Stat 425 HW1 Solutions

Fritz Scholz

Chapter 1, Section 1, Problem 8.:
In a study of a new approach to elementary mathematics. 10 children are selected at random from a group of 21 to be taught by the new method with the other 11 serving as controls. Find the probability that by chance the treatment group (i) consists of the 10 most intelligent children; (ii) contains the most intelligent child; (iii) contains the three most intelligent children.

Solution: (i)

$$\begin{aligned} P(\text{treatment group contains the 10 most intelligent children}) &= \frac{1}{\binom{21}{10}} \\ &= 1/\texttt{choose}(21,10) = 1/352716 \\ &= \texttt{2.835142e}-\texttt{06}. \end{aligned}$$

(ii)

$$P(\text{treatment group contains the most intelligent child}) = \frac{\binom{1}{1}\binom{20}{9}}{\binom{21}{10}}$$

$$= \texttt{choose}(1,1) * \texttt{choose}(20,9)/\texttt{choose}(21,10) = 0.4761905.$$

(iii)

$$P(\text{treatment group contains the 3 most intelligent children}) = \frac{\binom{3}{3}\binom{18}{7}}{\binom{21}{10}}$$

$$= \texttt{choose}(3,3) * \texttt{choose}(18,7)/\texttt{choose}(21,10) = 0.09022556.$$

Chapter 1, Section 1, Problem 15.:
Continue the enumeration (1.11) to the values $W_s = 20$ and $W_s = 21$, and verify the probabilities $P_H(W_s \le 20)$ and $P_H(W_s \le 21)$ given in the text.

Solution:

$$\begin{array}{lcllcl} 1+2+3+4+10 &=& 20 & 1+2+4+6+7 &=& 20 \\ 1+2+3+5+9 &=& 20 & 1+3+4+5+7 &=& 20 \\ 1+2+3+6+8 &=& 20 & 2+3+4+5+6 &=& 20 \\ 1+2+4+5+8 &=& 20 & & & \end{array}$$

$$\Longrightarrow \quad P_{H_0}(V_s \le 20) = \frac{12+7}{252} = 0.07539683 = \texttt{pwilcox}(20-15,5,5)$$

$$
\begin{array}{rcl|rcl}
 & & & 1+2+4+6+8 & = & 21 \\
1+2+3+5+10 & = & 21 & 1+2+5+6+7 & = & 21 \\
1+2+3+6+9 & = & 21 & 1+3+4+5+8 & = & 21 \\
1+2+3+7+8 & = & 21 & 1+3+4+6+7 & = & 21 \\
1+2+4+5+9 & = & 21 & 2+3+4+5+7 & = & 21
\end{array}
$$

$$
\Longrightarrow \quad P_{H_0}(V_s \leq 21) = \frac{12+7+9}{252} = 0.1111111 = \texttt{pwilcox}(21-15,5,5)
$$

Chapter 1, Section 1, Problem 22.:
From a group of nine rats available for a study of the transfer of learning, five were selected at random and were trained to imitate leader rats in a maze. They were then placed together with four untrained control rats in a situation where imitation of the leaders enabled them to avoid receiving an electric shock. The results (the number of trials required to obtain ten correct responses in ten consecutive trials) were as follows:

Trained rats:	78	64	75	45	82
Controls:	110	70	53	51	

Find the significance probability of these results when the Wilcoxon test is used.
Solution:

```
rats =function(){
rats.t=c(78,64,75,45,82)
rats.c=c(110,70,53,51)
m=length(rats.c)
n=length(rats.t)
W.s= sum(rank(c(rats.t,rats.c))[1:5])
out1=wilcox.test(rats.t,rats.c,alternative="less")
out2=mean(combn(1:(m+n),n,FUN=sum)<=W.s)
list(out1=out1,out2=out2)
}
```

The argument `alternative="less"` indicates that under the alternative to H_0 the first argument (`rats.t`) is expected to be generally less than the second argument (`rats.c`).
The following call produced

```
> rats()
$out1

Wilcoxon rank sum test

data:  rats.t and rats.c
W = 11, p-value = 0.6349
alternative hypothesis: true location shift is less than 0


$out2
[1] 0.6349206
```

i.e., the significance probability is 0.6349206.

Chapter 1, Section 1, Problem 25.:
Under the conditions of the preceding problem, suppose that $m = n = 9$ and the observed recovery times are (in days)

Controls:	20	21	24	30	32	36	40	48	54
Treatment:	19	22	25	26	28	29	34	37	38

Find the significance probability of these results when the Wilcoxon rank-sum test is used.

Solution: Since one would hope that the new treatment results in faster recovery times we would look for low values of W_s being significant.

```
postsurgical = function(){
control=c(20 ,  21 , 24 , 30 , 32 , 36 , 40 , 48 , 54)
treatment=c(19 , 22 , 25 , 26 , 28 , 29 , 34 , 37 , 38)
wilcox.test(treatment,control,alternative="less")
}
```

where the `alternative="less"` indicates that (under the alternative) the first argument is expected to be generally smaller than the second, which is what we want. We get the following result when calling this function.

```
> postsurgical()

Wilcoxon rank sum test

data:  treatment and control
W = 31, p-value = 0.2181
alternative hypothesis: true location shift is less than 0
```

The result does not speak strongly for the new surgical procedure.