Stat 425 HW9

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1. This problem is about comparing the exact power of the sign test (in the case of normal shift alternatives) with the two normal approximations given for that power on slides 15 (with continuity correction) and 19. Assume that the sign test rejects for large values of S_N and that the critical point $c = c_{\text{crit}}$ is determined such that $P_{H_0}(S_N \geq c) \leq \alpha$ and $P_{H_0}(S_N \geq c-1) > \alpha$. To determine c make use of `qbinom`.

Write a function `signtest.power=function(N,alpha)`$\{\ldots\}$ that does this evaluation of the exact power and its approximations for a spectrum of alternatives

$$\Delta/\sigma \in \texttt{Delta.sig} = \texttt{seq(0,(3+qnorm(1-alpha))*sqrt(pi/(2*N)),length.out = 20)}$$

and plots the exact power and both its approximation against `Delta.sig`, using different line types and colors, as indicated by an appropriate legend (using `legend(...)`). Superimposed on this comparison of the exact power function and its approximations plot the power function of the t-test

$$1 - \texttt{pt}(\texttt{qt}(1-\alpha_c, \texttt{N}-1), \texttt{N}-1, \texttt{sqrt(N)*Delta.sig[i]})$$

and also the same power function of the t-test with N reduced to $N' = N \times 2/\pi$. Use different line types for these last two power functions and add them to your legend (which should show 5 lines with different line types). Put the legend in the upper left corner at (0,1). Add a horizontal line at the nominal level α. On this plot annotate the values of N, α, c_{crit}, and the achieved level α_c when using $c = c_{\text{crit}}$. Make sure that you use α_c in the second normal approximation.
Provide the code of `signtest.power` and the resulting plots for $\alpha = .05$ and $N = 15$ and $N = 50$.

2. Repeat the above by comparing the power curve of the Wilcoxon signed rank test (rejecting H_0 for $V_s \geq c$) with both of its normal approximations. Here you want to use `PowerSignRank` (as posted on the class web site). It estimates the exact power by using `Nsim` simulations, and it also gives the two approximation values.
Call the function that does this comparison

$$\texttt{signedranktest.power = function(N = 20, alpha = .05, Nsim = 10000)}\{\ldots\}$$

As plotting grid use here

$$\texttt{Delta.sig = seq(0,(3+qnorm(1-alpha))*sqrt(pi*N*(N+1)*(2*N+1)/24)/}$$
$$\texttt{(N*(N-1)/2+N/sqrt(2)),length.out = 20)}$$

and evaluate the power and its approximations using the appropriate elements of the fifth vector in the list returned by

$$\texttt{PowerSignRank(..., Delta = Delta.sig[i], scale = 1, ...)}$$

While developing this function you may want to use `Nsim=100` or `Nsim=1000`. Superimposed on this comparison of the simulated power function and its normal approximations plot the power function of the t-test

$$1-\texttt{pt}(\texttt{qt}(1-\alpha_c,\texttt{N}-1),\texttt{N}-1,\texttt{sqrt}(\texttt{N})*\texttt{Delta.sig}[\texttt{i}])$$

and also the same power function of the t-test with N reduced to $N' = N \times 3/\pi$ (note the different factor $3/\pi$ here as compared tp $2/\pi$ in problem 1). Use different line types for these last two power functions and add them to your legend (which should show 5 lines). Put the legend in the upper left corner at (0,1). Add a horizontal line at the nominal level α. On this plot annotate the values of N, N_{sim}, α, c_{crit}, and the achieved level α_c when using $c = c_{\text{crit}}$.
Give the code of your `signedranktest.power` and give plots for $\alpha = .05$ and $N = 10$ and $N = 20$, using $\texttt{Nsim} = 10000$.