The Boeing
Math Group

Statistics in Aviation

Celebrating 100 Years of Flight

Fritz Scholz

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Applied Statistics Group

Mathematics & Computing Technology

The Boeing Company

Structure of this Talk

Early Importance of Data (Wright Brothers/Lindbergh)

Statisticians connected to aviation

Statistics within Boeing, BSRL, Reliability as Field

How safe is flying? Statistics about aviation.

Into Space, the New Frontier

Statistical challenges in aviation

Odds and Ends

Wilbur and Orville Wright

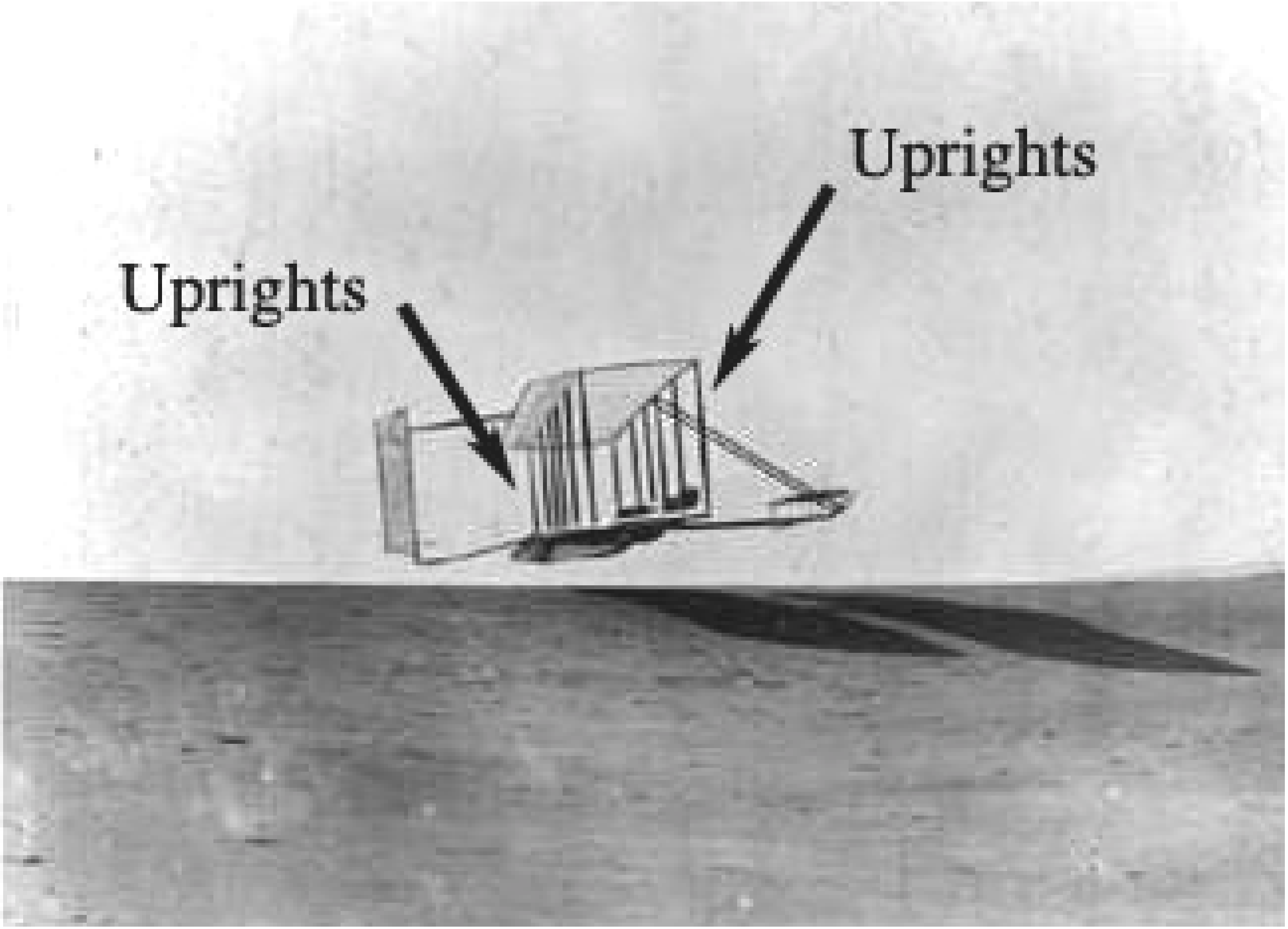
Wright Flyer at 10:35am on Dec 17, 1903

On Dec 14 Wilbur won the coin toss, made the first attempt and stalled, but Orville made the first flight on Dec. 17, 12 seconds & 120 ft

Wind Tunnel Data Important from Start

Replica of the 1901 Wright Wind Tunnel
(constructed with assistance from Orville Wright).

Aerodynamics of Uprights



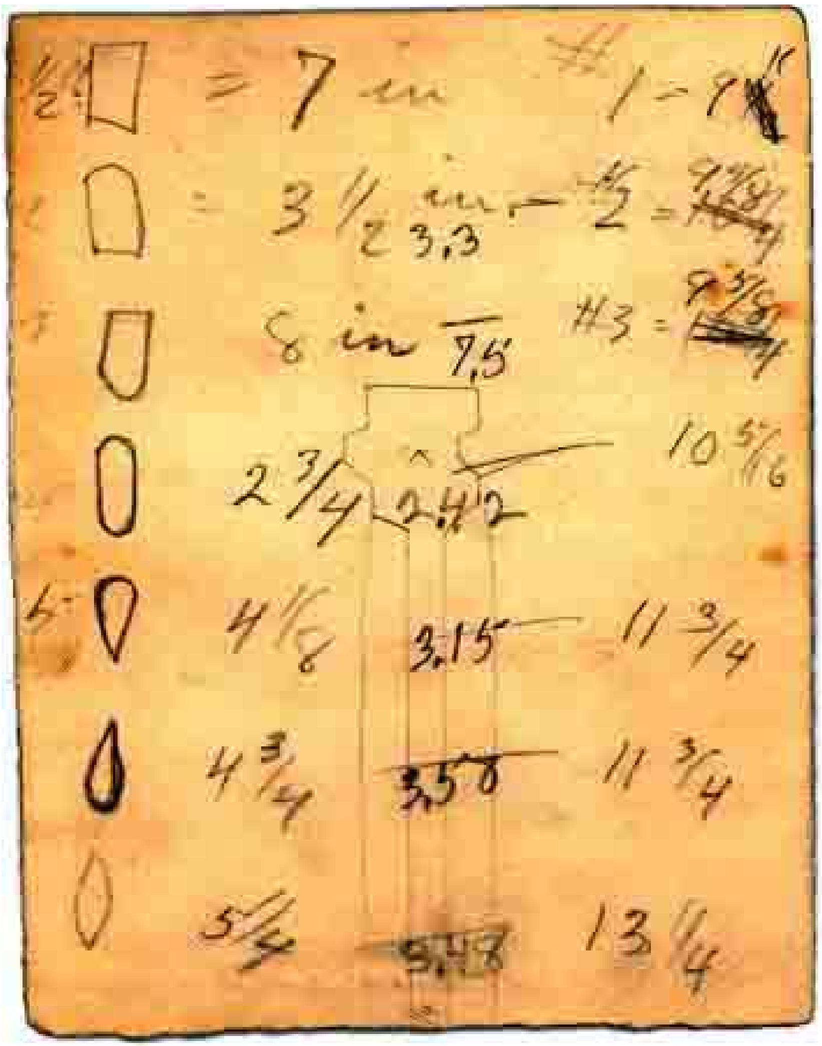
Experimenting for Flight

During January, 1903 the Wrights began to investigate the shape of the uprights (the long posts which separated the upper and lower wings).

Initially, a rectangular shape was used.

However, from their experiments on wing shapes, the Wrights believed a shape with more curvature on the sides and without the sharp edges of the rectangle would be more aerodynamic.

These charts from the Franklin Institute Science Museum are in Wilbur Wright's handwriting. You can see the different shapes the Wrights examined.



Charles Lindbergh, NY-Paris, May 20, 1927
after 33 ½ hours of flight

As the plane took off, the plane’s landing gear
missed a set of telephone wires by a mere 20 feet.

Take-Off Distance and Gross Weight

There were concerns over the take-off distance of a fully loaded plane.

They could not test it because the landing gear might not support a landing at that weight.

They did not want to fly it for hours to burn off fuel.

Thus they tested it at lower weights and extrapolated.

TECHNICAL NOTES

NATIONAL ADVISORY COMMITTEE FOR AERONAUTICS

No. 257

TECHNICAL PREPARATION OF THE AIRPLANE "SPIRIT OF ST. LOUIS"

Written for the

National Advisory Committee for Aeronautics

By Donald A. Hall
Chief Engineer, Ryan Airlines, Inc.

FILE COPY
To be returned to
the files of the Langley
Memorial Aeronautical
Laboratory

Washington
July, 1927

General Dimensions and Specifications (Cont.)

Take-Off Distances--

Tests made at Camp Kearney near San Diego, California, at 600 ft. altitude. Oil = 4 gallons.

Gal. Gas	Gross Wt. lb.	Approx. Head Wind Velocity M.P.H.	Take-Off Distance ft.
36	2600	7	229
71	2800	9	287
111	3050	9	389
151	3300	6	483
201	3600	4	615
251	3900	2	800
301	4200	0	1023

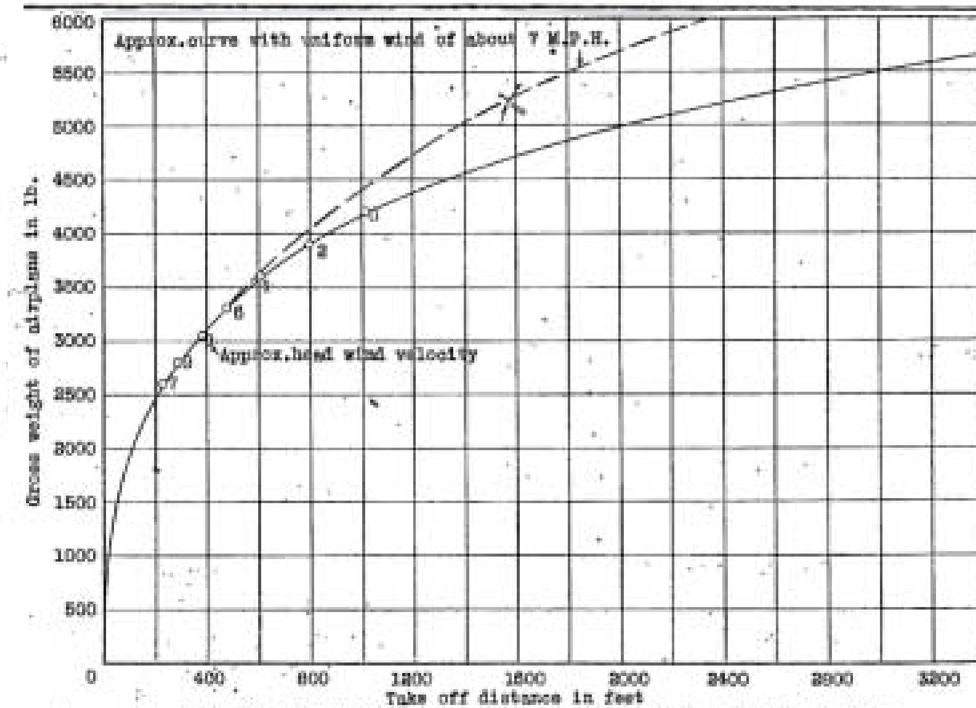
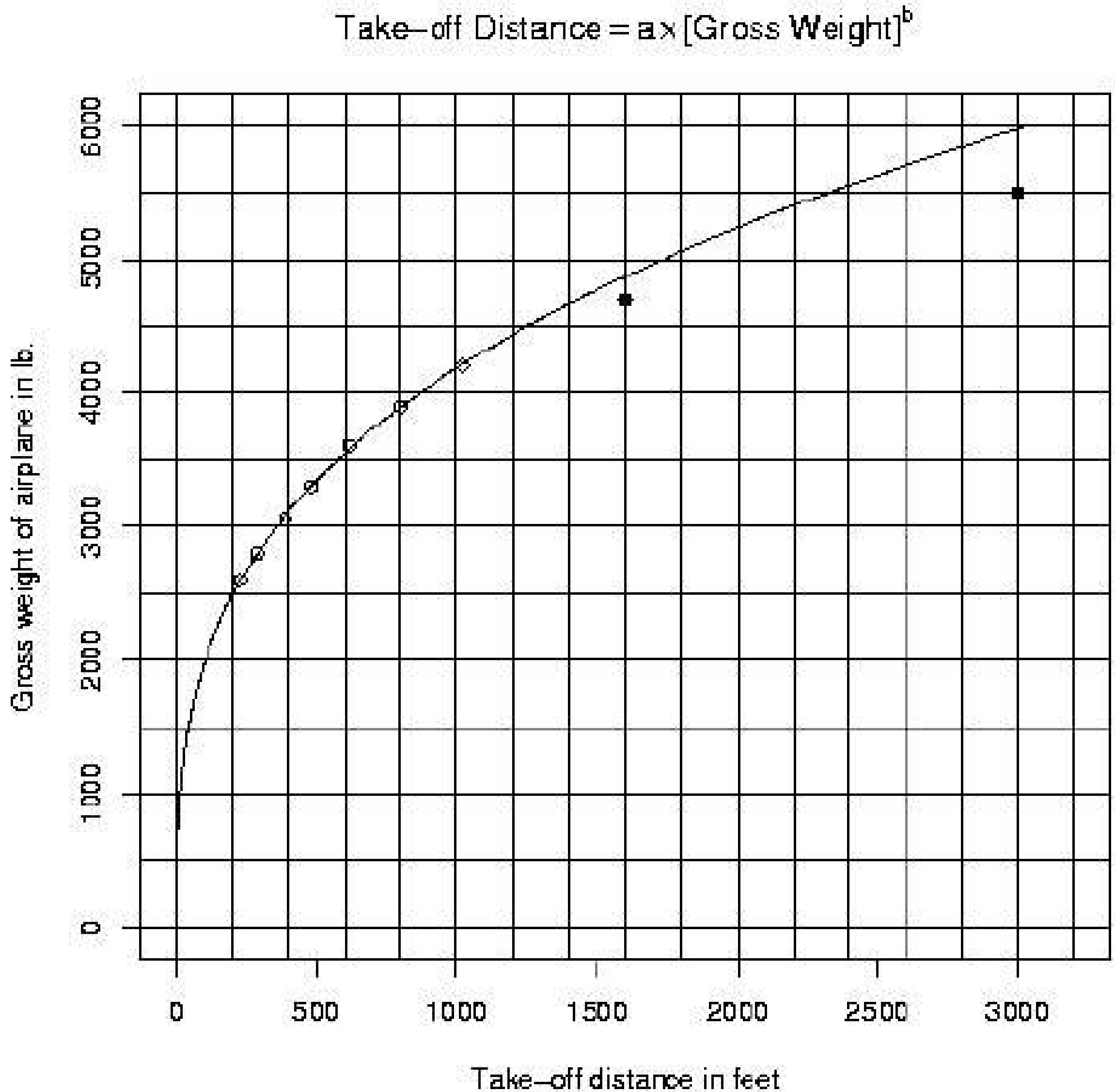
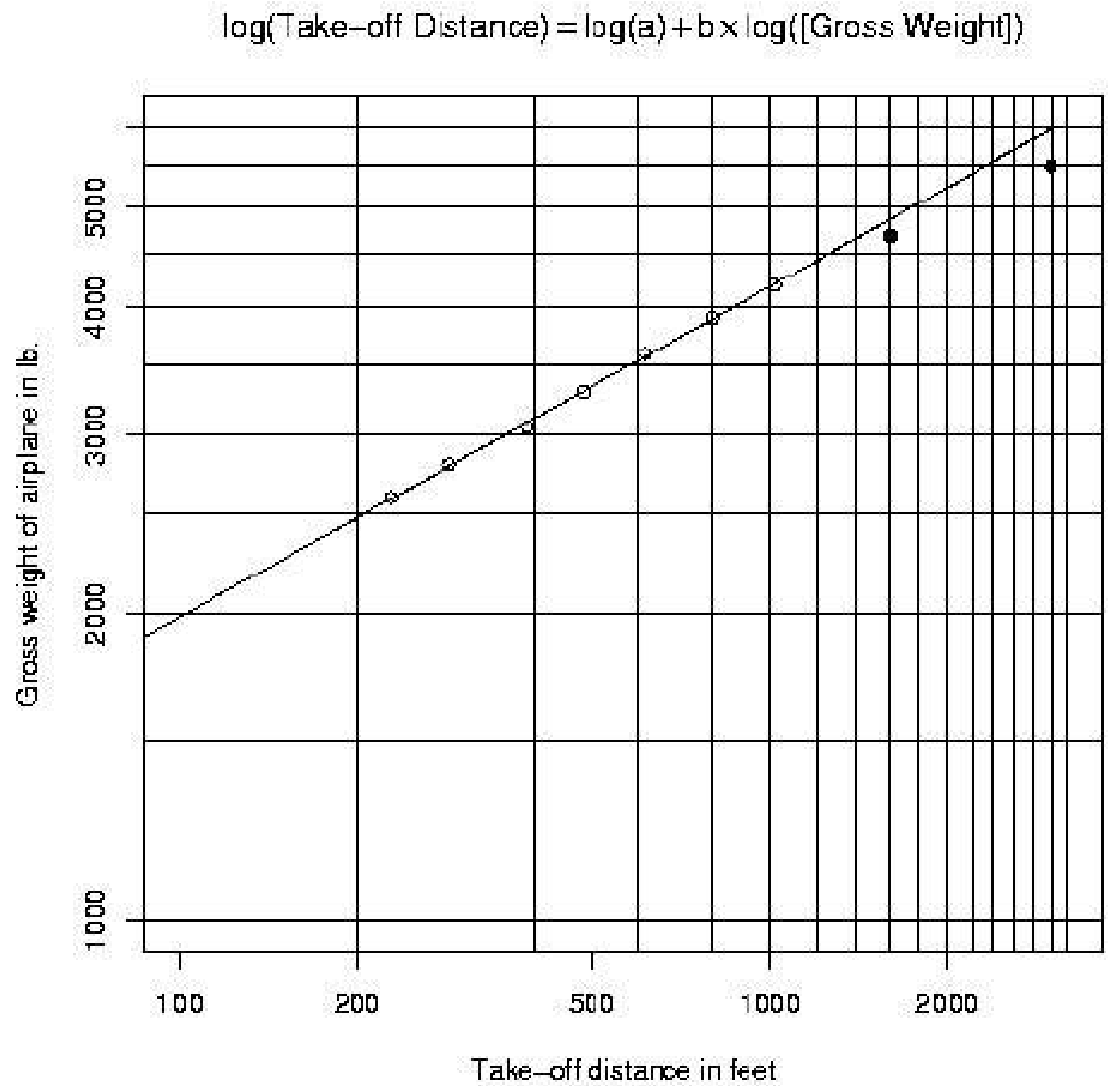
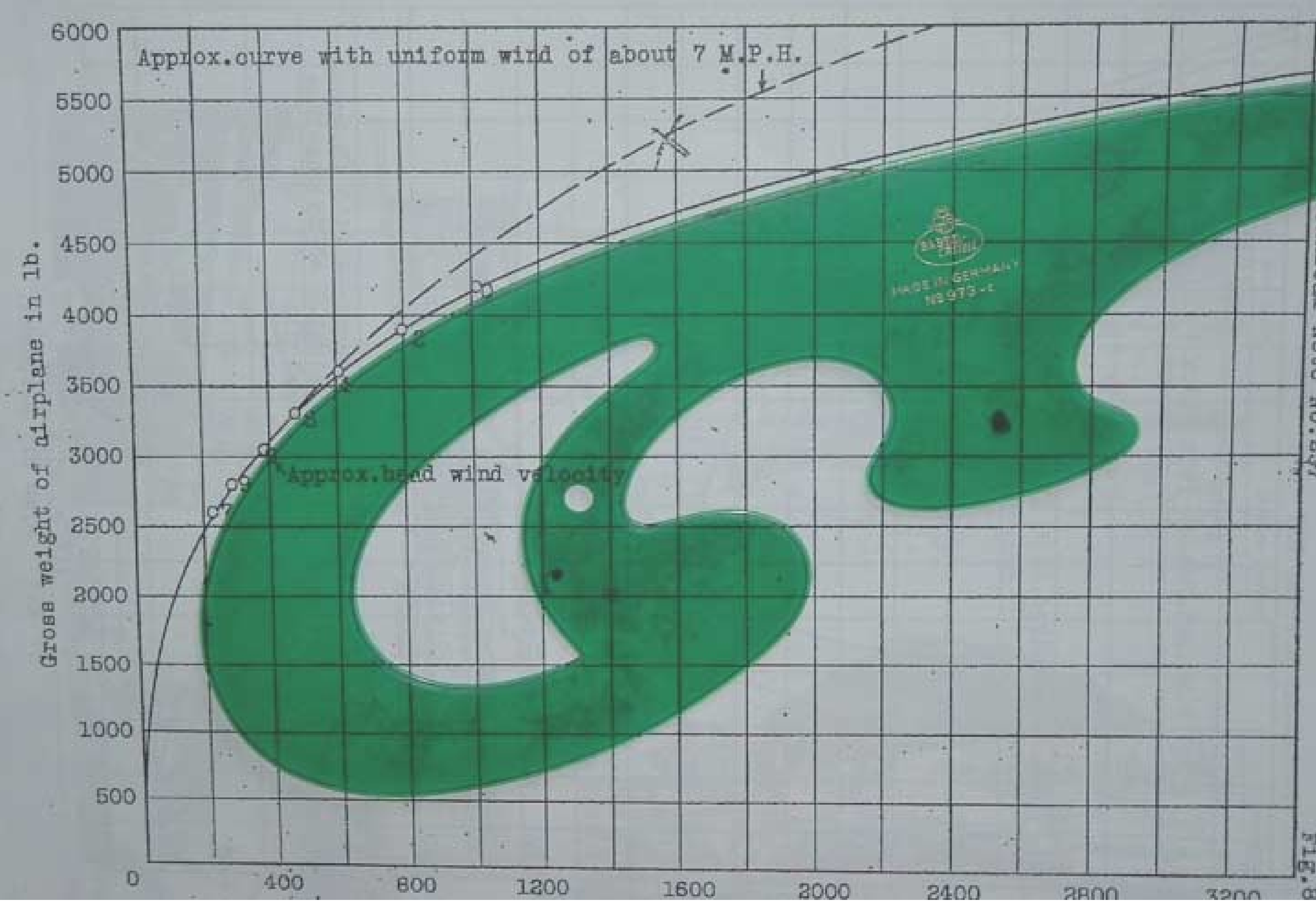


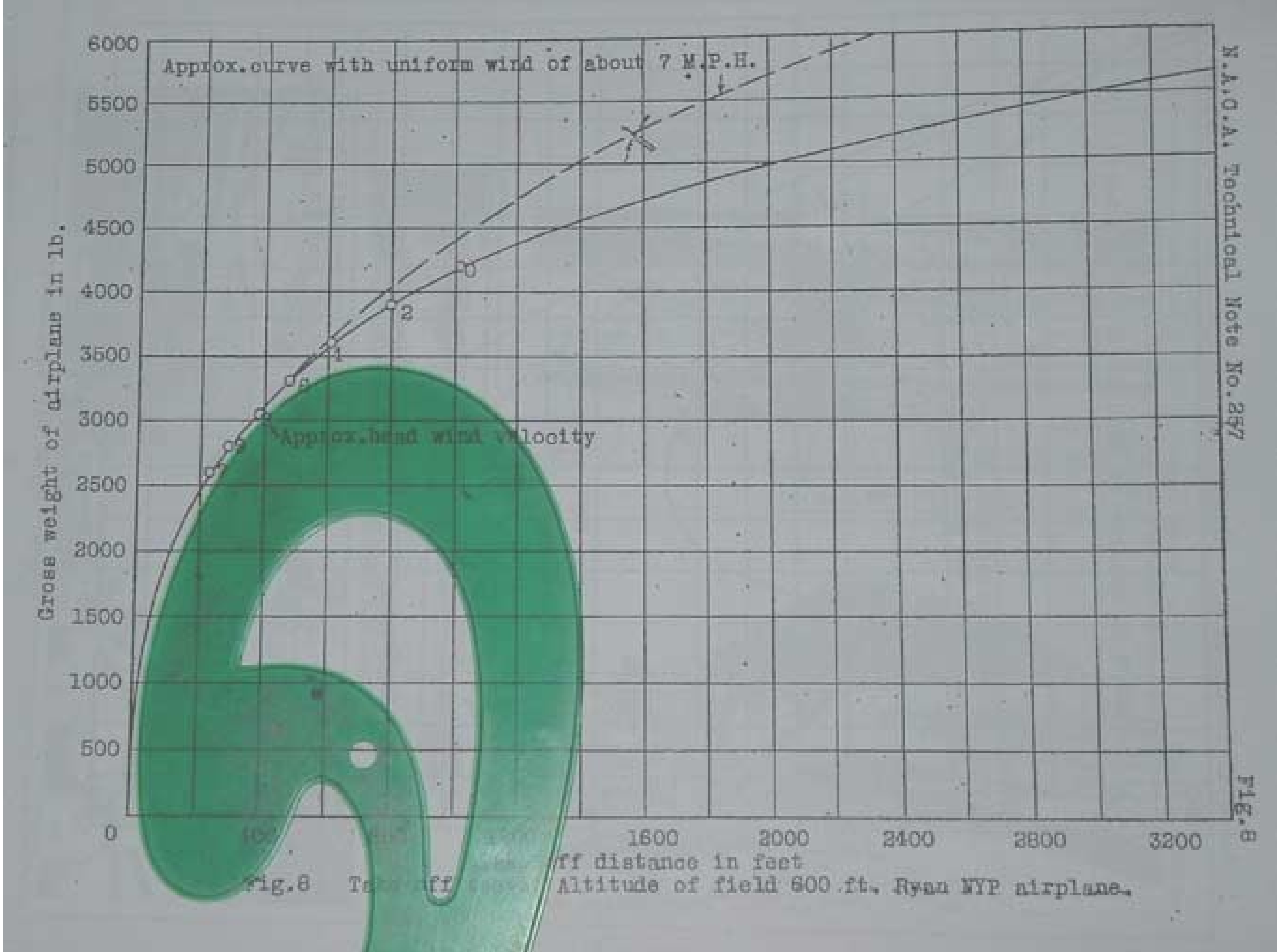
Fig.8 Take off tests. Altitude of field 600 ft. Ryan NYP airplane.


Take–off Distance = a x [Gross Weight]^b
Gross weight of airplane in lb.
0
1000
2000
3000
4000
5000
6000
0
500
1000
1500
2000
2500
3000
Take–off distance in feet


log(Take−off Distance) = log(a) + b × log([Gross Weight])
Gross weight of airplane in lb.
1000
2000
3000
4000
5000
100
200
500
1000
2000
Take−off distance in feet



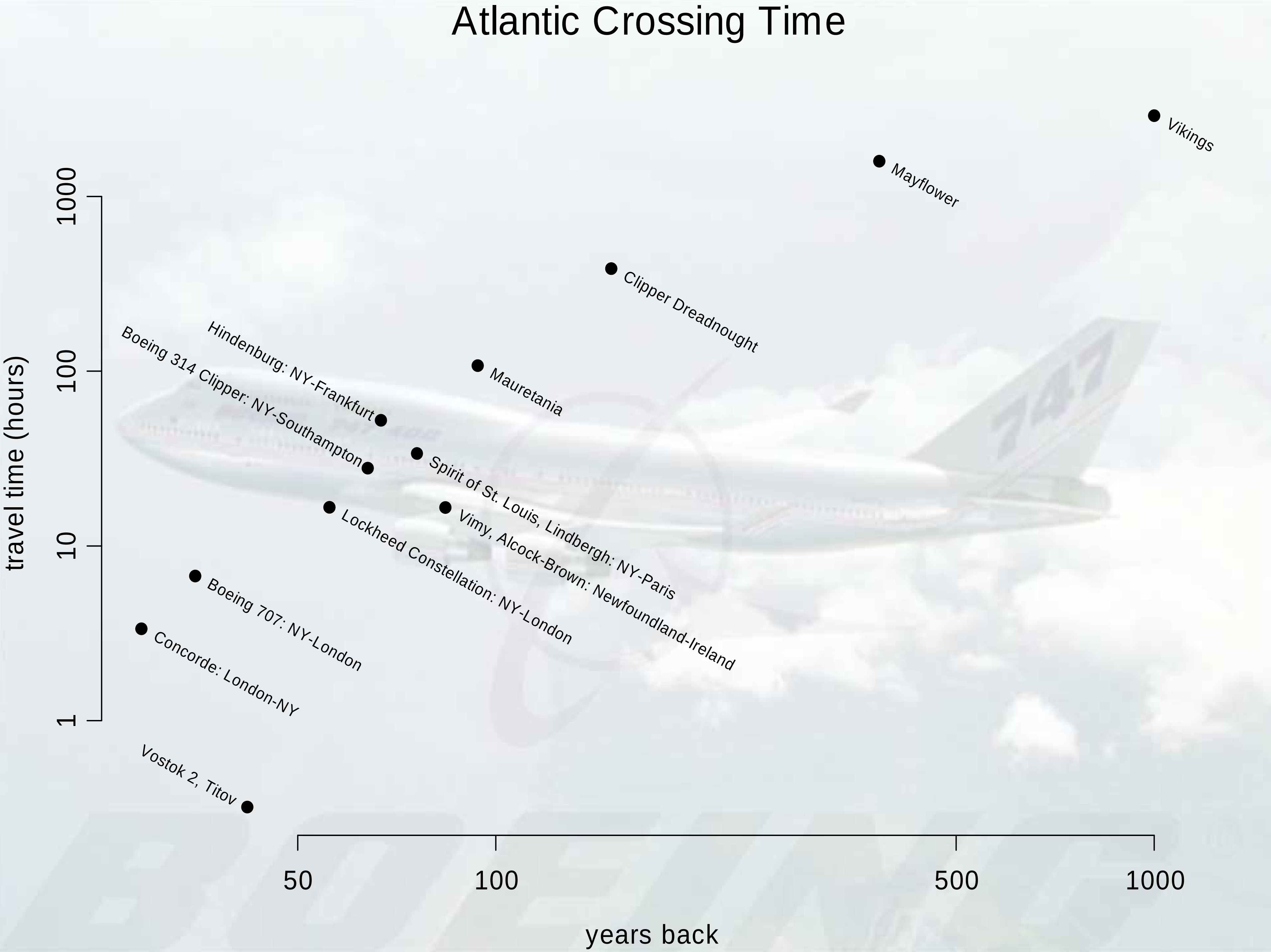
Plausible Curve Fit (part 1). Note the use of a German French Curve


Approx. curve with uniform wind of about 7 M.P.H.
Approx. head wind velocity
Gross weight of airplane in lb.
6000
5500
5000
4500
4000
3500
3000
2500
2000
1500
1000
500
0
1600
2000
2400
2800
3200
ff distance in feet
N.A.C.A. Technical Note No. 257
Fig.8
Fig.8
Altitude of field 600 ft. Ryan NYP airplane.

(part 2)

Designer – Circa 1970

A Physical Spline



Atlantic Crossing Time
travel time (hours)
1000
100
10
1
years back
50
100
500
1000
Vikings
Mayflower
Clipper Dreadnought
Mauretania
Hindenburg: NY-Frankfurt
Boeing 314 Clipper: NY-Southampton
Spirit of St. Louis, Lindbergh: NY-Paris
Lockheed Constellation: NY-London
Vimy, Alcock-Brown: Newfoundland-Ireland
Boeing 707: NY-London
Concorde: London-NY
Vostok 2, Titov

Statisticians in Aviation

Richard Martin Edler von Mises, 1883-1953

One of the greatest applied mathematicians of 20th century.

Gave 1st university course in powered flight (1913, Strasburg).

At beginning of World War I he joined the Flying Corps of the Austro-Hungarian Army & acquired a pilot’s license.

Was recalled from field service to become technical instructor in flight theory to German & Austrian Officers.

Founded: Zeitschrift für Angewandte Mathematik und Mechanik 1921

Some of von Mises' Legacy

Theory of Flight (1959)

Fluid Dynamics (1971)

Probability, Statistics and Truth (1981)

von Mises foundation of probability (Kollektiv)

von Mises expansions, von Mises functional

von Mises (directional) distribution,

Cramer-von Mises test

Extreme value theory: von Mises form of distribution, von Mises conditions

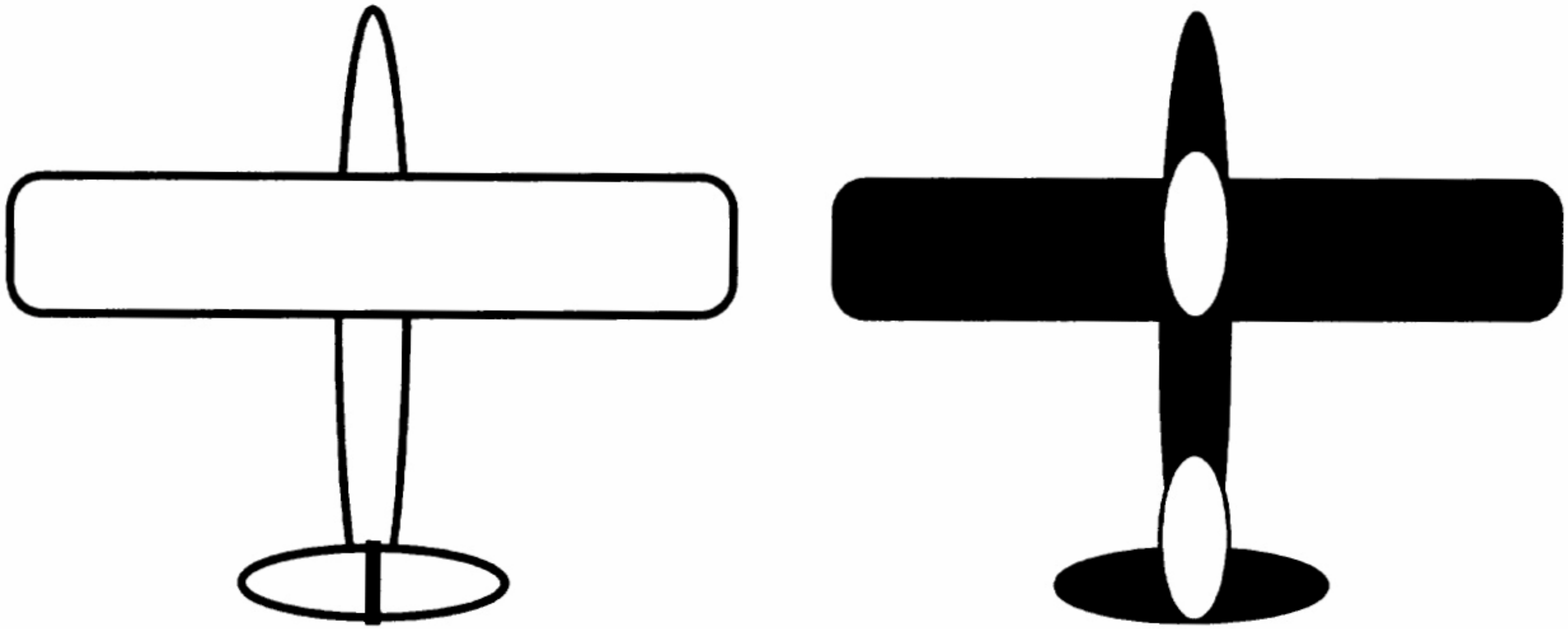
Abraham Wald
1902-1950

Father of

Decision Theory &
Sequential Analysis

During WWII and later in Korea and Vietnam, the U.S. Navy and Air Force studied bullet-hole patterns on returning aircraft to determine where to reinforce the aircraft against ground fire. Abraham Wald (a statistician at U.S. Center for Naval Analyses) worked on this problem from 1941. Wald dryly noted better information would have been obtained from the planes that hadn't returned. He nevertheless managed to construct statistical models which gave a useful insight into the vulnerability of different parts of the aircraft.

Wald died in an aircraft crash over India in 1950



An outline of a plane.

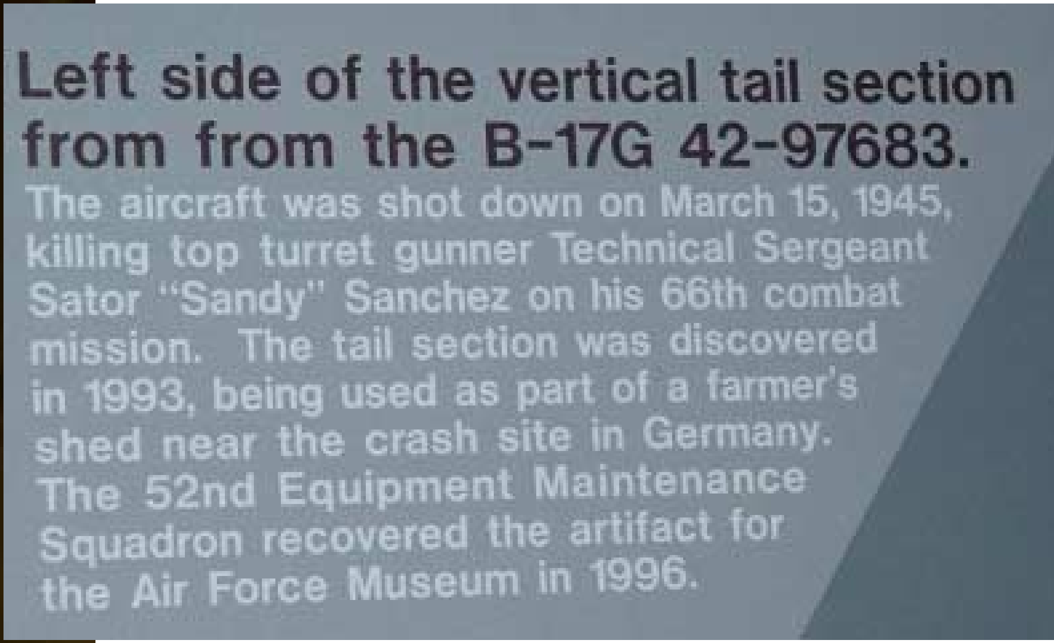
A depiction of a plane with shading indicating where returning planes had been shot.

Figure 6. A schematic representation of Abraham Wald's ingenious scheme to investigate where to armor aircraft.

Wainer, Palmer and Bradlow

Chance, 11, 2, 1998

United States Air Force Museum



United States
Air Force
Museum

Boeing Scientific Research Laboratories (BSRL)

George S. Schairer, acting head of the newly created Boeing Scientific Research Laboratories in 1958:

"If you're considering manned spacecraft applications, you need basic answers to a lot of questions....We're talking about temperatures only science-fiction writers talked about a few years ago. Our new research organization will give us one of the spearheads for taking steps further into the future than we've been able to do before."

A Bell Labs of the West Coast (Ron Pyke)

Statisticians/Mathematicians associated with BSRL, 1958-1969

R.E. Barlow,
Z.W. Birnbaum,
T.A. Bray,
G.B. Crawford,
G. Dantzig,
J.D. Esary (B),
L.C. Hunter,
V. Klee,
N.R. Mann,
G. Marsaglia (B),
A.W. Marshall (B),
J.M. Myhre,
I. Olkin,
F. Proschan (B)
R. Pyke
S.C. Saunders (B),
R. Van Slyke,
D.W. Walkup (B),
R. Wets

(B) Boeing employee, others were visitors/consultants

The Birth of Reliability Theory

Barlow (MMR 2002, Trondheim):

“it was not until 1961 with the publication of the Birnbaum, Esary and Saunders paper on coherent structures that reliability theory began to be treated as a separate subject.”

“The Boeing 707 was under development at the time the de Haviland Comets were crashing. It was partly for this reason that the Boeing Scientific Research Laboratories in Seattle began to emphasize reliability theory in their mathematics division.”

⟶ **Mathematical Theory of Reliability** (1965),

by Barlow & Proschan (w. contributions by Hunter)

Mathematical Methods of Reliability Theory (1965)

by Gnedenko, Belyayev, and Solovyev

BOEING SCIENTIFIC RESEARCH LABORATORIES

Multi-Component Systems and Structures and Their Reliability

Z. W. Birnbaum

J. D. Esary

S. C. Saunders

Mathematics Research

Technometrics, 1961, 55-77

Frank Proschan (1963), Theoretical Explanation of Observed Decreasing Failure Rate, *Technometrics*

This paper presents the famous air conditioner failure data from a fleet of Boeing 720 planes.

Pooled failure time data do not appear to be exponential, in fact they seem to indicate a *decreasing failure rate*.

Failure data from individual planes appear to be exponential.

Proschan used this to illustrate that a mixture of exponentials has a decreasing failure rate and suggested to be aware of this possibility for any DFR appearance.

This data set has since been much reanalyzed.

It is one of the few data sets that got away.

TABLE 1

Intervals between failures

Plane												
7907	7908	7909	7910	7911	7912	7913	7914	7915	7916	7917	8044	8045
194	413	90	74	55	23	97	50	359	50	130	487	102
15	14	10	57	320	261	51	44	9	254	493	18	209
41	58	60	48	56	87	11	102	12	5		100	14
29	37	186	29	104	7	4	72	270	283		7	57
33	100	61	502	220	120	141	22	603	35		98	54
181	65	49	12	239	14	18	39	3	12		5	32
	9	14	70	47	62	142	3	104			85	67
	169	24	21	246	47	68	15	2			91	59
	447	56	29	176	225	77	197	438			43	134
	184	20	386	182	71	80	188				230	152
	36	79	59	33	246	1	79				3	27
	201	84	27	**	21	16	88				130	14
	118	44	**	15	42	106	46					230
	**	59	153	104	20	206	5					66
	34	29	26	35	5	82	5					61
	31	118	326		12	54	36					34
	18	25			120	31	22					
	18	156			11	216	139					
	67	310			3	46	210					
	57	76			14	111	97					
	62	26			71	39	30					
	7	44			11	63	23					
	22	23			14	18	13					
	34	62			11	191	14					
		**			16	18						
		130			90	163						
		208			1	24						
		70			16							
		101			52							
		208			95							

(** major overhaul)

Boeing data set probably most analyzed in statistical community

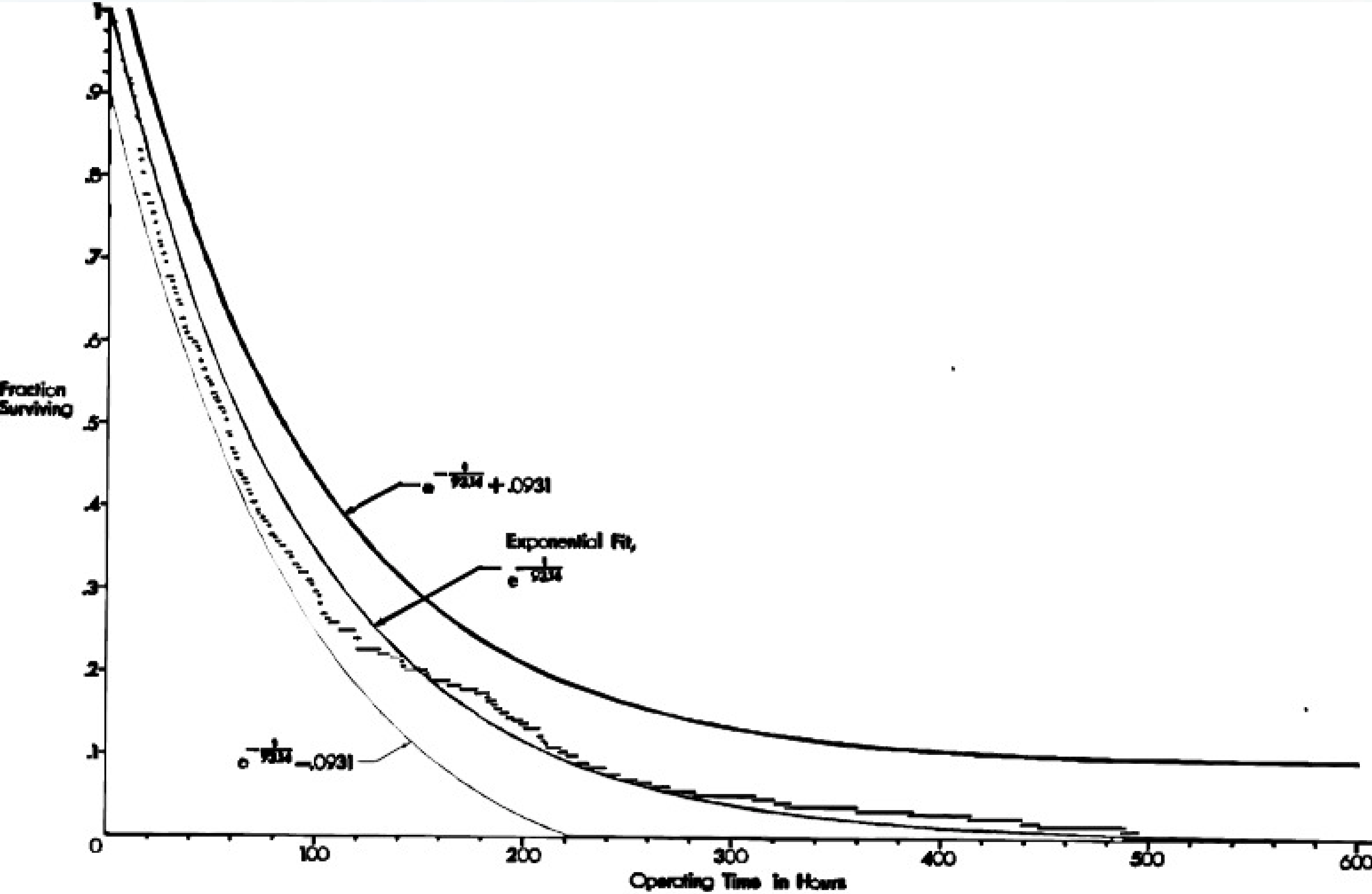

Fraction Surviving
Operating Time in Hours
Exponential Fit,
+.0931
−.0931
0
100
200
300
400
500
.1
.2
.3
.4
.5
.6
.7
.8
.9

FIGURE 1

Ron Pyke (1965), Spacings, with Discussion**,** *JRSS(B)*

This landmark paper, partially supported by The Boeing Company through BSRL, was presented before the Royal Statistical Society

In the spirit of such presentations Ron felt he had to show a data analysis application to the theory, although he admits to not having much experience in data analysis.

He had observed that aircraft accidents seemed to come in clusters of 3, **speculating** that the first would lead to preventive maintenance actions, possibly leading to screw-ups and more accidents.

He put this to the test for data from US and British carriers and found by various metrics: Accidents happen randomly over time.

The discussion confirmed that, although some criticized rightly that calendar time was probably not appropriate. Number of flights would have been better.

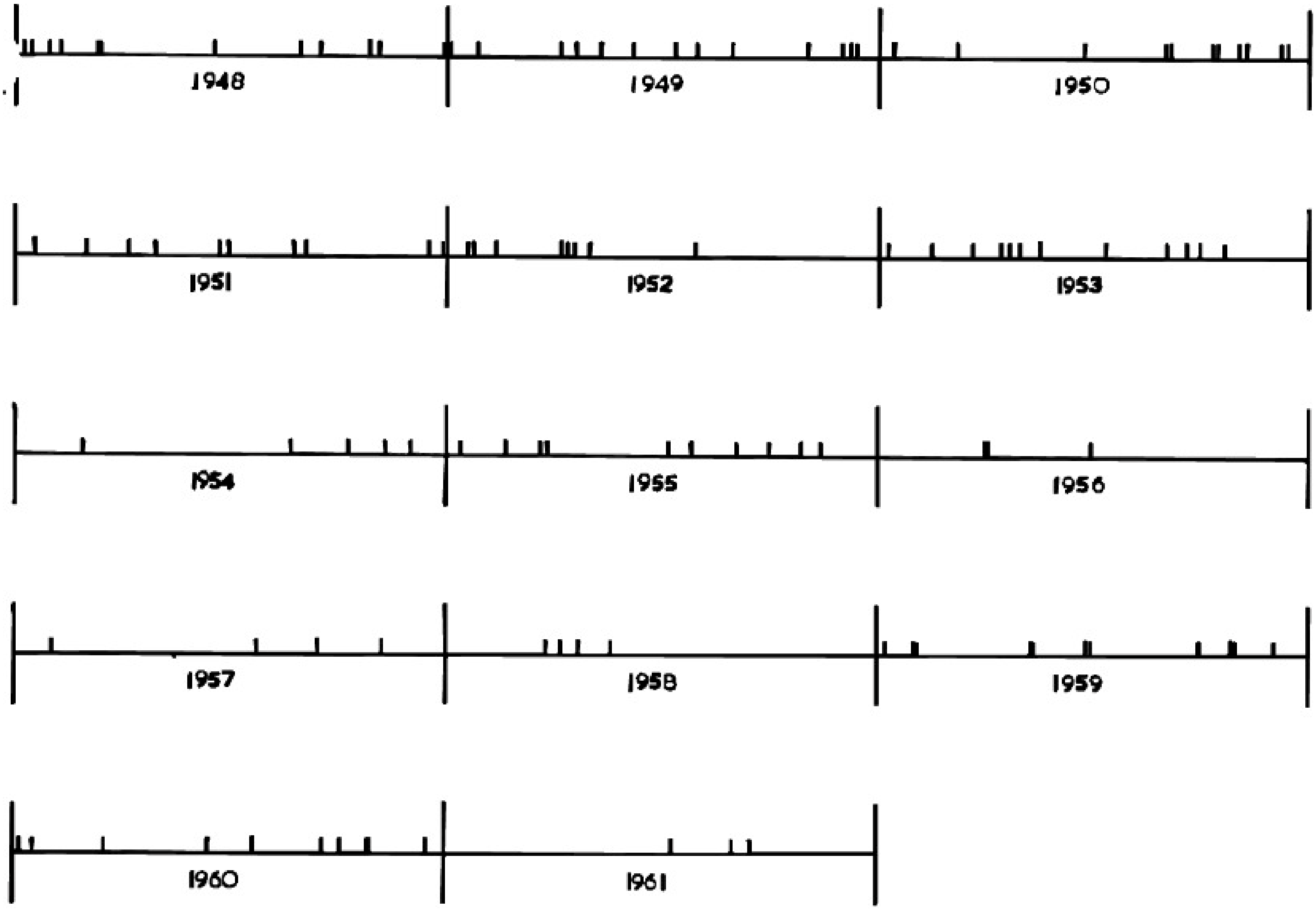


FIG. 1. Occurrences of fatal accidents to scheduled American domestic-operated passenger aircraft, 1948–61.

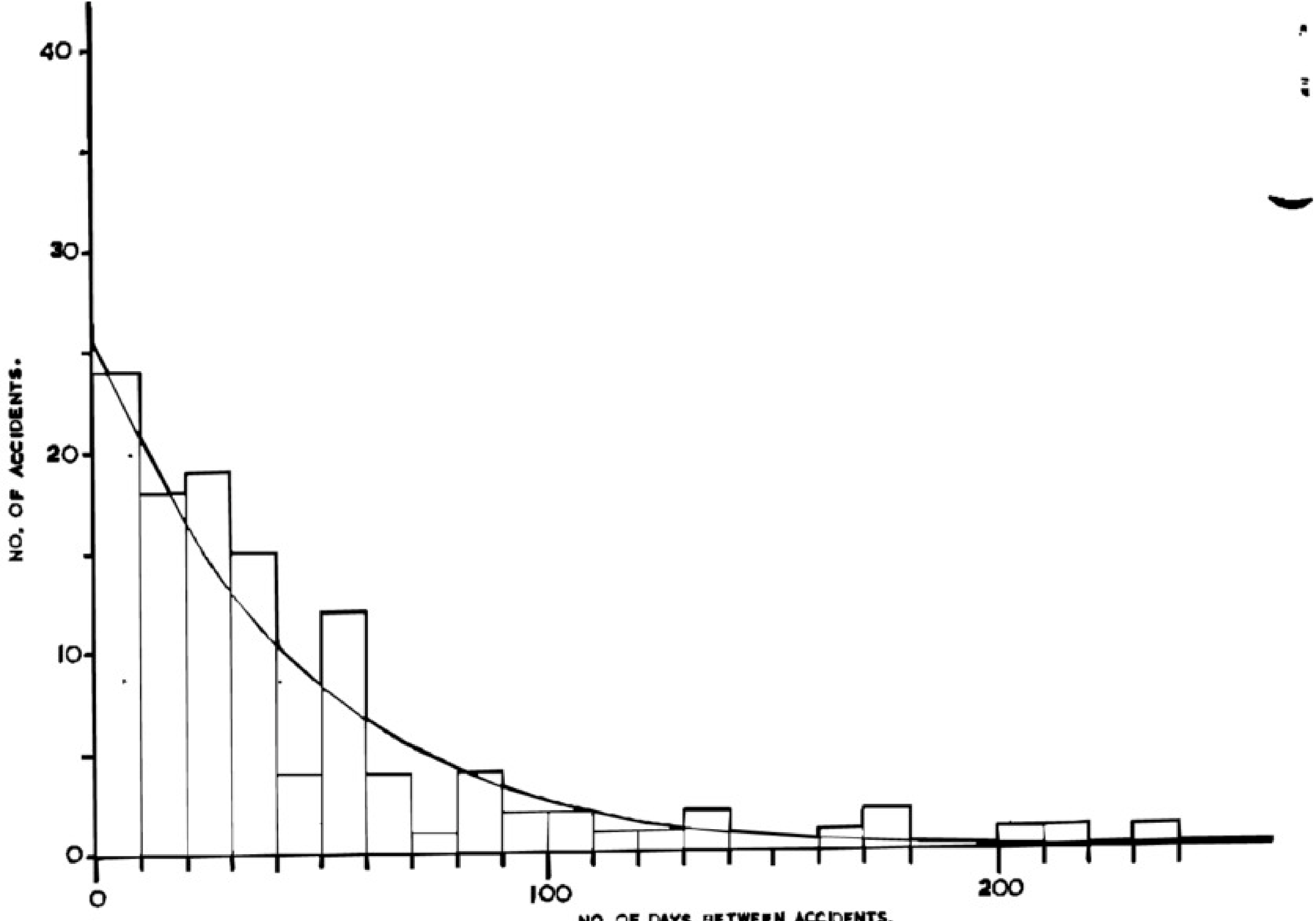


FIG. 3. Histogram of the inter-accident times for the American data. The smooth curve is 115 $\exp(-\lambda x)$ where $\lambda = 0{\cdot}0230$. A χ^2-test was performed using groupings: 0–10, 11–20, 21–30, 31–40, 41–50, 51–60, 61–80, 81–100, 101–130, >130; for these data, $\chi^2 = 12{\cdot}232$, which is approximately the 85-percentile.

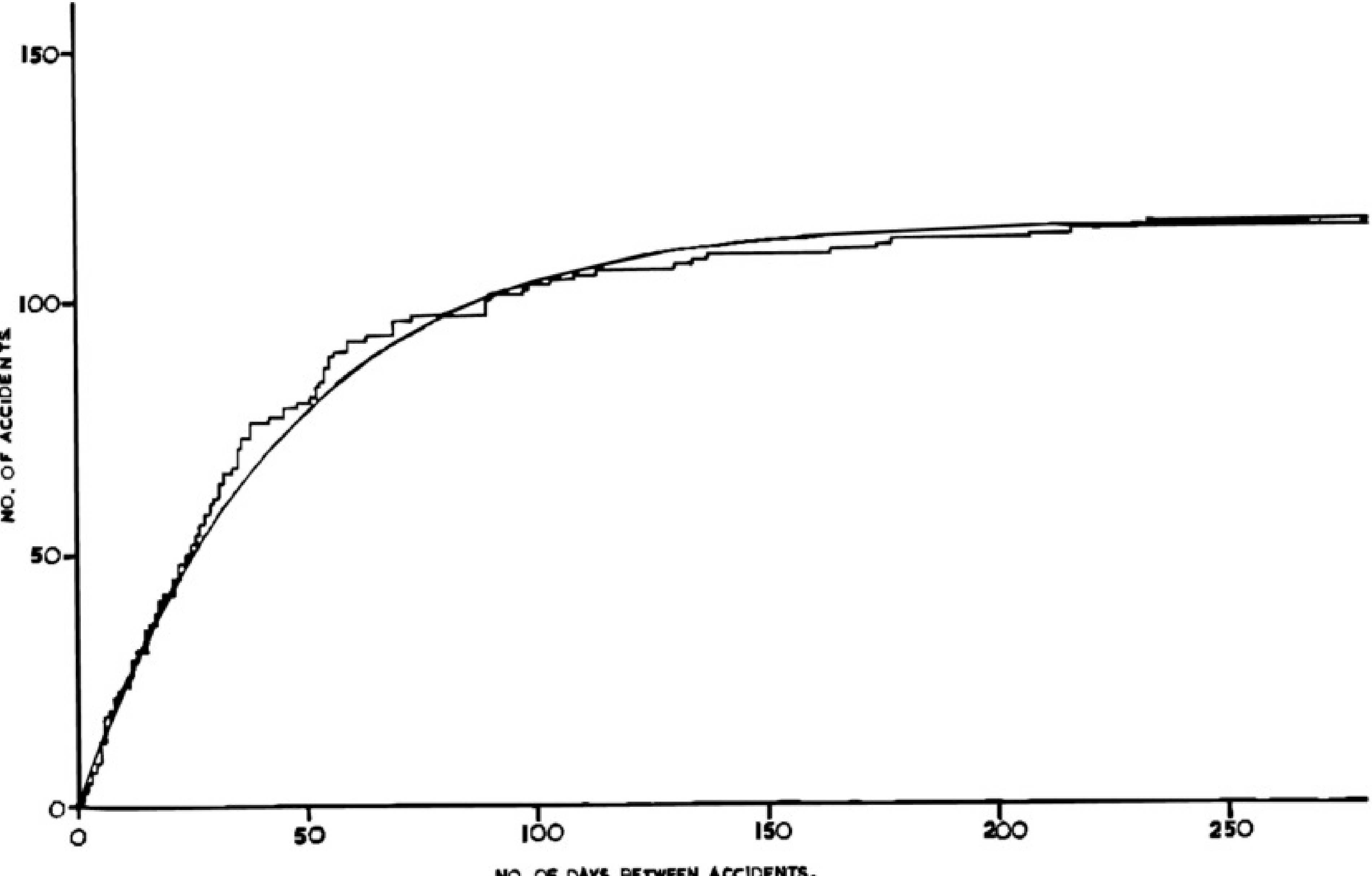


FIG. 5. The empirical distribution function of the inter-accident times for American data. The smooth curve is $115\{1-\exp(-\lambda x)\}$ with $\lambda = 0{\cdot}0230$. The Kolmogorov–Smirnov distance equals 9·00, which is approximately the 52-percentile.

The Boeing Bust

Over 86,000 employees were laid off in 1969-71

Boeing employment reached a low of 56,300

BSRL was closed, some found refuge elsewhere in Boeing, some went into academia

The economic downturn (Boeing was the major employer, no Microsoft, etc) inspired the billboard below.

It also led to the demise of the planned UW Statistics Department

According to Ron Pyke it led to my arrival at the UW Math Department in 1972.

Dean Beckman made sure that the next open position would go to a statistician.

The Applied Statistics Group of the Boeing Math Group has 17 members

Roberto Altschul, Shobbo Basu, **Andrew Booker**, Bill Fortney, Roman Fresnedo, **Stephen Jones**, I-Li Lu, Martin Meckesheimer, Ranjan Paul, Julio Peixoto, **Fritz Scholz**, Shuguang Song, **Winson Taam**, Valeria Thompson, **Rod Tjoelker**, Tom Tosch, Virginia Wheway

It is part of the **Math & Computing Technology** which is the closest successor organization to BSRL within Boeing

We do some research but mostly consult within Boeing, with occasional outside contract work

There are many more statisticians and mathematicians clustered throughout the company

How safe is flying?

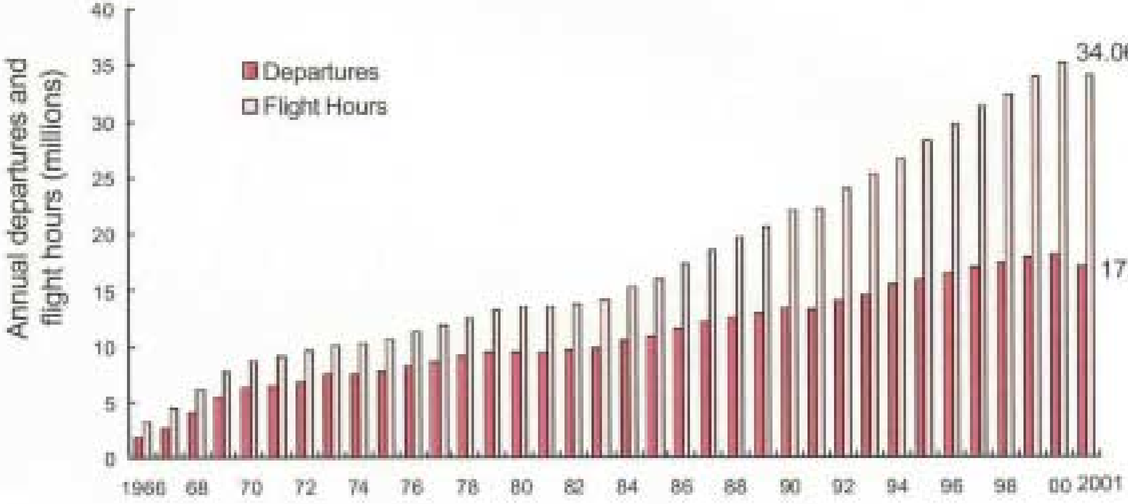
Since accidents do happen the answer is given statistically

For more definitive information see

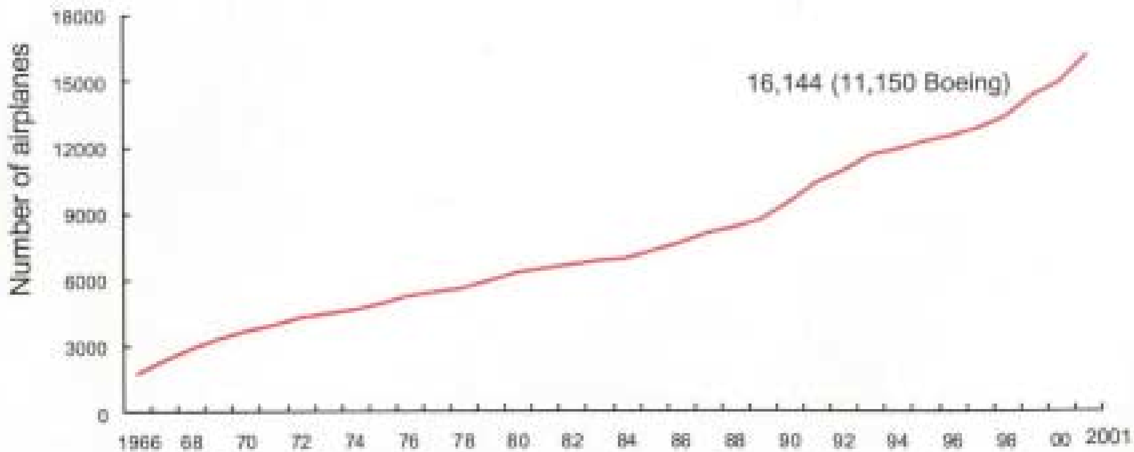
http://www.boeing.com/news/techissues/pdf/statsum.pdf

Departures, Flight Hours, and Jet Airplanes in Service*

Worldwide Operations 1966 to 2001



- 395.8 million cumulative departures (330.0 million on Boeing airplanes)
- 644.5 million cumulative flight-hours (546.5 million on Boeing airplanes)
- 7 Manufacturers - 33 significant types (13 Boeing) in service as of 12/31/2001

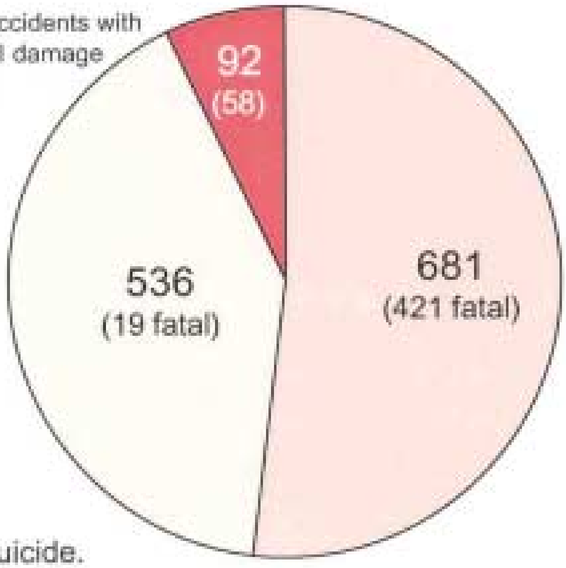


*Certified jet airplanes greater than 60,000 pounds maximum gross weight, including those in temporary non-flying status and those in use by non-airline operators. Excluded are military airplanes and CIS-manufactured airplanes.

Accident Summary by Damage and Injury

All Accidents - Worldwide Commercial Jet Fleet - 1959 through 2001

1,307 accidents worldwide

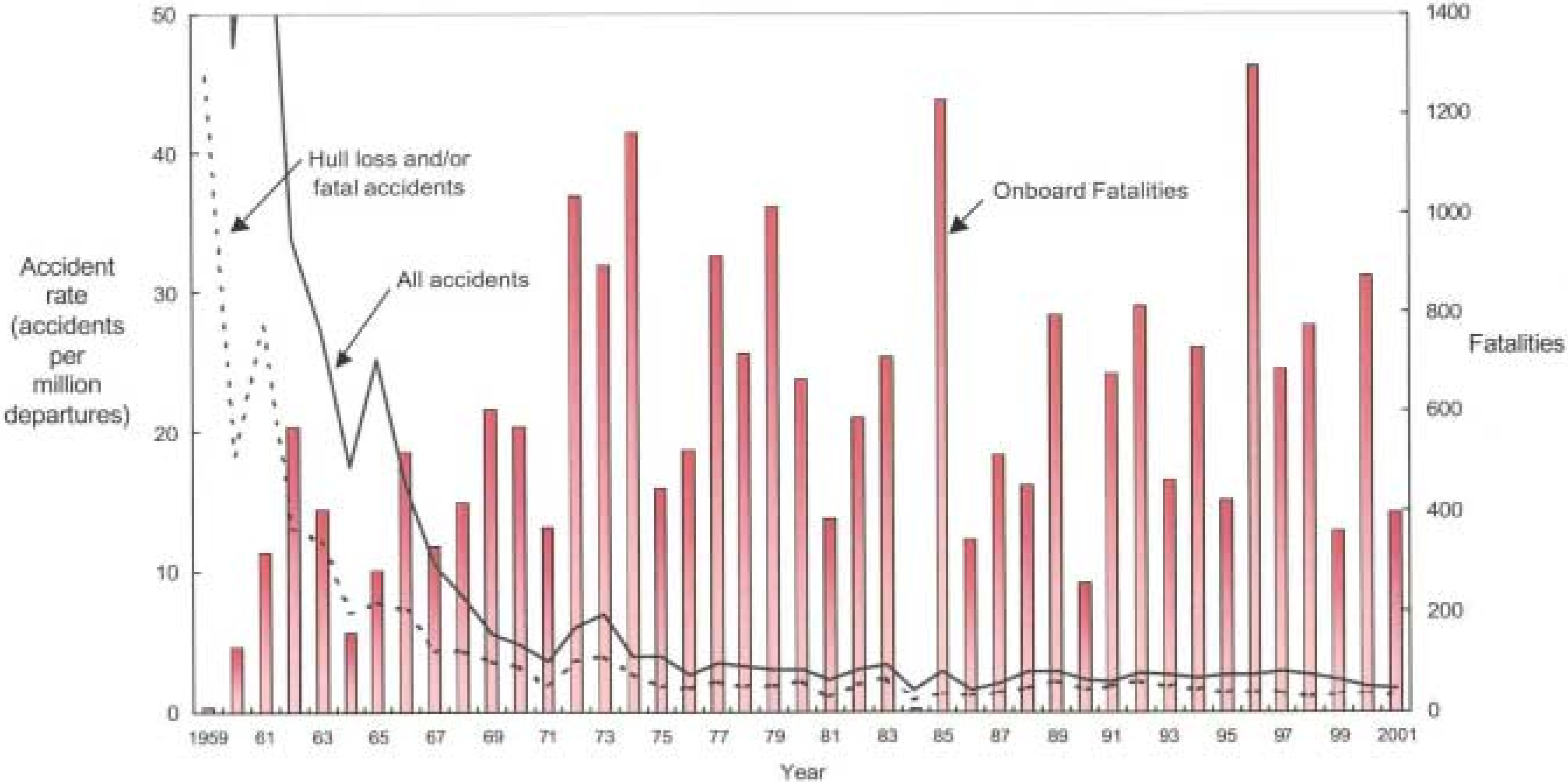


Excludes:

- Fatal injuries from natural causes, or suicide.
- Experimental test flights.
- Military airplanes.
- Sabotage, hijacking, terrorism, or military action.
- Non-fatal injuries involving:
  - Atmospheric turbulence, maneuvering, or loose objects.
  - Boarding, disembarking, or evacuation.
  - Maintenance or servicing.
  - Persons not onboard the airplane.

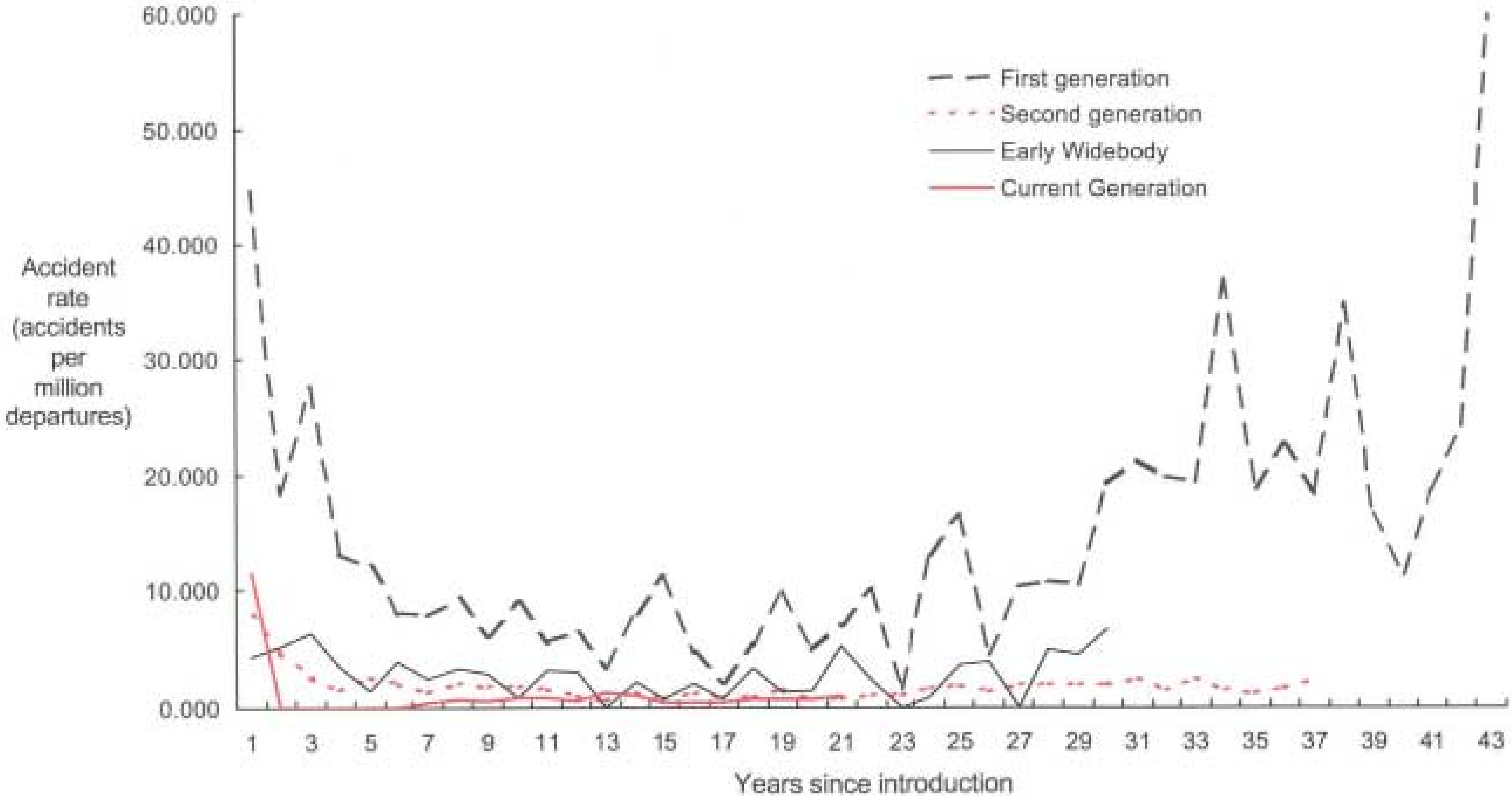
Accident Rates and Fatalities by Year

All Accidents - Worldwide Commercial Jet Fleet - 1959 through 2001



Accident Rates by Years Following Introduction

Hull Loss and/or Fatal accidents - Worldwide Commercial Jet Fleet - 1959 through 2001



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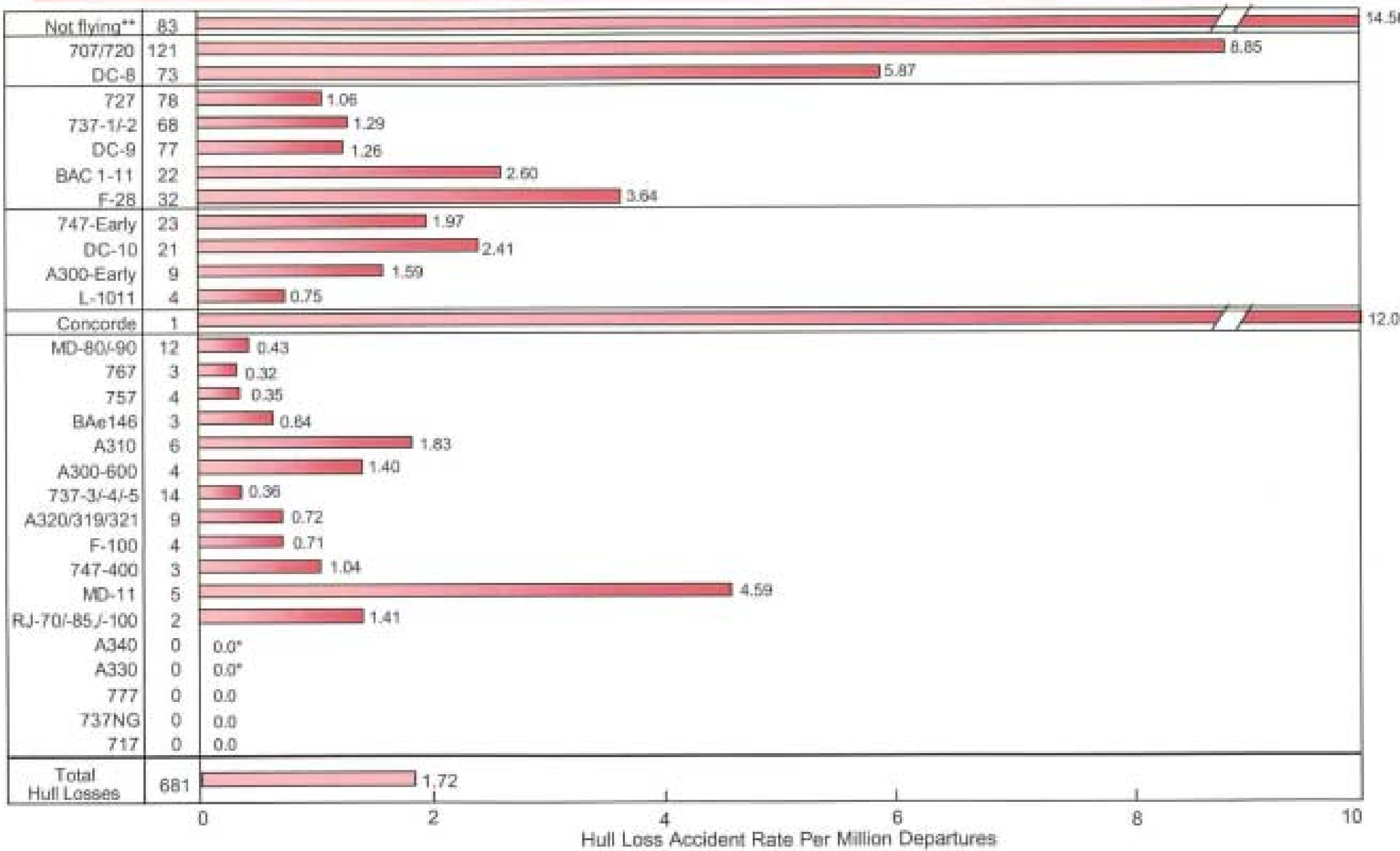
Accident Rates by Airplane Type

Hull Loss Accidents - Worldwide Commercial Jet Fleet - 1959 through 2001



** The Comet, CV880/990, Caravelle, Trident & VC-10 are no longer in commercial service, and are combined in the "Not Flying" bar.

* These types have accumulated fewer than 1 million departures.

Accident Categories by Airplane Generation

All accidents - Worldwide Commercial Jet Operations - 1992 through 2001

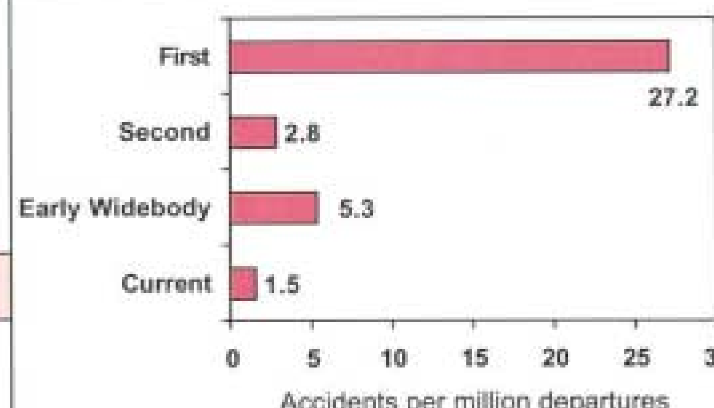
Generation	Controlled flight into terrain	Loss of control	Mid - air collision	In-flight fire	Fuel tank explosion	Landing: Off end on landing	Landing: Offside on landing	Landing: Hard landing	Landing: Landed short	Gear collapse/failure	Ice/snow	Fuel management/exhaustion	Windshear	Takeoff configuration	Refused takeoff - off end	Offside on takeoff	Runway incursion vehicle/people	Wing strike	Engine failure/separation	Ground collision	Ground crew injury	Boarding/deplaning	Turbulence fatality	Miscellaneous*	Fire on ground	Aircraft structure	Unknown	Total
First	5	7		1		6	3	3	4	8		1		1	1			2	3					1	1		2	49
Second	15	8	1	4		18	22	15	10	11	2	2	1	1	6	1	1	1	1	2	1			1	1	2	3	130
Early widebody	4	1	1	1	1	4	3	5	1	4	1	1	1		3	3	1		5	3	1	1	1	2	3	2		53
Current	12	16		1	1	24	13	35	2	14	1		1	1	4	3	8		3	4	1	3	1	3	2	2	4	161
Total	36	32	2	7	2	52	41	58	18	37	4	4	3	3	14	7	10	3	12	9	3	4	2	7	7	6	9	393

Generation	
First	Comet 4, 707/720, DC-8, CV-880/-990, Caravelle, Mercure
Second	727, Trident, VC-10, BAC 1-11, DC-9, 737-100/-200, F-28
Early widebody	747-100/-200/-300/SP, DC-10, L-1011, A300
Current	MD-80, -90, 767, 757, A310, Bae 146, A300-600, 737-300/-400/-500, F-70, F-100, A320, 319, 321, 747-400, MD-11, RJ-70, -85, -100, A340, A330, 777, 737NG, 717

***Miscellaneous Accidents**

- Coffee maker explosion
- Instrument Error
- Flight attendant fell from door
- Jet blast
- Pilot incapacitated
- RTO – off end
- Taxied across ditch

10-Year Accident Rate



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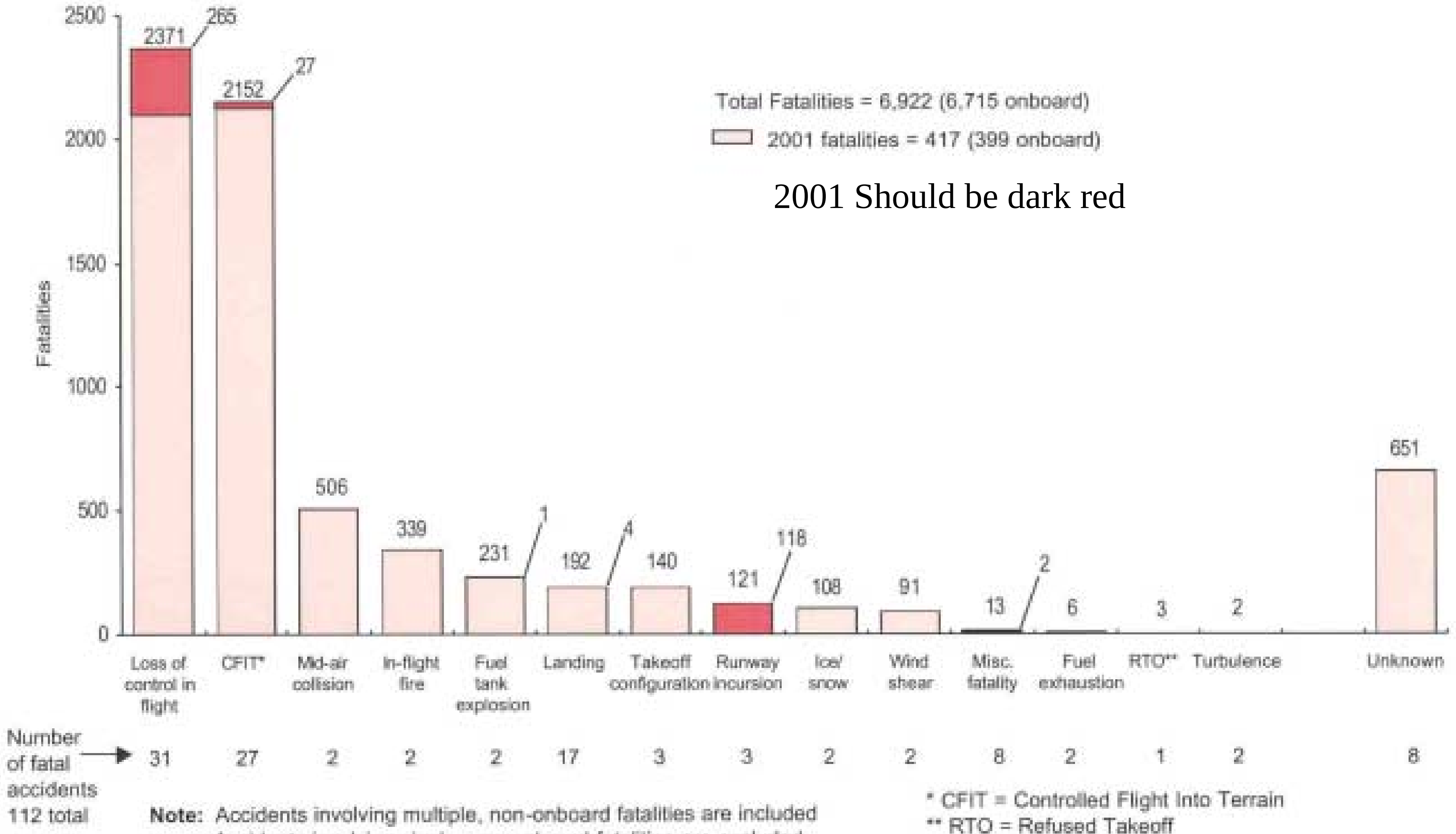
Fatalities by Accident Categories

Fatal Accidents - Worldwide Commercial Jet Fleet - 1992 through 2001



Note: Accidents involving multiple, non-onboard fatalities are included
Accidents involving single, non-onboard fatalities are excluded

* CFIT = Controlled Flight Into Terrain
** RTO = Refused Takeoff

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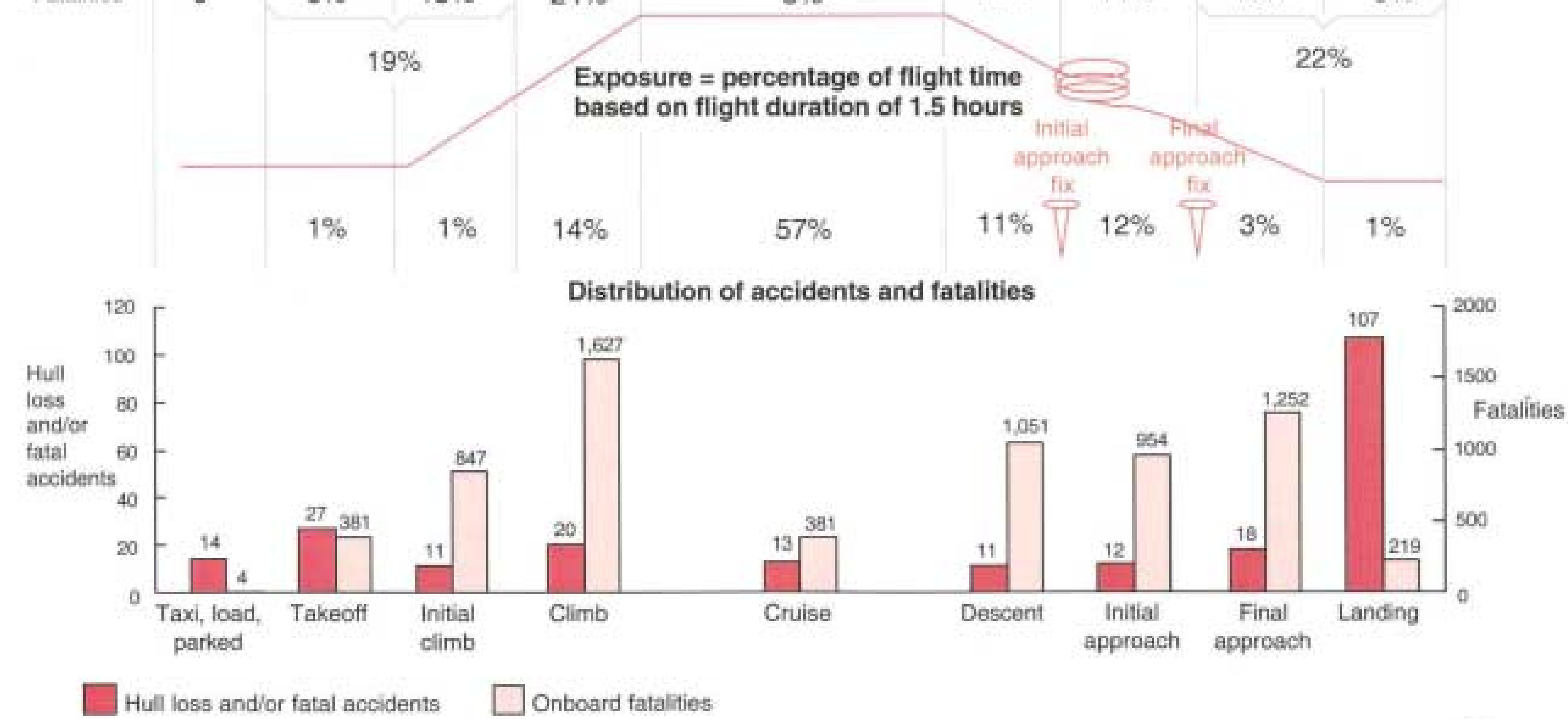
Accidents and Onboard Fatalities by Phase of Flight

Hull Loss and/or Fatal Accidents - Worldwide Commercial Jet Fleet - 1992 - 2001

Percentage of accidents/fatalities

	Taxi, load, parked	Takeoff	Initial climb	Climb (flaps up)	Cruise	Descent	Initial approach	Final approach	Landing
Accidents	6%	12%	5%	9%	6%	5%	5%	8%	46%
Fatalities	0	6%	13%	24%	6%	16%	14%	19%	3%

Takeoff + Initial climb: 17% of accidents, 19% of fatalities. Final approach + Landing: 54% of accidents, 22% of fatalities.



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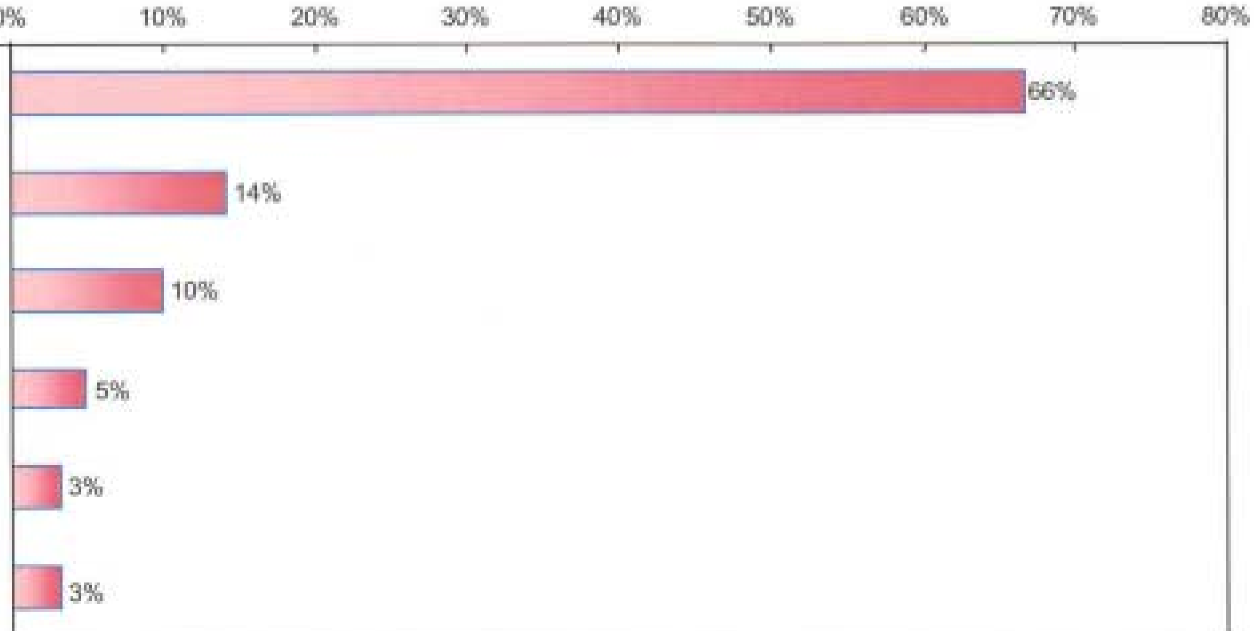
Accidents by Primary Cause*

Hull Loss - Worldwide Commercial Jet Fleet - 1992 through 2001

Flight Crew	98
Airplane	21
Weather	14
Misc./Other	8
Maintenance	4
Airport/ATC	4
Total with known causes	149
Unknown or awaiting reports	61
Total	210



* As determined by the investigating authority

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Off into Space, the New Frontier

International Space Station (ISS)

Probabilistic Design was used to balance penetration risk and cost

Penetration by space debris and meteoroids

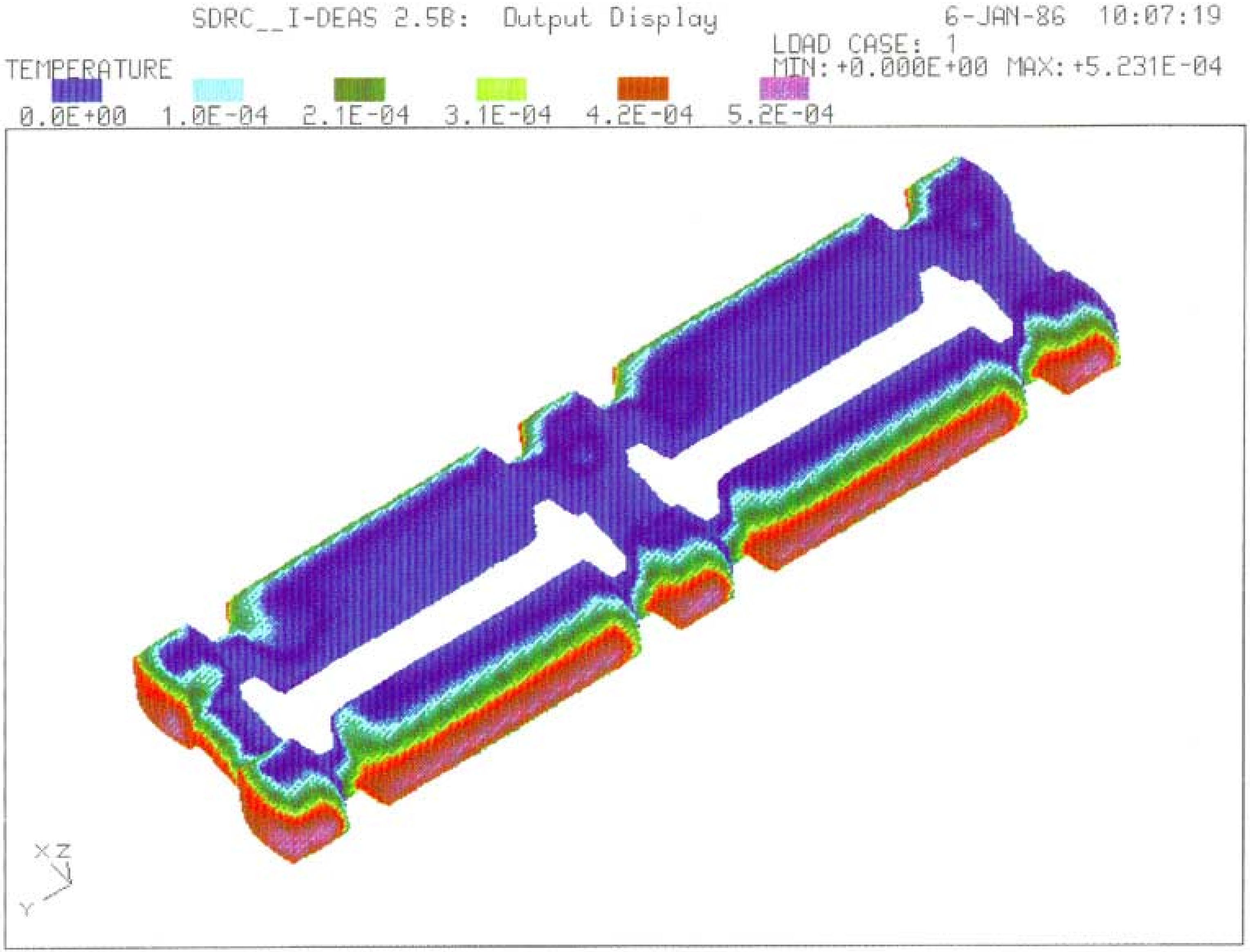
Marked Poisson process, frequency, mass, angle, and velocity, combined with engineering models of wall design and strength

Initially the risk of penetration was aimed at 5% over 10 years NASA TM-82585

Each surface element of the ISS was modeled for its risk

The Challenger disaster caused delays and increases in costs

The latest risk found: 24% risk of at least one penetration over 10 years. (Aircraft Survivability-Fall 2000)


SDRC__I-DEAS 2.5B: Output Display
6-JAN-86 10:07:19
LOAD CASE: 1
TEMPERATURE
MIN: +0.000E+00 MAX: +5.231E-04
0.0E+00
1.0E-04
2.1E-04
3.1E-04
4.2E-04
5.2E-04
X Z
Y

Some Statistical Challenges in Aircraft Industry

High reliability requirements will yield data that are highly censored.

Tracking 30,000 instances of a part in the field could easily yield just 47 failures, the rest still functioning and thus censored.

Very expensive parts only allow small sample sizes for testing.

In both cases large sample asymptotics need to be treated with care.

Meaning of 95% upper confidence bound of 2.3×10^{-8} on a risk?

How do we bring two such disparate chances under one hat?

How to regulate maintenance for large and small fleets. One adverse event in a large fleet makes for a small/acceptable rate. One adverse event in a small fleet (it has to happen somewhere) will cause a flap.

Acceptance Sampling: The c = 0 Issue

Accept shipment or lot as long as number D of defects in a sample of size n does not exceed $c = 0$, i.e., accept when $D \leq c$.

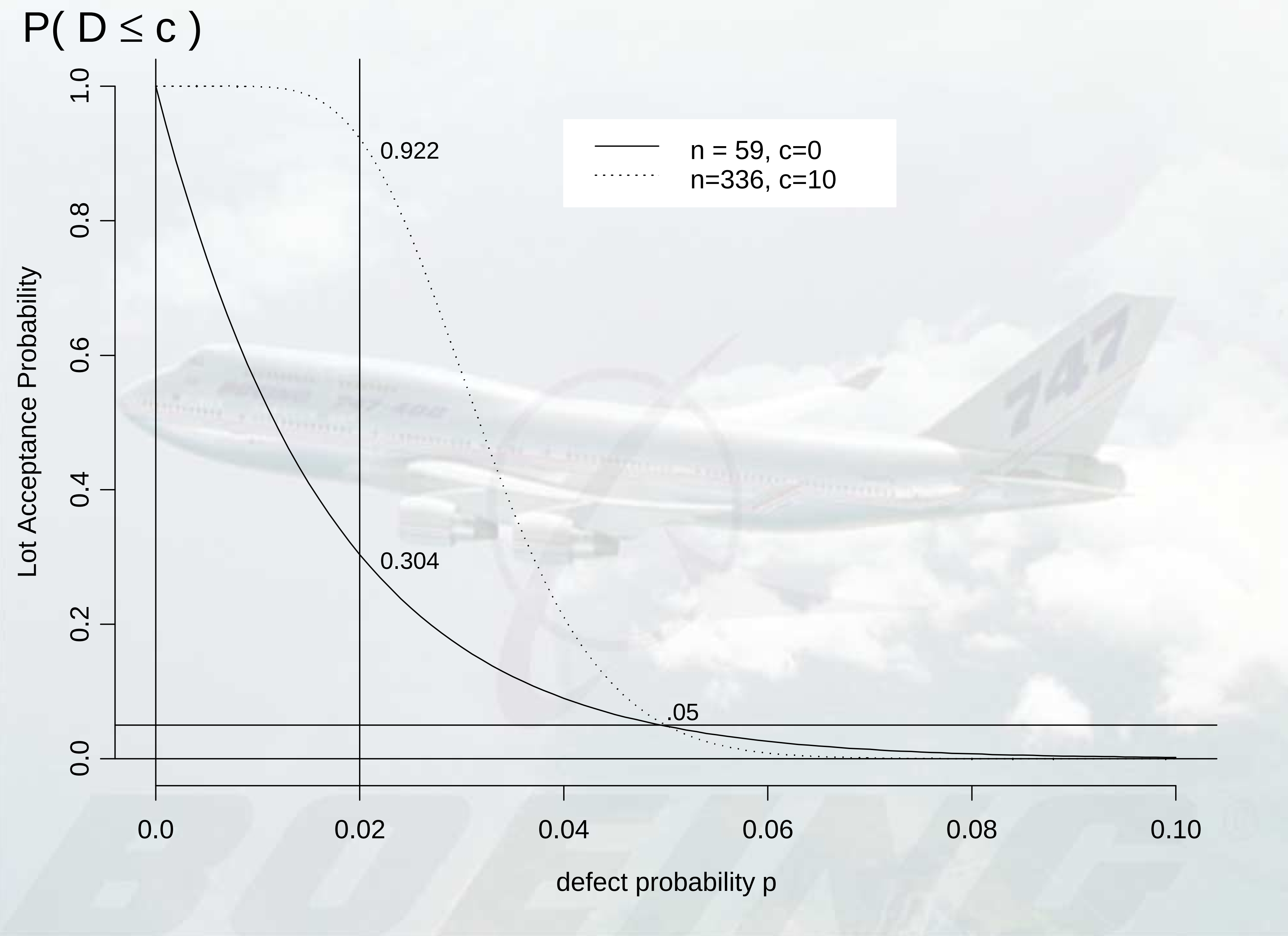
This leads to sample sizes n which guarantee a specified risk α of false lot acceptance when the true defect rate $p > p_0$

It also leads to high lot rejection rates ($\approx 1 - \alpha$) when the defect rate $p < p_0$ ($p \approx p_0$)

It is very hard to get across that a cutoff $c > 0$ with a higher n leads to much better operating characteristics.

For some people it is very difficult to accept a lot when some defects are found.

“Rejection” of lot most often means 100% inspection, high cost.



P(D ≤ c)
Lot Acceptance Probability
1.0
0.8
0.6
0.4
0.2
0.0
0.922
0.304
.05
n = 59, c=0
n=336, c=10
0.0
0.02
0.04
0.06
0.08
0.10
defect probability p

Maintenance Schedules

Strongly linked to probability of crack detection and crack growth curves.

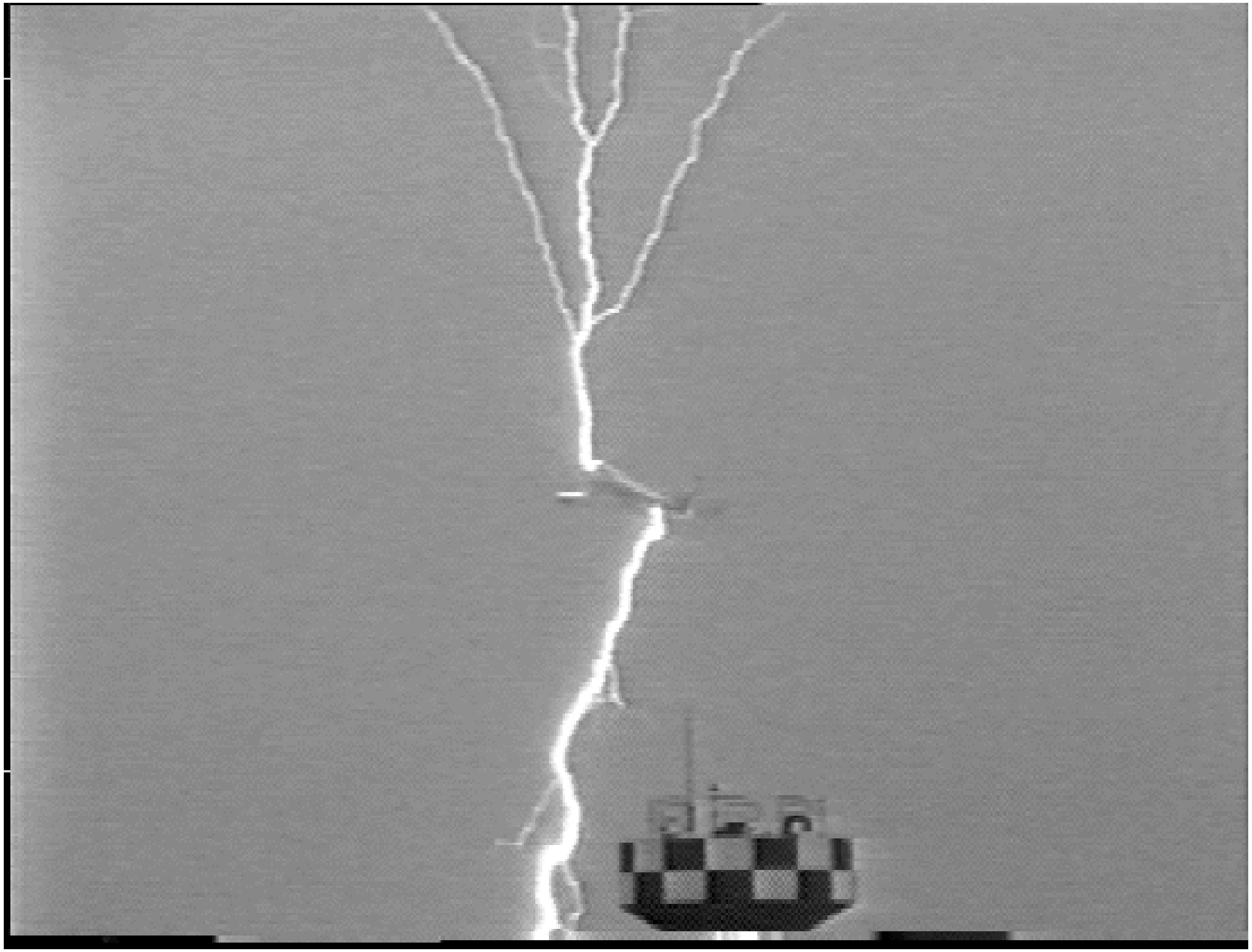
Micro-cracks are always present and it is important to catch them before they get too large.

The chance of missing them on inspection is factored in.

The fleet leaders will give warnings for the rest of the fleet when new trouble spots arise.

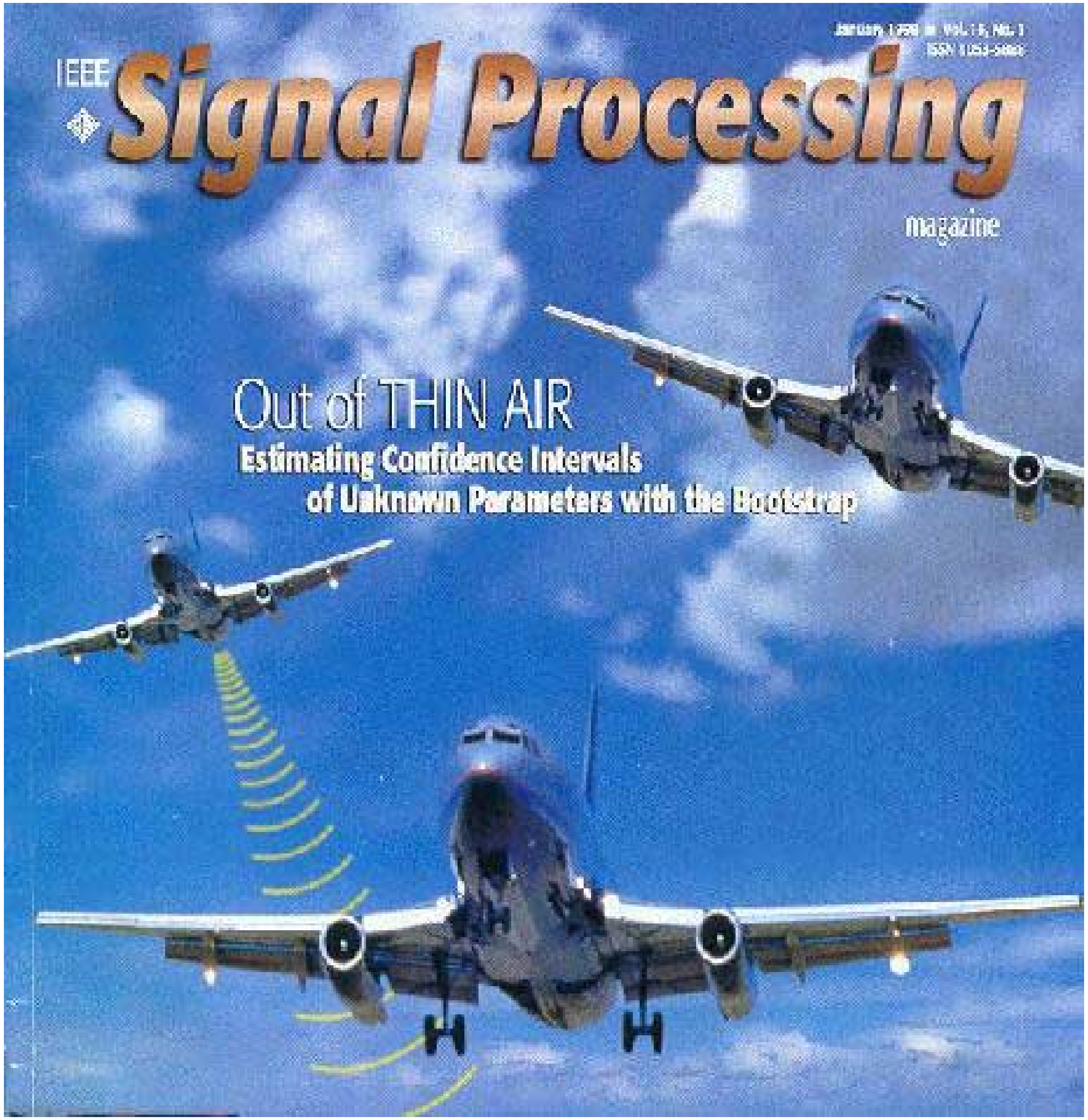
Similar strategies play a role for other wear phenomena.

Lightning strikes do happen and they present mostly a risk for composite material technology for non-conductivity reasons.



Efron's Bootstrap is 25 Years Old

It gave wings to statistics



The Weibull Distribution

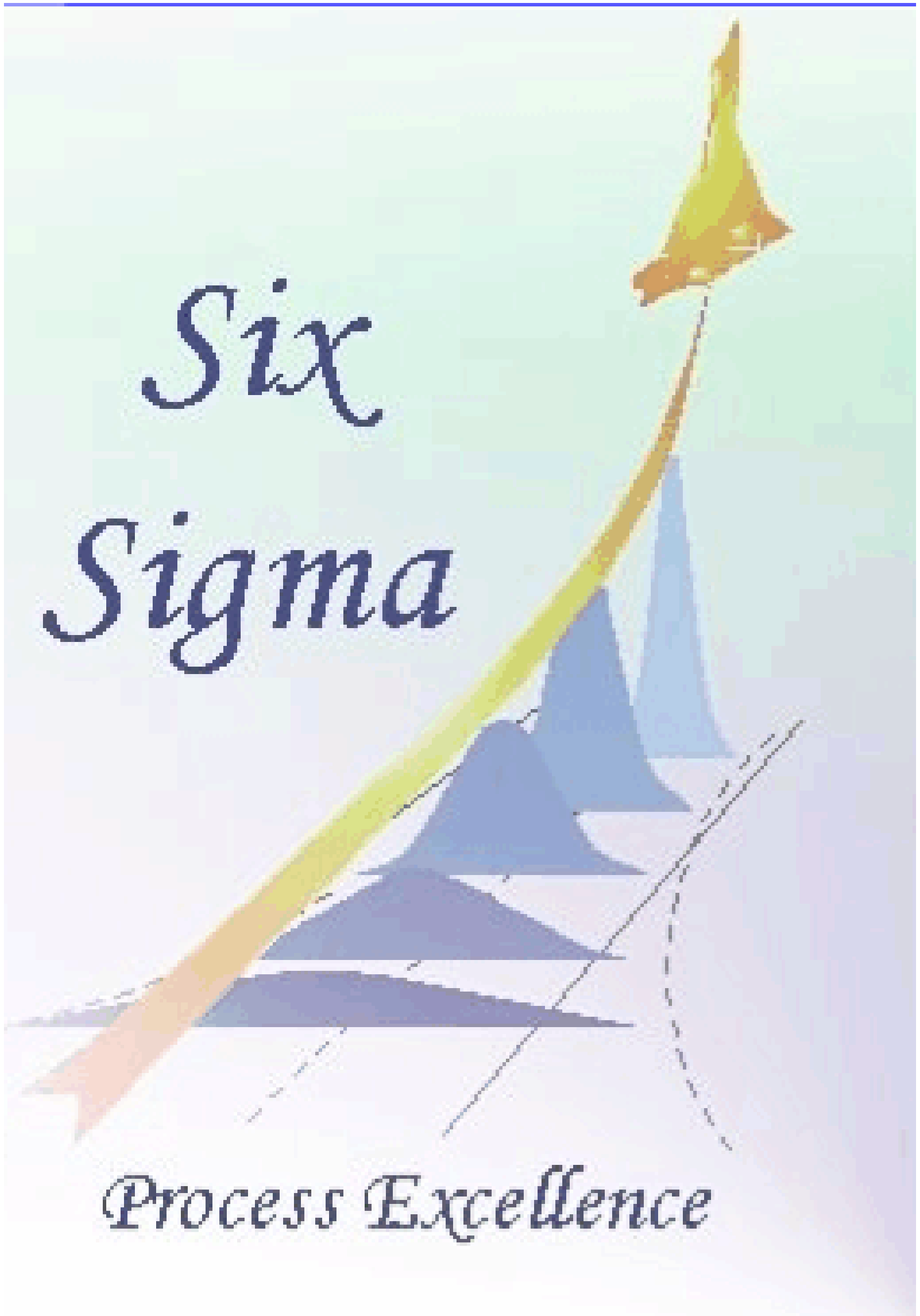
Plays a dominant role in aviation reliability.

The **Weibull Analysis Handbook** by Abernethy, Breneman, Medlin, and Reinman was originally created at Pratt & Whitney Aircraft.

Before joining Boeing I knew little about it, not exponential family.

One of my first projects led to a program for computing A- and B-Allowables for the 3-parameter Weibull distribution. It was incorporated into MIL-HDBK-5.

A- and B-Allowables are 95% lower confidence bounds for the .01-quantile and .10-quantile of a population, a double probability statement.



The six sigma program is alive and well inside Boeing.

Shobbo Basu in our group has a black belt

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BOEING SCIENTIFIC RESEARCH LABORATORIES

Tables Of The Distribution Of Quadratic Forms Of Ranks Two And Three

George Marsaglia

August, 1960

Tables of Percentiles of

$$Q_2(s,r) = P(X^2 + s^2Y^2 \le r^2)$$

where X and Y are i.i.d. $\mathcal{N}(0,1)$

and

$$Q_3(s,v,r) = P(X^2 + s^2Y^2 + v^2Z^2 \le r^2)$$

where X, Y, Z are i.i.d. $\mathcal{N}(0,1)$

Peter Hall (2002), Statistical Smoothing and the Investigation of Flight 587, *Chance 15, 4, 25-26.*

Discusses the smoothing that pilots see and as it is recorded on the Flight Data Recorder (FDR).

The current (old) smoothers flatten irregular signals, Hall advocates wavelets, a relatively new methodology.

It can handle irregular signals, which might be important to the pilots.

In 1994 the NTSB recommended that unsmoothed data be fed to FDR, and the FAA accepted this recommendation in 1997.

Hall also recommends that some smoothed data be recorded with the raw data. Presumably to be sure of what the pilot saw on the display in case of an accident.