Case Study in Statistical Tolerancing

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THE PROBLEM OF ATTACHING A CARGO DOOR TO AN AIRPLANE BODY WITH A SERIES OF TEN HINGES IS SIMPLIFIED BY USING STATISTICAL TOLERANCING. AT ISSUE IS WHETHER THE GAPS AND LUGS OF THE HINGES ASSEMBLE SUCCESSFULLY WHEN ALLOWING FOR VARIATION IN LUG WIDTHS AND TRANSLATIONS, AND IN THE POSITIONING OF THE HINGES ON THE DOOR AND BODY OF THE AIRPLANE. THE PROBLEM IS TREATED ONLY IN THE HINGE AXIS DIRECTION, AND IS FIRST ATTACKED IN THE CONTEXT OF AN UNFASTENED HINGE PAIR. IT CAN THEN BE GENERALIZED TO THE WHOLE SEQUENCE OF FASTENED HINGES AND SOLVED BY VIEWING THE TWO SEQUENCES EACH AS COMPOSITE HINGE HALVES.

TOLERANCING & METROLOGY

PART 2

CASE STUDY

A cargo door is to be attached to an airplane body with a sequence of ten hinges. Figure 1 shows a stylized version of such a hinge together with its clearance criterion, C. We address some tolerancing aspects that need to be considered for fitting the door to the body properly. The tolerancing problem will only be treated in a one-dimensional (1D) setting, namely for variations along the hinge axis direction. Such variations cover the widths of lugs and gaps, their translations (positioning of lugs and gaps), and the positions of hinge pairs on door and body. Figure 2 shows an example of the tolerance specifications in the design of one hinge half.

The problem is first treated for a single hinge pair not fastened to the door or the body. Of interest is the assembly clearance criterion, C, which must be positive for proper assembly of the two hinge halves to occur, as illustrated in Fig. 1. Suppose we were able to make the fit criterion function, C, linear in terms of the input variations, in other words, we could write:

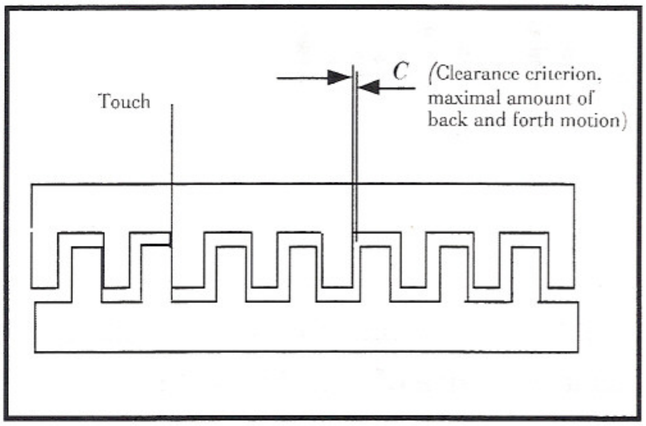

Fig. 1. Stylized hinge and its clearance criterion, C.

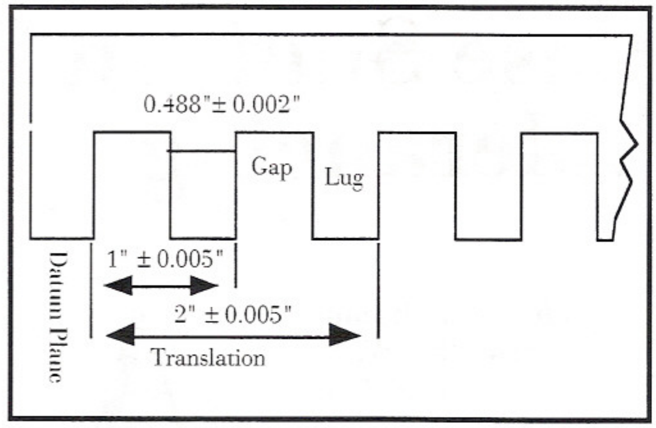

Fig. 2. The tolerance specifications in the design of one hinge half.

$$C = f(X_1, K, X_n) \approx \alpha_0 + \sum_{i=1}^{n} \alpha_i(X_i - \mu_i),$$

where the μ_i are the nominal dimensions, and the α_i are known coefficients. X_i are the actual dimensions that are treated as varying randomly around their mean values, μ_i. Then we have two benefits. The first is that we can easily compute approximate values for the mean and variance of C. The second is that we can appeal to the Central Limit Theorem of Probability Theory to treat C as approximately normal even if the perturbations $X_i - \mu_i$ are themselves not normal. This gives an easy and compact way of describing the variability of the output criterion C. Because of the linearization, one refers to the above type of tolerance analysis as linear tolerance stacking. This paradigm is very pervasive in the statistical tolerancing literature, see [1–4]. However, the above linearization requires that the function f be sufficiently smooth, that is, it has first order partial derivatives, which is not the case in this situation. Nevertheless, a good description of the distribution of C is possible by simulation. We can generate the random inputs $X_1, ..., X_n$, evaluate f to get one value of C, and repeat this process many times over.

After resolving the hinge assembly problem, proceeding to the door to body assembly problem is a simple conceptual step. Here we deal with assembling the hinges fastened to the door with the hinges on the airplane body, with no other issues concerning proper fitting of door and body. The idea is to treat the door and its fastened hinges as one half of a composite hinge pair, and likewise to treat the body and its fastened hinges as the corresponding half of the composite hinge pair. The remainder of the problem is mainly one of notation and tracking the various sources of variation and dependencies, but that is easily dealt with in the simulation.

SINGLE HINGE PAIR ASSEMBLY CRITERION

To start, we address the assembly of a single hinge pair that has not yet been fastened to the door or the airplane body, that is, the two halves can be moved freely in the hinge axis direction until the lugs and the corresponding gaps are aligned correctly. This assumes that lugs and gaps have proper widths and translations.

It is also assumed that a fixed reference point on the hinge axis of each hinge half exists. This reference point defines the primary datum plane perpendicular to the hinge axis. As noted above, we will only deal with this problem in the direction of the hinge axis. All lug positions can be described relative to this reference point. For example, one could specify the position of the right edge of each lug with respect to this point (baseline dimensioning). This would avoid tolerance stack buildup in machining the right lug edges. A buildup could occur if the position of the right edge of a lug was specified as the distance from the right edge of the previous lug (chain dimensioning). The lug widths would typically be specified as the distance of the left lug edge from the established right lug edge. Specifying the positions of the right lug edges and the lug widths automatically establishes the dimensions of the gaps by complementation. The specifications are nominal dimensions with bilateral tolerances, as illustrated in Fig. 2.

In order to properly describe the assembly criterion, we introduce the notion of "left and right clearances," shown in Fig. 3. It is assumed that the two hinge halves are positioned so that the left edge of the leftmost lug in the bottom hinge is aligned with the left edge of the leftmost gap of the top hinge. This common position is shown to coincide with the datum plane in Fig. 3.

Usually the right facing edge of a lug on the bottom hinge half has to clear the left facing edge of the corresponding lug on the top hinge half. Let the positions of these two interacting edges be $N + X$ and $N' + X'$, respectively. Here N and N' stand for the nominal values and X and X' are the random variables that express the variation from nominal. They typically would be assumed to have mean zero and their probable range should cover the specified ± tolerance. They could also be composed as a sum of other random variables. The latter specification could occur if the relevant lug edge is established by measuring off the other lug edge and then measuring the width of the lug to get to the edge in question. In that case two independent variations would be compounded in an additive fashion. The difference, $L = N' + X' - (N + X)$ is referred to as a *left clearance.* If this clearance is positive, we could still move the upper hinge half to the left (hence the name left clearance) by an amount L before interference occurs at that interface.

Similarly, the left facing edge of a lug on the bottom hinge half has to clear the right facing edge of the corresponding lug on the top hinge half. Let the positions of these two interacting edges be $M + Y$ and $M' + Y'$, respectively. Here M and M' stand for the nominal values and Y and Y' are random variables that express the variation from nominal. The difference $R = M + Y - (M' + Y')$ is referred to as a *right clearance.* If this clearance is positive, we could still move the upper hinge half to the right (hence the name right clearance) by an amount R before interference occurs at that interface.

In the design implied by Figs. 1 and 2 there are n left clearances $L_1, \ldots, L_n$, with $L_i = N_i' + X_i' - (N_i + X_i)$, and $m = n - 1$ right clearances $R_1, \ldots, R_m$ with $R_i = M_i + Y_i - (M_i' + Y_i')$. As long as all left clearances are positive we can still move the upper hinge half to the left by the amount

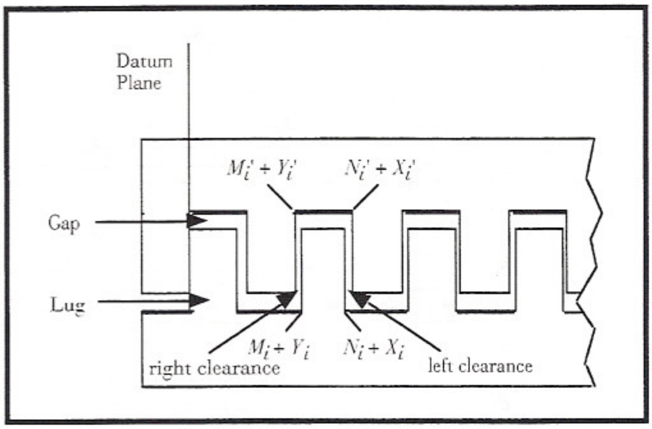

Fig. 3. Example of left and right clearances.

$$T_L = \min(L_1, \ldots, L_n)$$

without causing interference at any of these left interfaces. Similarly, as long as all right clearances are nonnegative, we have

$$T_R = \min(0, R_1, \ldots, R_m) \geq 0,$$

we have no interference problem at any of these right interfaces. The zero in the definition of T_R accounts for the alignment at the datum plane. Finally, if $C = T_L + T_R$ is positive, we have enough "play," namely the amount C, in moving the top hinge half left or right, so that no interferences occur at any of the hinge interfaces. Requiring that C be positive for clearance does not mean that both T_L and T_R must be positive. For example, we could have $T_R < 0$ but T_L is sufficiently large to be positive. In that case we could make $T_R > 0$ as well through left motion of the upper hinge half, but in the process we also reduce T_L. Of course, it is assumed that these movements are so minute that no other interference issues arise.

In typical hinge design the nominal left and right play is constant from lug to lug, $N_i' - N_i = \Delta$ and $M_i - M_i' = \delta = 0$ (again the zero results from the alignment at the datum plane), so that

$$T_L = \Delta + \min(X_1' - X_1, \ldots, X_n' - X_n)$$

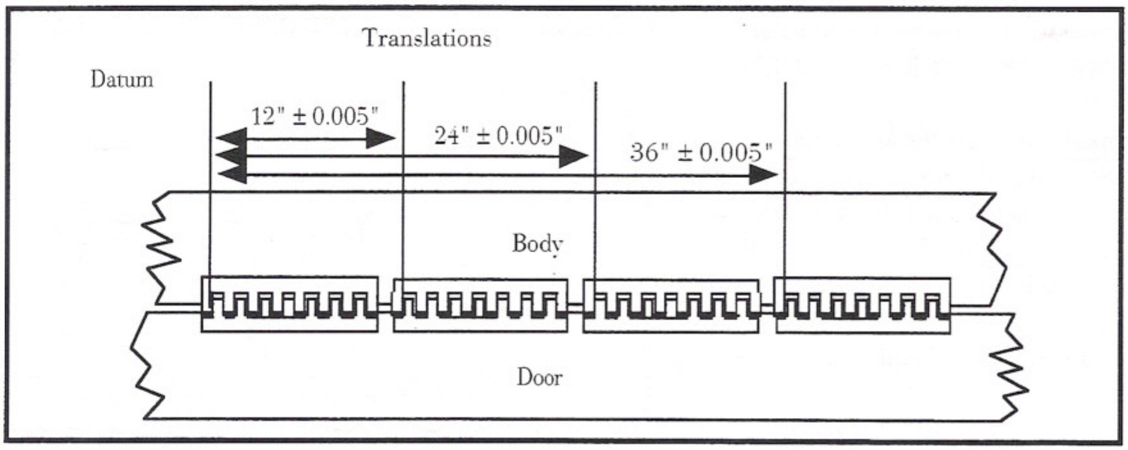

Fig. 4. Attaching the door to the airplane body.

and

$$T_R = \delta + \min(Y_1 - Y_1', \ldots, Y_m - Y_m').$$

This shows that neither T_L nor T_R is differentiable with respect to X_i, X_i', Y_i, Y_i' at their nominal values zero. Also, it is not easy to derive analytically the distribution of T_L, T_R and thus C, because the X's and Y's are not all independent. This dependence is due to the carry-over effect of the error in determining one lugedge and measuring off the lug width to obtain the position of the other edge. Thus, left and right clearances can be affected by the same source of variation. However, such dependencies are mild, because their respective scopes are limited to one lug at a time, and they are easily handled when simulating the distribution of C in order to assess what proportion of simulated hinge pairs have positive clearance with the chosen design.

DOOR TO BODY ASSEMBLY CRITERION

Having dealt with the single hinge pair, we can now carry over this approach to the assembly of the door (with ten attached hinges) to the airplane body (with its corresponding attached hinges). This is illustrated in Fig. 4. As noted above, the problem is solved by treating door and body as two composite hinge halves, but with greater variety of lug/gap structure. The only added complication is that now the notation and analysis must be carried out with respect to some pair of fixed reference points on the door and body. We also need to account for the additional variation from nominal in the positioning of the reference points of the individual hinges within the composite hinge structure. This time we have many more left and right clearances, say $L_1, \ldots, L_I$ and $R_1, \ldots, R_J$. Using the same reasoning as in the single example above, we have as the overall clearance criterion

$$C_T = \min(L_1, \ldots, L_I) + \min(0, R_1, \ldots, R_J),$$

where the zero again accounts for alignment at the datum plane.

MODELING SOURCES OF VARIATION

A position error in one particular hinge affects all the left and right clearances coming from this hinge in the same way. It provides a common random effect on those clearances, and as such introduces strong dependencies or correlations among these clearances. Superimposed on this effect are the variations within that hinge, namely the lug width errors and the translation errors that are typically independent from lug to lug, but which may, as previously discussed, introduce some additional mild dependencies in the left and right clearances from that hinge. The position error of the hinges depends heavily on how the positioning is accomplished. For example, we may use a special tool that shows the positions for each of the hinges on the body. These positions may be in error, but one of the positions may serve as a reference plane for another lug and, thus, has no error. These tool errors would then act as a consistent bias in positioning the hinges. However, even this error can change if the tool is recalibrated occasionally. In addition to this type of error, we must also allow for the hinge positioning and fastening error relative to the tool.

Similarly, we may use another tool for positioning the hinges on the door. This tool may have had its hinge position indicators established with error, either relative to an absolute reference point or relative to the tool used for positioning the hinges on the body. This error also acts as a random bias, unless the tool is recalibrated occasionally.

Writing out the probabilistic model for the various sources of variation entering the clearance criterion, C_T, while omitted here, is tedious yet straightforward. Fortunately, simulating the distribution of C_T poses no difficulty.

CONCLUSION

In translating the various tolerances for statistical tolerancing one may use normal distributions with a $\pm 3\sigma$ range over the given tolerance intervals for lug width, translation, and positioning of hinges on doors and body. All these variations are assumed to be independent, but can lead to significant dependence among the various left and right clearances. Alternatively, one can try uniform variations over the tolerance ranges, with naturally higher likelihood of assembly failure. In the particular example for which the above approach was executed we found that with normal variation about 99.4 percent of the doors would have assembled properly, but with uniform variation only 14.2 percent assembled.

Such simulations should be quite useful in deciding how to set tolerances and design clearances. In particular, simulation allows the effects of tool errors to be studied. Since many factors can be varied, namely hinge positioning/fastening tolerances, tool tolerances, lug translation and width tolerances, and design clearances, it may be useful to employ experimental design techniques to sort out the best design factors. However, as the difference in clearances under normal and uniform variation has already indicated, it is of much greater value to use real data, and to find answers to the following questions: How good is the positioning of the hinges? What tolerances can be achieved in producing the hinges? It is of little use to design something and then be unable to hold the stated tolerances. The real variation in a process ultimately drives the failure rate.

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