Mathematical and Scientific and "Miscalculations" in Lucretius
De Rerum Natura, Book I

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I. INTRODUCTION

It should not seem surprising that a two-thousand year old treatise on the workings of the universe should in our day be found to contain a few mathematical and scientific flaws. It may even appear bullish on our part, with the advantage of possessing two millennia worth of data since the time of Lucretius (c. 100?–55? B.C.E.), to point out the errors of someone who so long ago did the best he could, given the information available to him. How could he have understood that wind is the result of air pressure differences (see his denial of air compression in 391–397)? We have only recently begun to understand the nature of electro-magnetic radiation and the possibility of conversion from mass to energy (via Einstein's famous $E = mc^2$), through which we have become aware of a "third kind of nature," a concept rejected by Lucretius in 430–448. How could he have known about the possibility of space-time curvature which might answer his objection (see 958ff.) to the idea of a finite universe?

In at least two points of error, however, Lucretius was at variance with those who came centuries before his time. Was Lucretius not familiar with the work of his predecessors? Did he not understand their work or recognize the validity of earlier proofs that conflicted with his theories? Did he find their work simply irrelevant to his own philosophical pursuits? Was there something wrong with his methodology?

This paper first analyzes two errors of Lucretius which one might consider inexcusable on the grounds that in his time sufficient information was available through which he could have avoided the errors. Afterwards principles of Lucretius' methodology and possible causes of his errors are briefly discussed.

II. MATHEMATICAL ERROR REGARDING INFINITIES

In 599–634 Lucretius sets forth arguments to prove that there are "least parts" of atoms. If there is no pre-set limit to the successive dividing in half of matter, each atom could be said to consist of an infinite number of parts. According to Lucretius the universe itself contains an infinite number of parts. In his mind the idea of infinite divisibility of an atom led to the paradox of making each atom equal to the whole universe, since both equal infinity. Lucretius is so moved by the force of this paradox that in emotional tones he says: "But since true reasoning cries out and denies that the mind can believe it, you must admit to defeat and now accept that there are things which no longer consist endowed with any parts and are the smallest nature" (623–626).

Historians of Mathematics note that although Aristotle (c. 384–322 B.C.E.) had difficulty with certain aspects of infinity, it is clear that he at least understood the concept of infinitesimals as used by mathematicians of his day, as seen in expressions such as: "for that which is continuous is divisible without limit" (lit. "unto what is boundless," *Physics*, 185b); "from the divisibility among magnitudes (which the mathematicians treat as without limit)" (*Physics*, 203b, see also 206b for a further statement followed by a mathematical illustration).

Archimedes (c. 287–212 B.C.E.) says that it was Emulous (c. 408–355 B.C.E.) who actually used the concept of infinitesimals in the so-called "method of exhaustion" in mathematical proofs to derive the relationship between pyramids and prisms, and between cones and cylinders (in each case the former contains one-third the volume of the latter), and also that circles are to one another as the squares of the diameters. (*The Thirteen Books of Euclid's Elements*, transl. by Sir Thomas L. Heath, Vol. III, Cambridge, 1926, see "Historical Note" beginning Book XII, pg. 365). These principles are given mathematical treatment in Euclid's *Elements* (c. 300 B.C.E.; circles in xii, 2; pyramids in xii, 7; cones in xii, 10).

Lucretius' problem was of a conceptual nature. Mathematicians did not argue the feasibility of actual, physical infinite division of objects (which, of course, would take an infinite amount of time to accomplish). What Lucretius found unbelievable was a very useful mathematical concept that had long been in use to solve practical problems in determining surface areas and volumes of solids.

III. SCIENTIFIC ERROR REGARDING THE EARTH

Lucretius argues against the centripetal nature of matter in 1052–1113 and thereby rejects what is considered scientifically true today.

The Pythagoreans believed the earth was spherical by analogy with other heavenly bodies, but it was Aristotle who put forth clear arguments to prove that the earth was a sphere. He stated firmly, "It necessarily has a spherical form" (*De caelo*, 297a). One argument was based on the curvature of the earth's shadow as seen on the moon during a lunar eclipse—it is always the same curvature regardless of the exact configuration of sun, earth, and moon. Only a spherical shape for the earth would account for this observation. Secondly, Aristotle noted that different stars could be seen from different latitudes, which indicated not only that the earth was spherical, but also that its size could not be overly large, since a fairly small change in latitude produced a noticeable effect on star observations. He stated further that mathematicians had been able to measure the circumference of the earth (see *De caelo*, 297a–298b).

Eratosthenes (c. 276–194 B.C.E.) calculated the circumference of the earth based on observations similar to those of Aristotle, and he was much more accurate than earlier mathematicians. Noting the angular difference of the sun's rays falling on the surface of the earth at two points (almost) due north and south of each other (Alexandria and Syene, modern Aswan), and knowing the distance between the two points, it was a straightforward matter to solve geometrically for the circumference of the sphere of the earth, assuming (and accurately so) the sun's rays to be practically parallel to each other over the whole surface of the earth.

It is interesting to note that the Phoenicians recorded observations showing that the earth was spherical. In recounting how the Phoenicians had circumnavigated Africa, Herodotus relates a report (although he himself does not believe it) that the Phoenicians, in sailing around "Libya" on a westerly course, had the sun on their right, which would be in the north (*Histories,* IV, 42). This is no great surprise to those who live in the southern hemisphere.

Although Lucretius was right in denying that the earth is the center of the universe, he was wrong in his treatment of the earth itself, taking into consideration neither convincing mathematical demonstrations nor first-hand accounts of experienced sailors.

IV. "OBJECTIVITY" IN LUCRETIUS

The vocabulary of Lucretius emphasizes a methodology of "objectivity" in the sense of facts or perceptions expressed undistorted by personal emotions, prejudices, or traditional viewpoints. Lucretius places considerable stress on *ratio* and related terms as the mental basis upon which explanations of the universe are to be made. *Ratio* is used in various senses, but its primary significance is "reasoning, rational system of thought or explanation." Things are to be confirmed (*confirmare*) by the "reasoning of the mind" (*animi ratione,* 425) and are to be seen (*videndum*) with "keen reasoning" (*ratione sagaci,* 130–131). The arguments Lucretius presents are sufficient for "a keen mind" (*animo sagaci,* 402). One needs to be open-minded, which involves "empty ears" (*vacuas auris,* 50), being "removed from cares" (*semotum a curis, 51),* freedom from oppressive religion (62–101), freedom from fear, particularly fear of death (102ff.). The ultimate explanation must be comprehensive (*omne immensum peragravit mente animoque,* 74) and should be based on evidence presented to the human senses, particularly that of sight (422–425, 699–700). A serious truth-seeker will not allow himself to fall away from "true reasoning" (*vera ratione*) by twisted words (*inversis verbis)* that touch the ears nicely (*belle tangere auris)* and are "painted" ("tricked out") with a charming (seductive) sound (*lepido quae sunt fucata sonore*) (635–644).

In keeping with these principles, Lucretius makes frequent use of inferential conjunctions throughout his work to emphasize the logical flow and force of his arguments. Most common among these is *igitur;* others include *ergo, quapropter* and the emphatic *quare etiam atque etiam*.

Lucretius was aware of many other theorists. Besides Epicurus he refers specifically to Heraclitus (c. 540–480 B.C.E.; I,638), Empedocles of Acragas (c. 493–433 B.C.E.; I, 716ff.), Anaxagoras (c. 500–428 B.C.E.; I,830ff.) and Democritus (5th cent. B.C.E.; III,371). He mentions and refutes the ideas of others whom he does not name. He seems to have been thorough in his research.

V. CONFIDENCE OF LUCRETIUS

Based on an assessment of his own rational principles and comprehensiveness of study, Lucretius is thoroughly convinced of the correctness of his views, which are set forth with loyal devotion (*studio disposta fideli,* 52). Though in his view Latin compared with Greek shows grave deficiencies in this endeavor, Lucretius' concentrated efforts through sleepless nights made it possible for him to present to our minds clear lights (*clara lumina*) so that we can gain understanding (136–145).

Lucretius, like Epicurus, is *victor* (75; compare 62–79 with 921–950). He does not always leave the arguments to speak for themselves. Sometimes he wants us to know when it is necessary for us to admit their truthfulness (*necessest confiteare,* 269–270; *fatendumst,* 205; *victus fateare necessest,* 624).

VI. POSSIBLE REASONS FOR THE ERRORS

What went wrong? Why did Lucretius make at least two errors that must have seemed ludicrous to the more scientifically knowledgeable even of his day? The short answer is that humans are fallible, including Lucretius. Specifically in what respect did Lucretius' "humanness" fail him? Several explanations could be pursued. (1) Lucretius simply liked essential Epicurean ideas, many of which he found intellectually superior to other ideas, and accepted a "package," so to speak, without carefully examining all the contents. (2) Lucretius was not as unemotionally calm and rational as he claims to be and requires of others. He has strong emotional aversions to religious abuses and teachings of his day. There is a tendency to over-react in such cases, and arguments tending toward an opposite extreme become increasingly less objective. (3) He was an egotist. He not only compares himself to Epicurus, the greatest of all, he excels him. His pre-occupation with himself and his claim to fame got in the way of critical self-examination. (4) He was not comprehensive enough. He did not, or could not, become familiar with all work relating to his treatise, and it was his misfortune that he did not obtain access to better sources of information. (5) He did not understand many of the things he wrote about: a great poet-philosopher may not necessarily be a great mathematician.

Whatever the reasons, anyone who today will attempt with great boldness to declare the final, ultimate explanation of the nature of things must be aware that factuality is not necessarily correlated with emphasis on rationality and confident assertion. Ideas must be demonstrated to be true by a number of tests, one of which sometimes lies beyond our reach—the test of time.

APPENDIX: HISTORICAL NOTE ON "INFINITIES"

The commentaries by Bailey (1966, Vol. II, pg. 703) and Leonard & Smith (1942, pp. 262–263) note that Lucretius' argument is proved to be fallacious by Isaac Newton in a passage quoted by Munro. The quotation itself is cited in Leonard & Smith and comes from Newton's second letter to Richard Bentley, a cleric of St. Martin-in-the-Fields, written on 17 January 1692/3. The pertinent sections follow:

> But ye argue in ye next paragraph of your letter that every particle of matter in an infinite space has an infinite quantity of matter on all sides & by consequence an infinite attraction every way & therefore must rest *in equilibrio* because all infinites are equal… The generality of mankind consider infinites no other ways then definitely, & in this sense they say all infinites are equal, though they would speak more truly if they should say they are neither equal nor unequal nor have any certain difference or proportion one to another… So when men argue against ye infinite divisibility of magnitude, by saying that if an inch may be divided into an infinite number of parts, ye sum of those parts will be an inch, & if a foot may be divided into an infinite number of parts ye sum of those parts must be a foot, & therefore since all infinites are equal those summs must be equal, that is an inch equal to a foot. The falsness of ye conclusion shews an error in ye premisses, & ye error lies in ye position that all infinites are equal. There is therefore another way of considering infinites used by Mathematicians…
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> (*The Correspondence of Isaac Newton*, Vol. III (1688–1694), ed. H.W. Turnbull, Cambridge, 1961, pg. 239).

Newton was concerned that Bentley might not comprehend his explanation. After dealing with other matters, Newton returns to the subject of infinities at the close of the letter:

> I fear what I have said of infinites will seem obscure to you: but it is enough if you understand that infinites when considered absolutely without any restriction or limitation, are neither equal nor unequal nor have any certain proportion to one another, & therefore ye principle that all infinites are equal is a precarious one. Sr I am
>
> Is. Newton.
>
> (ibid., pg. 240).

Newton's fear was well-founded. Bentley responded with objections, not unlike those of Lucretius, to which Newton again replied in a letter dated 25 February 1692/3 with further illustrations.

Newton went on as the first (or concurrently with Leibnitz) to rigorously develop the infinitesimal calculus, but, as shown above, the essentials of infinitesimals were already worked out by Greek mathematicians even before the time of Lucretius.

QUOTATIONS FROM LATIN AND GREEK SOURCES:

Latin quotations from Lucretius are from:
Brown, P. Michael, *Lucretius, De Rerum Natura I,* 1984.
Translations are my own.

Greek quotations from Aristotle are from the Loeb editions.
Translations are my own.

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