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Statistical Analysis of Cumulative Shipper-Receiver Data

Prepared by D. M. Rose, F. W. Scholz

Boeing Computer Services Company

Prepared for
U.S. Nuclear Regulatory Commission

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Prepared by
D. M. Rose, F. W. Scholz

Boeing Computer Services Company
565 Andover Park West
Tukwila, WA 98188

Prepared for
Division of Facility Operations
Office of Nuclear Regulatory Research
U.S. Nuclear Regulatory Commission
Washington, D.C. 20555
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ABSTRACT

This report presents a method of constructing a test for the detection of nuclear material diversions over a series of n shipments. The available data are the shipper-receiver differences (SRD's) which are assumed to be multivariate normally distributed with known covariance matrix. Given a specific diversion scenario the most powerful test is derived. The "least favorable" scenario with respect to NRC detection is identified and the most powerful test against this scenario is found. The power function of this test depends only on the total amount diverted and not on the particular scenario chosen. This test maximizes among all level α tests, the minimum power over all scenarios with total diversion at least m. For various diversion scenarios, assuming independent SRDs, the power of the max-min test is compared to that of tests that are specific for each of these scenarios. Assuming the SRD's are independent with unknown variances, the max-min test is modified using Satterthwaite's approximation. The quality of this latter approximation is investigated for a range of parameter sets.

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1. EXECUTIVE SUMMARY

The shipper-receiver difference problem is concerned with nuclear material accounting discrepancies between a pair of licensees. The shipper sends a group of containers, a shipment, with special nuclear material to the receiver. The shipper measures the amount of material sent, and estimates the variability in the measurement. The receiver similarly measures the amount of material received and estimates the measurement variability. This process is repeated for a number of shipments.

The shipper-receiver difference problem is:

> The shipper and receiver measurements differ. Is this difference due to measurement error or loss/diversion of special nuclear material?

A statistical test is desired which is powerful in detecting material loss. A test could be applied separately for each shipment, or aggregately for the time history between the licensee pair.

If a large amount of material is taken from a single shipment (block loss), a separate analysis of each shipment yields a high probability of detection. If a small amount of material is lost from each shipment (trickle loss), separate analyses of the shipments yield a low probability of detection. A cumulative test on the shipper-receiver differences, however, has a much greater chance of detecting trickle loss.

A statistical test is desired which is powerful in detecting material loss for a diverse set of material loss/diversion scenarios.

In this study we have restricted attention to a fixed set of n shipments. A test is developed in Section 3 for correlated measurement errors with a known covariance matrix. This test, called the max-min test, is optimal against the least favorable diversion scenario as identified in Section 3.3. This test is evaluated for a number of diversion scenarios identified by NRC, and compared to optimal tests which require prior information on the diversion scenario. The max-min test is found to perform well relative to these alternative tests.

Usually the measurement variances and covariances are unknown, but estimates of these terms may be available. The max-min test is modified in Section 4 under the assumption of independent errors to incorporate variance estimates. The distribution of the resultant test, T_S, has no well-behaved analytic solution. Of the procedures investigated for approximating this distribution, the Satterthwaite procedure is shown to perform best in controlling the test's false alarm rate. This conclusion is arrived at after performing a Monte Carlo study over a range of shipment parameters. It is shown analytically that the power of the max-min test, assuming either known or estimated variances, depends on the diversion scenario only through the total diversion over all shipments. The only other primary factor affecting the power of the test is the total measurement variation of the shipper-receiver differences. The Monte Carlo program developed permits the user to compute the probability of detecting material diversion for any specified diversion scenario. The power of the test is evaluated over a range of shipment parameters, including number of shipments and allocation of measurement variability among shipments. It is found that these shipment parameters have little effect on the power.

Implementation of the T_S test is discussed in Section 5. Information requirements and assumptions are specified for the case of independent errors and for a specified structure of correlated errors. Necessary research for a sequential extension of this procedure is discussed in Section 5. A sequential procedure permits controlling the overall false alarm rate while allowing for decisions on material loss at earlier points in time.

2. STATEMENT OF PROBLEM

2.1. Background

Nuclear material is sent by a shipper to a receiver. Each shipment is measured by the shipper and the receiver, and the separate measurement by each is recorded. The shipments may be sent at regular time intervals, or at irregular intervals. The shipments are sequentially labeled by shipment number, with a number of containers comprising each shipment. Attention in this study is restricted to only those containers with similar nuclear material.

Based on the amount of material measured and recorded separately by the shipper and the receiver, one is interested in detecting loss or diversion of nuclear material from the shipment. If the shipper records amount S_i shipped on the ith shipment and the receiver records amount R_i received, then

$$D_i = S_i - R_i \quad \text{for} \quad i = 1, \ldots, n$$

represents the discrepancy between the shipper and receiver measurements for the ith shipment. D_i is the shipper-receiver difference for shipment i. There are n such shipments between the shipper and receiver pair. If $D_i > 0$, then one might conclude that some of the nuclear material has been lost or diverted. However, one might also attribute the non-zero D_i value to measurement errors in the separate analyses performed by the shipper and the receiver. Statistical methods provide a mechanism for deciding whether the observed D_i may be attributed to random measurement error, or whether it is too large to have been a likely value sampled from the measurement error distribution. In order to make this type of assessment one must have:

i) A realistic mathematical model of the process (see 2.3),

ii) Knowledge or estimates of the parameters of the model (see 2.4, 3.0, 4.0).

One needs to determine the acceptable false-alarm rate, the probability of incorrectly stating that material has been lost, when no material has been lost. For this study, false-alarm rates of .10, .05, and .01 have been used in the analysis. In order to evaluate the performance of a statistical procedure for detecting material loss, a mathematical representation of the loss mechanism is determined (see 3.1, 3.3 and 3.6).

In current practice, S_i, R_i and their limits of error are reported. The limit of error of S_i (R_i), LES_i (LER_i), is twice the standard deviation of S_i (R_i). A normal (Gaussian) error structure is assumed. From this one may construct the limit of error of D_i

$$LED_i = (LES_i^2 + LER_i^2)^{\frac{1}{2}}$$

If $D_i > LED_i$ then one may conclude that there is evidence of potential nuclear material diversion and the matter should be investigated. If

$$D_i \leq LED_i$$

there is no significant statistical evidence of material loss. However, material loss is still possible. For example, if a diverter were to take small amounts of material from each shipment such that each observed D_i is, with a high probability, less than its limit of error, individual tests on D_i would not be very effective in detecting the material loss. The following statistic, called the cumulative shipper-receiver difference

$$CD_i = \sum_{j=1}^{i} D_j$$

is sensitive to detecting small amounts of material diverted over several shipments, as long as the total amount diverted is not negligible compared to the standard deviation of CD_i. For example this would be the case if the total amount diverted from i shipments is of order i and the standard deviation of CD_i is of order $\sqrt{i}$. As with D_i, one compares CD_i to its limit of error

$$LECD_i = (\sum_{j=1}^{i} (LED_j)^2)^{\frac{1}{2}}$$

2.2. Problem to be Addressed

The statistical procedure developed is, as specified by NRC, to be powerful in detecting the following three material diversion scenarios:

i) Diversions proportional to shipment size
ii) A monotonic increasing sequence of diversions
iii) A cyclic pattern of diversions

A statistical procedure is derived for correlated errors with known covariance matrix in 3.3. The performance of this statistical procedure is compared to diversion specific tests for these three scenarios in 3.6 for the restricted case of independent errors. Block loss is also of interest, and is addressed in 3.6. The known variance procedure, developed in 3.3, is independent of diversion scenario, depending only on total amount of material diverted, not on the pattern of diversion. This procedure when extended to independent errors with unknown variances, Section 4.1, is again dependent only on total material diversion. No diversion specific tests for unknown variances are developed for power comparisons. The results would, most likely, be similar to results for the known variance situation. Power results for the unknown variance procedure are presented in 4.3.4.

An alternative to a fixed framework of n shipments is to view the process sequentially. Sequential procedures are inherently more complex than fixed sample size procedures. Typically they are some modification of fixed sample size procedures, the statistical properties of the latter usually being well understood. Since the fixed sample problem (fixed number of shipments) in the present context has not yet been fully addressed it appears to us appropriate to limit investigation to the fixed sample problem. Extensions of the procedure developed and additional research are discussed in 5.5.

In developing a statistical procedure which is powerful for detecting diversion according to any of the above scenarios, we generally restrict attention to the basic information contained in the data from n successive shipments. The data requirements are briefly addressed in 4.1, and discussed in detail in 5.2 and 5.3.

The approach taken provides a simple and natural extension to the single shipper-receiver difference framework. Within this framework it is possible to detect small diversions over several shipments which are not effectively detectable within the single shipper-receiver difference framework. The diversion sensitivity of any reasonable procedure based on n shipper-receiver differences will increase with n if the standard deviation is of order $\sqrt{n}$. Thus it should be possible to adjust n in accordance with deemed critical amounts of diversion per shipment in order to obtain prespecified detection power without compromising the false alarm rate.

2.3 Mathematical Model

For the ith shipment, the shipper receiver difference D_i is the difference between the amount reported shipped S_i and the amount reported received R_i

$$D_i = S_i - R_i$$

In this study, we assume that

$$D_i = \mu_i + Z_i \quad \text{for } i=1,\ldots,n$$

where

μ_i = true amount of material loss
Z_i = measurement error

and $\underline{Z}' = (Z_1,\ldots,Z_n)$ is assumed to have an n-variate normal distribution with mean vector $\underline{0}$ and non-singular covariance matrix C.

In matrix notation this model for the n shipper receiver differences reads:

$$\underline{D} = \underline{\mu} + \underline{Z} \quad , \quad \underline{Z} \sim N_n(\underline{0},C)$$

with $\underline{D}' = (D_1,\ldots,D_n)$ and $\underline{\mu}' = (\mu_1,\ldots,\mu_n)$ the ' denoting the matrix transpose.

This representation for $\underline{D}$ is derived in the following sections. In the rest of this report, the analysis is performed on this model. In Section 5.2 information required by this approach is addressed.

2.3.1 Model Derivation

The ith shipment between the shipper-receiver pair is composed of N_i containers. The shipper measurement, S_{ij}, of material in the jth container of shipment i is

$$S_{ij} = \delta_{ij} + e_{ij}$$

where

δ_{ij} = true amount of material in container j from shipment i when measured by the shipper.

e_{ij} = aggregate sum of all contributing measurement errors affecting container j from shipment i.

In order to understand the aggregate sum nature of e_{ij} and the correlation structure of the e_{ij} for varying i and j, we propose the following generic model: In the course of all the measurements performed by the shipper during the n shipments many distinct elemental errors, $E_1,\ldots,E_L$, arise. Some of the elemental errors combine to form the aggregate error e_{ij} for container j from shipment i. A different set of elemental errors will typically combine to form the aggregate error $e_{i'j'}$ for container j' from shipment i' with $(i,j) \neq (i',j')$. However, there may be some overlap in both sets of elemental errors, i.e., some particular elemental errors may appear in both aggregates e_{ij} and $e_{i'j'}$. Such overlap is the result of systematic errors. For example, a particular elemental error may be a calibration error for a certain instrument. Between calibrations such error, denote it by E_ℓ , affects all container measurement errors in the same fashion, i.e., the elemental error term E_ℓ will appear in all the aggregate error terms e_{ij} for those containers.

This model is more clearly stated in mathematical terms. Let $E_1,\ldots,E_L$ be independent random variables with mean zero and respective variances $\omega_1^2,\ldots,\omega_L^2$. To each (i,j) there corresponds an index set $I_{ij} \subset \{1,\ldots,L\}$ for container j from shipment i. This index set I_{ij} indicates which of the elemental errors in $\underline{E}' = (E_1,\ldots,E_L)$ contribute to the total measurement error e_{ij} for container j from shipment i, i.e.

$$(2.3.1) \qquad e_{ij} = \sum_{\ell \in I_{ij}} E_\ell$$

or $$e_{ij} = \underline{u}'_{ij} \cdot \underline{E}$$

where $\underline{u}'_{ij} = (u_{ij1},\ldots,u_{ijL})$ with $u_{ij\ell} = 1$ if $\ell \in I_{ij}$

and $u_{ij\ell} = 0$ if $\ell \notin I_{ij}$.

Note that in (2.3.1) we assume that the elemental errors aggregate in an additive fashion.

According to the above model e_{ij} and $e_{i'j'}$ will be independent whenever $I_{ij} \cap I_{i'j'} = \phi$ i.e., there are no common (systematic) elemental errors contributing to both e_{ij} and $e_{i'j'}$.

In a similar fashion the receiver measurement, R_{ij}, of material in the jth container from shipment i is modeled as follows:

$$R_{ij} = \Delta_{ij} + f_{ij}$$

where Δ_{ij} = true amount of material in container j from shipment i when measured by the receiver.

f_{ij} = aggregate sum of all contributing measurement errors affecting container j from shipment i.

Similarly as e_{ij} was expressed in terms of elemental independent errors $E_1,\ldots,E_L$ we express f_{ij} as follows:

$$f_{ij} = \sum_{\ell \in J_{ij}}' F_\ell = \underline{v}'_{ij} \cdot \underline{F}$$

where $\underline{F}' = (F_1,\ldots,F_{L'})$ are L' independent elemental error random variables with mean zero and variances $\lambda_1^2,\ldots, \lambda_{L'}^2$ respectively. The index set $J_{ij} \subset \{1,\ldots,L'\}$ indicates which of the elemental errors in $\underline{F}' = (F_1,\ldots,F_L')$ contribute to the total measurement error f_{ij} for container j from shipment i and $\underline{v}_{ij}' = (v_{ij1},\ldots,v_{ijL'})$ with $v_{ij\ell} = 1$ if $\ell \in J_{ij}$ and $v_{ij\ell} = 0$ if $\ell \notin J_{ij}$.

Combining shipper and receiver measurement over the ith shipment we have:

$$S_i = \sum_{j=1}^{N_i} S_{ij} = \sum_{j=1}^{N_i} \delta_{ij} + \sum_{j=1}^{N_i} e_{ij}$$

$$= \delta_i + \sum_{j=1}^{N_i} \underline{u}'_{ij} \cdot \underline{E}$$

$$= \delta_i + \underline{U}'_i \cdot \underline{E}$$

and

$$R_i = \sum_{j=1}^{N_i} R_{ij} = \sum_{j=1}^{N_i} \Delta_{ij} + \sum_{j=1}^{N_i} f_{ij}$$

$$= \Delta_i + \sum_{j=1}^{N_i} \underline{v}'_{ij} \cdot \underline{F}$$

$$= \Delta_i + \underline{V}'_i \cdot \underline{F}$$

where

$$\delta_i = \sum_j \delta_{ij} \qquad \Delta_i = \sum_j \Delta_{ij}$$

$$\underline{U}_i = \sum_j \underline{u}_{ij} \qquad \underline{V}_i = \sum_j \underline{v}_{ij} \quad .$$

Thus the shipper receiver difference D_i for the ith shipment is:

$$D_i = S_i - R_i$$

$$= \delta_i - \Delta_i + \underline{U}'_i \underline{E} - \underline{V}'_i \underline{F}$$

$$= \mu_i + Z_i$$

with $\mu_i = \delta_i - \Delta_i$

and $Z_i = \underline{U}'_i \underline{E} - \underline{V}'_i \underline{F}$

with $U = \begin{pmatrix} \underline{U}'_1 \\ \\ \underline{U}'_n \end{pmatrix}$ and $V = \begin{pmatrix} \underline{V}'_1 \\ \\ \underline{V}'_n \end{pmatrix}$

we can write

(2.3.2) $\underline{Z} = U \underline{E} - V \underline{F}$

Making the reasonable assumption that the elemental error sets $\underline{E}$ and $\underline{F}$ of shipper and receiver are independent we can conclude that $\underline{Z}$ has mean zero and covariance matrix

(2.3.3)
$$\begin{aligned} C &= U \operatorname{diag}(\omega_1^2, \ldots, \omega_L^2) U' \\ &+ V \operatorname{diag}(\lambda_1^2, \ldots, \lambda_{L'}^2) V' \end{aligned}$$

where $\operatorname{diag}(X_1, .., X_k)$ denotes a k by k square diagonal matrix with diagonal elements $X_1, .., X_k$. Note that by the structure of U,V and (2.3.3) all elements of C are nonnegative.

The matrices U and V are structure matrices which indicate which elemental errors contribute with what multiplicity to each shipper receiver difference error term Z_i.

Let us illustrate this with a very concrete example in which we assume that n=2 for notational convenience.

Let, $\underline{E}' = (E_1,..,E_{N_1}, E_{N_1+1},..,E_{N_1+N_2}, E_{N_1+N_2+1}, E_{N_1+N_2+2})$

and

$$\underline{F}' = (F_1,..,F_{N_1}, F_{N_1+1},.., F_{N_1+N_2}, F_{N_1+N_2+1}, F_{N_1+N_2+2}).$$

The index sets I_{ij} and J_{ij} are defined as follows:

$$I_{1j} = J_{1j} = j, N_1 + N_2 + 1 \qquad j=1,..,N_1$$

$$I_{2j} = J_{2j} = j, N_1 + N_2 + 2 \qquad j = N_1 + 1,.., N_1 + N_2,$$

i.e., the error term e_{ij} (and similarly f_{ij}) for container j from shipment i is the sum of two components

$$E_j + E_{N_1 + N_2 + i}$$

the first of which, E_j, changes from container to container within the same shipment whereas the second, $E_{N_1 + N_2 + i}$, remains the same for all containers within the same shipment. Thus the error term for each container contains a random component and a systematic component. However, the systematic components in different shipments are independent of each other. The structure matrices U and V have therefore the following form:

$$U = V = \begin{pmatrix} 1, ..., 1, 0, ..., 0, N_1, 0 \\ 0, ..., 0, 1, ..., 1, 0, N_2 \end{pmatrix}$$

Writing E_i^* and F_i^* for $E_{N_1 + N_2 + i}$ and $F_{N_1 + N_2 + i}$ respectively we can write the error terms Z_i for the shipper receiver differences as follows:

$$Z_1 = \sum_{j=1}^{N_1} (E_j - F_j) + N_1 (E_1^* - F_1^*)$$

$$Z_2 = \sum_{j=N_1+1}^{N_1+N_2} (E_j - F_j) + N_2 (E_2^* - F_2^*)$$

If in addition we assume that

$$\mathrm{var}(E_j - F_j) = \sigma_e^{\,2} \qquad j=1,\ldots,N_1 + N_2$$

and $\mathrm{var}\,(E_i^* - F_i^*) = \sigma_s^2$

we find: $\mathrm{var}\, Z_i = N_i \sigma_e^2 + N_i^2 \ \sigma_s^2 \qquad i=1,2$

and $\mathrm{cov}(Z_1, Z_2) = 0$,

i.e., the covariance matrix of $\underline{Z}' = (Z_1, Z_2)$

is

$$C = \begin{pmatrix} N_1 \ \sigma_e^2 + N_1^2 \ \sigma_s^2 & 0 \\ 0 & N_2 \sigma_e^2 + N_2^2 \sigma_s^2 \end{pmatrix}$$

The significance of the presence of systematic error terms is underscored by the multiplier N_i^2 in front of σ_s^2 in contrast to N_i as multiplier of σ_e^2.

2.3.2 The Normality Assumption

Finally, let us address the normality assumption concerning the distribution of $\underline{Z}$. Any such distributional assumption can at best be of approximate nature. At issue is the correctness and quality of this approximation. There are two arguments supporting the normality assumption for $\underline{Z}$. The first is that the elemental measurement errors by their nature are approximately normally distributed which by equation (2.3.2) entails approximate normality of $\underline{Z}$. The second argument is based on the central limit theorem and uses the fact that by equation (2.3.2) each Z_i is the aggregate sum of several if not many elemental errors which are independent. Therefore the Z_i are approximately normally distributed even if the elemental errors were not.

2.4. Approach

In this study we address the shipper-receiver difference problem for a fixed number of shipments. Using the mathematical model in Section 2.3, we first develop a test procedure for correlated errors with known covariance matrix, Section 3.3. The performance of the procedure is evaluated for several diversion scenarios under the restriction of independent errors between shipments.

We next address the fixed number of shipments problem with independent errors and variances unknown, Section 4. The known variance test procedure is modified to incorporate variance estimates. A Monte Carlo simulation program is used to determine the power of the test. This program permits power calculations, the probability of detection, for any diversion scenario, Section 4.3.

Areas for further work are discussed in Section 5.5.

3. RESULTS WITH KNOWN COVARIANCE MATRIX

3.1. Hypothesis Testing Framework

The observable data consists of n shipper-receiver differences $D_1,\ldots,D_n$ which are assumed to follow the following statistical model (see ?.3).

$$D_i = \mu_i + Z_i \qquad \text{for } i=1,\ldots,n$$

where $Z_1,\ldots,Z_n$ represent the measurement errors and $\mu_1,\ldots,\mu_n$ represent the true losses or diversions. It is assumed that $\underline{Z}' = (Z_1,\ldots,Z_n)$ has an n-variate normal distribution with mean vector $\underline{0}$ and known nonsingular covariance matrix C. The unknown vector $\underline{\mu}' = (\mu_1,\ldots,\mu_n)$ will appropriately be called a diversion (or loss) scenario since it specifies how the material diversions (or losses) are allocated over the n shipments. In this report we will speak of diversions but that will mean either material diversions or losses.

Our primary objective is to find a test that has good detection power against a variety of diversion scenarios. To accomplish this goal we cast the problem in an appropriate hypothesis testing framework and introduce some relevant notation. Let

$$A = \left\{ \underline{\mu} \in R^n : \sum_{i=1}^{n} \mu_i > 0 \right\}$$

be the collection or set of all possible diversion scenarios.

Alternatively, A may be represented as follows:

$$A = \bigcup_{\underline{e} \in E} A(\underline{e})$$

where

$$A(\underline{e}) = \{m\underline{e} : m > 0\}$$

and $$E = \left\{\underline{e} \in R^n : \sum_{i=1}^{n} e_i = 1\right\}$$

E may be thought of as representing all diversion scenarios which result in a total diversion of one unit over the n shipments. The different elements of E distinguish themselves from each other by the allocation of that one unit total diversion to the various shipments. We may think of $\underline{e}$ as indicating the "direction" of the diversion. The factor m in the term $m\underline{e}$ indicates the total amount of material diverted over n shipments. The diversion parameter is thus $\underline{\mu} = m\underline{e}$.

As a notational convenience we shall write Πx_i and Σx_i for

$$\prod_{i=1}^{n} x_i \quad \text{and} \quad \sum_{i=1}^{n} x_i$$

respectively, as long as there is no cause for confusion.

3.2. The UMP Test Against a Specified Diversion Scenario (Direction $\underline{e}$)

In this section we will show that there is a uniformly most powerful (UMP) level α test for testing the hypothesis of no material diversion H: $\underline{\mu} = \underline{0}$ against the alternative K($\underline{e}$): $\underline{\mu} \in A(\underline{e})$.

By the Neyman-Pearson lemma, Lehmann (1959) p. 65, the most powerful level α test of the simple hypothesis H: $\underline{\mu} = \underline{0}$ against the simple alternative K: $\underline{\mu} = m\underline{e}$ $(m > 0)$ rejects when

$$\Lambda = \frac{(2\pi)^{-\frac{1}{2}n}(\det C)^{-\frac{1}{2}} \exp(-\frac{1}{2}(\underline{D} - m\underline{e})'C^{-1}(\underline{D} - m\underline{e}))}{(2\pi)^{-\frac{1}{2}n}(\det C)^{-\frac{1}{2}} \exp(-\frac{1}{2}\underline{D}'C^{-1}\underline{D})}$$

$$= \exp\left(-\tfrac{1}{2} m^2 \underline{e}' C^{-1} \underline{e} + m \underline{e}' C^{-1} \underline{D}\right)$$

is too large or equivalently (since $m > 0$) when the test statistic

$$T(\underline{e}) = \underline{e}' C^{-1} \underline{D} > k_\alpha$$

where k_α is chosen such that $P(T(\underline{e}) > k_\alpha \mid \underline{\mu} = \underline{0}) = \alpha$.

Since under $H: \underline{\mu} = \underline{0}$,

$$T(\underline{e}) \sim N(0, h(\underline{e}))$$

it follows that

$$k_\alpha = z_{1-\alpha} \; h^{\frac{1}{2}}(\underline{e}),$$

where $z_{1-\alpha}$ is the $1-\alpha$ quantile point of the standard normal distribution and

$$h(\underline{e}) = \underline{e}' C^{-1} \underline{e} .$$

Since the above test does not depend on the value of $m > 0$, it follows that the same test is the most powerful level α test for testing $H: \underline{\mu} = 0$ against any $K: \underline{\mu} = m\underline{e}$ with $m > 0$. Hence the test which rejects when $T(\underline{e}) > k_\alpha$ is a UMP level α test for testing $H: \underline{\mu} = \underline{0}$ versus $K(\underline{e}): \underline{\mu} \in A(\underline{e})$.

From the form of the test statistic $T(\underline{e})$ above, it is clear that it intrinsically depends on the diversion scenario direction $\underline{e} \in E$. Thus there cannot exist a uniformly most powerful level α test for testing $H: \underline{\mu} = \underline{0}$ against $K: \underline{\mu} \in A$.

Next we derive the power function of the test based on $T(\underline{e})$. Note that since

$$\underline{D} \sim N_n(\underline{\mu}, C)$$

the test statistic

$$T(\underline{e}) \sim N(\underline{e}'C^{-1}\underline{\mu}\ ,\ h(\underline{e})),$$

so that the power function is

$$\beta_{\underline{e}}\ (\underline{\mu}) = P(T(\underline{e}) > z_{1-\alpha} h^{\frac{1}{2}}(\underline{e})\ \Big|\ \underline{\mu})$$

$$= P\left(\frac{T(\underline{e}) - \underline{e}'C^{-1}\underline{\mu}}{h^{\frac{1}{2}}(\underline{e})} > z_{1-\alpha} - \frac{\underline{e}'C^{-1}\underline{\mu}}{h^{\frac{1}{2}}(\underline{e})}\ \Bigg|\ \underline{\mu}\right)$$

$$= 1 - \Phi(z_{1-\alpha} - \underline{e}'C^{-1}\underline{\mu}\ /\ h^{\frac{1}{2}}(\underline{e}))$$

where Φ is the standard normal distribution function. $\beta_{\underline{e}}(\underline{\mu})$ is the probability of detecting a diversion when the diversion scenario is $\underline{\mu}$. In the particular case when $\underline{\mu} = m\underline{e}$ $(m > 0)$, i.e., a scenario against which the test is most powerful, then the power function simplifies to

(3.2.1) $$\beta'_{\underline{e}}\ (m) = \beta_{\underline{e}}\ (m\underline{e}) = 1 - \Phi(z_{1-\alpha} - m\ h(\underline{e})^{\frac{1}{2}})$$

which is an increasing function of m, the total amount diverted in the direction of $\underline{e}$.

3.3. The Least Favorable Scenario and the Optimal Test S Against It

In the previous section we derived the optimal test against any diversion scenario direction and we found that the optimal test changed with the direction, i.e., we could not find an overall UMP test against all directions (all diversion scenarios). It is thus natural to ask whether there is a least favorable direction in the sense that it would be the hardest diversion scenario for NRC to detect. To make this question meaningful let us look at diversion scenarios which have a fixed total amount m of diversion, i.e., $\underline{\mu} = m\underline{e}$ with $\underline{e} \in E$. The best power that can be achieved against $\underline{\mu} = m\underline{e}$ is

$$\beta_{\underline{e}}(m) = 1 - \Phi(z_{1-\alpha} - m\, h^{\frac{1}{2}}(\underline{e}))$$

and this power is minimized by that direction $\underline{e}$ for which $h(\underline{e})$ is minimized. To accomplish this minimization of $h(\underline{e})$ over $\underline{e} \in E$ let

$$\underline{e}^* = C\,\underline{1}/(\underline{1}'\,C\,\underline{1}) \text{ where } \underline{1}' = (1,\ldots,1) \text{ (note } \underline{e}^* \in E)$$

then

$$h(\underline{e}) = \underline{e}'C^{-1}\underline{e} = (\underline{e}-\underline{e}^*)'C^{-1}(\underline{e}-\underline{e}^*) + 2\,\underline{e}^{*\prime}C^{-1}\underline{e} - \underline{e}^{*\prime}C^{-1}\underline{e}^*$$

$$= (\underline{e}-\underline{e}^*)'C^{-1}(\underline{e}-\underline{e}^*) + \frac{1}{\underline{1}'C\underline{1}} \geq \frac{1}{\underline{1}'C\underline{1}} = h(\underline{e}^*)$$

which clearly is minimized by $\underline{e} = \underline{e}^*$.

Thus when the material is diverted from the shipments according to the scenario

$$\underline{\mu} = m\,\underline{e}^*$$

NRC will have the least chance of detecting the diversion. For this least favorable (to NRC) scenario, the UMP level α test for testing $H:\underline{\mu}=\underline{0}$ against $K(\underline{e}^*):\underline{\mu} \in A(\underline{e}^*)$ rejects when

$$T(\underline{e}^*) = \underline{e}^{*\prime}C^{-1}\underline{D} = \Sigma D_i/\underline{1}'C\underline{1}$$

exceeds

$$k_\alpha = z_{1-\alpha}h^{\frac{1}{2}}(\underline{e}^*) = z_{1-\alpha}(\underline{1}'C\underline{1})^{-\frac{1}{2}}$$

or equivalently which rejects when

$$S = \frac{\Sigma D_i}{(\underline{1}'C\underline{1})^{\frac{1}{2}}} > z_{1-\alpha}$$

The UMP test S for $\underline{\mu} = \underline{0}$ against the least favorable scenario $\underline{\mu} = m\underline{e}^*$ is a CUSUM test. A characteristic property of this test is that its power over A does not depend on the scenario direction $\underline{e}$ but only on the total amount m diverted. If $\underline{\mu} = m\underline{e}$ with $\underline{e} \in E$ and $m > 0$, then

$$S \sim N(m\,(\underline{1}'C\underline{1})^{-\frac{1}{2}},\, 1)$$

Hence the power function $\beta_{\underline{e}^*}(\underline{\mu})$, $(\underline{\mu}=m\underline{e})$, of the level α test based on S is

$$(3.3.1) \qquad \beta_{\underline{e}^*}(\underline{\mu}) = \beta_S(m) = 1-\Phi(z_{1-\alpha} - m\,(\underline{1}'C\underline{1})^{-\frac{1}{2}}).$$

3.4. The Max-Min Property of the S-Test

The test based on S was derived by minimizing the maximum power, namely for each direction $\underline{e}$ and given total diversion $m > 0$ the test based on $T(\underline{e})$ maximized the power against $\underline{\mu} = m\underline{e}$. This maximal power was then minimized for the direction $\underline{e}=\underline{e}^*$ and the optimal test against it is based on S. Because of the property that the power of the test based on S does not depend on the scenario direction $\underline{e}$ it is also possible to show that the test based on S is a max-min test for testing H: $\underline{\mu} = \underline{0}$ vs. K: $\underline{\mu} \in A$.

Theorem:

For any fixed $m > 0$ the level α test based on S maximizes the minimum power over $A_m = \{\underline{\mu}: \underline{\mu} = r\underline{e},\ r \geq m,\ \underline{e} \in E\}$ among all level α tests, and this maximized minimum power is given in the previous section, equation (3.3.1), by $\beta_S(m)$.

Proof:

Let ϕ denote an arbitrary test and C_α the class of all level α tests for testing H: $\underline{\mu} = \underline{0}$ vs. K: $\underline{\mu} \in A$. Let $\beta_\phi(\underline{\mu})$ denote the power function of the test ϕ. ϕ_S is the level α test based on S derived in 3.3.

The theorem above is then equivalent to

$$(3.4.1) \qquad \max_{\phi\in C_\alpha} \min_{\underline{\mu}\in A_m} \beta_\phi(\underline{\mu}) = \min_{\underline{\mu}\in A_m} \max_{\phi\in C_\alpha} \beta_\phi(\underline{\mu}) = \beta_S(m).$$

The second equality in (3.4.1) comes directly from the derivation of ϕ_S in Section 3.3. To show the first equality in (3.4.1) we note that:

$$(3.4.2) \qquad \max_{\phi\in C_\alpha} \min_{\underline{\mu}\in A_m} \beta_\phi(\underline{\mu}) \le \min_{\underline{\mu}\in A_m} \max_{\phi\in C_\alpha} \beta_\phi(\underline{\mu}) = \beta_S(m)$$

since

$$\beta_\phi(\underline{\mu}) \le \max_{\phi\in C_\alpha} \beta_\phi(\underline{\mu})$$

for all $\phi \in C_\alpha$ and for all $\underline{\mu} \in A_m$, and thus

$$\min_{\underline{\mu}\in A_m} \beta_\phi(\underline{\mu}) \le \min_{\underline{\mu}\in A_m} \max_{\phi\in C_\alpha} \beta_\phi(\underline{\mu})$$

for all $\phi\in C_\alpha$, from which (3.4.2) follows immediately.

To show (3.4.2) with the inequality reversed, thus proving (3.4.1), we note:

$$\min_{\underline{\mu}\in A_m} \beta_{\phi_S}(\underline{\mu}) = \beta_S(m)$$

which implies

$$\max_{\phi\in C_\alpha} \min_{\underline{\mu}\in A_m} \beta_\phi(\underline{\mu}) \ge \min_{\underline{\mu}\in A_m} \beta_{\phi_S}(\underline{\mu}) = \beta_S(m)$$

and

$$\beta_S(m) = \max_{\phi \in C_\alpha} \min_{\underline{\mu} \in A_m} \beta_\phi(\underline{\mu})$$

which proves the stated result (3.4.1).

This theorem and the results leading up to it were derived independently of Avenhaus and Jaech (1981) whose priority is hereby acknowledged.

The nature of the max-min test is best understood in the context of a game between two opponents A and B, see Lehmann (1959), chapter 8. Opponent A tries to divert SNM according to a scenario $\underline{\mu}$ which he may choose from a set A_m of scenarios. Opponent B in turn tries to detect such diversion by using a test ϕ which he may choose from the set C_α of level α tests.

The set A_m consists of all those scenarios for which the total amount diverted is m or more. The set C_α consists of all those tests which have a false alarm rate of α or less. For a particular scenario $\underline{\mu}$ in A_m and test ϕ in C_α let $M(\underline{\mu}, \phi)$ denote the probability of detecting the diversion $\underline{\mu}$ with the test ϕ ($\beta_\phi(\underline{\mu})$ in the hypothesis testing notation). The objective of opponent A is to minimize M regardless of the test chosen by B. Opponent B on the other hand will try to maximize M whatever diversion scenario A chooses.

If A knows which test ϕ B will employ then clearly the best action for A to take is to choose that scenario $\underline{\mu} = \underline{\mu}_\phi$ which minimizes $M(\underline{\mu}, \phi)$ over $\underline{\mu}$ for this particular test ϕ, i.e.,

$$M(\underline{\mu}_\phi, \phi) = \min_{\underline{\mu} \in A_m} M(\underline{\mu}, \phi)$$

The best way B may protect himself against this is to choose a test ϕ for which $M(\underline{\mu}_\phi, \phi)$ is as large as possible. Denote the test that accomplishes this by $\tilde{\phi}$, i.e.,

$$M(\underline{\mu}_{\tilde{\phi}}, \tilde{\phi}) = \max_{\phi \epsilon C_\alpha} M(\underline{\mu}_{\phi}, \phi) .$$

Choosing $\tilde{\phi}$ B takes the best action in the face of the contingency that A may know which test ϕ B will choose.

On the other hand, if B knows which scenario $\underline{\mu}$ in A_m opponent A will choose then clearly the best action for B is to choose that test $\phi = \phi_{\underline{\mu}}$ in C_α which maximizes $M(\underline{\mu}, \phi)$ over ϕ for this scenario $\underline{\mu}$, i.e.,

$$M(\underline{\mu}, \phi_{\underline{\mu}}) = \max_{\phi \epsilon C_\alpha} M(\underline{\mu}, \phi) .$$

The best way A may counter this is to choose a scenario $\underline{\mu}$ for which $M(\underline{\mu}, \phi_{\underline{\mu}})$ is as small as possible. Denote such a scenario by $\underline{\mu}^*$ (previously called the least favorable scenario), i.e.,

$$M(\underline{\mu}^*, \phi_{\underline{\mu}^*}) = \min_{\underline{\mu} \epsilon A_m} M(\underline{\mu}, \phi_{\underline{\mu}}) .$$

In choosing the scenario $\underline{\mu}^*$ A takes the best action in the face of the contingency that B may know which scenario $\underline{\mu}$ A will choose. The choices $\underline{\mu}^*$ for A and $\tilde{\phi}$ for B both represent conservative attitudes on the part of each player. As it turns out the two strategies of A and B lead to the same result, namely the same probability of detection, i.e.,

$$M(\underline{\mu}_{\tilde{\phi}}, \tilde{\phi}) = M(\underline{\mu}^*, \phi_{\underline{\mu}^*}) .$$

In the language of decision theory one then says that the game has a value. A further consequence of the max-min property proved above is that opponent A may gain no advantage by taking a so-called randomized strategy, i.e., a strategy that randomly chooses one strategy $\underline{\mu}$ from the set A_m. Random diversion is addressed in Section 3.6.5.

B could improve his choice of ϕ if he had knowledge of A choosing a strategy $\underline{\mu}$ different from the least favorable strategy $\underline{\mu}^*$. Similarly opponent A could improve his choice of $\underline{\mu}$ if he had knowledge of B choosing a test ϕ different from $\tilde{\phi}$. Without such advantageous knowledge each opponent will have to settle on the strategies outlined above to optimize his payoff.

3.5. Definition of Measure of Efficiency $R(\underline{e})$

We have shown that the test ϕ_S based on S is optimal against the least favorable scenario and that it enjoys the max-min property as developed in the previous section. Nevertheless it is of interest to investigate how much power this test loses potentially for scenarios $\underline{e}$ other than the least favorable scenario $\underline{e}^*$. If we had known the diversion scenario was $\underline{e}$ and not $\underline{e}^*$ we could have used the op[illegible]al test $T(\underline{e})$ derived in 3.2.

To facilitate the comparison of the power of the tests based on S and $T(\underline{e})$, respectively, we recall their power functions for $\underline{\mu} = m\underline{e}$ are given by (3.2.1) and (3.3.1) as

$$\beta'_{\underline{e}}(m) = 1-\Phi(z_{1-\alpha} - m\,h(\underline{e})^{\frac{1}{2}}) \quad \text{for } T(\underline{e})$$

and

$$\beta_S(m) = 1-\Phi(z_{1-\alpha} - m\,(\underline{1}'C\underline{1})^{-\frac{1}{2}}) \quad \text{for S.}$$

One meaningful comparison (independent of the chosen α level) of these two power curves is as follows: Suppose we wish to have a certain amount of power β_0 of detection with either of the above tests. One may then ask: For what amount m of material diverted would one achieve this power with the test based on $T(\underline{e})$ and for what m would one achieve this same power with the max-min test based on S?

Denote the two resulting m-values by $m(\underline{e})$ and m_S, respectively.

The ratio

$$(3.5.1) \quad R(\underline{e}) = R = \frac{m_s - m(\underline{e})}{m(\underline{e})} = \frac{m_s}{m(\underline{e})} - 1$$

may then be considered as a measure of relative efficiency loss of the max-min test S relative to the UMP test $T(\underline{e})$ for diversion scenario $\underline{e}$. As an illustration, suppose $R(\underline{e})=.3$ and the UMP test against the diversion direction $\underline{e}$ detects a total diversion $m=5$ with probability $\beta_0=.9$. Then the max-min test would require a 30% larger diversion of $m=6.5$ to yield the same probability of detection. The larger the value of R, the larger the efficiency loss when using S as the test statistic. Another way to interpret the efficiency loss measure R is as follows: If the total diversion over n shipments is m, that would amount to $\Delta = m/n$ diversion per shipment. This Δ does not mean that the same amount Δ is taken out of each shipment, rather Δ should be interpreted as some kind of rate. Suppose now that this rate Δ of diversion per shipment is fixed, then the number of shipments required to achieve a total m is $n=m/\Delta$.

Thus

$$n(\underline{e}) = m(\underline{e})/\Delta \quad \text{and } n_s = m_s/\Delta$$

could be interpreted as the respective number of shipments required to yield the same power β_0 under a fixed diversion rate Δ with the two tests considered above.

Thus

$$R(\underline{e}) = \frac{n_s - n(\underline{e})}{n(\underline{e})} = \frac{n_s}{n(\underline{e})} - 1$$

Hence with a fixed diversion rate Δ per shipment and with a value of R=.3 the max-min test would require a string of n_s shipments against the string of $n(\underline{e})$ shipments necessary for the UMP test to achieve the same power β_0, and n_s would be 30% larger than $n(\underline{e})$.

The measure R of efficiency loss is particularly convenient since it is easily computed:

$$\beta_0 = 1 - \Phi(z_{1-\alpha} - m(\underline{e})\, h(\underline{e})^{1/2})$$

$$= 1 - \Phi(z_{1-\alpha} - m_s\, (\underline{1}'C\underline{1})^{-1/2})$$

Thus

$$m(\underline{e})\, h(\underline{e})^{1/2} = m_s(\underline{1}'C\underline{1})^{-1/2}$$

and

$$(3.5.2) \quad R(\underline{e}) = R = (h(\underline{e})\, \underline{1}'C\underline{1})^{1/2} - 1.$$

Note that R, the efficiency loss, is independent of the level of the test α, and the power β_0.

By the Cauchy-Schwarz inequality we further see that

$$(h(\underline{e})\, \underline{1}'C\underline{1})^{1/2} = (\underline{e}'C^{-1}C\, C^{-1}\underline{e} \cdot \underline{1}'C\underline{1})^{1/2} \geq \underline{e}'C^{-1}C\underline{1} = 1$$

Thus $R(\underline{e}) \geq 0$ with equality if and only if $\underline{e} = \underline{e}^*$.

3.6. Performance of the Max-Min Test S for Specified Diversion Scenarios

In discussing the performance of the S-test with respect to specific scenarios we make the following simplifying assumption. The shipper-receiver differences $D_1,\ldots,D_n$ are independently distributed with unknown variances $\sigma_1^2,\ldots,\sigma_n^2$, i.e., the covariance matrix C is a diagonal matrix with σ_i^2 as the ith diagonal element.

If measurement processes are recalibrated between shipments the above independence assumption is a reasonable one and entails mathematical simplicity in studying the performance of various tests against specific diversion scenarios of interest. Without this independence assumption the range of possible covariance matrix structures becomes unwieldy and is not conducive to the understanding of the effect of various diversion scenarios of interest to NRC. Aside from the diagonal structure of C, no other specific structures for C which hold specific interest could be singled out for special attention. Thus the remaining discussion in this section will assume the above independence.

One criterion that the statistical procedure developed must satisfy is that it is to be powerful in detecting the following three diversion scenarios specified by NRC:

i) Diversions proportional to shipment size
ii) Monotone increasing sequence of diversions
iii) Cyclic pattern of diversions

These scenarios are characterized, and the performance of the max-min test is assessed in Sections 3.6.1, 3.6.2 and 3.6.4, respectively. The max-min test should also have reasonable performance for other diversion scenarios. The block loss scenario arises naturally when investigating the worst case for efficiency loss, and is addressed in 3.6.3. A random diversion pattern is discussed in 3.6.5.

The max-min test is to perform well for the scenarios considered. Its distribution is seen, in 3.3, to be independent of the diversion pattern. Another compelling reason for employing the max-min test is its simplicity. Furthermore, it is relatively easy and straightforward to modify the max-min test to accommodate the situation when the σ_j^2 need to be estimated. On the other hand to modify the UMP test as given in (3.6.1) by replacing the σ_i^2 by estimates s_i^2 presents serious problems. We address the problem of unknown variances in Section 4.1.

3.6.1 Diversions Proportional to Shipment Size

If one removes a small amount of material from each container, then the amount of material diverted per shipment is proportional to the size of the shipment. Assuming that all containers are of the same size, this can be characterized by

$$\mu_i = c\ N_i \qquad \text{for } i=1,\ldots, n \text{ and } c > 0$$

where N_i measures the size of the ith shipment. From Section 2.3, the mathematical model characterization, the variance σ_i^2 of the shipper receiver difference D_i is of the form:

$$\sigma_i^2 = N_i\ \sigma_e^2 + N_i^2\ \sigma_s^2$$

where σ_e^2 is the random error variance (combined for shipper and receiver) per container measurement and σ_s^2 is the systematic error variance (combined) per container measurement. N_i denotes the number of containers in the ith shipment.

If σ_s^2 is quite small as compared to σ_e^2, as would be the case through proper and regularly applied calibration procedures, then the term $N_i^2\ \sigma_s^2$ may be deemed negligible and one might say that

$$\sigma_i^2 \simeq N_i\ \sigma_e^2$$

i.e., σ_i^2 is proportional to the shipment size N_i. Then the scenario μ_i=c N_i is approximately the least favorable one, where

$$c \simeq \sigma_i^2 / \Sigma\ \sigma_i^2$$

Thus the max-min test is approximately the best test that one can employ against the diversion scenario $\mu_i = c\ N_i$. Of course the best test against this scenario is, according to Section 3.2 to reject $H: \underline{\mu} = \underline{0}$ when

$$(3.6.1) \qquad \Sigma \frac{N_i D_i}{\sigma_i^2} > z_{1-\alpha} \left(\Sigma \left(\frac{N_i}{\sigma_i} \right)^2 \right)^{1/2}$$

Against the scenario $\mu_i = c\ N_i$ this test may be somewhat better than the max-min test. Of course this will depend largely on how negligible the systematic term in σ_i^2 really is. Let us quantify this in terms of the relative efficiency loss measure developed in Section 3.5.

The scenario $\mu_i = c\ N_i$, $i=1,\ldots,n$, corresponds to the following scenario direction $\underline{e}$:

$$e_i = N_i/N \quad i=1,\ldots,n \quad \text{where } N = \Sigma N_i.$$

Let

$$x_i = N_i\ \sigma_s^2/\sigma_e^2 \qquad i=1,\ldots,n$$

then

$$\sigma_i^2/N_i = \sigma_e^2\ (1 + x_i) \qquad i=1,\ldots,n$$

and

$$h(\underline{e}) = \Sigma\, (\frac{e_i}{\sigma_i})^2 = \Sigma e_i\, (N\sigma_e^2\, (1 + x_i))^{-1}$$

$$\Sigma \sigma_i^2 = \Sigma N e_i\, \sigma_e^2\, (1 + x_i)$$

so that, from (3.5.2),

$$(3.6.2) \quad R(\underline{e}) = (\, \Sigma\, (\frac{e_i}{\sigma_i}\,)^2 \;\; \Sigma\, \sigma_i^2\,)^{\frac{1}{2}} -1$$

$$= (\, \Sigma\;\; e_i(1 + x_i)^{-1} \;\; \Sigma\; e_i(1 + x_i))^{\frac{1}{2}} -1$$

$$\leq (\, \Sigma\;\; e_i\;\; (1-x_i+x_i{}^2) \;\; \Sigma\;\; e_i(1+x_i))^{\frac{1}{2}} -1$$

$$= ((1 - \;\Sigma\;\; e_i x_i + \;\; \Sigma\, e_i x_i{}^2)\; (1 + \;\; \Sigma\; e_i x_i))^{\frac{1}{2}} -1$$

$$= (1 + \Sigma\; e_i x_i{}^2 - (\;\Sigma\; e_i x_i)^2 + \;\; \Sigma\;\; e_i x_i{}^2 \;\; \Sigma\;\; e_i x_i)^{\frac{1}{2}} -1$$

$$\leq (\;\; \Sigma\, e_i x_i^2 \; - \; (\;\; \Sigma\, e_i x_i)^2 \; + \;\; \Sigma\;\; e_i x_i^2 \;\; \Sigma\;\; e_i x_i)/2$$

where the first inequality above follows from

$$(1+x)^{-1} \leq 1-x + x^2 \quad \text{for } x \geq 0$$

and the second inequality follows from

$$(1+x)^{\frac{1}{2}} \leq 1 + x/2 \qquad \text{for } x \geq 0.$$

Let $k = \max(x_1,\ldots,x_n)$ then (3.6.2) yields

$$R(\underline{e}) \leq k^2/8 + k^3/2$$

Thus if $N_i \ \sigma_s^2 \leq .1 \ \sigma_e^2$ then $R(\underline{e}) \leq (.1)^2/8 + (.1)^3/2 = .00175$

We see that the relative efficiency loss of the max-min test relative to the best test against the scenario $\mu_i = cN_i$ is governed by the quantity k which measures the size of the systematic component $N_i \sigma_s^2$ against the random component σ_e^2. For k-values which are reasonably small the efficiency loss may be considered negligible. Then one may employ the max-min test in that situation with a minimal detection penalty.

In summary, the scenario $\mu_i = cN_i$ is close to being least favorable. It is a reasonable scenario for an intelligent diverter to choose since such a person is likely to divert twice as much from a shipment that is twice as large, i.e., the diverter will make his diversions proportional to the shipment size. The max-min test is nearly optimal against such a strategy and is further recommended by its simplicity.

3.6.2 Monotone Increasing Sequence of Diversions

The scenario of monotonic increasing diversions is mathematically characterized by

$$\mu_i = ci\sigma_i^2 \quad \text{for } i=1,\ldots,n \qquad \text{and } c > 0.$$

This scenario, far from being least favorable as a diversion scenario, may be a reasonable loss scenario. The cause of such loss might be some general deterioration in the measurement process; e.g., the calibration of instruments drifts slowly out of control.

The scenario direction $\underline{e}$ corresponding to this model is

$$e_i = i\sigma_i^2 / \sum_j j\sigma_j^2$$

and the optimal test ϕ_L according to Section 3.2 rejects $H: \underline{\mu} = \underline{0}$ when

$$\frac{\Sigma\ i\ D_i}{(\Sigma\ i^2\sigma_i^2)^{\frac{1}{2}}} > z_{1-\alpha}$$

Since the scenario direction $\underline{e}$ is far from being least favorable one might expect that ϕ_L is much better than the max-min test against this direction $\underline{e}$. To quantify this difference we compute the relative efficiency loss of the max-min test for this situation. We have, substituting into equation (3.5.2),

$$R(\underline{e}) = \left(\frac{\Sigma\ (i\ \sigma_i)^2}{(\Sigma\ i\sigma_i^2)^2}\Sigma\,\sigma_i^2\right)^{\frac{1}{2}} - 1$$

It is difficult to assess this efficiency loss for general choices for the σ_i^2. To gain some insight into the magnitude of $R(\underline{e})$ let us assume that $\sigma_i^2=\sigma^2$ for $i=1,\ldots,n$. Then $R(\underline{e})$ reduces to

$$R(\underline{e}) = \left(\frac{\Sigma\ i^2}{(\Sigma\ i)^2}\ n\right)^{\frac{1}{2}} - 1$$

$$= \left(\frac{4n+2}{3n+3}\right)^{\frac{1}{2}} - 1$$

Below we give a table of R values for various values of n:

n	2	3	4	5	7	10	15	20	∞
$R(\underline{e})$	.054	.080	.095	.106	.118	.128	.137	.141	.155

It is seen that the efficiency loss deteriorates from .054 to .155, the latter being a strict upper bound for all n. It appears then that the inefficiency when we use the max-min test is minimal. At most 15.5% more material diversion is required by the max-min test to have the same power as the UMP test against the considered scenario $\underline{e}$. However, we would have to know the diversion scenario in advance to use the UMP test. It should be pointed out that these efficiency losses are computed under the assumption of equal variances. If the variances are nearly homogeneous one might expect similar results.

3.6.3 Block Loss

A natural question to ask is the following: For what kind of diversion scenario will the max-min test incur its largest efficiency loss? To answer this question let the measurement error variance $\sigma_{j_o}^2$ be the smallest among $\sigma_1^2, \ldots, \sigma_n^2$. Then $R(\underline{e})$ is obviously maximized by $\underline{e} = \underline{e}_o$ where

$$e_{oj_o} = 1 \text{ and } e_{oi} = 0 \text{ for } i \neq j_o.$$

For that case

$$R(\underline{e}_o) = \left(\sum_j \sigma_j^2 / \sigma_{j_o}^2 \right)^{\frac{1}{2}} - 1$$

and if all variances are roughly equal, then $R(\underline{e}_o) \simeq \sqrt{n} - 1$, which at first glance may seem unacceptably large. The diversion scenario direction $\underline{e}_o$ amounts to a block loss in that shipment for which the measurement error variance is smallest. Note that this particular block loss scenario constitutes the least favorable scenario for the diverter if the usual individual SRD test is performed.

The efficiency loss of the max-min test against block loss in the ith shipment, denote this scenario by $\underline{e}_i$, is

$$R\ (\underline{e}_i) = \left(\sum_j \sigma_j^2 / \sigma_i^2 \right)^{\frac{1}{2}} - 1$$

which still appears unacceptably large.

The fact that the max-min test incurs such a large efficiency loss against block loss diversions relative to the UMP test for that block loss is deceiving, since the application of the UMP test based on $1(\underline{e}_i)$ presupposes knowledge of the shipment label i during which block loss takes place.

Of course this criticism may be blunted somewhat by using a test that guards specifically against block loss in any shipment without assuming in which shipment the block loss takes place. An intuitive test which accomplishes that, rejects when

$$M = \max \left(\frac{D_1}{\sigma_1}, \ldots, \frac{D_n}{\sigma_n}\right) > k_\alpha$$

where k_α is determined by

$$\alpha = P(M > k_\alpha \mid \underline{\mu} = \underline{0}) = 1 - P(M \leq k_\alpha \mid \underline{\mu} = \underline{0})$$

$$= 1 - \Pi\, P(\frac{D_i}{\sigma_i} \leq k_\alpha \mid \mu_i = 0)$$

$$= 1 - \Phi^n (k_\alpha)$$

Thus

$$k_\alpha = \Phi^{-1}((1-\alpha)^{1/n}).$$

The power function of this test, denoted by ϕ_M, against any diversion $\underline{\mu} \neq \underline{0}$ is

$$\beta_{\phi_M} (\underline{\mu}) = 1 - \Pi\, P(\frac{D_i}{\sigma_i} \leq k_\alpha \mid \mu_i)$$

$$= 1 - \Pi\, \Phi(k_\alpha - \frac{\mu_i}{\sigma_i})$$

In particular against a block loss diversion of size m in shipment i, i.e., $\underline{\mu}_i (m) = m\, \underline{e}_i = m(0,\ldots,0, 1, 0,\ldots,0)$ (with a 1 in the ith position), the power is

$$(3.6.3) \quad \beta_{\phi_M}(\underline{\mu}_i(m)) = 1 - (1-\alpha)^{\frac{n-1}{n}} \Phi(k_\alpha - \frac{m}{\sigma_i})$$

whereas the power of the max-min test β_S for this block loss diversion is

$$(3.6.4) \quad \beta_S(m) = 1 - \Phi(z_{1-\alpha} - m(\Sigma\ \sigma_j^2)^{-1/2})$$

On the other hand the power function of ϕ_M against the least favorable scenario $\underline{\mu} = m\ \underline{e}^*$ is

$$(3.6.5) \quad \beta_{\phi_M}(m\underline{e}^*) = 1 - \prod_{i=1}^{n} \Phi(k_\alpha - \frac{m\sigma_i}{\sum_j \sigma_j^2})$$

whereas the power of ϕ_S is the same as given in (3.6.4). $\beta_S(m)$ remains unchanged since it depends only on m.

A comparison of these power functions (3.6.3, 3.6.4 and 3.6.5) for the case n=10, α =.05, and $\sigma_1^2 = \ldots = \sigma_{10}^2 = 1$ is given in Figure 3.1. As is evident from this figure the result is mixed. On the one hand the ϕ_M test performs much better than the max-min test ϕ_S against block loss alternatives and on the other hand the ϕ_M test performs much worse than the ϕ_S test against the least favorable scenario $\underline{e}^*$.

An attempt to combine the two tests in some fashion would result in a degradation of $\beta\phi_M(\underline{\mu}_i(m))$ but improvement of $\beta_S(m)$ in the case of block loss alternatives whereas in the case of the least favorable scenario the power function of the combined test would lie between β_S and $\beta\phi_M(m\ \underline{e}^*.)$ What this comparison tells us and what is evident from the max-min property of the test ϕ_S is that we cannot improve over the power properties of ϕ_S without some loss of power against the least favorable alternative.

It should also be noted that a block loss scenario was not one of the identified scenarios against which the chosen test should perform well. The reason the block loss scenario was considered here was that it came up

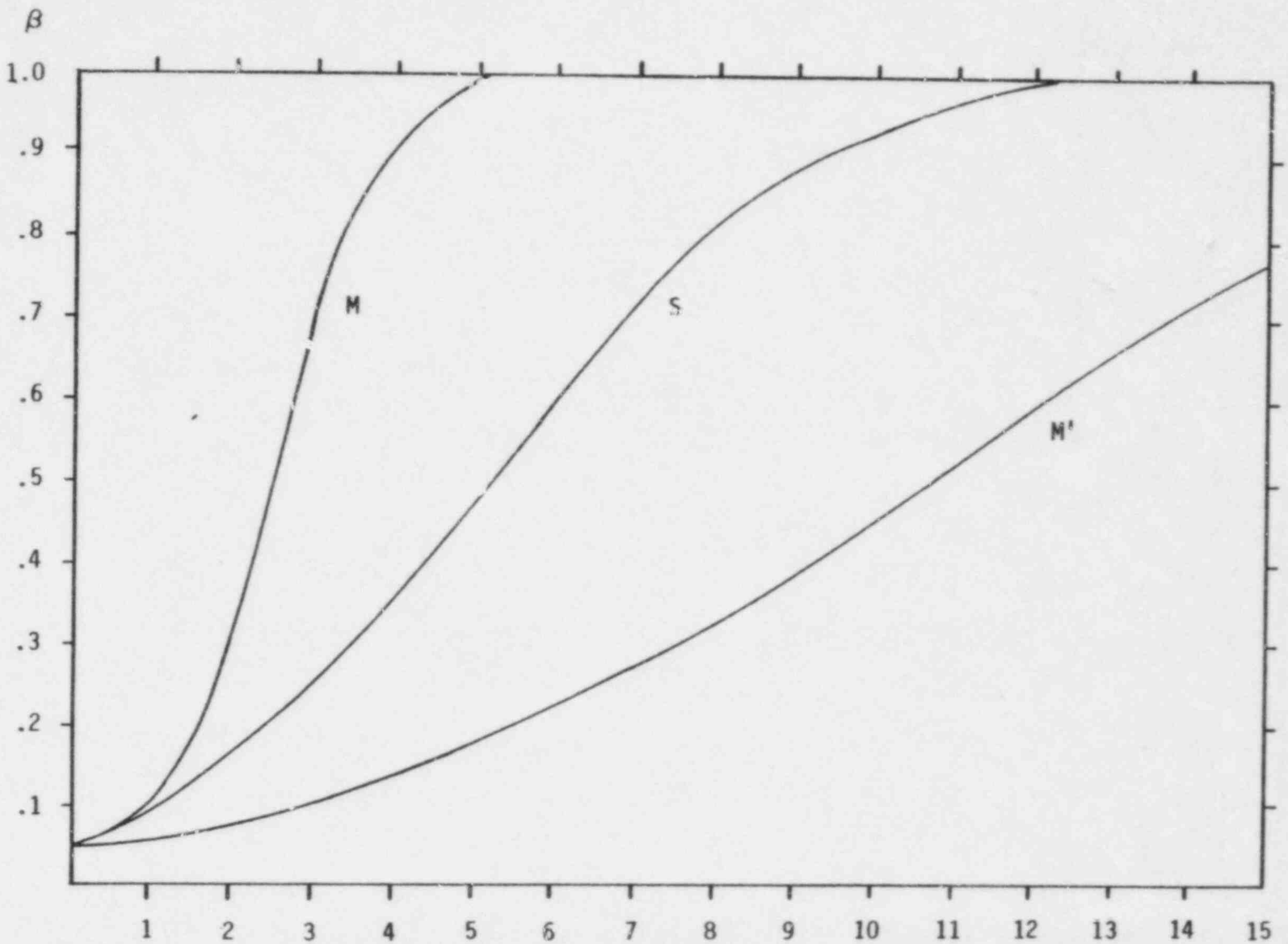

Figure 3.1 Power Curves β for Block Loss Test φ_M for Block Loss (M) and Least Favorable Scenario (M'), and for Max-Min Test (S)

naturally when we identified the case of highest efficiency loss of the max-min test. The remainder of the discussion of the block loss scenario as given above should serve as an illustration of the problems involved when trying to improve on the max-min test.

3.6.4 Cyclic Pattern of Diversions

Suppose that material is diverted only every kth shipment starting with the ith shipment. To be precise, assume the following general periodic scenario:

$$\mu_j = a_j \quad \text{for } j=i,\ i+k,\ldots,\ i+(r-1)k$$

$$\mu_j = 0 \quad \text{for all other } j = 1,\ldots,n$$

Here it is assumed that n=rk and the a_j represent unknown diversions in the jth shipment if such diversion takes place. The above starting index i is some number between 1 and k.

If one knows the period k and the starting index i then one may as well restrict attention to the sufficient statistic

$$(D_i,\ D_{i+k},\ldots,D_{i+(r-1)k})$$

for the unknown parameter $(a_i,\ a_{i+k},\ldots,\ a_{i+(r-1)k})$. With that reduction from n shipper receiver differences to just r differences we basically are back in the original situation with no assumed structure for the diversion scenario. Drawing on the results in the previous sections we may conclude that the least favorable scenario with total diversion of m for this reduced problem is as follows:

$$\underline{\mu}_i = m\, \underline{e}^*_i$$

where

$$e^*_{ij} = \sigma^2_j / c_i \quad \text{for } j=i,\ i+k, \ldots, i+(r-1)k\ ,\ c_i > 0$$

$$= 0 \text{ else}$$

$$\text{with } c_i = \sum_{j=o}^{r-1} \sigma^2_{i+jk}\ .$$

The UMP test against this scenario direction $\underline{e}^*_i$ rejects $H: \underline{\mu} = \underline{o}$ when

$$T_i = \frac{\sum_{j=o}^{r-1} D_{i+jk}}{\sqrt{c_i}} > z_{1-\alpha}$$

The power function of the test based on T_i depends on the a_i, $a_{i+k}, \ldots, a_{i+(r-1)k}$ only through their sum, namely

$$\beta_{T_i}(\underline{\mu}) = 1 - \Phi(z_{1-\alpha} - m\, c_i^{-\frac{1}{2}})$$

We see that this test uses specifically the knowledge of the starting point i as well as the cycle length k which puts it at some advantage over the max-min test based on S. It is more reasonable to compare the max-min test S to a statistic based on knowledge of k, but not on the starting point i. One may look at $T_1, \ldots, T_k$ simultaneously.

The test statistics $T_1, \ldots, T_k$ are independent since they are formed from nonoverlapping D_i. A very plausible way to combine the test statistics $T_1, \ldots, T_k$ is to reject $H: \underline{\mu} = \underline{0}$ when

$$M_k = \max(T_1, \ldots, T_k) > k_\alpha$$

where k_α is determined by

$$\alpha = P(M_k > k_\alpha \mid \underline{\mu} = \underline{0}) = 1 - P(M_k \le k_\alpha \mid \underline{\mu} = \underline{0})$$

$$= 1 - \prod_{i=1}^{k} P(T_i \le k_\alpha \mid \underline{\mu} = \underline{0})$$

$$= 1 - \Phi(k_\alpha)^k$$

Thus

$$k_\alpha = \Phi^{-1}((1-\alpha)^{1/k})$$

The power function of this test against a periodic diversion of period k commencing with the ith shipment and with a total diversion amount m is

$$\beta_{M_k}(m) = 1 - \prod_{i=1}^{k} P(T_i \le k_\alpha \mid \underline{\mu})$$

$$= 1 - \Phi(k_\alpha)^{k-1} \quad \Phi(k_\alpha - m\, c_i^{-\frac{1}{2}})$$

$$= 1 - (1-\alpha)^{\frac{k-1}{k}} \Phi(k_\alpha - m\, c_i^{-\frac{1}{2}})$$

The power function for S as given in (3.3.1) does not depend on $\underline{e}$. For illustration purposes assume n=24 which allows for the following possible periods: k=2(r=12), k=3 (r=8), k=4 (r=6), k=6 (r=4), k=8 (r=3), k=12 (r=2). Further assume $\alpha=.05$ and $\sigma_1=\ldots=\sigma_n=1$, thus $c_i=r$.

Then

$$\beta_{M_k}(m) = 1 - (.95)^{\frac{k-1}{k}} \Phi(k_{.05} - m/\sqrt{r})$$

where $k_{.05} = \Phi^{-1}(.95^{1/k})$

Figure 3.2 presents these power curves for M_k for k=2, 3, 4, 6, 8, 12 when the diversion scenario is cyclic. The M_k test is more powerful than the

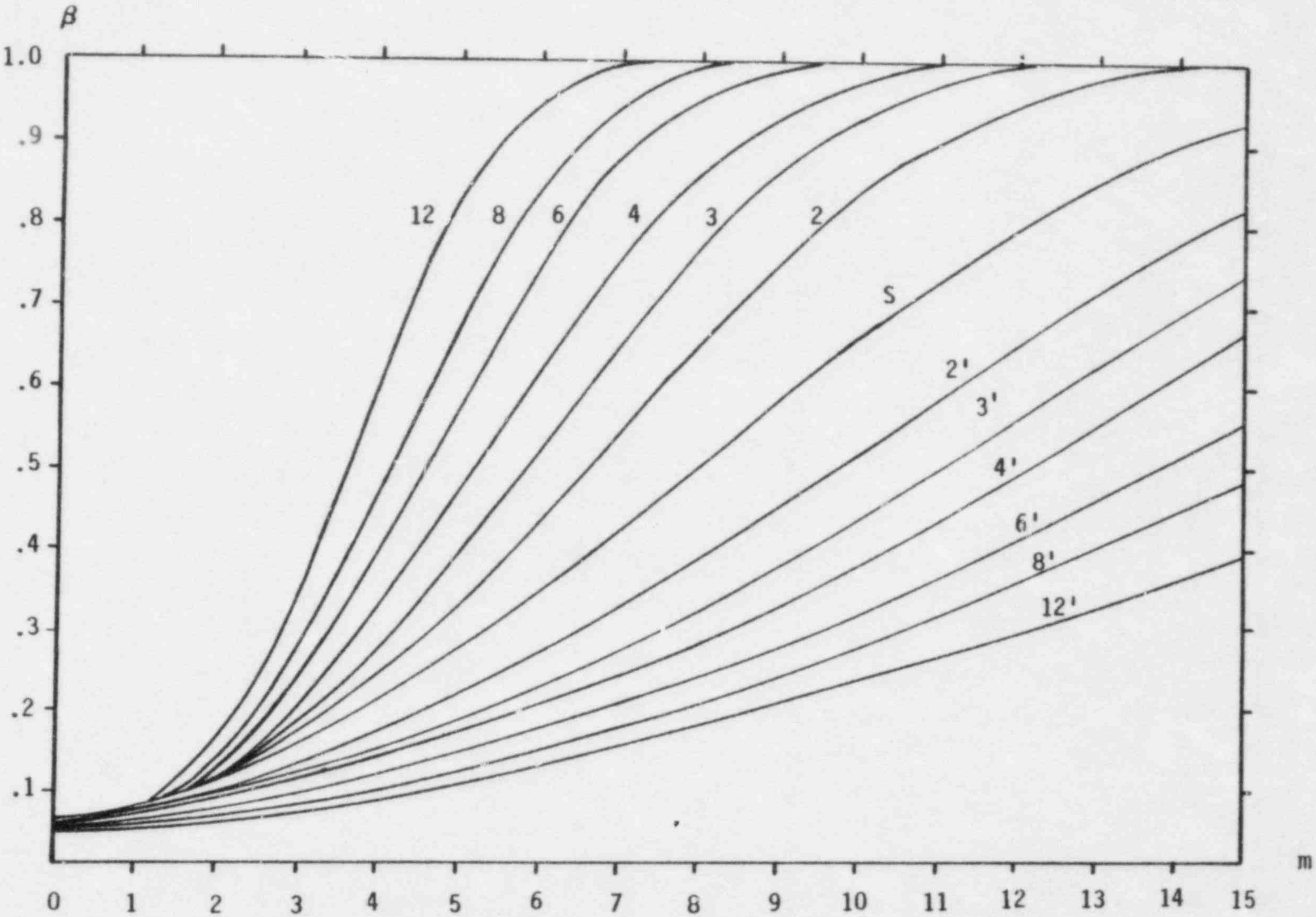

Figure 3.2 Power Curves β for Periodic Loss Test M_K When Loss Pattern is Cyclic (K) and Least Favorable (K') and for Max-Min Test (S)

max-min test for all k. Thus if you know that the diversion is cyclic, and you know the cycle length k, the M_k test is preferable to S. If k is very large, the M_k test does much better than the max-min test. This is not surprising, for when k=12, we know that all but two of the 24 shipments have no diversion. For the case when k=24, the cyclic scenario becomes the block loss, where the max-min test does worst (see Section 3.6.3). At k=2, we know that every other μ_i is zero. At k=1 we are back at unconstrained diversion, and as we saw in 3.3 and 3.4, the max-min test is the best one can do with no information about the diversion strategy.

From Figure 3.2, one also notes that when the diversion scenario is the least favorable, as defined in 3.3, the M_k test does increasingly worse as k increases. This also is not surprising, since as k increases, one is basing the test on a larger set of parameter assumptions.

3.6.5 Random Pattern of Diversion

The last diversion scenario to be discussed is that of random diversions. Consider again our basic statistical model

(3.6.6) $$D_i = \mu_i + Z_i \qquad i=1,\ldots,n$$

where the unknown parameters $\mu_1, \ldots, \mu_n$ (≥ 0) represent the diversions during the n shipments and $Z_1, \ldots, Z_n$ are independent normally distributed measurement errors with mean zero and variances $\sigma_1^2, \ldots, \sigma_n^2$.

In a random diversion scenario one considers the diversions $\mu_1, \ldots, \mu_n$ as nonnegative random variables. To emphasize this different interpretation we write M_i in place of μ_i. Our statistical model becomes

(3.6.7) $$D_i = M_i + Z_i \qquad i=1,\ldots,n$$

where the $Z_1, \ldots, Z_n$ are as before and $M_1, \ldots, M_n$ are nonnegative random variables which are distributed independently of $Z_1, \ldots, Z_n$. The $M_1, \ldots, M_n$ are allowed to be dependent random variables. For example, it may be that the total diversion m is fixed by the diverter, i.e., $m = M_1 + \ldots + M_n$ so that only the allocation of the amount m over the n shipments is random. Let G denote the joint distribution of $M_1, \ldots, M_n$.

If ϕ is any test then its power function under model (3.6.6) is denoted by $\beta_\phi(\underline{\mu})$ whereas under model (3.6.7) the power function of ϕ is computed as

(3.6.8) $$\bar{\beta}_\phi(G) = E_G(\beta_\phi(\underline{M}))$$

where E_G denotes the expectation or average over the random vector $\underline{M} = (M_1, \ldots, M_n)$, which is distributed according to G.

For example, for the max-min test ϕ_S we have:

(3.6.9) $$\beta_{\phi_S}(\underline{\mu}) = 1 - \Phi(z_{1-\alpha} - \Sigma \mu_i (\Sigma \sigma_j^2)^{-\frac{1}{2}})$$

whereas

(3.6.10) $$\bar{\beta}_{\phi_S}(G) = E_G(1 - \Phi(z_{1-\alpha} - \Sigma M_i(\Sigma \sigma_j^2)^{-\frac{1}{2}})).$$

Let us discuss the distinction between $\beta_\phi(\underline{\mu})$ and $\bar{\beta}_\phi(G)$ in detail. $\beta_\phi(\underline{\mu})$ represents the long run frequency with which the test ϕ would detect a diversion when applied repeatedly under the same diversion scenario $\underline{\mu}$. $\bar{\beta}_\phi(G)$ also represents a long run frequency with which the test ϕ would detect a diversion when applied repeatedly, however this time under randomly changing diversion scenarios.

During any given cycle of n shipments the diverter selects randomly the diversion $M_1, \ldots, M_n$ according to some distribution G. Denote the particular realization of $M_1, \ldots, M_n$ for this cycle by $m_1, \ldots, m_n$. For all practical purposes the $m_1, \ldots, m_n$ are unknown numbers and the probability of detecting a diversion during this particular cycle using the test ϕ is $\beta_\phi(\underline{m})$. Depending on the values of $\underline{m} = (m_1, \ldots, m_n)$ this probability $\beta_\phi(\underline{m})$ may either be small, moderate, or very large. In (3.6.8), by averaging over the values of $\underline{m}$ according to G, we average the good performance of ϕ for some $\underline{m}$ with the bad performance of ϕ for some other $\underline{m}$. This is not so much a fault of the test ϕ but points to a flaw in measuring the performance of a test ϕ in the face of random diversions by the criterion (3.6.8).

To clarify this further consider the max-min test and suppose that the joint distribution G of $\underline{M}$ stipulates that $\underline{M}$ will be chosen randomly such that $\Sigma M_i = 0$ with probability $1 - p$ and that $\Sigma M_i = a$ with probability p. The average total diversion is then

$$E_G(\Sigma M_i) = 0(1 - p) + a \cdot p = ap \; .$$

By choosing a very large and p very small we can have this average total diversion take on any value we like. However, according to (3.6.10), the probability of detecting diversion is

$$\bar{\beta}_{\phi_S}(G) = (1-p)(1 - \Phi(z_{1-\alpha})) + p(1 - \Phi(z_{1-\alpha} - a(\Sigma \sigma_j^2)^{-\frac{1}{2}}))$$
$$\leq (1-p)\alpha + p$$

which for small p may become arbitrarily close to α, the false alarm rate. We thus have the following situation: In the long run the expected total diversion per cycle of n shipments can be quite sizable whereas the diversion detection probability is hardly better than the false alarm rate. This example should make clear that it doesn't make much sense to consider (3.6.8) as a reasonable test performance measure when we average over random total diversions which may take on very small and very large values.

If, however, we restrict ourselves to random diversions $\underline{M}$ for which the total diversion ΣM_i is fixed at m, i.e., $M_1, \ldots, M_n$ are random but

$$P(\Sigma M_i = m) = 1,$$

then the power of the max-min test is not affected at all since it only depends on the total diversion and not on the way the diversions are allocated over the n shipments. This explains the comment made earlier that the diverter may gain nothing against the max-min test by selecting his diversion scenario randomly from A_m.

4. RESULTS WITH UNKNOWN VARIANCES

The max-min test is extended in Section 4.1, under the restriction of independent errors between shipments, to incorporate variance estimates. The resulting test statistic is shown to be a generalized version of the Behrens-Fisher type. A correlated error problem with unknown variances is addressed in Section 5.3. The Satterthwaite procedure is shown in Section 4.3 to provide a critical value for the test statistic with an α level closest to that intended. The α level and power of the test are shown to be relatively unaffected by changes in population parameter values, Section 4.3.

4.1. Problem Background

The max-min test is developed in Section 3.3 under the assumption of known covariance matrix C. The framework of Section 3.1, with

$$D_i = \mu_i + Z_i \qquad \text{for } i=1,\dots, n$$

is still assumed, where μ_i are the unknown true material losses, $\mu_i \geq 0$, except

$$Z_i \text{ are independent } N(0, \sigma_i^2) \quad \text{for } i=1,\dots n.$$

The null hypothesis H of no material diversion, and the alternative K of some diversion only involve $\underline{\mu}$:

(4.1.1) $H: \underline{\mu} = \underline{0} \qquad \text{vs} \qquad K: \underline{\mu} \in A$

and $\sigma_1^2, \dots, \sigma_n^2$ are nuisance parameters which are not necessarily equal. We assume that estimates s_i^2 of the σ_i^2 are available such that

i) $f_i \, s_i^2/\sigma_i^2$ is a chi-squared variable with f_i degrees of freedom, with f_i known, for i=1, ..., n

ii) $s_1^2, \ldots, s_n^2, D_1, \ldots, D_n$ are mutually independent

For n=2, with the unknown variances not assumed equal, the problem above (4.1.1) is referred to as the Behrens-Fisher problem. With unknown nuisance parameters σ_i^2, the straightforward Neyman-Pearson theory used in 3.2 no longer applies. This problem has defied attempts at an exact solution. Linnik (1968) has shown that no well behaved exact solution is possible. The problem has received considerable attention, with much of the research focused on obtaining practical solutions to the problem. This is the approach we also have taken.

For known covariance matrix C the max-min test S appears to be the best test. Under independence the S test rejects H: $\underline{\mu} = \underline{0}$ when

$$\frac{\Sigma D_i}{(\Sigma \sigma_i^2)^{\frac{1}{2}}} > z_{1-\alpha}$$

The obvious modification of this test is to replace the σ_i^2 by their estimates s_i^2. The test is then to reject H when

$$(4.1.2) \quad T_S = \frac{\Sigma D_i}{(\Sigma s_i^2)^{\frac{1}{2}}} > t_\alpha$$

This modification is in the spirit of the practical solutions for the Behrens-Fisher problem (n=2). The place where these practical solutions differ is in the choice of the critical value t_α. For a discussion of the approaches see G. K. Robinson (1982), Lee and Gurland (1975) and Scheffé (1970). The basic problem is that the distribution of the statistic T_S depends on the values of the unknown nuisance parameters $\sigma_1^2, \ldots, \sigma_n^2$. Even if the variances are known, the distribution of T_S would be difficult, if not impossible, to derive. Thus we have focused attention on determining the critical value t_α such that

$$(4.1.3) \quad P(T_S > t_\alpha \mid \underline{\mu} = \underline{0}) \simeq \alpha$$

over a range of parameters n, $f_1, \ldots, f_n$, and $\sigma_1^2, \ldots, \sigma_n^2$.

Note that from (4.1.2) we have

$$(4.1.4) \quad T_S = \frac{\Sigma D_i}{(\Sigma s_i^2)^{1/2}} = \frac{D/\sigma}{\left(\Sigma\left(\frac{\sigma_i^2}{\sigma^2}\ 1/f_i\right) \chi_{f_i}^2\right)^{1/2}}$$

where

$$\sigma^2 = \Sigma \sigma_i^2$$

$$D = \Sigma D_i$$

Under H (4.1.1) the numerator of T_S, D/σ, is a N(0,1) random variable. The denominator, however, involves an algebraic function of independent chi-squared variates, which is not in general a chi-squared variate. Thus T_S is not a Student's t variable, and (4.1.2) is an n population extension of the Behrens-Fisher problem, necessitating an approximate solution to (4.1.3).

4.2. Procedures Investigated

The statistical literature was searched for practical solutions for determining t_α in (4.1.2) for $n > 2$. Based on this review, four procedures were selected for evaluation. Each of the procedures involves straightforward computations. The first three involve using the s_i^2 and f_i for determining t_α. The fourth procedure was originally thought of as a "quick and dirty" procedure which permits benchmarking the Monte Carlo program, Section 4.3. The procedures given in 4.2.1-4.2.4 are:

i) Satterthwaite
ii) Banerjee
iii) Cochran
iv) Sum

4.2.1 Satterthwaite

The Satterthwaite procedure, see Brownlee (1965), is based on approximating the distribution of an algebraic function of chi-squared variates $f_i\, s_i^2/\sigma_i^2$ by a chi-squared variate divided by its degrees of freedom f_S. Substituting s_i^2 for the unknown σ_i^2 gives

$$f_S = \frac{(\Sigma\, s_i^2)^2}{\Sigma(s_i^4/f_i)}$$

where

$$\min f_i \leq f_S \leq \Sigma f_i.$$

Note that f_S will usually be non-integer. Using this approximation in 4.1.4 we have under H: $\underline{\mu} = \underline{0}$ that T_S is approximately a Student's t variate with fractional degrees of freedom f_S. Under K: $\underline{\mu} = m\underline{e}$ then T_S is approximately a non-central t with f_S degrees of freedom and non-centrality parameter

$$(4.2.1) \quad \lambda = m/(\Sigma\sigma_i^2)^{\frac{1}{2}} = m/\sigma.$$

The Satterthwaite critical value $t_{S,\alpha}$ for testing H (4.1.1) is $t_{f_S,\alpha}$ the $1-\alpha$ quantile for Student's t with f_S degrees of freedom. To compute critical values for fractional degrees of freedom, inverse interpolation is used between neighboring integers f_o, f_1, which gives a reasonable approximation:

$$t_{S,\alpha} = t_{f_S,\alpha} \simeq t_{f_1,\alpha} + \frac{(1/f_S - 1/f_1)}{(1/f_o - 1/f_1)} \quad (t_{f_o,\alpha} - t_{f_1,\alpha})$$

where

$$f_o < f_S < f_1$$

$$f_1 = f_o + 1.$$

4.2.2 Banerjee

Banerjee (1960) proposes comparing T_S to $t_{B,\alpha}$ where

$$t_{B,\alpha} = (\Sigma(s_i^2/\Sigma s_j^2)\ t_{f_i,\alpha}^2)^{\frac{1}{2}}$$

This procedure is guaranteed to have level α or less, which isn't true for Satterthwaite or Sum and hasn't been shown for Cochran. Note that

$$(4.2.2) \quad \min t_{f_i,\alpha} \leq t_{B,\alpha} \leq \max t_{f_i,\alpha}$$

4.2.3 Cochran

In the Cochran procedure, Snedecor and Cochran (1976), T_S is compared to $t_{C,\alpha}$ where:

$$t_{C,\alpha} = \Sigma(s_i^2/\Sigma s_j^2)\ t_{f_i,\alpha}$$

This uses as a critical value for T_S, a weighted average of the critical values for testing each of the D_i. Note that $t_{C,\alpha}$ satisfies a similar inequality to $t_{B,\alpha}$ (4.2.3)

$$\min t_{f_i,\alpha} \leq t_{C,\alpha} \leq \max t_{f_i,\alpha}$$

When the f_i are all equal, the Banerjee and Cochran procedures are identical, using the common degrees of freedom f_o for the critical value, $t_{f_o,\alpha}$.

4.2.4 Sum

A fourth procedure included in the Monte Carlo evaluation, Sum, compares T_S to $t_{A,\alpha}$ where A is the sum of the degrees of freedom, $A = \Sigma f_i$. If the n variances are equal, and the f_i are all equal to f_o, then T_S is a Student's t variate with nf_o degrees of freedom. This follows directly from equation (4.1.4) since a sum of chi-squared variables is chi-squared.

Note that the denominator in 4.1.4 is always an estimate of the standard deviation of D_i. The variance estimates are not being pooled.

The Sum procedure is "quick and dirty" in that the critical value $t_{A,\alpha}$ is not a function of the s_i^2. This procedure permits an accuracy check of the Monte Carlo program, 4.3, since the distribution of T_S is known for $\sigma_i^2 = \sigma_o^2$ and $f_i = f_o$, $i=1,\ldots,n$, both when $\underline{\mu} = \underline{0}$ and when $\underline{\mu} \neq \underline{0}$.

4.3. Monte Carlo Analysis

The distribution of the test statistic T_S (4.1.2) does not have a well behaved analytic solution, as mentioned in 4.1. Four procedures for computing a critical value t_α for T_S are given in 4.2. In order to evaluate the performance of these procedures a Monte Carlo simulation program has been developed, Section 4.3.1 and Appendix A. This program generates realizations of T_S for the mathematical model given in 2.3 and 4.1. Shipper-receiver system parameters are inputs to the program; α levels or power values for the test procedures are outputs.

4.3.1 Model Implemented

The Monte Carlo simulation program generates realizations of the shipper-receiver model described in 2.3 and 4.1. The user describes system parameters. These are:

i) The number of shipments n (number of variance estimates s_i^2)
ii) The true variance values $\sigma_1^2, \ldots, \sigma_n^2$
iii) The degrees of freedom values for the n variance estimates, $f_1, \ldots, f_n$
iv) The true material diversions $\mu_1, \ldots, \mu_n$

The user also specifies N, the number of samples to generate for estimating the desired probabilities.

The test statistic T_S, as given by (4.1.4), is

(4.3.1) $$T_S = \frac{\Sigma D_i}{(\Sigma s_i^2)^{1/2}} = \frac{D/\sigma}{\left(\Sigma(\sigma_i^2/(\sigma^2 f_i))\chi^2_{f_i}\right)^{1/2}}$$

The numerator, D/σ, is a single normal variate with mean $\lambda = \Sigma \mu_i/\sigma$ and variance 1, where $\sigma^2 = \Sigma \sigma_i^2$. The denominator is an algebraic function of n chi-squared variates. To generate a single realization of T_S we thus need to generate a single normal variate and n chi-squared variates and combine them appropriately. To compute critical values $t_{S,\alpha}$, $t_{B,\alpha}$ and $t_{C,\alpha}$ the chi-squared variates are again used (see Sections 4.2.1, 4.2.2 and 4.2.3). The realization of T_S is then compared to the critical values for each of the procedures. This process is repeated N times.

The fraction of times where T_S exceeds the critical value is then an unbiased estimate of the probability

$$P(T_S > t_{X,\alpha})$$

for each of the procedures. This is done for α = .10, .05, .025, .01 and .005. If $\underline{\mu} = \underline{0}$, this probability is, hopefully, close to α, the intended false alarm rate. If $\underline{\mu} = m\underline{e}$, this probability is the power of the test.

For each parameter set evaluated, N=40,000 realizations are used to estimate α. If the true α is .10, .05, .01, or .005, the accuracy of the estimates (two standard deviations) are

α	.1000	.0500	.0250	.0100	.0050
$2\sqrt{\alpha(1-\alpha)/40{,}000}$	.0030	.0022,	.0016	.0010	.0007

4.3.2 Parameter Sets Investigated

The model parameters input to the Monte Carlo simulation program are n, $(\sigma_1^2, \ldots, \sigma_n^2)$, $(f_1,\ldots,f_n)$ and $(\mu_1,\ldots,\mu_n)$. The simulation permits

i) Evaluation of the relative performance of the four procedures given in 4.2 for determining critical values of size α.

ii) Determination of the power of each of the four procedures

4.3.2.1 Parameter Sets for n=2

For n=2, Lee and Gurland (1975) evaluated the Satterthwaite, Banerjee and Cochran procedures for α=.05, f_1=4, f_2,=8 and a range of values for $\sigma_1^2/(\sigma_1^2 + \sigma_2^2)$. We evaluated all four procedures for variance parameters

$$\sigma_1^2/(\sigma_1^2 + \sigma_2^2) = .05, .1, .2, \ldots, .8, .9, .95$$

with observed alpha values close to those obtained by Lee and Gurland, giving some verification of the Monte Carlo program.

4.3.2.2 Parameter Sets for n > 2

We compared the four procedures of Section 4.2 for n = 6, 12 and 18 and for a range of the other parameters. The degrees of freedom f_i used for s_i^2 were 4 (low), 10 (medium), and 20 (high). The choices for degrees of freedom were:

i) $f_i = \ldots = f_n = 4$ (all low)

ii) $f_1 = \ldots = f_n = 10$ (all medium)

iii) $f_1 = \ldots = f_n = 20$ (all high)

iv) $f_1 = \ldots = f_{n/2} = 4$, $f_{n/2+1} = \ldots = f_n = 20$ (first half of shipments low degrees of freedom, second half high)

v) $f_1 = \ldots = f_{n/3} = 4$ (first third low degrees of freedom,

$f_{n/3+1} = \ldots = f_{2n/3} = 10$ second third medium,

$f_{2n/3+1} = \ldots = f_n = 20$ last third high)

Examining (4.1.4) we note that the behavior of T_S is only a function of

$$a_i = \sigma_i^2/\sigma^2$$

where

$$\sigma^2 = \Sigma\sigma_i^2.$$

Thus we need only specify the relative magnitude a_i of the variances. The cases evaluated were:

i) $a_1 = \ldots = a_n$ (all variances equal)

ii) $a_1 = \ldots = a_{n/2} = 1/2n$ (variance of second half 3 times variance of first half)

$a_{n/2+1} = \ldots = a_n = 3/2n$

iii) $a_1 = \ldots = a_{n/2} = 3/2n$ (same as ii but with roles reversed)

$a_{n/2+1} = \ldots = a_n = 1/2n$

iv) $a_1 = \ldots = a_{n/3} = 1/2n$ (variances in ratio of 1 to 2 to 3 for first third, second third, and last third of the shipments)

$a_{n/3+1} = \ldots a_{2n/3} = 2/2n$

$a_{2n/3+1} = \ldots = a_n = 3/2n$

v) $a_1 = \ldots = a_{n/3} = 3/2n$ (same as iv with roles

$a_{n/3+1} = \ldots = a_{2n/3} = 2/2n$ reversed)

$a_{2n/3+1} = \ldots = a_n = 1/2n$

It was felt unlikely that one shipper-receiver difference D_i would have more than three times the variability of another shipment between the same pair of facilities. For the hypothesis of no material diversion, $H:\underline{\mu}=\underline{0}$, the true alpha values are estimated, for N=40,000 runs. The test plan, Figure 4.1, comprised of 15 parameter sets for a given n, was run for n=6, 12, and 18. The computer output is given in Tables B.1 - B.45 and discussed in Section 4.3.2.

		Variance Structure i	ii	iii	iv	v
Degrees	i	X	X		X	
of	ii	X	X		X	
Freedom	iii	X	X		X	
	iv	X	X	X		
	v	X			X	X

Figure 4.1 Test plans for comparing procedures, repeated for n=6, 12 and 18

4.3.3 Comparison of Procedures

4.3.3.1 General Remarks

The Satterthwaite procedure gives an α level closest to the intended value. Based on the test plan in 4.3.2, Satterthwaite performs best of the four procedures, showing minimal effect with changes of number of shipments, degrees of freedom and variance pattern.

The Satterthwaite and Sum procedures compute degrees of freedom f and then use the appropriate Student's t critical value $t_{f,\alpha}$. Banerjee and Cochran, however, take a function of n Student t values, $t_{f_1,\alpha}$, ..., $t_{f_n,\alpha}$, for their critical values (see Section 4.2). For a set of s_i^2 and f_i, we can take the Banerjee and Cochran critical values, $t_{B,\alpha}$ and $t_{C,\alpha}$ and find the degrees of freedom f_B and f_C which would give these critical values. This permits comparing the four procedures on the same scale. Let f_S and f_A be the degrees of freedom for Satterthwaite and Sum. Then

$$\min f_i \leq f_B \leq f_C \leq \max f_i \leq f_A$$

and

$$\min f_i \leq f_S \leq f_A$$

Thus

$$t_{B,\alpha} \geq t_{C,\alpha} \geq t_{A,\alpha}$$

and

$$t_{S,\alpha} \geq t_{A,\alpha}$$

Thus the observed alpha levels will satisfy for all α

$$\alpha_B \leq \alpha_C \leq \alpha_A$$

and

$$\alpha_S \leq \alpha_A$$

We believe, but have not shown, that

$$f_B \leq f_C \leq f_S \leq f_A$$

which would imply

$$\alpha_B \leq \alpha_C \leq \alpha_S \leq \alpha_A$$

All cases investigated support this.

4.3.3.2 Results n=2

The case n=2 is interesting for several reasons. The variance σ_i^2 of D_i is the sum of the shipper variance $\sigma_{S,i}^2$ and the receiver variance $\sigma_{R,i}^2$. To test a single D_i, we need to combine estimates of these two variances. We then use the test (4.1.2) with n=2 in the denominator. As mentioned earlier, the results for n=2 have been studied for the first three procedures of 4.2, Lee and Gurland (1975), permitting verification of the simulation results. The results for α = .005 to .05, f_1=4, f_2=8 are given in Figures B.46-B.55 and summarized in Figure 4.2 for α = .05. The first variance estimate, s_1^2, is less precise then the second variance estimate, s_2^2, giving unsymmetric results with respect to $C=\sigma_1^2/(\sigma_1^2 + \sigma_2^2)$.

From Figure 4.2, Satterthwaite performs less favorably when the true variances are equal, C=.5, or when 95% of the variation of D is from the sample with the smaller degrees of freedom, C = .95. The true α level oscillates around .05, giving a slightly conservative test for C=.5 and a liberal test at C=.95.

The Banerjee and Cochran procedures give nearly identical results. The observed α level is always less than .05. These procedures do better for C close to 0 or 1, performing less favorably at C=.5. When C is close to 0 or 1, the test (4.1.2) converges to the one-sample t-test, and the Banerjee and Cochran critical values converge to the correct degrees of freedom. These procedures degrade and give a less precise approximate test, as α decreases.

Sum has the least accuracy at C=.1 and C=.95, the extreme cases run. At C=.95 Sum gives the largest divergence, with a false alarm rate α 40% larger than intended. As mentioned above, as C goes to 0 or 1 the test (4.1.2) converges to a one-sample t-test and f_1 or f_2 degrees of freedom should be used for the critical value, whereas Sum uses $f_1 + f_2$. Sum performs best at C=.5.

In comparing the four procedures over this restricted parameter set, we conclude that Satterthwaite performs best overall. The observed α level is closest to the target value over the range of C, and the α values studied. Banerjee and Cochran give virtually equivalent results. The observed α level is always less than the target value, and is always less accurate than the Satterthwaite procedure. Furthermore, as α decreases the performance of Banerjee and Cochran decreases. Sum performs slightly better than Satterthwaite for C=.3, .4 and .5. Satterthwaite, however, performs considerably better over the other C values.

C	Satterthwaite	Banerjee	Cochran	Sum
.1	.0507	.0447	.0448	.0538
.2	.0503	.0411	.0413	.0523
.3	.0478	.0384	.0387	.0497
.4	.0492	.0382	.0385	.0513
.5	.0470	.0348	.0350	.0503
.6	.0475	.0361	.0363	.0525
.7	.0488	.0372	.0375	.0546
.8	.0501	.0401	.0402	.0587
.9	.0509	.0425	.0426	.0650
.95	.0521	.0470	.0471	.0713

Figure 4.2 Comparison of observed α values when intended value is .05, for n=2, f_1=4, f_2=8 and $C = \sigma_1^2/(\sigma_1^2 + \sigma_2^2)$

4.3.3.3 Results $n > 2$

Equal variances and equal degrees of freedom

The equal variances and equal degrees of freedom correspond to the first three cases in column 1 of Figure 4.1. The corresponding output is given by Figures B.1-B.9 and summarized in Figure 4.3.

	Sum and α	Satterthwaite f=4	Satterthwaite f=10	Satterthwaite f=20	Banerjee-Cochran f=4	Banerjee-Cochran f=10	Banerjee-Cochran f=20
	.10	.1000	.1002	.0113	.0704	.0872	.0943
n=6	.05	.0480	.0492	.0503	.0214	.0371	.0442
	.01	.0082	.0101	.0099	.0005	.0040	.0059
	.10	.0988	.0988	.0998	.0656	.0847	.0933
n=12	.05	.0481	.0495	.0505	.0192	.0367	.0432
	.01	.0091	.0097	.0104	.0001	.0032	.0064
	.10	.0987	.0991	.0993	.0664	.0856	.0929
n=18	.05	.0491	.0503	.0483	.0177	.0357	.0415
	.01	.0095	.0103	.0095	.0002	.0031	.0060

Figure 4.3 Comparison of α values for equal variances and equal degrees of freedom. Sum is exact, agreeing with α desired

The Satterthwaite procedure shows no significant effect as n increases. The observed α values show some slight improvement as f_i increases.

The Banerjee and Cochran procedures are identical when the f_i are equal (see 4.2.3). They give very conservative results for $f_i=4$, doing appreciably worse with decreasing α. As n increases they show some decrease in size. The effect of f_i is quite significant, with the conservatism decreasing drastically as f_i goes from 4 to 20.

The Sum procedure is exact for these cases, see 4.2.4. Thus its performance is best for equal variances and equal degrees of freedom. The Satterthwaite procedure gives good agreement with the target values, while the Banerjee and Cochran procedures are unduly conservative unless the degrees of freedom are large.

Equal variances, degrees of freedom not equal

The equal variances but unequal degrees of freedom correspond to the last two cases in the first column of the test plan, Figure 4.1. The computer results are given by Figures B.10-B.15 and summarized in Figure 4.4.

Satterthwaite shows no appreciable effect as n increases, or the f_i pattern changes. Again, it performs best in attaining an α level closest to the desired level. Sum, performing next best, shows a slight improvement with an increase in n. Banerjee and Cochran again exhibit extreme conservatism for $\alpha=.01$. This conservatism increases with n.

		Degrees of Freedom Pattern iv (f_i 4 and 20)				Degrees of Freedom Pattern v (f_i 4,10 and 20)			
	α	Satterthwaite	Banerjee	Cochran	Sum	Satterthwaite	Banerjee	Cochran	Sum
n=6	.10	.1009	.0831	.0835	.1034	.1006	.0833	.0836	.1023
	.05	.0520	.0323	.0333	.0545	.0505	.0344	.0350	.0522
	.01	.0097	.0016	.0018	.0111	.0103	.0023	.0027	.0116
n=12	.10	.0991	.0786	.0791	.1005	.1019	.0849	.0852	.1031
	.05	.0509	.0300	.0307	.0522	.0510	.0314	.0319	.0523
	.01	.0102	.0014	.0016	.0109	.0092	.0016	.0018	.0099
n=18	.10	.1008	.0776	.0780	.1018	.0987	.0795	.0799	.0995
	.05	.0488	.0271	.0278	.0498	.0495	.0296	.0302	.0502
	.01	.0094	.0010	.0012	.0101	.0090	.0013	.0016	.0096

Figure 4.4 Comparison of α values for equal variances, unequal degrees of freedom patterns iv and v (see Figure 4.1)

Variances unequal (ratio 1 to 3)

Figure 4.5 summarizes the performance of the four procedures for degrees of freedom pattern iv, variance patterns ii and iii. The first half of the variance estimates have $f_i=4$, the second half have $f_i=20$. The true variances are in the ratio of 1 to 3 in the first case, with the larger degrees of freedom associated with the larger variances estimated. In the second case, variances 3 to 1, the larger variances are estimated less precisely, have the smaller f_i. Figure 4.6 presents the results for variances in the ratio of 1 to 3, pattern iii, for equal degrees of freedom. The complete results are given in Figures B.16-B.30.

The Satterthwaite procedure slightly outperforms Sum, with Banerjee and Cochran continuing to show conservatism which increases dramatically with decreasing α. After examining Figure 4.5, Satterthwaite shows no appreciable effect from flip-flopping the degrees of freedom. Sum appears to give a larger α value as the f_i are interchanged, while Banerjee and Cochran perform considerably worse when the larger variance has the smaller degrees of freedom.

The Satterthwaite procedure, from Figure 4.6, appears to be relatively unaffected by increasing n or f_i. It again slightly outperforms Sum. Banerjee and Cochran again show extreme conservatism for small f_i and small α. As f_i increases, the observed α approaches the desired level.

		Variance Pattern ii (σ_i^2 ratio 1 to 3)				Variance Pattern iii (σ_i^2 ratio 3 to 1)			
	α	Satterthwaite	Banerjee	Cochran	Sum	Satterthwaite	Banerjee	Cochran	Sum
	.10	.0959	.0833	.0836	.0965	.0981	.0769	.0771	.1049
n=6	.05	.0465	.0339	.0344	.0470	.0486	.0279	.0285	.0552
	.01	.0092	.0028	.0032	.0095	.0096	.0014	.0017	.0134
	.10	.0991	.0848	.0851	.0994	.0992	.0730	.0735	.1033
n=12	.05	.0504	.0362	.0369	.0506	.0500	.0246	.0251	.0540
	.01	.0105	.0026	.0030	.0106	.0092	.0010	.0011	.0115
	.10	.1005	.0856	.0861	.1010	.0990	.0706	.0712	.1012
n=18	.05	.0511	.0355	.0360	.0511	.0471	.0219	.0223	.0500
	.01	.0100	.0026	.0028	.0102	.0087	.0006	.0007	.0103

Figure 4.5 Comparison of α values for degrees of freedom pattern iv, f_i=4 and 20, for variance ratios 1 to 3, pattern ii, and 3 to 1, pattern iii

	α	Sat. ($f_i = 4$)	Banerjee Cochran ($f_i = 4$)	Sum ($f_i = 4$)	Sat. ($f_i = 10$)	Banerjee Cochran ($f_i = 10$)	Sum ($f_i = 10$)	Sat. ($f_i = 20$)	Banerjee Cochran ($f_i = 20$)	Sum ($f_i = 20$)
	.10	.0994	.0723	.1029	.0998	.0878	.1009	.1018	.0962	.1022
n=6	.05	.0486	.0231	.0523	.0500	.0393	.0509	.0501	.0433	.0507
	.01	.0095	.0008	.0115	.0108	.0041	.0115	.0093	.0066	.0096
	.10	.0981	.0671	.1006	.0990	.0865	.0996	.0990	.0928	.0993
n=12	.05	.0489	.0202	.0513	.0502	.0370	.0508	.0503	.0430	.0504
	.01	.0091	.0002	.0104	.0101	.0036	.0104	.0102	.0064	.0102
	.10	.0980	.0641	.0995	.1055	.0908	.1057	.0963	.0899	.0965
n=18	.05	.0475	.0179	.0493	.0514	.0377	.0518	.0482	.0410	.0483
	.01	.0089	.0003	.0097	.0105	.0032	.0108	.0097	.0063	.0097

Figure 4.6 Comparison of α values for variance pattern iii, ratio of 1 to 3, for equal degrees of freedom, patterns i, ii and iii

Variances unequal (ratio 1 to 2 to 3)

Figure 4.7 summarizes the results for degrees of freedom pattern v and variance patterns iv and v. The first third of the variance estimates have $f_i=4$, the second third $f_i=10$ and the last third $f_i=20$. The variances σ_i^2 are in the ratio of 1 to 2 to 3 where the larger variances have the larger degrees of freedom in the first case, and the smaller degrees of freedom in the second case. The complete simulation results are given in Figures B.31-B.45.

The Satterthwaite procedure is slightly favored over Sum. Banerjee and Cochran show a decrease in α level as the larger variances are associated with the smaller degrees of freedom, consistent with the results from Figure 4.5.

	α	Variance Pattern iv (σ_i^2 in ratio 1, 2, 3) Satterthwaite	Banerjee	Cochran	Sum	Variance Pattern v (σ_i^2 in ratio 3, 2, 1) Satterthwaite	Banerjee	Cochran	Sum
	.10	.0999	.0871	.0873	.1006	.1008	.0821	.0825	.0160
n=6	.05	.0501	.0373	.0377	.0506	.0507	.0329	.0332	.0549
	.01	.0099	.0034	.0036	.0101	.0096	.0018	.0021	.0124
	.10	.0987	.0849	.0852	.0990	.0991	.0760	.0763	.1017
n=12	.05	.0500	.0344	.0348	.0502	.0490	.0275	.0281	.0512
	.01	.0099	.0026	.0029	.0100	.0096	.0012	.0015	.0114
	.10	.1011	.0868	.0872	.1012	.0968	.0750	.0754	.0985
n=18	.05	.0483	.0337	.0341	.0486	.0485	.0258	.0262	.0500
	.01	.0098	.0025	.0027	.0099	.0092	.0012	.0013	.0102

Figure 4.7 Comparison of α values for degrees of freedom pattern v, f_i=4, 10, 20, for variance ratios 1 to 2 to 3, iv, and 3 to 2 to 1, v

4.3.4 Power Results

Power results are only presented for the Satterthwaite procedure. For all the parameter sets we have investigated we have found

$$\alpha_B \leq \alpha_C \leq \alpha_S \leq \alpha_A$$

It is not appropriate to compare the power of two procedures if they don't have the same α value. The four procedures are all trying to achieve the desired α value, but differ greatly in their achieved level.

The test statistic (4.1.2)

$$T_S = \frac{\Sigma D_i}{(\Sigma s_i^2)^{1/2}}$$

is seen from (4.3.1) to be of the form

$$T_S = \frac{Z + \lambda}{\left(\Sigma(\sigma_i^2/(\sigma^2 f_i))\, \chi^2_{f_i}\right)^{1/2}}$$

when the diversion is $\underline{\mu}$, where Z is normal (0,1) and

$$\lambda = \Sigma \mu_i/\sigma = (\Sigma\ \mu_i)/(\Sigma\ \sigma_i^2)^{1/2}$$

The power, as has been noted earlier, is a function of the total diversion, and not the diversion pattern. For a given α level, the power is determined by the total diversion, $\Sigma \mu_i$, and the total variation, $\Sigma \sigma_i^2$, which are combined to form λ. If we double n, the number of shipments, have the σ_i^2 for the second n shipments repeat the pattern of the first n shipments, we must increase the total diversion by 41% for λ to remain constant and have the same probability of detection.

The power of the Satterthwaite T_S test is evaluated for the 45 parameter sets given in Figure 4.1, for α =.10, .05 and .01. The general shape for the power curves is given in Figure 4.8. The results are summarized in Figures 4.9-4.17. The power is seen to be a function of only the α level desired, and λ, the standardized cumulative diversion parameter. The power is virtually unaffected by changes in n, $\underline{f}$ and $\underline{\sigma}^2$. Thus Figure 4.8 may be used for computing approximate power of the test for a diverse set of parameter values, much larger than the set investigated.

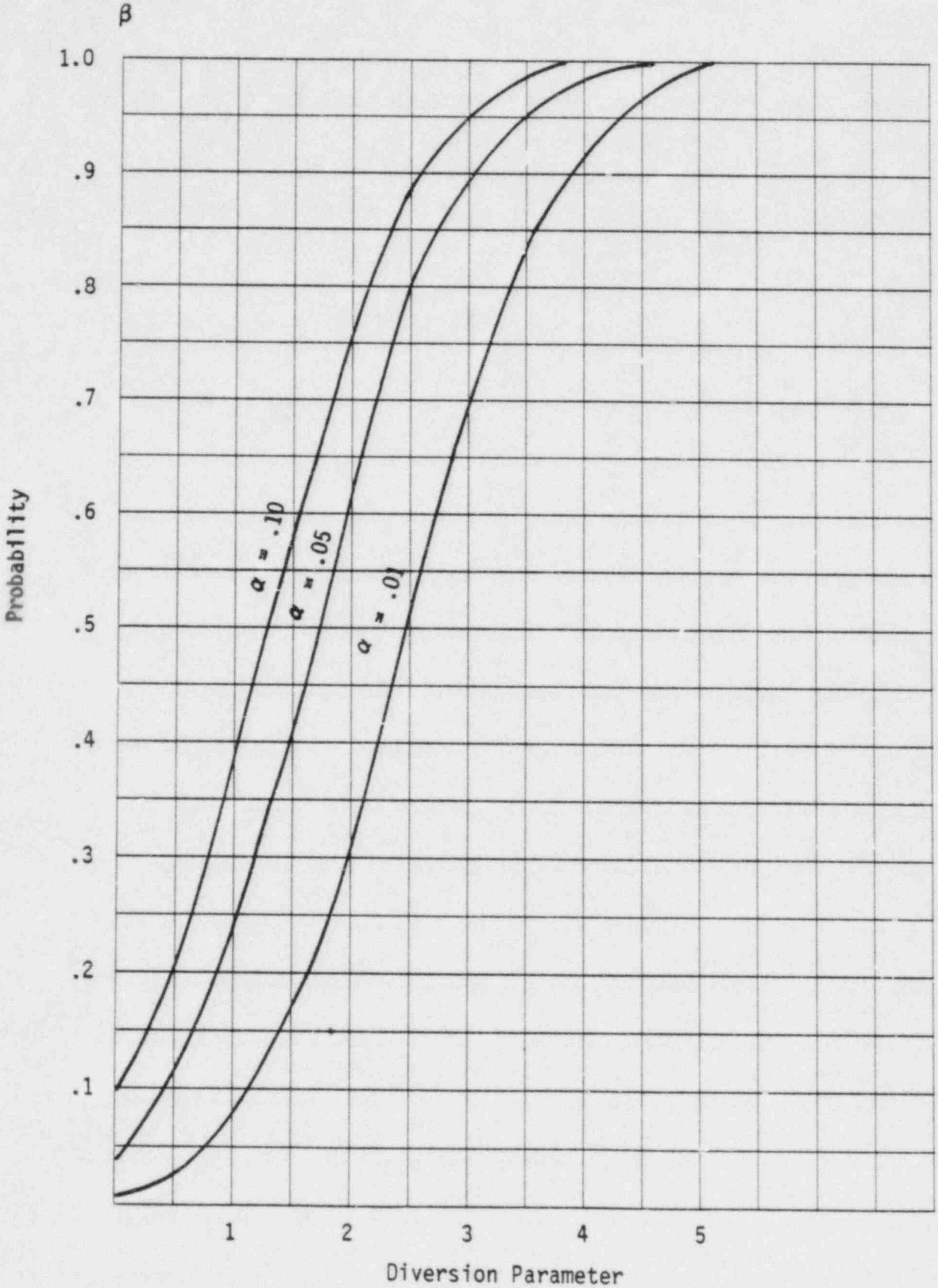

Figure 4.8 Power curves β for Satterthwaite procedure with $n = 6$, $\underline{f} = \underline{4}$ and equal variances

n	f_i	pattern	σ_i^2	pattern	0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
					λ- material diversion parameter										
6	I	4	I	EQUAL	.1000	.2130	.3783	.5733	.7504	.8773	.9503	.9840	.9959	.9992	.9999
6	II	10	I	EQUAL	.1002	.2154	.3884	.5855	.7636	.8887	.9585	.9873	.9972	.9995	.9998
6	III	20	I	EQUAL	.1003	.2183	.3892	.5838	.7641	.8861	.9564	.9868	.9964	.9992	.9999
6	IV	4,20	I	EQUAL	.1009	.2146	.3843	.5774	.7523	.8795	.9523	.9846	.9963	.9994	1.0000
6	V	4,10,20	I	EQUAL	.1006	.2174	.3880	.5815	.7570	.8814	.9533	.9854	.9961	.9993	.9999
12	I	4	I	EQUAL	.0988	.2134	.3826	.5780	.7535	.8801	.9523	.9844	.9959	.9993	.9999
12	II	10	I	EQUAL	.0988	.2144	.3867	.5820	.7584	.8839	.9552	.9858	.9964	.9992	.9999
12	III	20	I	EQUAL	.0998	.2168	.3878	.5837	.7622	.8878	.9565	.9866	.9968	.9993	.9999
12	IV	4,20	I	EQUAL	.0991	.2127	.3872	.5799	.7595	.8841	.9563	.9863	.9968	.9995	.9999
12	V	4,10,20	I	EQUAL	.1019	.2190	.3885	.5839	.7607	.8854	.9578	.9862	.9960	.9993	.9999
18	I	4	I	EQUAL	.0987	.2118	.3819	.5786	.7577	.8833	.9553	.9856	.9965	.9992	.9999
18	II	10	I	EQUAL	.0991	.2127	.3858	.5816	.7592	.8854	.9560	.9863	.9962	.9993	.9999
18	III	20	I	EQUAL	.0993	.2173	.3908	.5883	.7642	.8889	.9569	.9867	.9969	.9994	1.0000
18	IV	4,20	I	EQUAL	.1008	.2192	.3916	.5897	.7631	.8874	.9546	.9864	.9967	.9994	.9999
18	V	4,10,20	I	EQUAL	.0987	.2153	.3848	.5807	.7599	.8840	.9549	.9866	.9962	.9991	.9998

Figure 4.9 Power values for Satterthwaite procedure for .10 α level, equal variances

n	f_i pattern		σ_i^2 pattern		λ- material diversion parameter 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	II	1,3	.0994	.2104	.3753	.5706	.7462	.8738	.9493	.9833	.9958	.9992	.9999
6	II	10	II	1,3	.0998	.2175	.3889	.5815	.7568	.8842	.9554	.9851	.9964	.9994	.9998
6	III	20	II	1,3	.1018	.2199	.3920	.5904	.7633	.8870	.9559	.9867	.9963	.9992	.9999
6	IV	4,20	II	1,3	.0959	.2136	.3845	.5797	.7603	.8851	.9548	.9855	.9961	.9993	1.0000
6	IV	4,20	III	3,1	.0981	.2127	.3779	.5749	.7507	.8808	.9522	.9844	.9961	.9991	.9999
12	I	4	II	1,3	.0981	.2143	.3797	.5739	.7525	.8794	.9509	.9843	.9957	.9991	.9998
12	II	10	II	1,3	.0990	.2189	.3889	.5861	.7617	.8850	.9551	.9863	.9967	.9993	.9999
12	III	20	II	1,3	.0990	.2166	.3878	.5858	.7622	.8888	.9573	.9858	.9965	.9994	.9999
12	IV	4,20	II	1,3	.0991	.2138	.3851	.5831	.7594	.8833	.9542	.9854	.9962	.9993	.9999
12	IV	4,20	III	3,1	.0992	.2123	.3813	.5779	.7586	.8848	.9545	.9857	.9956	.9990	.9998
18	I	4	II	1,3	.0980	.2142	.3818	.5774	.7587	.8846	.9539	.9855	.9964	.9993	.9999
18	II	10	II	1,3	.1055	.2210	.3893	.5874	.7632	.8882	.9568	.9859	.9964	.9994	.9999
18	III	20	II	1,3	.0963	.2128	.3837	.5837	.7605	.8868	.9573	.9866	.9962	.9993	.9999
18	IV	4,20	II	1,3	.1005	.2170	.3849	.5798	.7609	.8885	.9569	.9868	.9968	.9995	1.0000
18	IV	4,20	III	3,1	.0990	.2150	.3858	.5806	.7589	.8839	.9549	.9856	.9962	.9991	.9998

Figure 4.10 Power values for Satterthwaite procedure for .10 α level, unequal variances

n	f_i pattern		σ_i^2 pattern		λ - material diversion parameter 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	IV	1,2,3	.0990	.2122	.3756	.5672	.7446	.8747	.9497	.9843	.9952	.9988	.9998
6	II	10	IV	1,2,3	.1018	.2191	.3872	.5806	.7599	.8853	.9558	.9860	.9963	.9992	.9999
6	III	20	IV	1,2,3	.0968	.2148	.3857	.5840	.7591	.8849	.9554	.9855	.9965	.9993	.9999
6	V	4,10,20	IV	1,2,3	.0999	.2151	.3834	.5802	.7569	.8853	.9545	.9855	.9962	.9991	.9998
6	V	4,10,20	V	3,2,1	.1008	.2165	.3816	.5742	7507	.8801	.9509	.9839	.9956	.9990	1.0000
12	I	4	IV	1,2,3	.1014	.2120	.3812	.5750	.[illegible]554	.8842	.9534	.9855	.9964	.9994	.9999
12	II	10	IV	1,2,3	.1000	.2169	.3880	.5858	.7594	.8841	.9552	.9857	.9967	.9992	.9999
12	III	20	IV	1,2,3	.0999	.2171	.3886	.5866	.7614	.8867	.9555	.9857	.9967	.9991	1.0000
12	V	4,10,20	IV	1,2,3	.0987	.2144	.3855	.5834	.7592	.8843	.9562	.9874	.9966	.9994	.9999
12	V	4,10,20	V	3,2,1	.0991	.2123	.3832	.5786	.7547	.8840	.9535	.9853	.9957	.9993	.9999
18	I	4	IV	1,2,3	.0989	.2141	.3841	.5811	.7557	.8834	.9544	.9857	.9964	.9994	.9999
18	II	10	IV	1,2,3	.0994	.2161	.3887	.5870	.7628	.8887	.9578	.9868	.9964	.9995	.9999
18	III	20	IV	1,2,3	.0996	.2150	.3881	.5857	.7632	.8873	.9574	.9869	.9967	.9992	.9998
18	V	4,10,20	IV	1,2,3	.1011	.2182	.3897	.5870	.7639	.8883	.9571	.9862	.9967	.9995	1.0000
18	V	4,10,20	V	3,2,1	.0968	.2139	.3818	.5791	.7584	.8829	.9548	.9861	.9966	.9991	1.0000

Figure 4.11 Power values for Satterthwaite procedure for .10 α level, unequal variances

n	f_i pattern		σ_i^2 pattern		λ - material diversion parameter 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	I	EQUAL	.0480	.1213	.2454	.4192	.6095	.7775	.8935	.9572	.9862	.9967	.9995
6	II	10	I	EQUAL	.0492	.1237	.2534	.4376	.6335	.7992	.9101	.9677	.9904	.9980	.9997
6	III	20	I	EQUAL	.0503	.1259	.2608	.4391	.6333	.8018	.9095	.9670	.9903	.9974	.9995
6	IV	4,20	I	EQUAL	.0520	.1236	.2517	.4281	.6203	.7838	.8994	.9610	.9878	.9973	.9995
6	V	4,10,20	I	EQUAL	.0505	.1252	.2559	.4323	.6252	.7888	.9018	.9629	.9887	.9974	.9995
12	I	4	I	EQUAL	.0481	.1209	.2487	.4294	.6206	.7863	.8997	.9614	.9880	.9970	.9995
12	II	10	I	EQUAL	.0495	.1247	.2556	.4371	.6344	.7981	.9065	.9662	.9893	.9973	.9995
12	III	20	I	EQUAL	.0505	.1253	.2611	.4406	.6338	.8009	.9118	.9679	.9907	.9978	.9996
12	IV	4,20	I	EQUAL	.0509	.1225	.2545	.4354	.6295	.7958	.9072	.9659	.9903	.9979	.9997
12	V	4,10,20	I	EQUAL	.0510	.1261	.2585	.4383	.6327	.7959	.9081	.9672	.9900	.9970	.9996
18	I	4	I	EQUAL	.0491	.1226	.2505	.4300	.6262	.7944	.9058	.9652	.9893	.9975	.9996
18	II	10	I	EQUAL	.0503	.1245	.2541	.4367	.6319	.7981	.9081	.9667	.9903	.9976	.9995
18	III	20	I	EQUAL	.0483	.1253	.2604	.4447	.6381	.8041	.9117	.9675	.9905	.9980	.9995
18	IV	4,20	I	EQUAL	.0488	.1267	.2601	.4425	.6388	.8006	.9092	.9661	.9902	.9978	.9996
18	V	4,10,20	I	EQUAL	.0495	.1248	.2551	.4362	.6303	.7967	.9076	.9654	.9897	.9974	.9994

Figure 4.12 Power values for Satterthwaite procedure for .05 α level, equal variances

					λ - material diversion parameter										
n	f_1 pattern		σ_i^2 pattern		0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	II	1,3	.0486	.1186	.2402	.4130	.6047	.7712	.8879	.9552	.9847	.9964	.9992
6	II	10	II	1,3	.0500	.1256	.2573	.4360	.6262	.7898	.9049	.9645	.9886	.9973	.9996
6	III	20	II	1,3	.0501	.1282	.2597	.4447	.6378	.8015	.9078	.9659	.9898	.9972	.9994
6	IV	4,20	II	1,3	.0465	.1210	.2513	.4332	.6277	.7952	.9063	.9644	.9893	.9972	.9996
6	IV	4,20	III	3,1	.0486	.1198	.2453	.4165	.6103	.7767	.8951	.9588	.9864	.9966	.9992
12	I	4	II	1,3	.0489	.1206	.2502	.4232	.6169	.7853	.8980	.9604	.9879	.9967	.9993
12	II	10	II	1,3	.0502	.1267	.2597	.4390	.6346	.7982	.9084	.9656	.9900	.9979	.9995
12	III	20	II	1,3	.0503	.1239	.2581	.4395	.6371	.8007	.9125	.9673	.9902	.9979	.9996
12	IV	4,20	II	1,3	.0504	.1245	.2553	.4381	.6335	.7981	.9071	.9646	.9895	.9973	.9996
12	IV	4,20	III	3,1	.0500	.1213	.2479	.4276	.6215	.7903	.9031	.9625	.9892	.9965	.9992
18	I	4	II	1,3	.0475	.1202	.2504	.4287	.6235	.7939	.9054	.9638	.9886	.9974	.9995
18	II	10	II	1,3	.0514	.1293	.2608	.4395	.6379	.8008	.9109	.9673	.9898	.9976	.9996
18	III	20	II	1,3	.0482	.1208	.2559	.4373	.6346	.8005	.9094	.9673	.9902	.9976	.9997
18	IV	4,20	II	1,3	.0511	.1278	.2573	.4354	.6329	.8001	.9120	.9677	.9904	.9979	.9996
18	IV	4,20	III	3,1	.0471	.1229	.2542	.4308	.6271	.7954	.9049	.9648	.9891	.9973	.9994

Figure 4.13 Power values for Satterthwaite procedure for .05 α level, unequal variances

					λ - material diversion parameter										
n	f_i pattern		σ_i^2 pattern		0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	IV	1,2,3	.0482	.1195	.2408	.4110	.6012	.7705	.8888	.9567	.9862	.9958	.9988
6	II	10	IV	1,2,3	.0511	.1266	.2583	.4360	.6251	.7942	.9076	.9649	.9890	.9973	.9994
6	III	20	IV	1,2,3	.0482	.1229	.2548	.4378	.6343	.7965	.9079	.9655	.9898	.9975	.9995
6	V	4,10,20	IV	1,2,3	.0501	.1241	.2526	.4311	.6291	.7940	.9064	.9648	.9895	.9975	.9992
6	V	4,10,20	V	3,2,1	.0507	.1244	.2506	.4224	.6130	.7799	.8960	.9581	.9871	.9964	.9993
12	I	4	IV	1,2,3	.0503	.1235	.2491	.4253	.6192	.7887	.9034	.9630	.9886	.9974	.9995
12	II	10	IV	1,2,3	.0508	.1257	.2589	.4389	.6331	.7964	.9070	.9659	.9902	.9977	.9996
12	III	20	IV	1,2,3	.0510	.1252	.2572	.4412	.6362	.7996	.9110	.9661	.9902	.9977	.9996
12	V	4,10,20	IV	1,2,3	.0500	.1243	.2550	.4368	.6363	.7978	.9078	.9671	.9909	.9976	.9995
12	V	4,10,20	V	3,2,1	.0490	.1223	.2509	.4289	.6207	.7910	.9044	.9629	.9886	.9969	.9995
18	I	4	IV	1,2,3	.0493	.1240	.2512	.4314	.6271	.7911	.9048	.9647	.9893	.9977	.9996
18	II	10	IV	1,2,3	.0487	.1244	.2572	.4393	.6349	.7997	.9120	.9681	.9907	.9976	.9997
18	III	20	IV	1,2,3	.0495	.1259	.2564	.4395	.6375	.8014	.9102	.9683	.9907	.9977	.9995
18	V	4,10,20	IV	1,2,3	.0483	.1268	.2592	.4424	.6368	.8014	.9108	.9670	.9904	.9978	.9996
18	V	4,10,20	V	3,2,1	.0485	.1227	.2535	.4314	.6275	.7935	.9044	.9654	.9900	.9973	.9992

Figure 4.14 Power values for Satterthwaite procedure for .05 α level, unequal variances

				λ - material diversion parameter										
n	f_i pattern		σ_i^2 pattern		0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	I	EQUAL	.0082	.0278	.0760	.1694	.3096	.4903	.6721	.8197	.9160	.9678	.9896
6	II	10	I	EQUAL	.0101	.0320	.0868	.1913	.3519	.5459	.7284	.8647	.9461	.9820	.9954
6	III	20	I	EQUAL	.0099	.0336	.0908	.2004	.365[illegible]	.5567	.7393	.8716	.9472	.9837	.9952
6	IV	4,20	I	EQUAL	.0097	.0329	.0861	.1859	.3384	.5242	.7035	.8421	.9315	.9750	.9924
6	V	4,10,20	I	EQUAL	.0103	.0332	.0851	.1899	.3464	.5355	.7126	.8494	.9351	.9775	.9932
12	I	4	I	EQUAL	.0091	.0306	.0826	.1830	.3376	.5272	.7055	.8473	.9344	.9765	.9934
12	II	10	I	EQUAL	.0097	.0333	.0887	.1957	.3597	.5536	.7335	.8681	.9469	.9824	.9954
12	III	20	I	EQUAL	.0104	.0337	.0907	.2003	.3678	.5604	.7431	.8753	.9515	.9839	.9962
12	IV	4,20	I	EQUAL	.0102	.0338	.0874	.1926	.3546	.5446	.7291	.8628	.9439	.9812	.9954
12	V	4,10,20	I	EQUAL	.0092	.0333	.0912	.1969	.3586	.5493	.7313	.8653	.9464	.9826	.9943
18	I	4	I	EQUAL	.0095	.0319	.0855	.1876	.3448	.5376	.7230	.8606	.9426	.9804	.9943
18	II	10	I	EQUAL	.0103	.0336	.0904	.1973	.3626	.5557	.7380	.8714	.9492	.9837	.9956
18	III	20	I	EQUAL	.0095	.0325	.0912	.2026	.3719	.5667	.7469	.8776	.9513	.9846	.9963
18	IV	4,20	I	EQUAL	.0094	.0317	.0891	.1992	.3646	.5586	.7375	.8696	.9455	.9827	.9955
18	V	4,10,20	I	EQUAL	.0090	.0327	.0884	.1967	.3598	.5519	.7360	.8678	.9462	.9832	.9952

Figure 4.15 Power values for Satterthwaite procedure for .01 α level, equal variances

n	f_i pattern		σ_i^2 pattern		λ - material diversion parameter 0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	II	1,3	.0095	.0285	.0744	.1631	.2986	.4739	.6516	.8013	.9024	.9593	.9863
6	II	10	II	1,3	.0108	.0340	.0865	.1922	.3532	.5395	.7185	.8552	.9389	.9788	.9938
6	III	20	II	1,3	.0093	.0318	.0921	.2014	.3665	.5597	.7382	.8700	.9453	.9827	.9952
6	IV	4,20	II	1,3	.0092	.0313	.0837	.1915	.3500	.5427	.7274	.8619	.9419	.9808	.9947
6	IV	4,20	III	3,1	.0096	.0297	.0773	.1698	.3108	.4875	.6653	.8116	.9122	.9642	.9880
12	I	4	II	1,3	.0091	.0310	.0823	.1798	.3281	.5151	.6987	.8413	.9285	.9736	.9925
12	II	10	II	1,3	.0101	.0332	.0888	.1986	.3618	.5542	.7339	.8665	.9457	.9819	.9954
12	III	20	II	1,3	.0102	.0339	.0907	.1992	.3670	.5628	.7420	.8754	.9514	.9836	.9959
12	IV	4,20	II	1,3	.0105	.0348	.0894	.1956	.3601	.5551	.7372	.8689	.9460	.9824	.9951
12	IV	4,20	III	3,1	.0092	.0314	.0832	.1825	.3373	.5228	.7054	.8494	.9345	.9765	.9927
18	I	4	II	1,3	.0089	.0294	.0829	.1858	.3427	.5317	.7184	.8563	.9392	.9787	.9940
18	II	10	II	1,3	.0105	.0348	.0951	.2031	.3650	.5609	.7425	.8727	.9495	.9831	.9952
18	III	20	II	1,3	.0097	.0323	.0884	.1970	.3642	.5623	.7429	.8746	.9516	.9842	.9955
18	IV	4,20	II	1,3	.0100	.0346	.09.3	.2017	.3622	.5577	.7404	.8756	.9504	.9843	.9959
18	IV	4,20	III	3,1	.0087	.0296	.0848	.1876	.3479	.5368	.7196	.8572	.9400	.9793	.9938

Figure 4.16 Power values for Satterthwaite procedure for .01 α level, unequal variances

n	f_i pattern		σ_i^2 pattern		λ - material diversion parameter										
					0.0	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.0	4.5	5.0
6	I	4	IV	1,2,3	.0086	.0281	.0756	.1628	.3024	.4739	.6551	.8049	.9062	.9635	.9873
6	II	10	IV	1,2,3	.0109	.0334	.0884	.1936	.3510	.5393	.7202	.8589	.9421	.9795	.9942
6	III	20	IV	1,2,3	.0091	.0315	.0878	.1962	.3594	.5554	.7354	.8681	.9461	.9820	.9952
6	V	4,10,20	IV	1,2,3	.0099	.0335	.0877	.1922	.3508	.5424	.7245	.8621	.9418	.9804	.9945
6	V	4,10,20	V	3,2,1	.0096	.0327	.0831	.1808	.3275	.5046	.6822	.8279	.9206	.9683	.9899
12	I	4	IV	1,2,3	.0094	.0315	.0855	.1804	.3333	.5195	.7024	.8469	.9342	.9762	.9933
12	II	10	IV	1,2,3	.0097	.0344	.0904	.1978	.3616	.5557	.7335	.8652	.9455	.9818	.9955
12	III	20	IV	1,2,3	.0097	.0337	.0913	.2016	.3663	.5637	.7433	.8732	.9496	.9831	.9955
12	V	4,10,20	IV	1,2,3	.0099	.0324	.0888	.1956	.3607	.5535	.7358	.8683	.9478	.9842	.9957
12	V	4,10,20	V	3,2,1	.0096	.0312	.0850	.1854	.3428	.5333	.7136	.8542	.9369	.9777	.9931
18	I	4	IV	1,2,3	.0099	.0315	.0853	.1879	.3459	.5355	.7163	.8569	.9408	.9797	.9946
18	II	10	IV	1,2,3	.0090	.0313	.0901	.1994	.3645	.5605	.7410	.8730	.9502	.9836	.9956
18	III	20	IV	1,2,3	.0102	.0327	.0903	.2006	.3687	.5649	.7455	.8756	.9512	.9845	.9960
18	V	4,10,20	IV	1,2,3	.0098	.0323	.0920	.2022	.3668	.5616	.7422	.8740	.9502	.9833	.9959
18	V	4,10,20	V	3,2,1	.0092	.0307	.0857	.1909	.3496	.5419	.7262	.8616	.9416	.9808	.9949

Figure 4.17 Power values for Satterthwaite procedure for .01 α level, unequal variances

5. CONCLUDING REMARKS

The test T_S given in Section 4.1 may be directly used to test n shipper-receiver differences. In Section 5.1 we address the impact of incorrectly applying the T_S assuming independent errors when a correlated structure exists. Section 5.2 gives the information required for the T_S test when the errors are independent, and Section 5.3 modifies this for correlated errors. Section 5.4 addresses issues raised in a recent technical journal. Extensions and further research are discussed in Section 5.5.

5.1 Correlated Error Structure

In Section 3 the max-min test was derived under general correlated error structure for the vector $\underline{D}$ of shipper receiver differences, i.e.

$$\underline{D} \sim N_n(\underline{\mu}, C) \quad .$$

If σ_{ij} represents the ijth element of C and assuming the error structure presented in Section 2.3.1, we recall that $\sigma_{ij} \geq 0$ for all ij as was pointed out in connection with equation (2.3.3).

The max-min test rejects the hypothesis of no diversion when

$$\frac{\sum_i D_i}{\left(\sum_{ij} \sigma_{ij}\right)^{\frac{1}{2}}} > z_{1-\alpha} \quad .$$

If instead we ignore the correlation aspect and falsely assume that $\sigma_{ij} = 0$ $i \neq j$ then we would reject when

$$\frac{\sum_i D_i}{\left(\sum_i \sigma_{ii}\right)^{\frac{1}{2}}} > z_{1-\alpha} \quad .$$

Since $\sigma_{ij} \geq 0$ for all ij implies

$$\frac{\sum_i D_i}{\left(\sum_i \sigma_{ii}\right)^{1/2}} \geq \frac{\sum D_i}{\left(\sum_{ij} \sigma_{ij}\right)^{1/2}}$$

we see that the false assumption of zero correlation would lead to a higher false alarm rate than indicated by α. How serious this increase in the false alarm rate is, depends directly on the relative discrepancy between $\sum_i \sigma_{ii}$ and $\sum_{ij} \sigma_{ij}$, i.e., on

$$RD = \frac{\sum_{ij} \sigma_{ij} - \sum_i \sigma_{ii}}{\sum_{ij} \sigma_{ij}}$$

If RD is reasonably close to 0 not much harm is done in assuming an uncorrelated error structure. If RD is significantly larger than 0 the false assumption of uncorrelated errors will lead to a false alarm rate considerably greater than that intended. This may diminish confidence in the safeguards process. It should be noted that unidentified positive correlations between container measurements within a single shipment will create an identical problem for a SRD test on a single item, giving an inflated false alarm rate.

5.2 Information Required, Independent Errors

The Satterthwaite test T_S may be directly implemented to test for special nuclear material diversion. The test procedure developed in Section 4 for independent errors is to reject the hypothesis of no material diversion or loss $H: \underline{\mu} = \underline{0}$ when

$$T_S = \frac{\sum D_i}{(\sum s_i^2)^{1/2}} > t_{f_S, \alpha}$$

Recall from 2.3 that D_i is the difference between the shipper and receiver reported values

$$D_i = S_i - R_i \quad .$$

The unknown variance σ_i^2 of D_i is the sum of the variance of S_i and the variance of R_i, since S_i and R_i are independent

$$\sigma_i^2 = \sigma_{S,i}^2 + \sigma_{R,i}^2$$

In 4.1 we assumed that independent estimates s_i^2 of the σ_i^2 are available. If there are n shipments, we now assume that there are n pairs of estimates, u_i^2 and v_i^2, with f_i and g_i degrees of freedom. Let

$$\left.\begin{array}{l} u_i^2 \text{ estimate } \sigma_{S,i}^2 \\ v_i^2 \text{ estimate } \sigma_{R,i}^2 \end{array}\right. \qquad \text{for } i=1, \ldots, n$$

These u_i^2 and v_i^2 are still assumed to be mutually independent. We assume that $f_i \; u_i^2/\sigma_{S,i}^2$ is a chi-squared variable with f_i degrees of freedom, with a similar result for v_i^2.

Thus the numerator of T_S is a sum over the n differences and the denominator involves a sum over 2n terms

$$T_S = \frac{\Sigma \, D_i}{(\Sigma(u_i^2 + v_i^2))^{1/2}}$$

In the analysis, the numerator has been treated as a univariate, with investigation focusing on behavior of the test over ranges of the σ_i^2, f_i and the number n of variance estimates. Thus our conclusions are still applicable.

In order to apply the test procedure for a set of n shipments, we need:

From the shipper

i) $S_1,\ldots,S_n$ - the shipper reported values for the amount shipped for the n shipments

ii) $u_1^2,\ldots,u_n^2$ - independent estimates of the variance of the n S_i such that $f_i u_i^2/\sigma_{S,i}^2$ is approximately a chi-squared variable with f_i degrees of freedom

iii) $f_1,\ldots,f_n$ - the degrees of freedom associated with the estimates u_i^2

From the receiver

i) $R_1,\ldots, R_n$ - the receiver reported values for the amount received for the n shipments

ii) $v_1^2, \ldots,v_n^2$ - independent estimates of the variance of the n R_i such that $g_i v_i^2/\sigma_{R,i}^2$ is a chi-squared variable with g_i degrees of freedom

iii) $g_1,\ldots,g_n$ - the degrees of freedom associated with the estimates v_i^2

5.3 Information Required, Correlated Errors

In Section 4 the max-min test for assumed known variances and uncorrelated error structure for the shipper receiver differences D_i was modified to allow for using estimates of the variances still retaining the uncorrelated error structure. It was shown that Satterthwaite's procedure gave the most satisfactory modification procedure of those that were examined.

Here we wish to point out that these results also have a direct bearing on the more general case when the correlated error structure cannot be ignored. Recall from Section 2.3 that the error covariance matrix C is of the following form:

$$C = U \operatorname{diag}(\omega_1^2, \ldots, \omega_L^2) U'$$

$$+ V \operatorname{diag}(\lambda_1^2, \ldots, \lambda_{L'}^2) V'$$

where U and V are certain known structural matrices. Let $\underline{U}'\underline{1} = \underline{a} = (a_1, \ldots, a_L)'$ and $\underline{V}'\underline{1} = \underline{b} = (b_1, \ldots, b_{L'})'$ then

$$\underline{1}' C \underline{1} = \underline{a}' \operatorname{diag}(\omega_1^2, \ldots, \omega_L^2) \underline{a}$$

$$+ \underline{b}' \operatorname{diag}(\lambda_1^2, \ldots, \lambda_{L'}^2) \underline{b}$$

$$= \sum_i a_i^2 \omega_i^2 + \sum_i b_i^2 \lambda_i^2$$

Thus if we have independent unbiased estimates w_i^2 and ℓ_i^2 of the elemental error variances (with known degrees of freedom) we may again estimate $\underline{1}' C \underline{1}$ by

$$\sum_i a_i^2 w_i^2 + \sum_i b_i^2 \ell_i^2 = \tau^2$$

and approximate τ^2 by an appropriate multiple of a chi-squared variable with appropriate degrees of freedom, i.e, approximate

$$\tau^2 \text{ by } k \cdot \chi^2_{f_s}$$

such that $$E(\tau^2) = k \cdot f_s$$

and $$\operatorname{var}(\tau^2) = k^2 \cdot 2 f_s \quad ,$$

which is nothing but Satterthwaite's procedure.

Assuming $w_i^2 \sim \omega_i^2 \; \chi^2_{f_i}/f_i$ and $\ell_i^2 \sim \lambda_i^2 \; \chi^2_{g_i}/g_i$.

we have

$$f_s = \frac{\left(\Sigma\, a_i^2 w_i^2 \; + \; \Sigma\, b_i^2 \ell_i^2\right)^2}{\Sigma a_i^4 \; w_i^4/f_i + \; \Sigma b_1^4 \ell_i^4/g_i}$$

after having replaced ω_i, λ_i by w_i and ℓ_i respectively.

The resulting modified max-min test rejects the hypothesis of no diversion when

$$\frac{\Sigma\, D_i}{\tau} > t_{f_s,\alpha} \quad .$$

In order to apply the test procedure for n shipments we need the information specified in 5.2 with w_i^2 and ℓ_i^2 replacing u_i^2 and v_i^2 and we must specify the error structure matrices U and V.

5.4 Rejoinder

As pointed out previously the max-min theorem was derived independently by Avenhaus and Jaech (1981) in a different context. Here we draw attention to the discussion by Littell and Downing (1982) of Goldman et al. (1982). There Littell and Downing, with reference to Avenhaus and Jaech (1981), propose two tests as alternatives to the max-min S-test. One is based on Fisher's method for combining P-values and the other is the likelihood ratio test. Both these tests show remarkably little power loss against the least favorable scenario, but show much power gain relative to the S-test against block loss. This gain against block loss is to be viewed, however, against the continuing practice of performing individual tests for each shipper-receiver difference regardless of whether a cumulative procedure is implemented or not.

In defense of the S-test it should be pointed out that a correlated error structure poses no problems and due to the central limit theorem effect on ΣD_i the false alarm rate as well as the power are reasonably robust to violations of the normality assumption for the error structure. Fisher's method depends crucially on the independence and the normality assumption.

The likelihood ratio test may be modified to account for correlations posing, however, some fairly non-trivial computational problems, cf. Barlow et al. (1972) Chapter 4, and it is not clear how it is affected by violations of the normality assumption.

5.5 Extensions and Further Research

So far the problem addressed and solved concerns shipper-receiver differences during a fixed window of n shipments. The size n of the window is a matter of choice. A long window allows good detection power against a low diversion rate over a larger period of time. A short window has the same detection power only against a more substantial diversion rate. However, the advantage of a short window is the possibility of a more timely response in case of an alarm. This suggests that the size n of the window should not be fixed in advance but should be influenced by the prevailing diversion rate. This points to a dynamic or sequential view of the problem and invites the use of methods in Sequential Analysis as developed by Abraham Wald. For example, among all tests with the same bounds on the false alarm rate and false no-alarm rate the sequential probability ratio test (SPRT) has on average the shortest window length, see Ghosh (1970). The SPRT deals with the problem of testing a simple hypothesis against a simple alternative and corresponds in spirit to the Neyman-Pearson test in the fixed window case.

One sequential approach to the SRD problem could exploit the above analogy by trying to parallel the fixed window solution path. One complication is that the power functions of SPRT's are not very tractable and one may need to use approximations instead.

Another sequential approach could be based on the fact that the max-min test for a fixed window of size n is based on the cumulative sum

$$C_n = \sum_{i=1}^{n} D_i.$$

Cumulative sum tests which monitor successive sums C_n, n=1, 2, ... are described in Van Dobben de Bruyn (1968). There the variances of the SRD's D_i are assumed to be homogeneous. Thus further work is needed to extend that methodology to the case of nonhomogeneous variances.

In any case it is advised to solve the problem as far as possible for the case of known variances just as was done in the case of the fixed window. The performance of any method wherein known variances are estimated by sample variances should then be assessed through computer simulations.

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APPENDIX A

Monte Carlo Simulation Program

INTRODUCTION

The Monte Carlo Simulation program is an implementation of the mathematical model described in Section 2.3. Shipper-receiver system parameters are input to the program. They are the following:

a) The number of shipments n
b) The true variance values $\sigma_1^2, \ldots, \sigma_n^2$
c) The degree of freedom values $f_1, \ldots, f_n$ for the variance estimates
d) The true material diversions $\mu_1, \ldots, \mu_n$ (versions of the program were also written with internally-supplied relative diversion λ)

The user also specifies N, the number of samples to generate (simulation replications) for estimating the desired probabilities. Input files are discussed in A.3.

The test statistic T_S is calculated for each simulated shipment scenario and the probability

$$P\ (T_S > t_{X,\alpha})$$

is estimated for each procedure (X = Satterthwaite, Banerjee, Cochran, or Sum) by performing N simulation replications. The details are presented in Section A.2.

Output files are described in Section A.4. Section A.5 contains source listings. Section A.6 serves as a User's Guide.

A.1 Program Description

An overall description is given first, then the Monte Carlo Simulation and the output statistics are described.

A list of the subprograms used is given in Figure A.1.

Module	Purpose
PROGRAM SRD	Main program. Contains the simulation loop and writes simulation results.
SUBROUTINE GETDAT	Reads shipment scenario from TAPE5.
FUNCTION CHISQR	Generates chi-squared random variate for even degrees of freedom.
FUNCTION FRACDF	Computes fractional degrees of freedom for Satterthwaite procedure.
SUBROUTINE COMPAR	Compares test statistic to critical values.
FUNCTION RANFN	Generates random variate from a normal distribution

Figure A.1 Subprograms in SRD Programs

A.1.1 Overview of Program Flow

The function of the SRD programs is to perform repeated Monte Carlo simulations (Figure A.2) for each input parameter case. The program reads the number N of simulation replications and the shipment scenario.

It then simulates N (measured) shipper-receiver difference scenarios. Each simulated SRD consists of n measured differences

$$D_i \sim N(\mu_i, \sigma_i^2)$$

and variance estimates

$$s_i^2 \sim \frac{\sigma_i^2}{f_i} \cdot \chi_{f_i}^2 \quad ,$$

where n = the number of shipments, $\underline{\mu}$ = true diversion, $\underline{\sigma}^2$ = measurement variances, and $\underline{f}$ = degrees of freedom for the variance estimates.

The test statistic is computed as

$$T_S = \Sigma D_i / (\Sigma s_i^2)^{1/2}$$

and then compared to the critical value $t_{X,\alpha}$ of each test procedure. The fraction of times T_S exceeds the critical value is then an unbiased estimate of the probability $P(T_S > t_{X,\alpha})$.

A.1.2 SRD Simulation

To evaluate the performance of the test statistic T_S each shipment scenario is repeatedly simulated (Figure A.3). The input to the simulation step consists of the control parameter (N = number of replications) and the scenario data (n = number of shipments, $\underline{\mu}$ = true diversion, $\underline{\sigma}^2$ = measurement variances, and $\underline{f}$ = degrees of freedom for the variance estimates).

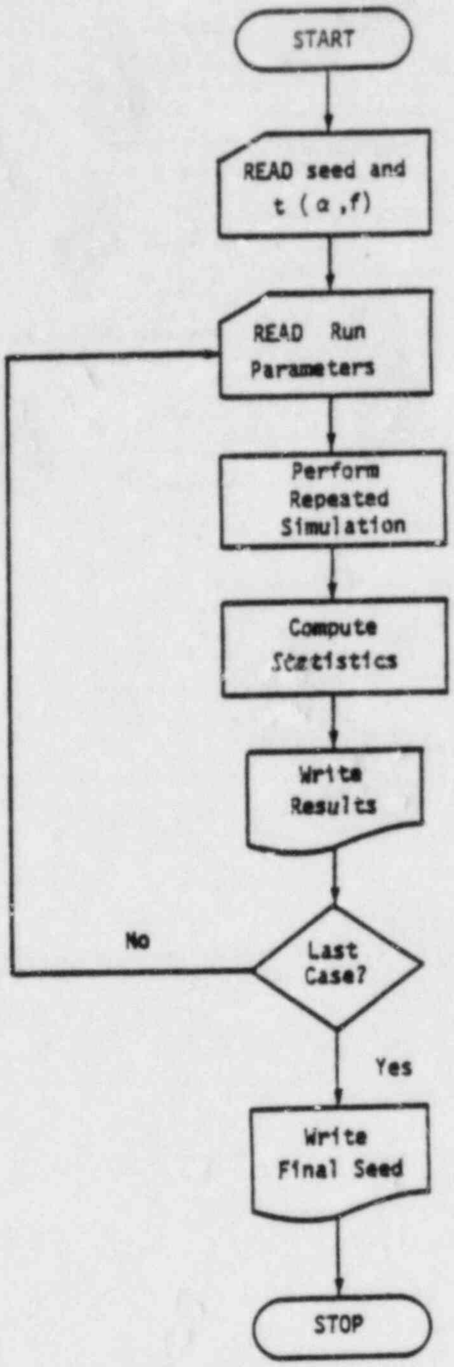

Figure A.2 Monte Carlo Simulation of SRD

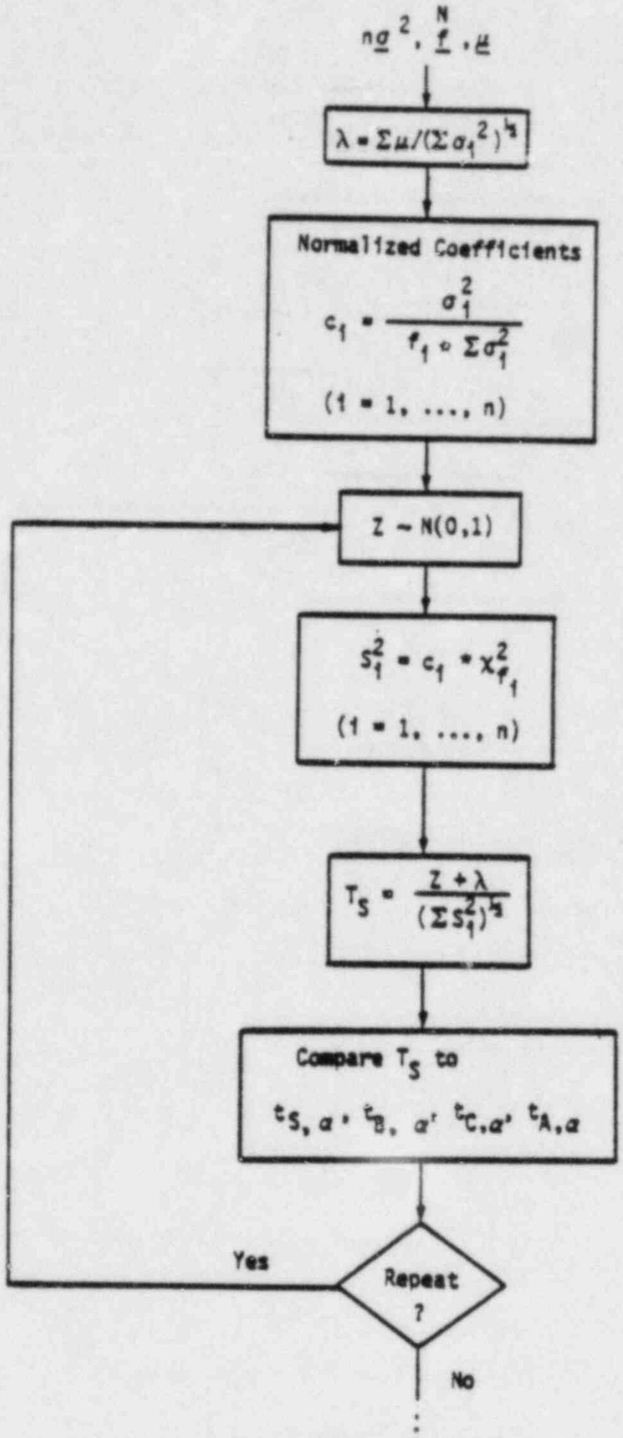

Figure A.3 Monte Carlo Simulation Loop

The program makes initial calculations for all repetitions and a loop of random perturbations of the test scenario comparing the test statistic T_S to the critical values $t_{\chi,\alpha}$.

The following representation of the test statistic is used for reasons of efficiency

$$T_S = \frac{\Sigma D_i}{(\Sigma s_i^2)^{1/2}} \qquad \text{Original definition}$$

$$= \frac{\Sigma N(\mu_i, \sigma_i^2)}{\left(\Sigma \frac{\sigma_i^2}{f_i} \cdot \chi^2_{f_i}\right)^{1/2}}$$

$$= \frac{\Sigma \mu_i + \Sigma N(0, \sigma_i^2)}{\left((\Sigma \sigma_i^2)(\Sigma c_i \, \chi^2_{f_i})\right)^{1/2}} \qquad \text{where} \quad c_i = \frac{\sigma_i^2}{f_i \cdot \Sigma \sigma_i^2} \quad \text{are normalized coefficients}$$

$$= \frac{\Sigma \mu_i / (\Sigma \sigma_i^2)^{1/2} + N(0,1)}{\left(\Sigma c_i \cdot {}^2_{f_i}\right)^{1/2}}$$

$$= \frac{\lambda + N(0,1)}{(\Sigma c_i \cdot \chi^2_{f_i})^{1/2}} \qquad \text{where} \qquad \lambda = \frac{\Sigma \mu_i}{\left(\Sigma \sigma_i^2\right)^{1/2}}$$

Using this equation, values of λ and $\underline{c} = (c_1, \ldots, c_n)$ are determined before entering the loop, and 1 normal variate and n chi-squared variates are generated for each replication.

The test statistic T_S is compared to the critical values (Subroutine COMPAR) which increments entries of the array NOVER(α-index, test index) if T_S exceeds the relevant critical value $t_{test,\alpha}$.

The output from the simulation step is the array NOVER of the number of times the test statistic of a simulation exceeded the various critical values.

A.1.3 Compare Test Statistic to Critical Values

Subroutine COMPAR is called each pass through the simulation loop. The test statistic for the simulated shipment scenario is given as input, with data necessary to calculate critical values for the test procedures. Output from COMPAR is the array NOVER of the number of times the test statistic exceeded the respective critical value (Figure A.4).

The critical values $t_{S,\alpha}$, $t_{B,\alpha}$, $t_{C,\alpha}$, $t_{A,\alpha}$ are defined in Section 4.2. The algorithms used to implement them are described in the following Section A.2.

A.2 Algorithms and Functions

A.2.1 Critical Values for Procedures

The critical values for each procedure are defined in Section 4.2. The algorithms used to implement them are described here.

Satterthwaite

The critical value for the Satterthwaite procedure is a Student's t variate with fractional degrees of freedom

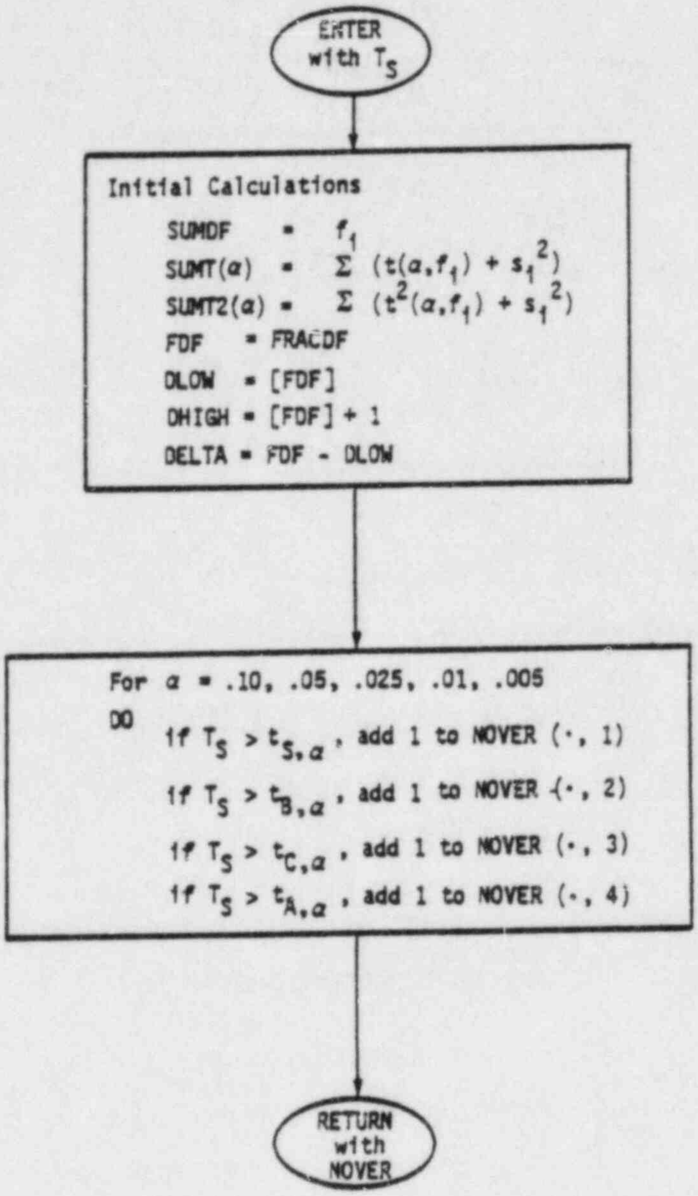

Figure A.4 Subroutine COMPAR

$$FDF = \frac{(\Sigma s_i^2)^2}{\Sigma(s_i^4/f_i)}$$

The value t(FDF,α) is approximated by inverse interpolation using neighboring integers

DLOW = INT(FDF)

DHIGH = DLOW + 1

and the array TCRIT(5,360) of input t-values. The approximation is as follows:

TLOW = TCRIT (IALPHA, DLOW)

THIGH = TCRIT (IALPHA, DHIGH)

TDELTA = THIGH + DLOW + (DHIGH/FDF - 1) · (TLOW-THIGH)

If FDF exceeds the largest input degrees of freedom 360, TDELTA = TCRIT(IALPHA, 360) is used.

Banerjee

The critical value for the Banerjee procedure is

$$t_{B,\alpha} = \left(\frac{\Sigma (t^2_{f_{i,\alpha}} \cdot s_i^2)}{\Sigma s_i^2} \right)^{\frac{1}{2}}$$

This is calculated as follows:

```
SUMT2 = 0
WHILE SHIPMENTS
DO
  SUMT2 = SUMT2 + TCRIT(IALPHA,DF(I))**2 · VAREST(I)
ENDDO
C2 = SQRT(SUMT2/TVARES)
```

Cochran

The critical value for the Cochran procedure is

$$t_{C,\alpha} = \frac{\Sigma\, t_{f_i,\alpha}\, s_i^2}{\Sigma\, s_i^2}$$

This is implemented as follows:

```
SUMT = 0
WHILE SHIPMENTS
DO
  SUMT = SUMT + TCRIT(IALPHA,DF(I)) * VAREST(I)
ENDDO
C3 = SUMT/TVARES
```

SUM

The critical value for the Sum procedure is

$$t_{f,\alpha}, \text{ where } f = \Sigma f_i$$

This is implemented as follows:

```
ISUMDF = 0
WHILE SHIPMENTS
DO
   ISUMDF = ISUMDF + DF(ISHIP)
ENDDO
ISUMDF = MIN(ISUMDF, 360)
C4 = TCRIT(IALPHA, ISUMDF)
```

A.2.2 <u>Function CHISQR</u> Given f_i
Yields $x^2_{f_i}$

Generates a $x^2_{f_i}$ random deviate with even degrees of freedom f_i using RANF to supply a U(0,1) variable,

Method: Step1 $x_j \sim U(0,1)$, $j=1, \ldots, \frac{f_i}{2}$

Step2 $P = \prod_{1}^{f_i/2} x_j$

Step3 $x^2_{f_i} = -2 * \ln P$

$= -2 * \Sigma \ln x_j$

This is implemented as follows:

```
          N = INT ( DEGFR / 2)
          PRODX = 1.0
      DO 100 I = 1, N
          PRODX = PRODX * ( RANF() )
100   CONTINUE
      CHISQR = -2.0 * ALOG ( PRODX )
```

A.2.3 Function FRACDF Given k, f_i (i=1,...,k),
s_i^2(i=1,...k), Σs_i^2
Yields FDF

Uses the Satterthwaite formula

$$FDF = (\Sigma s_i^2)^2 / (\Sigma \frac{s_i^4}{f_i})$$

Function FRACDF is entered with

NSHIP = n
DF(NSHIP) = $\underline{f}$
VAREST(NSHIP) = $\underline{s}^2$
TVARES = Σs_i^2

and returns FRACDF calculated by

```
      SUMD = 0.0
      DO 100 ISHIP = 1, NSHIP
         SUMD = SUMD + VAREST(ISHIP)*VAREST(ISHIP) / DF(ISHIP)
  100 CONTINUE
      FRACDF = TVARES * TVARES / SUMD
```

A.2.4 Random Number Generators

The subprograms RANF, RANGET and RANSET are vendor supplied. The source code for RANFN is given here and included in the program.

RANF, RANFN, RANGET, RANSET - Random Number Generators

PURPOSE — RANF and RANFN generate sequences of, respectively, uniformly and normally distributed random REAL numbers. RANGET and RANSET get and set the seed upon which the random number sequences depend.

METHOD

RANF uses the multiplicative congruential method to generate a sequence of uniformly distributed random numbers contained in the open interval (0,1). The sequence depends on two numbers referred to as the multiplier and the seed. The multiplier, which is fixed and equal to 1207264271730565 (octal), has been shown to have good statistical properties. RANF has a built-in initial seed or subroutine RANSET can be used to establish a seed. Each reference to function RANF yields the next number in the sequence and updates the seed. RANF is vendor supplied and is implemented with in-line code under FTN5.

RANFN uses RANF and the ratio of uniforms method, Kinderman and Monahan (1977), to generate a sequence of normally distributed random numbers with mean 0 and variance 1. Since RANFN uses RANF, the output from RANFN depends on the seed used by RANF and on whether RANF is used independently. Each reference to function RANFN yields the next number in the sequence.

Function RANF - Generate Uniformly Distributed Random Numbers

USAGE RU = RANF()

OUTPUT RU Next random number of the uniformly distributed sequence, $0 < RU < 1$.

Function RANFN - Generate Normally Distributed Random Numbers

USAGE RN = RANFN()

OUTPUT RN Next random number of the normally distributed sequence.

SOURCE CODE:

```
      REAL FUNCTION RANFN()
C    RANFN RETURNS A NORMAL (GAUSSIAN) RANDOM VARIABLE
C    WITH MEAN = 0 AND STANDARD DEVIATION = 1
C
C    THIS IS THE FTN5 VERSION.  THE UNDERLYING
C    UNIFORM GENERATOR IS RANF (VENDOR SUPPLIED).
C
C    REFERENCE - KINDERMAN AND MONAHAN, ACM TOMS
                 VOL. 3, NO. 3 (1977) PP. 257-260
C
C
   10 U = RANF()
      X = ( 1.7156*RANF() - .8578 )/U
      IF( X*X .GT. -4.*ALOG(U) )GO TO 10
      RANFN = X
      RETURN
      END
```

Subroutine RANGET - Get Current RANF Seed

USAGE	CALL RANGET (ISEED)	
OUTPUT	ISEED	Current seed which will be used by RANF to generate the next number in the uniform sequence. This value may be saved and later reset, using RANSET, to restart or continue a sequence.

Subroutine RANSET - Set RANF Seed

USAGE	CALL RANSET (ISEED)	
INPUT	ISEED	Quantity to become the new seed for RANF. ISEED should be either an odd integer in the range 1 to 2^{48}-1 or a value previously ob-

tained using RANGET. If ISEED is not odd, RANSET will force an odd value by adding 1. If ISEED is greater than $2^{48}-1$, the excess high order bits are ignored.

A.3 INPUT FILES

The input file of run parameters for SRD, the file with the seed for RANF and the file of critical values of the t-distribution are described.

A.3.1 Run Parameters with User-Specified Diversion

This file contains the simulation data for SRD including the number of simulation replications desired for statistical calculations and the shipment scenarios. Several cases may be input together by listing the run parameters sequentially.

The following format is required for each case:

Line	Data	Constraints	Format
1.	N = number of replications of simulation run	$0 < N < 100000$	I5
2.	n = number of shipments	$0 < n \leq 50$	I5
3. . . .	μ_i, i=1, ..., n = diversion of each shipment		10.F8.d continued on subsequent lines if $n > 10$
$4 + \left[\frac{n}{10}\right]$	σ_i^2, i=1, ..., n = variance of each shipment	$0 \leq \sigma_i^2$	same as above
$5 + \left[\frac{2n}{10}\right]$	f_i, i=1,...,n = degrees of freedom of each shipment	$2 \leq f_i \leq 360$	same as above

A.3.2 Seed for Random Deviate Generator RANF

This file contains 1 number on line 1. It is used by Subroutine RANSET to become the initial seed for RANF. It should be an odd integer in the range 1 to $2^{48}-1$ (if it is even, RANSET will add 1; if it is greater than $2^{48}-1$, the excess high order bits are ignored).

The format is as follows:

line 1	ISEED	2X,I18

See Section A.2.4 for discussion of Subroutines RANGET and RANSET.

A.3.3 Table of Critical Values for the t-distribution.

The values $t(\alpha,f)$ for one-tailed α values α = .10, .05, .025, .01, .005 and for degrees of freedom f = 1, 2, 3,..., 360 are included in this file.

The format expected for this file is

line		
1	$t(\alpha,f), \alpha=.10,.05,\ldots,.005$ f=1	5X,5F10.5
2	$t(\alpha,f), \alpha=.10,.05,\ldots,.005$ f=2	5X,5F10.5
.		.
.		.
.		.
360	$t(\alpha,f), \alpha=.10,.05,\ldots,.005$ f=360	5X,5F10.5

The values used were computed using BCSLIB Subroutine HSPIT.

A.4 Output Files

Program output consists of simulation results and the final seed for the random deviate generator RANF.

A.4.1 Simulation Results

For each input case 1 page of output is written. First is a header containing the run parameters. The estimated probability of the test statistic exceeding the critical value for each of the four procedures is listed next. For each procedure (Satterthwaite, Banerjee, Cochran, Sum) values are given for five values (.10, .05, .025, .01, .005). An example of the output is given in Figure A.7.

Also the initial seed for RANF is written on a first page and the final seed is written on a last page. The final seed is returned to the program through Subroutine RANGET. It may be used as the initial seed of a subsequent simulation run. Section A.2.4 discusses Subroutines RANGET and RANSET.

A.4.2 Seed for Random Deviate Generator RANF

The input file giving the initial seed (Section A.2.3) is rewound and the final seed is written in the same 2X, I18 format.

The file may be used to input the seed to a subsequent simulation run (Section A.3.3).

A.5 Source Listings

A copy of the source code for SRD program and the interactive input program are included. They all follow ANSI FORTRAN 77 standard (X3.9-1978) except for the PROGRAM card which is a CDC feature.

```
      PROGRAM SRDIN ( INPUT, OUTPUT, TAPE1, TAPE5=INPUT, TAPE6=OUTPUT )

C     PURPOSE: INTERACTIVE PROGRAM TO GET DATA FOR PROGRAM SRD
C     PUTS DATA ON TAPE1
      INTEGER NRUNS
      INTEGER NSHIP, ISHIP
      REAL    MEAN(50)
      REAL    VAR(50)
      CHARACTER*2 ANS,RESP
      INTEGER DF(50)
      INTEGER MAXDF

      ANS = 'NO'
      MAXDF = 360
      ICASE = 0

C READ NUMBER OF RUNS
10    CONTINUE
      PRINT *, ' --- CASE ', ICASE+1, ' ---'
      PRINT *, 'ENTER NUMBER OF SIMULATION RUNS DESIRED '
      READ(5,*,END=900) NRUNS
      IF ( NRUNS .LE. 0 ) THEN
          PRINT *, 'NUMBER OF RUNS MUST BE GREATER THAN 0'
          GO TO 10
      ENDIF

20    CONTINUE
      PRINT *, 'ENTER NUMBER OF SHIPMENTS - 1 TO 50'
      READ *, NSHIP
      IF ( NSHIP .LT. 1 .OR. NSHIP .GT. 50 ) THEN
          PRINT *, 'NUMBER OF SHIPMENTS ', NSHIP,
     1            ' MUST BE BETWEEN 1 AND 50'
          GO TO 20
      ENDIF

C READ SHIPMENT S-R DIFFERENCES
30    CONTINUE
      PRINT *, 'ENTER S-R DIFFERENCES OF ', NSHIP, ' SHIPMENTS'
      READ *, (MEAN(ISHIP), ISHIP=1,NSHIP )

C READ SHIPMENT VARIANCES
40    CONTINUE
      PRINT *, 'ENTER VARIANCES OF ', NSHIP, ' SHIPMENTS'
      READ *, ( VAR(ISHIP), ISHIP=1, NSHIP )
      DO 300 ISHIP = 1, NSHIP
         IF ( VAR(ISHIP) .LT. 0.0 ) THEN
            PRINT *, 'VARIANCE ', VAR(ISHIP), ' OF SHIPMENT ', ISHIP,
     1               ' MUST BE NON-NEGATIVE'
            GO TO 40
         ENDIF
300   CONTINUE

C READ DEGREES OF FREEDOM OF SHIPMENTS
50    CONTINUE
      PRINT *, 'ENTER DEGREES OF FREEDOM OF ', NSHIP, ' SHIPMENTS'
```

```
      READ *, ( DF(ISHIP), ISHIP=1,NSHIP )
      DO 400 ISHIP = 1, NSHIP
         IF ( DF(ISHIP).LT.2 .OR. DF(ISHIP).GT.MAXDF ) THEN
            PRINT *, 'DEG OF FREEDOM ', DF(ISHIP), ' OF SHIPMENT '
     1               ISHIP, ' MUST BE BETWEEN 2 AND ', MAXDF
            GO TO 50
         ENDIF
         IF ( DF(ISHIP) .NE. (DF(ISHIP)/2)*2 ) THEN
            PRINT *, 'DEG OF FREEDOM ', DF(ISHIP), ' OF SHIPMENT '
     1            ISHIP, ' MUST BE EVEN '
            GO TO 50
         ENDIF
400   CONTINUE


C WRITE TO TAPE1
      WRITE(1,1010) NRUNS
1010  FORMAT( I5 )

      WRITE(1,1010) NSHIP

      WRITE(1,1030) ( MEAN(ISHIP), ISHIP=1,NSHIP )
1030  FORMAT( 10 F8.4 )

      WRITE(1,1040) ( VAR(ISHIP), ISHIP=1,NSHIP )
1040  FORMAT( 10 F8.4 )

      WRITE(1,1050) ( DF(ISHIP), ISHIP=1,NSHIP )
1050  FORMAT( 10 I8 )
C  UPDATE CASE NUMBER AND CONTINUE INPUT
      ICASE = ICASE + 1
      PRINT *, 'DO YOU WISH TO CONTINUE INPUTTING DATA?'
      PRINT *, '         ( NO OR CR TO TERMINATE )'
      READ(5,1060,END=900)RESP
1060  FORMAT(A2)
      IF( RESP .EQ. ANS ) GO TO 900
      GO TO 10
900   CONTINUE
      PRINT *, ICASE,' PARAMETER CASES ENTERED'

      RETURN
      END
```

```
      PROGRAM SRD ( INPUT, OUTPUT, TAPE1, TAPE2,
     1                       TAPE5=INPUT, TAPE6=OUTPUT )

C INPUT
      INTEGER NRUNS, IRUN
      INTEGER NSHIP, ISHIP
      REAL    MEAN(50)
      REAL    VAR(50)
      INTEGER DF(50)
      REAL    TCRIT(5,360)
      INTEGER ISEED

C INTERMEDIATE CALCULATIONS
      REAL    TMEAN
      REAL    TVAR
      REAL    LAMBDA
      REAL    COEFF(50)
      REAL    Z
      REAL    VAREST(50)
      REAL    TVARES
      REAL    TESTST

C NUMBER OVER CRITICAL VALUE
C     5 ALPHA VALUES, 4 TESTS
      INTEGER NOVER(5,4)
      REAL    POVER(5,4)

      CHARACTER*18 TSTNAM(4)

C     5 ALPHA VALUES
      INTEGER NALPHA
      REAL    ALPHA(5)
      INTEGER MAXDF
      DATA ALPHA /.1, .05, .025, .01, .005 /
      DATA NALPHA /5/
      DATA MAXDF /360/

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C BEGIN EXECUTABLE CODE
      TSTNAM(1) = 'SATTERTHWAITE'
      TSTNAM(2) = 'BANERJEE'
      TSTNAM(3) = 'COCHRAN'
      TSTNAM(4) = 'SUM OF DEG FREEDOM'
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C READ SEED FOR RANF FROM TAPE2
      READ(2,2000) ISEED
2000  FORMAT( I20 )

      CALL RANSET ( ISEED )
      WRITE(6,6010) ISEED
6010  FORMAT ( 'OSEED FOR RANF = ', I 20 )

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
```

```
C READ CRITICAL VALUES OF T-DIST FROM TAPE1
      DO 20 IDF = 1, MAXDF
          READ(1,1020) (TCRIT( I, IDF ), I=1,NALPHA)
1020      FORMAT( 5X, 5 F10.5 )
20    CONTINUE

C READ INPUT DATA    GIVING MAXDF
      EOF = 0
1     CONTINUE
      CALL GETDAT ( NRUNS, NSHIP, MEAN, VAR, DF, MAXDF, EOF )
      IF( EOF .NE. 0 ) GO TO 900

      WRITE(6,6000)
6000  FORMAT(1H1, 'SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC' / )
          PRINT *, 'NUMBER OF RUNS = ', NRUNS
          PRINT *, 'NUMBER OF SHIPMENTS = ', NSHIP
          PRINT *, 'MEAN(.) = ', (MEAN(ISHIP), ISHIP=1,NSHIP)
          PRINT *, 'VARIANCES = ', (VAR(ISHIP), ISHIP=1,NSHIP)
          PRINT *, 'DF(.) = ', (DF(ISHIP), ISHIP=1,NSHIP)
          PRINT *, 'ALPHA(.) = ', (ALPHA(I),I=1,NALPHA)

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C NORMALIZE
          TMEAN = 0.
          TVAR = 0.
      DO 100 ISHIP = 1, NSHIP
          TMEAN = TMEAN + MEAN(ISHIP)
          TVAR  = TVAR  + VAR(ISHIP)
100   CONTINUE

      LAMBDA = TMEAN / SQRT(TVAR)
      DO 110 ISHIP = 1, NSHIP
          COEFF(ISHIP) = VAR(ISHIP) / ( DF(ISHIP)*TVAR )
110   CONTINUE

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C PERFORM NRUNS SIMULATIONS
C
C  INITIALIZE NOVER(..) = 0
      DO 120 I = 1, NALPHA
         DO 120 J = 1, 4
            NOVER(I,J) = 0
120   CONTINUE
      DO 200 IRUN = 1, NRUNS

C         GENERATE N(0,1)
          Z = RANFN()

C         VARIANCE ESTIMATE (VAR)*(CHISQR)/(DEG.FREEDOM)
C         NORMALIZED BY DIVIDING BY SUM(VARIANCES)
                TVARES = 0.
          DO 210 ISHIP = 1, NSHIP
```

```
            VAREST(ISHIP) = COEFF(ISHIP) * CHISQR(DF(ISHIP))
            TVARES = TVARES + VAREST(ISHIP)
210       CONTINUE

C         TEST STATISTIC
          TESTST = ( Z + LAMBDA ) / SQRT(TVARES)

          CALL COMPAR ( TESTST, NSHIP, DF, VAREST, TVARES,
     1                  TCRIT, NALPHA, MAXDF,
     2                  NOVER )

200   CONTINUE
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC

C CALCULATE RESULTS
      DO 300 I = 1, NALPHA
          DO 300 J = 1, 4
          POVER(I,J) = FLOAT(NOVER(I,J)) / FLOAT(NRUNS)
300   CONTINUE

      DO 400 ITEST = 1, 4
          WRITE(6,6400) ITEST, TSTNAM(ITEST)
6400      FORMAT('0PROCEDURE ',I2, 5X,A18 )
          WRITE(6,6410) (I, ALPHA(I), POVER(I,ITEST), I=1,NALPHA)
6410      FORMAT(1H , 10X, 'ALPHA', 10X, 'PROB. OF EXCEEDING'
     1             / , 11X, 'LEVEL', 10X, 'CRITICAL VALUE'
     2             / , ( 6X, I2, 2X, F6.4, 15X, F6.4 ) )
400   CONTINUE
      GO TO 1

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
900   CONTINUE

C WRITE FINAL ISEED FOR RANF ON TAPE2
      CALL RANGET ( ISEED )
          REWIND 2
      WRITE ( 2,2000 ) ISEED
      WRITE ( 6,6010 ) ISEED

      STOP
      END
```

```
      SUBROUTINE GETDAT ( NRUNS, NSHIP, MEAN, VAR, DF, MAXDF, EOF )

C PURPOSE. READS INPUT DATA FROM UNIT 5
C INPUT
      INTEGER MAXDF
C READS
      INTEGER NRUNS
      INTEGER NSHIP, ISHIP
      REAL    MEAN(50)
      REAL    VAR(50)
      INTEGER DF(50)

      INTEGER ERRFLG

      ERRFLG = 0
C READ NUMBER OF RUNS
      READ(5,5100,END=800) NRUNS
5100  FORMAT(I5)
      IF ( NRUNS .LT. 0 ) THEN
          PRINT *, 'NUMBER OF RUNS ', NRUNS, ' MUST BE GREATER THAN 0'
          ERRFLG = 1
      ENDIF

      READ(5,5200,END=900) NSHIP
5200  FORMAT(I5)
      IF ( NSHIP .LT. 1 .OR. NSHIP .GT. 50 ) THEN
          PRINT *, 'NUMBER OF SHIPMENTS ', NSHIP,
     1            'MUST BE BETWEEN 1 AND 50'
          ERRFLG = 1
      ENDIF

      READ(5,5300) ( MEAN(ISHIP), ISHIP=1, NSHIP )
5300  FORMAT(10 F8.0)

      READ(5,5300) ( VAR(ISHIP), ISHIP=1, NSHIP )
      DO 300 ISHIP = 1, NSHIP
          IF ( VAR(ISHIP) .LT. 0.0 ) THEN
             PRINT *, 'VARIANCE ', VAR(ISHIP), ' OF SHIPMENT ', ISHIP,
     1                 ' MUST BE NON-NEGATIVE'
             ERRFLG = 1
          ENDIF
300   CONTINUE

      READ(5,5400) ( DF(ISHIP), ISHIP=1,NSHIP )
5400  FORMAT(10I8)
      DO 400 ISHIP = 1, NSHIP
          IF ( DF(ISHIP).LT.2 .OR. DF(ISHIP).GT.MAXDF ) THEN
             PRINT *, 'DEG OF FREEDOM ', DF(ISHIP), ' OF SHIPMENT ',
     1                  ISHIP, ' MUST BE BETWEEN 2 AND ', MAXDF
             ERRFLG = 1
          ENDIF
          IF ( DF(ISHIP) .NE. (DF(ISHIP)/2)*2 ) THEN
             PRINT *, 'DEG OF FREEDOM ', DF(ISHIP), ' OF SHIPMENT ',
     1            ISHIP, ' MUST BE EVEN '
             ERRFLG = 1
```

```
          ENDIF
400    CONTINUE

       IF ( ERRFLG .EQ. 0 ) GO TO 910
C      IF ERRFLG = 1
          PRINT *, 'INPUT ERROR - STOPPING'
          STOP 'INPUT ERROR'
C END-OF-FILE ENCOUNTERED
800    CONTINUE
       WRITE(6,6800)
6800   FORMAT('1END OF FILE ON INPUT FILE - LAST CASE')
       EOF = 1
       GO TO 910
900    CONTINUE
          PRINT *, 'END-OF-FILE ON INPUT FILE - STOPPING'
          STOP 'INPUT ERROR'

910    CONTINUE
       RETURN
       END
```

```
      FUNCTION CHISQR ( DEGFR )

C GENERATE RANDOM VARIABLE FROM CHI SQUARED DISTRIBUTION
C     WITH EVEN DEGREES OF FREEDOM     DF = 2*N

C METHOD  X(.)                DISTRIBUTED U(0,1)
C         -LN(X(.))           DISTRIBUTED GAMMA(1,1)
C         -SUM(LN(X(.)))      DISTRIBUTED GAMMA(N,1)
C         -2*SUM(LN(X(.)))    DISTRIBUTED GAMMA(N,2)=CHISQR(DF=2*N)

C INPUT
      INTEGER DEGFR

      INTEGER N, I

      REAL    PRODX

          N = INT ( DEGFR / 2 )

          PRODX = 1.0
      DO 100 I = 1, N
          PRODX = PRODX * ( RANF() )
100   CONTINUE

      CHISQR = -2.0 * ALOG ( PRODX )

      RETURN
      END
```

```
      FUNCTION FRACDF ( NSHIP, DF, VAREST, TVARES )

C INPUT
      INTEGER NSHIP, ISHIP
      INTEGER DF(NSHIP)
      REAL    VAREST(NSHIP)
      REAL    TVARES

C INTERMEDIATE VALUES
      REAL    SUMD

C OUTPUT
      REAL FRACDF

      SUMD = 0.0
      DO 100 ISHIP = 1, NSHIP
         SUMD = SUMD + VAREST(ISHIP)*VAREST(ISHIP) / DF(ISHIP)
100   CONTINUE

      FRACDF = TVARES * TVARES / SUMD

      RETURN
      END
```

```
      SUBROUTINE COMPAR ( TESTST, NSHIP, DF, VAREST, TVARES,
     1                    TCRIT, NALPHA, MAXDF,
     2                    NOVER )

C COMPARE TEST STATISTIC TO T VALUE
C    4 PROCEDURES
C      1. SATTERTHWAITE
C      2. BANERJEE
C      3. COCHRAN
C      4. TOTAL DEGREES OF FREEDOM = SUM( DF(.) )

C INPUT
      REAL    TESTST
      INTEGER NSHIP
      INTEGER DF(NSHIP)
      REAL    VAREST(NSHIP)
      REAL    TVARES
      REAL    TCRIT(NALPHA,MAXDF)
      INTEGER NALPHA
      INTEGER MAXDF

C INTERMEDIATE VALUES
      REAL    FDF
      INTEGER DLOW, DHIGH
      REAL    DELTA
      REAL    TLOW, THIGH
      REAL    TDELTA
      REAL    SUMT(5)
      REAL    SUMT2(5)
      REAL    SUMV
      REAL    SUMDF
      REAL    C2, C3, C4

C OUTPUT
      INTEGER NOVER(NALPHA,4)

      INTEGER I

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C INITIAL CALCULATIONS FOR ALL 4
      SUMDF = 0.
      DO 10 IALPHA = 1, NALPHA
           SUMT(IALPHA) = 0.
           SUMT2(IALPHA) = 0.
10    CONTINUE
      DO 20 ISHIP = 1, NSHIP
           SUMDF = SUMDF + DF(ISHIP)
           DO 30 IALPHA = 1, NALPHA

                SUMT(IALPHA) = SUMT(IALPHA) +
     1               TCRIT(IALPHA,DF(ISHIP)) * VAREST(ISHIP)
                SUMT2(IALPHA) = SUMT2(IALPHA) +
     1               TCRIT(IALPHA,DF(ISHIP))**2 * VAREST(ISHIP)
30         CONTINUE
20    CONTINUE
```

```
      IF ( SUMDF .GT. MAXDF ) SUMDF = MAXDF

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C FOR 1. SATTERTHWAITE
      FDF = FRACDF ( NSHIP, DF, VAREST, TVARES )
      DLOW = INT ( FDF )
      DHIGH = DLOW + 1
      DELTA = FDF - DLOW

CCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCCC
C     4 PROCEDURES FOR EACH ALPHA VALUE
C
      DO 100 I = 1, NALPHA

C         1. SATTERTHWAITE
          IF ( DLOW .LT. MAXDF ) THEN
             TLOW = TCRIT( I, DLOW )
             THIGH = TCRIT( I, DHIGH )
             TDELTA = THIGH + DLOW * ( (DHIGH/FDF) - 1 )
     1                                * ( TLOW - THIGH )
          ELSE
             TDELTA = TCRIT ( I, MAXDF )
          ENDIF
          IF ( TESTST .GT. TDELTA ) NOVER( I, 1 ) = NOVER( I, 1 ) + 1

C         2. BANERJEE
            C2 = SQRT( SUMT2(I) / TVARES )
            IF ( TESTST .GT. C2 ) NOVER( I, 2 ) = NOVER( I, 2 ) + 1

C         3. COCHRAN
            C3 = SUMT(I) / TVARES
            IF ( TESTST .GT. C3 ) NOVER( I, 3 ) = NOVER(I,3) + 1

C         4. TOTAL DF
            C4 = TCRIT( I, SUMDF )
            IF ( TESTST .GT. C4 ) NOVER( I, 4 ) = NOVER(I,4) + 1

100   CONTINUE

      RETURN
      END
```

```
      REAL FUNCTION RANFN()
C RANFN RETURNS A NORMAL (GAUSSIAN) RANDOM VARIABLE
C WITH MEAN = 0 AND STANDARD DEVIATION = 1
C
C THIS IS THE FTN5 VERSION.  THE UNDERLYING
C UNIFORM GENERATOR IS RANF (VENDOR SUPPLIED).
C
C REFERENCE - KINDERMAN AND MONAHAN, ACM TOMS
C             VOL.3, NO.3 (1977) PP.257-260
C
   10 U = RANF()
      X = ( 1.7156*RANF() - .8578 )/U
      IF( X*X .GT. -4.*ALOG(U) )GO TO 10
      RANFN = X
      RETURN
      END
```

A.6 User's Guide

The shipper-receiver difference (SRD) program utilizes three files for input. These files' names and associated contents are: TAPE1 - table of critical values for the t-distribution, TAPE2 - seed for the random deviate generator, TAPE5 - run parameters. The first two files are generated once and used for subsequent executions of the SRD program. The last file is built by the user through the interactive input program (SRDIN).

Building the run parameter file through SRDIN and subsequent execution of the SRD program is accomplished by using the job control procedure file RUN which should be resident on the users account in the direct access file PROCFIL. This procedure performs three functions which are:

1) Output description header

2) Execute program SRDIN

3) Build file SRDRUN which contains the job control language necessary to execute the SRD program for the user specified input.

(See Figure A.6 for a copy of procedure RUN.) This procedure is executed at the interactive level via the command BEGIN,RUN. Following is an example of a terminal session during which the procedure RUN is used to build the procedure file SRDRUN which contains the necessary job control language and run parameters input to execute via batch the SRD program for two run parameter cases. User input is preceeded by either N > or I > .

In the following example a user is interested in evaluating a specified scenario of five shipments. A false alarm rate of 5%, alpha = .05, is to be evaluated. For Case 1 the error variances for the five shipper-receiver differences are .80, .40, .40, .40 and .40. The estimates of these variances have 4, 10, 10, 10 and 10 degrees of freedom. The analyst wants to know what is the probability that the shipper-receiver test T_S

will correctly identify a material loss of 3.0 units? The user must input five S-R differences which total 3.0. The size and order do not affect the test since T_S is a CUSUM test. Thus in Case 1, the problem has been specified with all the material diverted from shipment one.

Usually there will be a shipper error and an independent receiver error which make up the error for D_i. There will also be separate independent estimates for the shipper and receiver error variances (see 5.2), each with an associated degrees of freedom. In Case 2 the user evaluates the same scenario as Case 1 except the variances for D_i are divided equally into a shipper and a receiver term. Note the user must specify ten shipments in order to input the ten variance terms.

EXAMPLE:

```
N>BEGIN,RUN
 12.27.24.PROCEDURE RUN ON FILE PROCFIL.

      PROGRAM TO ANALYZE POWER OF CUSUM SHIPPER-RECEIVER TEST FOR
                      SPECIFIED SCENARIO

        ENTER RUN PARAMETERS FOR SIMULATION (NUMBER OF SIMULATION
        RUNS, NUMBER OF SHIPMENTS, SHIPPER-RECEIVER DIFFERENCES,
        VARIANCES AND DEGREES OF FREEDOM)
        ENTER PARAMETER VALUES SEPARATED BY BLANKS OR COMMAS
        REPEATED VALUES MAY BE ENTERED AS INTEGER*VALUE

 --- CASE 1 ---
 ENTER NUMBER OF SIMULATION RUNS DESIRED
I>10000
 ENTER NUMBER OF SHIPMENTS - 1 TO 50
I>5
 ENTER S-R DIFFERENCES OF 5 SHIPMENTS                 (0 ≤ Difference ≤ 999.999,
I>3.0  4*0        <------------   Difference less than .0001 truncated to 0)
 ENTER VARIANCES OF 5 SHIPMENTS
I> .8  4*.4       <-----------------------     (.0001 ≤ Variance ≤ 999.9999)
 ENTER DEGREES OF FREEDOM OF 5 SHIPMENTS
I>4  4*10        <---------------------   (2 ≤ DOF ≤ 99,999,999, DOF even only)
 DO YOU WISH TO CONTINUE INPUTTING DATA?
        ( NO OR CR TO TERMINATE )
I>YES
  --- CASE 2 ---
 ENTER NUMBER OF SIMULATION RUNS DESIRED
I>10000
 ENTER NUMBER OF SHIPMENTS - 1 TO 50
I>10
 ENTER S-R DIFFERENCES OF 10 SHIPMENTS
I>3.0  9*0
 ENTER VARIANCES OF 10 SHIPMENTS
I>.2*.4  8*.2
 ENTER DEGREES OF FREEDOM OF 10 SHIPMENTS
I>2*4  8*10
 DO YOU WISH TO CONTINUE INPUTTING DATA?
        ( NO OR CR TO TERMINATE )
I>NO
 2 PARAMETER CASES ENTERED
 12.28.59.  TO SUBMIT RUN ENTER    ROUTE,SRDRUN,DC=TN,UN=XXXX
```

Figure A.5 Input Example

```
.PROC,RUN,PRI=P06,OUT=SRDOUT.
TYPE.PROCEDURE RUN ON FILE PROCFIL.
RETURN, TAPE1.
IF(.NOT.FILE(SRDIN1B.LO))GET,SRDIN1B.   <------------   (SRDIN BINARY FILE)
COPY,INDAT,OUTPUT.
SRDIN1B,INPUT,OUTPUT,TAPE1.
REWIND,TAPE1.
PACK,TAPE1.
.*
REWIND,SRDRUN.
COPY,COM,SRDRUN.
COPY,TAPE1,SRDRUN.
TYPE.  TO SUBMIT RUN ENTER    ROUTE,SRDRUN,DC=IN,UN=XXXXXX.
.DATA,INDAT.

       PROGRAM TO ANALYZE POWER OF CUSUM SHIPPER-RECEIVER TEST FOR
                     SPECIFIED SCENARIO

         ENTER RUN PARAMETERS FOR SIMULATION  (NUMBER OF SIMULATION
         RUNS, NUMBER OF SHIPMENTS, SHIPPER-RECEIVER DIFFERENCES,
         VARIANCES AND DEGREES OF FREEDOM)
         ENTER PARAMETER VALUES SEPARATED BY BLANKS OR COMMAS
         REPEATED VALUES MAY BE ENTERED AS INTEGER*VALUE

.EOR
.DATA,COM.
SRDRUN,CM77000,T120,PRI.
USER,ACCOUNT,PASSWORD.
*
*SHIPPER-RECEIVER DIFFERENCE
*
GET,SRD1B.   <------------------------------------   (SRD BINARY FILE)
GET,TVALDAT. CRITICAL VALUES OF T DIST.
*            5 ALPHA VALUES, 360 DF
GET,SRDSEED. SEED FOR FUNCTION RANF
*
MAP=OFF.
SRD1B,INPUT,OUT,TVALDATA,SRDSEED.
*
EXIT,U.
REPLACE,OUT.
COPY,OUT,OUTPUT.
REPLACE,SRDSEED.  NEW SEED
*
EXIT,U.
DAYFILE,SRDDAY.
REPLACE,SRDDAY.
```

Figure A.6 Procedure Run

Following is a listing of the file SRDRUN built through the use of the procedure RUN for the example illustrated previously.

```
SRDRUN,CM77000, T120,P06.
USER,ACCOUNT,PASSWORD.
*
*SHIPPER-RECEIVER DIFFERENCE
*
GET,SRD1B.
GET,TVALDAT. CRITICAL VALUES OF T DIST.
*            5 ALPHA VALUES, 360 DF
GET,SRDSEED. SEED FOR FUNCTION RANF
*
MAP=OFF.
SRD1B,INPUT,SRDOUT,TVALDAT,SRDSEED.
*
EXIT,U.
REPLACE,SRDOUT.
COPY,SRDOUT,OUTPUT.
REPLACE,SRDSEED. NEW SEED
*
EXIT,U.
DAYFILE,SRDDAY.
REPLACE,SRDDAY.
 10000
     5
  3.0000 0.0000 0.0000 0.0000 0.0000
   .8000  .4000  .4000  .4000  .4000
       4     10     10     10     10
10000
   10
   3.0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000 .0000
    .4000 .4000 .2000 .2000 .2000 .2000 .2000 .2000 .2000 .2000
        4     4    10    10    10    10    10    10    10    10
```

To execute the SRD program for the user specified run parameters route the file SRDRUN to the batch input queue. See Figure A.7 for a list of the output generated by the SRD program for this example.

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 10000
NUMBER OF SHIPMENTS = 5
MEAN(.) = 3. 0. 0. 0. 0.
VARIANCES = .8 .4 .4 .4 .4
DF(.) = 4 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.7377
2	.0500	.5876
3	.0250	.4569
4	.0100	.3076
5	.0050	.2211

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.7034
2	.0500	.5149
3	.0250	.3366
4	.0100	.1597
5	.0050	.0783

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.7043
2	.0500	.5163
3	.0250	.3390
4	.0100	.1649
5	.0050	.0848

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.7451
2	.0500	.6039
3	.0250	.4788
4	.0100	.3336
5	.0050	.2473

Figure A.7 SRD Output

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 10000
NUMBER OF SHIPMENTS = 10
MEAN(.) = 3. .0 .0 .0 .0 .0 .0 .0 .0 .0
VARIANCES = .4 .4 .2 .2 .2 .2 .2 .2 .2 .2
DF(.) = 4 4 10 10 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.7444
2	.0500	.6043
3	.0250	.4769
4	.0100	.3265
5	.0050	.2403

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.6987
2	.0500	.5172
3	.0250	.3279
4	.0100	.1371
5	.0050	.0588

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.6989
2	.0500	.5193
3	.0250	.3313
4	.0100	.1426
5	.0050	.0654

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.7478
2	.0500	.6125
3	.0250	.4904
4	.0100	.3403
5	.0050	.2590

Figure A.7 SRD Output (Continued)

APPENDIX B. MONTE CARLO OUTPUT

The test T_S is used to test for material loss when variances are estimated. Four procedures are described in Section 4.2 for determining the critical value for T_S. These procedures are compared for the parameter sets given in Section 4.3.2. The Monte Carlo program described in Section 4.3.1 and Appendix A generated the following output in support of the analysis in Section 4.3.

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .1000
      2    .0500                   .0480
      3    .0250                   .0225
      4    .0100                   .0082
      5    .0050                   .0038

PROCEDURE  2     BANERJEE
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .0704
      2    .0500                   .0214
      3    .0250                   .0051
      4    .0100                   .0005
      5    .0050                   .0001

PROCEDURE  3     COCHRAN
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .0704
      2    .0500                   .0214
      3    .0250                   .0051
      4    .0100                   .0005
      5    .0050                   .0001

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .1022
      2    .0500                   .0503
      3    .0250                   .0247
      4    .0100                   .0096
      5     0050                   .0049
```

Figure B.1 Comparison of α Values, Equal Variances, $f_i = 4$, $n = 6$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 1. 1. 1.
DF(.) = 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1002
2	.0500	.0492
3	.0250	.0248
4	.0100	.0101
5	.0050	.0050

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0872
2	.0500	.0371
3	.0250	.0151
4	.0100	.0040
5	.0050	.0016

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0872
2	.0500	.0371
3	.0250	.0151
4	.0100	.0040
5	.0050	.0016

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1006
2	.0500	.0496
3	.0250	.0251
4	.0100	.0105
5	.0050	.0051

Figure B.2 Comparison of α Values, Equal Variances, $f_i = 10$, $n = 6$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 1. 1. 1.
DF(.) = 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1003
2	.0500	.0503
3	.0250	.0250
4	.0100	.0099
5	.0050	.0044

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0943
2	.0500	.0442
3	.0250	.0195
4	.0100	.0059
5	.0050	.0022

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0943
2	.0500	.0442
3	.0250	.0195
4	.0100	.0059
5	.0050	.0022

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1005
2	.0500	.0504
3	.0250	.0251
4	.0100	.0099
5	.0050	.0045

Figure B.3 Comparison of α Values, Equal Variances, $f_i = 20$, $n = 6$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                 .0988
      2    .0500                 .0481
      3    .0250                 .0240
      4    .0100                 .0091
      5    .0050                 .0043

PROCEDURE  2     BANERJEE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                 .0656
      2    .0500                 .0192
      3    .0250                 .0039
      4    .0100                 .0001
      5    .0050                 .0000

PROCEDURE  3     COCHRAN
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                 .0656
      2    .0500                 .0192
      3    .0250                 .0039
      4    .0100                 .0001
      5    .0050                 .0000

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                 .0999
      2    .0500                 .0498
      3    .0250                 .0253
      4    .0100                 .0101
      5    .0050                 .0050
```

Figure B.4 Comparison of α Values, Equal Variances, $f_i = 4$, $n = 12$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 10 10 10 10 10 10 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1    SATTERTHWAITE
           ALPHA          PROB. OF EXCEEDING
           LEVEL          CRITICAL VALUE
        1  .1000                .0988
        2  .0500                .0495
        3  .0250                .0247
        4  .0100                .0097
        5  .0050                .0048

PROCEDURE  2    BANERJEE
           ALPHA          PROB. OF EXCEEDING
           LEVEL          CRITICAL VALUE
        1  .1000                .0847
        2  .0500                .0367
        3  .0250                .0136
        4  .0100                .0032
        5  .0050                .0010

PROCEDURE  3    COCHRAN
           ALPHA          PROB. OF EXCEEDING
           LEVEL          CRITICAL VALUE
        1  .1000                .0847
        2  .0500                .0367
        3  .0250                .0136
        4  .0100                .0032
        5  .0050                .0010

PROCEDURE  4    SUM OF DEG.FREEDOM
           ALPHA          PROB. OF EXCEEDING
           LEVEL          CRITICAL VALUE
        1  .1000                .0990
        2  .0500                .0497
        3  .0250                .0249
        4  .0100                .0098
        5  .0050                .0050
```

Figure B.5 Comparison of α Values, Equal Variances, $f_i = 10$, $n = 12$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 20 20 20 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .0998
     2  .0500                 .0505
     3  .0250                 .0253
     4  .0100                 .0104
     5  .0050                 .0051

PROCEDURE  2     BANERJEE
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .0933
     2  .0500                 .0432
     3  .0250                 .0192
     4  .0100                 .0064
     5  .0050                 .0023

PROCEDURE  3     COCHRAN
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .0933
     2  .0500                 .0432
     3  .0250                 .0192
     4  .0100                 .0064
     5  .0050                 .0023

PROCEDURE  4     SUM OF DEG.FREEDOM
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .0998
     2  .0500                 .0506
     3  .0250                 .0255
     4  .0100                 .0104
     5  .0050                 .0051
```

Figure B.6 Comparison of α Values, Equal Variances, f_i = 20, n =12

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0987
2	.0500	.0491
3	.0250	.0234
4	.0100	.0095
5	.0050	.0044

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0644
2	.0500	.0177
3	.0250	.0033
4	.0100	.0002
5	.0050	0.0000

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0644
2	.0500	.0177
3	.0250	.0033
4	.0100	.0002
5	.0050	0.0000

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0996
2	.0500	.0504
3	.0250	.0246
4	.0100	.0099
5	.0050	.0048

Figure B.7 Comparison of α Values, Equal Variances, $f_i = 4$, $n = 18$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0991
2	.0500	.0503
3	.0250	.0258
4	.0100	.0103
5	.0050	.0051

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0856
2	.0500	.0357
3	.0250	.0143
4	.0100	.0031
5	.0050	.0008

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0856
2	.0500	.0357
3	.0250	.0143
4	.0100	.0031
5	.0050	.0008

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0993
2	.0500	.0504
3	.0250	.0260
4	.0100	.0104
5	.0050	.0052

Figure B.8 Comparison of α Values, Equal Variances, f_i = 10, n = 18

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0993
2	.0500	.0483
3	.0250	.0246
4	.0100	.0095
5	.0050	.0053

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0929
2	.0500	.0415
3	.0250	.0184
4	.0100	.0060
5	.0050	.0022

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0929
2	.0500	.0415
3	.0250	.0184
4	.0100	.0060
5	.0050	.0022

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0994
2	.0500	.0484
3	.0250	.0246
4	.0100	.0095
5	.0050	.0053

Figure B.9 Comparison of α Values, Equal Variances, f_i = 20, n =18

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1009
2	.0500	.0520
3	.0250	.0246
4	.0100	.0097
5	.0050	.0045

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0831
2	.0500	.0323
3	.0250	.0103
4	.0100	.0016
5	.0050	.0004

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0835
2	.0500	.0333
3	.0250	.0108
4	.0100	.0018
5	.0050	.0005

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1034
2	.0500	.0545
3	.0250	.0268
4	.0100	.0111
5	.0050	.0056

Figure B.10 Comparison of α Values, Equal Variances, $f_i = 4,20$, $n = 6$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 10 10 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .1006
      2    .0500                   .0505
      3    .0250                   .0253
      4    .0100                   .0103
      5    .0050                   .0052

PROCEDURE  2     BANERJEE
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .0833
      2    .0500                   .0344
      3    .0250                   .0124
      4    .0100                   .0023
      5    .0050                   .0009

PROCEDURE  3     COCHRAN
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .0836
      2    .0500                   .0350
      3    .0250                   .0129
      4    .0100                   .0027
      5    .0050                   .0009

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
      1    .1000                   .1023
      2    .0500                   .0522
      3    .0250                   .0269
      4    .0100                   .0116
      5    .0050                   .0057
```

Figure B.11 Comparison of α Values, Equal Variances, f_i = 4,10,20, n = 6

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1      SATTERTHWAITE
         ALPHA            PROB. OF EXCEEDING
         LEVEL            CRITICAL VALUE
      1  .1000                  .0991
      2  .0500                  .0509
      3  .0250                  .0256
      4  .0100                  .0102
      5  .0050                  .0052

PROCEDURE  2      BANERJEE
         ALPHA            PROB. OF EXCEEDING
         LEVEL            CRITICAL VALUE
      1  .1000                  .0786
      2  .0500                  .0300
      3  .0250                  .0089
      4  .0100                  .0014
      5  .0050                  .0003

PROCEDURE  3      COCHRAN
         ALPHA            PROB. OF EXCEEDING
         LEVEL            CRITICAL VALUE
      1  .1000                  .0791
      2  .0500                  .0307
      3  .0250                  .0096
      4  .0100                  .0016
      5  .0050                  .0003

PROCEDURE  4      SUM OF DEG.FREEDOM
         ALPHA            PROB. OF EXCEEDING
         LEVEL            CRITICAL VALUE
      1  .1000                  .1005
      2  .0500                  .0522
      3  .0250                  .0269
      4  .0100                  .0109
      5  .0050                  .0059
```

Figure B.12 Comparison of α Values, Equal Variances, f_i = 4,20, n = 12

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 10 10 10 10 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1019
2	.0500	.0510
3	.0250	.0249
4	.0100	.0092
5	.0050	.0050

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0849
2	.0500	.0314
3	.0250	.0089
4	.0100	.0016
5	.0050	.0004

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0852
2	.0500	.0319
3	.0250	.0101
4	.0100	.0018
5	.0050	.0005

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1031
2	.0500	.0523
3	.0250	.0259
4	.0100	.0099
5	.0050	.0053

Figure B.13 Comparison of α Values, Equal Variances, f_i = 4,10,20, n = 12

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4 4 4 4 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
         ALPHA           PROB. OF EXCEEDING
         LEVEL           CRITICAL VALUE
      1  .1000                 .1008
      2  .0500                 .0488
      3  .0250                 .0241
      4  .0100                 .0094
      5  .0050                 .0048

PROCEDURE  2     BANERJEE
         ALPHA           PROB. OF EXCEEDING
         LEVEL           CRITICAL VALUE
      1  .1000                 .0776
      2  .0500                 .0271
      3  .0250                 .0077
      4  .0100                 .0010
      5  .0050                 .0002

PROCEDURE  3     COCHRAN
         ALPHA           PROB. OF EXCEEDING
         LEVEL           CRITICAL VALUE
      1  .1000                 .0780
      2  .0500                 .0278
      3  .0250                 .0081
      4  .0100                 .0012
      5  .0050                 .0003

PROCEDURE  4     SUM OF DEG.FREEDOM
         ALPHA           PROB. OF EXCEEDING
         LEVEL           CRITICAL VALUE
      1  .1000                 .1018
      2  .0500                 .0498
      3  .0250                 .0248
      4  .0100                 .0101
      5  .0050                 .0054
```

Figure B.14 Comparison of α Values, Equal Variances, $f_i = 4,20$, $n = 18$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4. 4.
DF(.) = 4 4 4 4 4 4 10 10 10 10 10 10 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1    SATTERTHWAITE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                  .0987
       2   .0500                  .0495
       3   .0250                  .0246
       4   .0100                  .0090
       5   .0050                  .0043

PROCEDURE  2    BANERJEE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                  .0795
       2   .0500                  .0296
       3   .0250                  .0089
       4   .0100                  .0013
       5   .0050                  .0003

PROCEDURE  3    COCHRAN
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                  .0799
       2   .0500                  .0302
       3   .0250                  .0092
       4   .0100                  .0016
       5   .0050                  .0003

PROCEDURE  4    SUM OF DEG.FREEDOM
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                  .0995
       2   .0500                  .0502
       3   .0250                  .0252
       4   .0100                  .0096
       5   .0050                  .0045
```

Figure B.15 Comparison of α Values, Equal Variances, f_i = 4,10,20, n = 18

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 3. 3. 3.
DF(.) = 4 4 4 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0959
2	.0500	.0465
3	.0250	.0236
4	.0100	.0092
5	.0050	.0045

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0833
2	.0500	.0339
3	.0250	.0123
4	.0100	.0028
5	.0050	.0008

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0836
2	.0500	.0344
3	.0250	.0127
4	.0100	.0032
5	.0050	.0011

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0965
2	.0500	.0470
3	.0250	.0244
4	.0100	.0095
5	.0050	.0046

Figure B.16 Comparison of α Values, Variance Ratio 1 to 3, f_i = 4,20, n = 6

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 3. 3. 3. 1. 1. 1.
DF(.) = 4 4 4 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                   .0981
        2  .0500                   .0486
        3  .0250                   .0233
        4  .0100                   .0096
        5  .0050                   .0049

PROCEDURE  2     BANERJEE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                   .0769
        2  .0500                   .0279
        3  .0250                   .0091
        4  .0100                   .0014
        5  .0050                   .0003

PROCEDURE  3     COCHRAN
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                   .0771
        2  .0500                   .0285
        3  .0250                   .0094
        4  .0100                   .0017
        5  .0050                   .0003

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                   .1049
        2  .0500                   .0552
        3  .0250                   .0292
        4  .0100                   .0134
        5  .0050                   .0074
```

Figure B.17 Comparison of α Values, Variance Ratio 3 to 1, f_i = 4,20, n = 6

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3.
DF(.) = 4 4 4 4 4 4 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0991
2	.0500	.0504
3	.0250	.0263
4	.0100	.0105
5	.0050	.0051

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0848
2	.0500	.0362
3	.0250	.0129
4	.0100	.0026
5	.0050	.0006

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0851
2	.0500	.0369
3	.0250	.0135
4	.0100	.0030
5	.0050	.0008

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0994
2	.0500	.0506
3	.0250	.0267
4	.0100	.0106
5	.0050	.0052

Figure B.18 Comparison of α Values, Variance Ratio 1 to 3, f_i = 4,20, n = 12

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 3. 3. 3. 3. 3. 3. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0992
2	.0500	.0500
3	.0250	.0240
4	.0100	.0092
5	.0050	.0042

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0730
2	.0500	.0246
3	.0250	.0062
4	.0100	.0010
5	.0050	.0001

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0735
2	.0500	.0251
3	.0250	.0064
4	.0100	.0011
5	.0050	.0002

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1033
2	.0500	.0540
3	.0250	.0274
4	.0100	.0115
5	.0050	.0058

Figure B.19 Comparison of α Values, Variance Ratio 3 to 1, f_i = 4,20, n = 12

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3. 3. 3. 3.
DF(.) = 4 4 4 4 4 4 4 4 4 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .1005
     2  .0500                 .0511
     3  .0250                 .0259
     4  .0100                 .0100
     5  .0050                 .0050

PROCEDURE  2     BANERJEE
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .0856
     2  .0500                 .0355
     3  .0250                 .0122
     4  .0100                 .0026
     5  .0050                 .0006

PROCEDURE  3     COCHRAN
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .0861
     2  .0500                 .0360
     3  .0250                 .0131
     4  .0100                 .0028
     5  .0050                 .0008

PROCEDURE  4     SUM OF DEG.FREEDOM
        ALPHA            PROB. OF EXCEEDING
        LEVEL            CRITICAL VALUE
     1  .1000                 .1010
     2  .0500                 .0511
     3  .0250                 .0261
     4  .0100                 .0102
     5  .0050                 .0051
```

Figure B.20 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 4,20$, $n = 18$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 3. 3. 3. 3. 3. 3. 3. 3. 3. 1. 1. 1. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4 4 4 4 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .0990
      2  .0500                   .0471
      3  .0250                   .0226
      4  .0100                   .0087
      5  .0050                   .0044

PROCEDURE  2     BANERJEE
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .0706
      2  .0500                   .0219
      3  .0250                   .0053
      4  .0100                   .0006
      5  .0050                   .0000

PROCEDURE  3     COCHRAN
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .0712
      2  .0500                   .0223
      3  .0250                   .0056
      4  .0100                   .0007
      5  .0050                   .0001

PROCEDURE  4     SUM OF DEG.FREEDOM
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .1012
      2  .0500                   .0500
      3  .0250                   .0246
      4  .0100                   .0103
      5  .0050                   .0054
```

Figure B.21 Comparison of α Values, Variance Ratio 3 to 1, f_i = 4,20, n = 18

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 3. 3. 3.
DF(.) = 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0994
2	.0500	.0486
3	.0250	.0234
4	.0100	.0095
5	.0050	.0048

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0723
2	.0500	.0231
3	.0250	.0064
4	.0100	.0008
5	.0050	.0001

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0723
2	.0500	.0231
3	.0250	.0064
4	.0100	.0008
5	.0050	.0001

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1029
2	.0500	.0523
3	.0250	.0269
4	.0100	.0115
5	.0050	.0062

Figure B.22 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 4$, $n = 6$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 3. 3. 3.
DF(.) = 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                 .0998
       2   .0500                 .0500
       3   .0250                 .0261
       4   .0100                 .0108
       5   .0050                 .0051

PROCEDURE  2     BANERJEE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                 .0878
       2   .0500                 .0393
       3   .0250                 .0166
       4   .0100                 .0041
       5   .0050                 .0014

PROCEDURE  3     COCHRAN
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                 .0878
       2   .0500                 .0393
       3   .0250                 .0166
       4   .0100                 .0041
       5   .0050                 .0014

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
       1   .1000                 .1009
       2   .0500                 .0509
       3   .0250                 .0272
       4   .0100                 .0115
       5   .0050                 .0057
```

Figure B.23 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 10$, $n = 6$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 1. 3. 3. 3.
DF(.) = 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1018
2	.0500	.0501
3	.0250	.0236
4	.0100	.0093
5	.0050	.0047

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0962
2	.0500	.0433
3	.0250	.0193
4	.0100	.0066
5	.0050	.0027

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0962
2	.0500	.0433
3	.0250	.0193
4	.0100	.0066
5	.0050	.0027

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1022
2	.0500	.0507
3	.0250	.0239
4	.0100	.0096
5	.0050	.0049

Figure B.24 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 20$, $n = 6$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3.
DF(.) = 4 4 4 4 4 4 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0981
2	.0500	.0489
3	.0250	.0242
4	.0100	.0091
5	.0050	.0048

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0671
2	.0500	.0202
3	.0250	.0046
4	.0100	.0002
5	.0050	0.0000

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0671
2	.0500	.0202
3	.0250	.0046
4	.0100	.0002
5	.0050	0.0000

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1006
2	.0500	.0513
3	.0250	.0261
4	.0100	.0104
5	.0050	.0057

Figure B.25 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 4$, $n = 12$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3.
DF(.) = 10 10 10 10 10 10 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1      SATTERTHWAITE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                  .0990
        2  .0500                  .0502
        3  .0250                  .0252
        4  .0100                  .0101
        5  .0050                  .0050

PROCEDURE  2      BANERJEE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                  .0865
        2  .0500                  .0370
        3  .0250                  .0142
        4  .0100                  .0036
        5  .0050                  .0010

PROCEDURE  3      COCHRAN
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                  .0865
        2  .0500                  .0370
        3  .0250                  .0142
        4  .0100                  .0036
        5  .0050                  .0010

PROCEDURE  4      SUM OF DEG.FREEDOM
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
        1  .1000                  .0996
        2  .0500                  .0508
        3  .0250                  .0258
        4  .0100                  .0104
        5  .0050                  .0051
```

Figure B.26 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 10$, $n = 12$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3.
DF(.) = 20 20 20 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
       1   .1000                 .0990
       2   .0500                 .0503
       3   .0250                 .0251
       4   .0100                 .0102
       5   .0050                 .0051

PROCEDURE  2     BANERJEE
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
       1   .1000                 .0928
       2   .0500                 .0430
       3   .0250                 .0189
       4   .0100                 .0064
       5   .0050                 .0025

PROCEDURE  3     COCHRAN
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
       1   .1000                 .0928
       2   .0500                 .0430
       3   .0250                 .0189
       4   .0100                 .0064
       5   .0050                 .0025

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
       1   .1000                 .0993
       2   .0500                 .0504
       3   .0250                 .0253
       4   .0100                 .0102
       5   .0050                 .0051
```

Figure B.27 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 20$, $n = 12$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3. 3. 3. 3.
DF(.) = 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                   .0980
      2    .0500                   .0475
      3    .0250                   .0229
      4    .0100                   .0089
      5    .0050                   .0044

PROCEDURE  2     BANERJEE
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                   .0641
      2    .0500                   .0179
      3    .0250                   .0038
      4    .0100                   .0003
      5    .0050                   .0000

PROCEDURE  3     COCHRAN
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                   .0641
      2    .0500                   .0179
      3    .0250                   .0038
      4    .0100                   .0003
      5    .0050                   .0000

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA            PROB. OF EXCEEDING
           LEVEL            CRITICAL VALUE
      1    .1000                   .0995
      2    .0500                   .0493
      3    .0250                   .0245
      4    .0100                   .0097
      5    .0050                   .0049
```

Figure B.28 Comparison of α Values, Variance Ratio 1 to 3, $f_i = 4$, $n = 18$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3. 3. 3. 3.
DF(.) = 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1    SATTERTHWAITE
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .1055
      2  .0500                   .0514
      3  .0250                   .0258
      4  .0100                   .0105
      5  .0050                   .0049

PROCEDURE  2    BANERJEE
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .0908
      2  .0500                   .0377
      3  .0250                   .0146
      4  .0100                   .0032
      5  .0050                   .0010

PROCEDURE  3    COCHRAN
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .0908
      2  .0500                   .0377
      3  .0250                   .0146
      4  .0100                   .0032
      5  .0050                   .0010

PROCEDURE  4    SUM OF DEG.FREEDOM
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
      1  .1000                   .1057
      2  .0500                   .0518
      3  .0250                   .0264
      4  .0100                   .0108
      5  .0050                   .0051
```

Figure B.29 Comparison of α Values, Variance Ratio 1 to 3, $f_1 = 10$, $n = 18$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 1. 1. 1. 3. 3. 3. 3. 3. 3. 3. 3. 3.
DF(.) = 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
      1  .1000                    .0963
      2  .0500                    .0482
      3  .0250                    .0240
      4  .0100                    .0097
      5  .0050                    .0053

PROCEDURE  2     BANERJEE
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
      1  .1000                    .0899
      2  .0500                    .0410
      3  .0250                    .0176
      4  .0100                    .0063
      5  .0050                    .0027

PROCEDURE  3     COCHRAN
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
      1  .1000                    .0899
      2  .0500                    .0410
      3  .0250                    .0176
      4  .0100                    .0063
      5  .0050                    .0027

PROCEDURE  4     SUM OF DEG.FREEDOM
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
      1  .1000                    .0965
      2  .0500                    .0483
      3  .0250                    .0241
      4  .0100                    .0097
      5  .0050                    .0054
```

Figure B.30 Comparison of α Values, Variance Ratio 1 to 3, $f_1 = 20$, $n = 18$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 2. 2. 3. 3.
DF(.) = 4 4 10 10 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0999
2	.0500	.0501
3	.0250	.0253
4	.0100	.0099
5	.0050	.0048

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0871
2	.0500	.0373
3	.0250	.0135
4	.0100	.0034
5	.0050	.0009

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0873
2	.0500	.0377
3	.0250	.0140
4	.0100	.0036
5	.0050	.0011

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1006
2	.0500	.0506
3	.0250	.0258
4	.0100	.0101
5	.0050	.0050

Figure B.31 Comparison of α Values, Variance Ratio 1 to 2 to 3, f_i = 4,10,20, n = 6

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 2. 2. 2. 2. 3. 3. 3. 3.
DF(.) = 4 4 4 4 10 10 10 10 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0987
2	.0500	.0500
3	.0250	.0246
4	.0100	.0099
5	.0050	.0047

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0849
2	.0500	.0344
3	.0250	.0129
4	.0100	.0026
5	.0050	.0008

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0852
2	.0500	.0348
3	.0250	.0133
4	.0100	.0029
5	.0050	.0008

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0990
2	.0500	.0502
3	.0250	.0248
4	.0100	.0100
5	.0050	.0049

Figure B.32 Comparison of α Values, Variance Ratio 1 to 2 to 3, f_i = 4,10,20, n = 12

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 2. 2. 2. 2. 2. 2. 3. 3. 3. 3. 3. 3.
DF(.) = 4 4 4 4 4 4 10 10 10 10 10 10 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1011
2	.0500	.0483
3	.0250	.0249
4	.0100	.0098
5	.0050	.0050

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0868
2	.0500	.0337
3	.0250	.0122
4	.0100	.0025
5	.0050	.0006

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0872
2	.0500	.0341
3	.0250	.0127
4	.0100	.0027
5	.0050	.0007

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1012
2	.0500	.0486
3	.0250	.0251
4	.0100	.0099
5	.0050	.0051

Figure B.33 Comparison of α Values, Variance Ratio 1 to 2 to 3, f_i = 4,10,20, n = 18

SHIPPER - RECEIVER DIFFERENCE TEST STATIST--

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 3. 3. 2. 2. 1. 1.
DF(.) = 4 4 10 10 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1008
2	.0500	.0507
3	.0250	.0255
4	.0100	.0096
5	.0050	.0046

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0821
2	.0500	.0329
3	.0250	.0103
4	.0100	.0018
5	.0050	.0004

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0825
2	.0500	.0332
3	.0250	.0107
4	.0100	.0021
5	.0050	.0004

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1060
2	.0500	.0549
3	.0250	.0294
4	.0100	.0124
5	.0050	.0064

Figure B.34 Comparison of α Values, Variance Ratio 3 to 2 to 1, f_i = 4,10,20, n = 6

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 3. 3. 3. 3. 2. 2. 2. 2. 1. 1. 1. 1.
DF( ) = 4 4 4 4 10 10 10 10 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0991
2	.0500	.0490
3	.0250	.0241
4	.0100	.0096
5	.0050	.0048

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0760
2	.0500	.0275
3	.0250	.0083
4	.0100	.0012
5	.0050	.0002

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0763
2	.0500	.0281
3	.0250	.0087
4	.0100	.0015
5	.0050	.0002

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1017
2	.0500	.0512
3	.0250	.0261
4	.0100	.0114
5	.0050	.0059

Figure B.35 Comparison of α Values, Variance Ratio 3 to 2 to 1, f_i = 4,10,20, n = 12

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 3. 3. 3. 3. 3. 3. 2. 2. 2. 2. 2. 2. 1. 1. 1. 1. 1. 1.
DF(.) = 4 4 4 4 4 4 10 10 10 10 10 10 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0968
2	.0500	.0485
3	.0250	.0236
4	.0100	.0092
5	.0050	.0050

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0750
2	.0500	.0258
3	.0250	.0072
4	.0100	.0012
5	.0050	.0003

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0754
2	.0500	.0262
3	.0250	.0076
4	.0100	.0013
5	.0050	.0003

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0985
2	.0500	.0500
3	.0250	.0249
4	.0100	.0102
5	.0050	.0057

Figure B.36 Comparison of α Values, Variance Ratio 3 to 2 to 1, f_i = 4,10,20, n = 18

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 2. 2. 3. 3.
DF(.) = 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0990
2	.0500	.0482
3	.0250	.0236
4	.0100	.0086
5	.0050	.0040

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0713
2	.0500	.0228
3	.0250	.0056
4	.0100	.0005
5	.0050	.0001

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0713
2	.0500	.0228
3	.0250	.0056
4	.0100	.0005
5	.0050	.0001

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1028
2	.0500	.0514
3	.0250	.0260
4	.0100	.0105
5	.0050	.0054

Figure B.37 Comparison of α Values, Variance Ratio 1 to 2 to 3, $f_1 = 4$, $n = 6$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 2. 2. 2. 2. 3. 3. 3. 3.
DF(.) = 4 4 4 4 4 4 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1014
2	.0500	.0503
3	.0250	.0253
4	.0100	.0094
5	.0050	.0046

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0689
2	.0500	.0214
3	.0250	.0042
4	.0100	.0002
5	.0050	.0000

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0689
2	.0500	.0214
3	.0250	.0042
4	.0100	.0002
5	.0050	.0000

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1033
2	.0500	.0523
3	.0250	.0270
4	.0100	.0112
5	.0050	.0055

Figure B.38 Comparison of α Values, Variance Ratio 1 to 2 to 3, $f_1 = 10$, $n = 6$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 2. 2. 3. 3.
DF(.) = 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1018
2	.0500	.0511
3	.0250	.0262
4	.0100	.0109
5	.0050	.0055

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0896
2	.0500	.0390
3	.0250	.0166
4	.0100	.0043
5	.0050	.0014

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0896
2	.0500	.0390
3	.0250	.0166
4	.0100	.0043
5	.0050	.0014

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1029
2	.0500	.0519
3	.0250	.0269
4	.0100	.0114
5	.0050	.0058

Figure B.39 Comparison of α Values, Variance Ratio 1 to 2 to 3, f_i = 20, n = 6

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 6
VARIANCES = 1. 1. 2. 2. 3. 3.
DF(.) = 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0968
2	.0500	.0482
3	.0250	.0235
4	.0100	.0091
5	.0050	.0048

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0912
2	.0500	.0419
3	.0250	.0183
4	.0100	.0058
5	.0050	.0027

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0912
2	.0500	.0419
3	.0250	.0183
4	.0100	.0058
5	.0050	.0027

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0970
2	.0500	.0484
3	.0250	.0237
4	.0100	.0092
5	.0050	.0048

Figure B.40 Comparison of α Values, Variance Ratio 1 to 2 to 3, $f_i = 4$, $n = 12$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 2. 2. 2. 2. 3. 3. 3. 3.
DF(.) = 10 10 10 10 10 10 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1000
2	.0500	.0508
3	.0250	.0250
4	.0100	.0097
5	.0050	.0051

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0872
2	.0500	.0378
3	.0250	.0138
4	.0100	.0035
5	.0050	.0009

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0872
2	.0500	.0378
3	.0250	.0138
4	.0100	.0035
5	.0050	.0009

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1004
2	.0500	.0514
3	.0250	.0255
4	.0100	.0099
5	.0050	.0053

Figure B.41 Comparison of α Values, Variance Ratio 1 to 2 to 3, $f_i = 10$, $n = 12$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 12
VARIANCES = 1. 1. 1. 1. 2. 2. 2. 2. 3. 3. 3. 3.
DF(.) = 20 20 20 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE  1     SATTERTHWAITE
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
     1   .1000                   .0999
     2   .0500                   .0510
     3   .0250                   .0248
     4   .0100                   .0097
     5   .0050                   .0050

PROCEDURE  2     BANERJEE
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
     1   .1000                   .0933
     2   .0500                   .0431
     3   .0250                   .0185
     4   .0100                   .0059
     5   .0050                   .0026

PROCEDURE  3     COCHRAN
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
     1   .1000                   .0933
     2   .0500                   .0431
     3   .0250                   .0185
     4   .0100                   .0059
     5   .0050                   .0026

PROCEDURE  4     SUM OF DEG.FREEDOM
         ALPHA             PROB. OF EXCEEDING
         LEVEL             CRITICAL VALUE
     1   .1000                   .1001
     2   .0500                   .0512
     3   .0250                   .0250
     4   .0100                   .0098
     5   .0050                   .0051
```

Figure B.42 Comparison of α Values, Variance Ratio 1 to 2 to 3, f_i = 20, n = 12

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 2 2. 2. 2. 2. 2. 3. 3. 3. 3. 3. 3.
DF(.) = 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4 4
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0989
2	.0500	.0493
3	.0250	.0247
4	.0100	.0099
5	.0050	.0045

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0652
2	.0500	.0192
3	.0250	.0038
4	.0100	.0003
5	.0050	.0000

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0652
2	.0500	.0192
3	.0250	.0038
4	.0100	.0003
5	.0050	.0000

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1002
2	.0500	.0508
3	.0250	.0261
4	.0100	.0,09
5	.0050	.0051

Figure B.43 Comparison of α Values, Variance Ratio 1 to 2 to 3, $f_i = 4$, $n = 18$

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 2. 2. 2. 2. 2. 2. 3. 3. 3. 3. 3. 3.
DF(.) = 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10 10
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = 0.

PROCEDURE 1 SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0994
2	.0500	.0487
3	.0250	.0229
4	.0100	.0090
5	.0050	.0045

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0855
2	.0500	.0345
3	.0250	.0126
4	.0100	.0032
5	.0050	.0008

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0855
2	.0500	.0345
3	.0250	.0126
4	.0100	.0032
5	.0050	.0008

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0997
2	.0500	.0491
3	.0250	.0232
4	.0100	.0092
5	.0050	.0046

Figure B.44 Comparison of α Values, Variance Ratio 1 to 2 to 3, $f_i = 10$, $n = 18$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 18
VARIANCES = 1. 1. 1. 1. 1. 1. 2. 2. 2. 2. 2. 2. 3. 3. 3. 3. 3. 3.
DF(.) = 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20 20
ALPHA(.) = .1 .05 .025 .01 .005
TOTAL DIVERSION = LMBD = C.

PROCEDURE  1    SATTERTHWAITE
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
      1   .1000                  .0996
      2   .0500                  .0495
      3   .0250                  .0241
      4   .0100                  .0102
      5   .0050                  .0055

PROCEDURE  2    BANERJEE
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
      1   .1000                  .0921
      2   .0500                  .0421
      3   .0250                  .0183
      4   .0100                  .0062
      5   .0050                  .0025

PROCEDURE  3    COCHRAN
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
      1   .1000                  .0921
      2   .0500                  .0421
      3   .0250                  .0183
      4   .0100                  .0062
      5   .0050                  .0025

PROCEDURE  4    SUM OF DEG.FREEDOM
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
      1   .1000                  .0998
      2   .0500                  .0497
      3   .0250                  .0242
      4   .0100                  .0103
      5   .0050                  .0055
```

Figure B.45 Comparison of α Values, Variance Ratio 1 to 2 to 3, $f_1 = 20$, $n = 18$

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 1. 9.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    274560804800025909

PROCEDURE       1SATTERTHWAITE
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .1014
      2    .0500                     .0507
      3    .0250                     .0244
      4    .0100                     .0100
      5    .0050                     .005       1

PROCEDURE  2      BANERJEE
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .0959
      2    .0500                     .0447
      3    .0250                     .019       1
      4    .0100                     .0062
      5    .0050                     .0025

PROCEDURE  3      COCHRAN
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .0960
      2    .0500                     .0448
      3    .0250                     .0193
      4    .0100                     .0064
      5    .0050                     .0025

PROCEDURE  4      SUM OF DEG.FREEDOM
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .1042
      2    .0500                     .0538
      3    .0250                     .027       1
      4    .0100                     .0113
      5    .0050                     .0059

SEED FOR RANF =    274627395713817509
*WEOR
```

Figure B.46 Comparison of α Values, $n = 2$, $f_1 = 4$, $f_2 = 8$, Variance Ratio 1 to 9

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 1. 4.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    274560804800025909

PROCEDURE         1SATTERTHWAITE
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .1009
      2    .0500                     .0503
      3    .0250                     .0240
      4    .0100                     .0096
      5    .0050                     .0045

PROCEDURE  2      BANERJEE
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .0922
      2    .0500                     .041      1
      3    .0250                     .016      1
      4    .0100                     .0044
      5    .0050                     .0015

PROCEDURE  3      COCHRAN
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .0923
      2    .0500                     .0413
      3    .0250                     .0164
      4    .0100                     .0046
      5    .0050                     .0016

PROCEDURE  4      SUM OF DEG.FREEDOM
           ALPHA              PROB. OF EXCEEDING
           LEVEL              CRITICAL VALUE
      1    .1000                     .1027
      2    .0500                     .0523
      3    .0250                     .0253
      4    .0100                     .0107
      5    .0050                     .0053

SEED FOR RANF =    274627395713817509
*WEOR
```

Figure B.47 Comparison of α Values, n = 2, f_1 = 4, f_2 = 8, Variance Ratio 1 to 4

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 3. 7.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    2746273957138l7509

PROCEDURE        1SATTERTHWAITE
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
       1   .1000                   .0978
       2   .0500                   .0478
       3   .0250                   .0239
       4   .0100                   .0088
       5   .0050                   .0043

PROCEDURE   2     BANERJEE
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
       1   .1000                   .0873
       2   .0500                   .0384
       3   .0250                   .0148
       4   .0100                   .0038
       5   .0050                   .0014

PROCEDURE   3     COCHRAN
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
       1   .1000                   .0874
       2   .0500                   .0387
       3   .0250                   .0150
       4   .0100                   .0039
       5   .0050                   .0014

PROCEDURE   4     SUM OF DEG FREEDOM
           ALPHA             PROB. OF EXCEEDING
           LEVEL             CRITICAL VALUE
       1   .1000                   .0993
       2   .0500                   .0497
       3   .0250                   .0253
       4   .0100                   .0096
       5   .0050                   .0048

SEED FOR RANF =    274563478625296445
*WEOR
```

Figure B.48 Comparison of α Values, n = 2, f_1 = 4, f_2 = 8, Variance Ratio 3 to 7

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 2. 3.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    274563478625296445

PROCEDURE       1SATTERTHWAITE
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
       1  .1000                    .1002
       2  .0500                    .0492
       3  .0250                    .0242
       4  .0100                    .0095
       5  .0050                    .0046

PROCEDURE  2    BANERJEE
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
       1  .1000                    .0877
       2  .0500                    .0382
       3  .0250                    .0152
       4  .0100                    .0038
       5  .0050                    .0014

PROCEDURE  3    COCHRAN
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
       1  .1000                    .0878
       2  .0500                    .0385
       3  .0250                    .0155
       4  .0100                    .0039
       5  .0050                    .0015

PROCEDURE  4    SUM OF DEG.FREEDOM
          ALPHA             PROB. OF EXCEEDING
          LEVEL             CRITICAL VALUE
       1  .1000                    .1023
       2  .0500                    .0513
       3  .0250                    .0259
       4  .0100                    .0107
       5  .0050                    .005       1

SEED FOR RANF =    274680419755855045
*WEOR
```

Figure B.49 Comparison of α Values, n = 2, f_1 = 4, f_2 = 8, Variance Ratio 2 to 3

SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 1. 1.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF = 274680419755855045

PROCEDURE 1SATTERTHWAITE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0982
2	.0500	.0470
3	.0250	.0222
4	.0100	.0092
5	.0050	.0045

PROCEDURE 2 BANERJEE

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0861
2	.0500	.0348
3	.0250	.0134
4	.0100	.0038
5	.0050	.0014

PROCEDURE 3 COCHRAN

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.0862
2	.0500	.0350
3	.0250	.0137
4	.0100	.0040
5	.0050	.0015

PROCEDURE 4 SUM OF DEG.FREEDOM

	ALPHA LEVEL	PROB. OF EXCEEDING CRITICAL VALUE
1	.1000	.1012
2	.0500	.0503
3	.0250	.0240
4	.0100	.0105
5	.0050	.0052

SEED FOR RANF = 27448427269760866 1
*WEOR

Figure B.50 Comparison of α Values, $n = 2$, $f_1 = 4$, $f_2 = 8$, Variance Ratio 1 to 1

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 3. 2.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    274680419755855045

PROCEDURE       1SATTERTHWAITE
          ALPHA          PROB. OF EXCEEDING
          LEVEL          CRITICAL VALUE
      1   .1000                .0983
      2   .0500                .0475
      3   .0250                .0227
      4   .0100                .0096
      5   .0050                .0047

PROCEDURE  2    BANERJEE
          ALPHA          PROB. OF EXCEEDING
          LEVEL          CRITICAL VALUE
      1   .1000                .0862
      2   .0500                .036      1
      3   .0250                .0139
      4   .0100                .0039
      5   .0050                .0014

PROCEDURE  3    COCHRAN
          ALPHA          PROB. OF EXCEEDING
          LEVEL          CRITICAL VALUE
      1   .1000                .0863
      2   .0500                .0363
      3   .0250                .0142
      4   .0100                .004      1
      5   .0050                .0015

PROCEDURE  4    SUM OF DEG.FREEDOM
          ALPHA          PROB. OF EXCEEDING
          LEVEL          CRITICAL VALUE
      1   .1000                .1031
      2   .0500                .0525
      3   .0250                .0256
      4   .0100                .011      1
      5   .0050                .0060

SEED FOR RANF =    27448427269760866        1
*WEOR
```

Figure B.51 Comparison of α Values, n = 2, f_1 = 4, f_2 = 8, Variance Ratio 3 to 2

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 7. 3.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    27448427269760866      1

PROCEDURE        1SATTERTHWAITE
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
     1   .1000                    .0978
     2   .0500                    .0488
     3   .0250                    .0233
     4   .0100                    .0096
     5   .0050                    .0050

PROCEDURE   2    BANERJEE
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
     1   .1000                    .0864
     2   .0500                    .0372
     3   .0250                    .0145
     4   .0100                    .0044
     5   .0050                    .0018

PROCEDURE   3    COCHRAN
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
     1   .1000                    .0868
     2   .0500                    .0375
     3   .0250                    .0147
     4   .0100                    .0045
     5   .0050                    .0019

PROCEDURE   4    SUM OF DEG.FREEDOM
         ALPHA              PROB. OF EXCEEDING
         LEVEL              CRITICAL VALUE
     1   .1000                    .1041
     2   .0500                    .0546
     3   .0250                    .0282
     4   .0100                    .0120
     5   .0050                    .0068

SEED FOR RANF =    27452417757092022      1
*WEOR
```

Figure B.52 Comparison of α Values, $n = 2$, $f_1 = 4$, $f_2 = 8$, Variance Ratio 7 to 3

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 4. 1.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    27448427269760866       1

PROCEDURE        1SATTERTHWAITE
          ALPHA                PROB. OF EXCEEDING
          LEVEL                CRITICAL VALUE
      1   .1000                       .1000
      2   .0500                       .050       1
      3   .0250                       .0246
      4   .0100                       .0102
      5   .0050                       .0055

PROCEDURE   2     BANERJEE
          ALPHA                PROB. OF EXCEEDING
          LEVEL                CRITICAL VALUE
      1   .1000                       .0890
      2   [illegible]                 .040       1
      3   [illegible]                 .0157
      4   .0100                       .0047
      5   .0050                       .0023

PROCEDURE   3     COCHRAN
          ALPHA                PROB. OF EXCEEDING
          LEVEL                CRITICAL VALUE
      1   .1000                       .0891
      2   .0500                       .0402
      3   .0250                       .0159
      4   .0100                       .0049
      5   .0050                       .0024

PROCEDURE   4     SUM OF DEG.FREEDOM
          ALPHA                PROB. OF EXCEEDING
          LEVEL                CRITICAL VALUE
      1   .1000                       .1092
      2   .0500                       .0587
      3   .0250                       .0324
      4   .0100                       .0144
      5   .0050                       .0093

SEED FOR RANF =    27452417757092022       1
*WEOR
```

Figure B.53 Comparison of α Values, n = 2, f_1 = 4, f_2 = 8, Variance Ratio 4 to 1

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

NUMBER OF RUNS = 40000
NUMBER OF SHIPMENTS = 2
MEAN(.) = 0. 0.
VARIANCES = 9. 1.
DF(.) = 4 8
ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =    27452417757092022       1

PROCEDURE       1SATTERTHWAITE
          ALPHA            PROB. OF EXCEEDING
          LEVEL            CRITICAL VALUE
      1   .1000                  .0995
      2   .0500                  .0509
      3   .0250                  .0267
      4   .0100                  .0123
      5   .0050                  .0068

PROCEDURE  2     BANERJEE
          ALPHA            PROB. OF EXCEEDING
          LEVEL            CRITICAL VALUE
      1   .1000                  .0937
      2   .0500                  .0425
      3   .0250                  .0198
      4   .0100                  .0069
      5   .0050                  .0030

PROCEDURE  3     COCHRAN
          ALPHA            PROB. OF EXCEEDING
          LEVEL            CRITICAL VALUE
      1   .1000                  .0938
      2   .0500                  .0426
      3   .0250                  .0200
      4   .0100                  .007        1
      5   .0050                  .003        1

PROCEDURE  4     SUM OF DEG.FREEDOM
        . ALPHA            PROB. OF EXCEEDING
          LEVEL            CRITICAL VALUE
      1   .1000                  .1141
      2   .0500                  .0650
      3   .0250                  .0374
      4   .0100                  .0193
      5   .0050                  .0118

SEED FOR RANF =    274447949492177429
*WEOR

SEED FOR RANF =    274687318523598953
```

Figure B.54 Comparison of α Values, n = 2, f_1 = 4, f_2 = 8, Variance Ratio 9 to 1

```
SHIPPER - RECEIVER DIFFERENCE TEST STATISTIC

 NUMBER OF RUNS = 40000
 NUMBER OF SHIPMENTS = 2
 MEAN(.) = 0. 0.
 VARIANCES = .95 .05
 DF(.) = 4 8
 ALPHA(.) = .1 .05 .025 .01 .005

SEED FOR RANF =  274442845740728101

PROCEDURE  1     SATTERTHWAITE
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
        1  .1000                0.1023
        2  .0500                0.0521
        3  .0250                0.0278
        4  .0100                0.0120
        5  .0050                0.0068

PROCEDURE  2     BANERJEE
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
        1  .1000                0.0986
        2  .0500                0.0472
        3  .0250                0.0226
        4  .0100                0.0080
        5  .0050                0.0034

PROCEDURE  3     COCHRAN
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
        1  .1000                0.0987
        2  .0500                0.0473
        3  .0250                0.0228
        4  .0100                0.0081
        5  .0050                0.0036

PROCEDURE  4     SUM OF DEG.FREEDOM
           ALPHA           PROB. OF EXCEEDING
           LEVEL           CRITICAL VALUE
        1  .1000                0.1200
        2  .0500                0.0713
        3  .0250                0.0429
        4  .0100                0.0237
        5  .0050                0.0149
```

Figure B.55 Comparison of α Values, $n = 2$, $f_1 = 4$, $f_2 = 8$, Variance Ratio 19 to 1

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This report presents a method of constructing a test for the detection of nuclear material diversions over a series of n shipments. The available data are the shipper-receiver differences (SRD's) which are assumed to be multivariate normally distributed with known covariance matrix. Given a specific diversion scenario the most powerful test is derived. The "least favorable" scenario with respect to NRC detection is identified and the most powerful test against this scenario is found. The power function of this test depends only on the total amount diverted and not on the particular scenario chosen. This test maximizes among all level α tests, the minimum power overall scenarios with total diversion at least M. For various diversion scenarios, assuming independent SRD's, the power of the max-min test is compared to that of tests that are specific for each of these scenarios. Assuming the SRD's are independent with unknown variances, the max-min test is modified using Satterthwaite's approximation. The quality of this latter approximation is investigated for a range of parameter sets.

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