exists in a material, heat will be conducted through the material from the region of higher temperature to a region of lower temperature...a process called heat conduction. The thermal conductivity is the rate at which heat is transferred through a material per unit temperature gradient. For a gas the thermal conductivity $\lambda$ is

$$\lambda = \frac{5}{2} \left(\frac{\langle C_v \rangle}{m}\right)\eta$$

where the average heat capacity $\langle C_v \rangle = \frac{\partial \langle E \rangle}{\partial T}$ and $\eta$ is the viscosity of the gas. Calculate the thermal conductivity of helium at $P=1$ atm and $T=298\text{K}$. What are the units of thermal conductivity?
\[
\lambda = \frac{5}{2} \left( \frac{C_v}{m} \right) \eta = \frac{5}{2} \left( \frac{C_v}{N,m} \right) \eta = \frac{5}{2} \left( \frac{C_v}{MW} \right) \eta
\]

\[
C_v = \frac{\partial E}{\partial T} = \frac{\partial}{\partial T} \left( \frac{3RT}{2} \right) = \frac{3R}{2} = 12.47 J \cdot K^{-1} \cdot \text{mole}^{-1}
\]

\[
\therefore \lambda = (2.5) \left( \frac{12.47 J \cdot K^{-1} \cdot \text{mole}^{-1}}{0.004 \text{kg} \cdot \text{mole}^{-1}} \right) (2.08 \times 10^{-5} \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-1})
\]

\[
= 0.162 J \cdot K^{-1} \cdot \text{m}^{-1} \cdot \text{s}^{-1}
\]

**Note:** The units of thermal conductivity may seem obscure, but they make sense when you realize that thermal conductivity is the constant of proportionality that relates the heat flux \( h \) (Joules per m\(^2\) per second) to the thermal gradient \( dT/dx \) (degrees K per meter). The relationship is analogous to Fick’s First Law of Diffusion which relates the solute mass flux \( J_2 \) (kg per m\(^2\) per second) to the concentration gradient \( dC(x)/dx \) (kg/ m\(^3\) per m). In Fick’s First Law the constant of proportionality is the diffusion coefficient \( D_2 \). Therefore we can deduce the analogous heat flux equation...

\[
J_2 = -D_2 \frac{dC_2(x)}{dx} \Leftrightarrow h = -\lambda \frac{dT(x)}{dx}.
\]

From Fick’s First Law it is clear that \( D_2 \) must have units of m\(^2\)/s. From the heat flux equation it is clear that \( \lambda \) must have units of J \cdot K\(^{-1}\) \cdot m\(^{-1}\) \cdot s\(^{-1}\). Because of their units, fluxes like \( h \) and \( J_2 \) are sometimes called current densities...because they measure the amount of something that passes through a unit area per unit time. We will find that the viscosity coefficient \( \eta \) is similarly a constant of proportionality in a flux equation...