# Homework 7
# Due March 6, 2019

**Problem 1:** Let $X \sim \text{Bin}(n,p)$. Find a simple expression for $P(X \text{ is even})$. Hint: make use of the binomial expansions of $(p+q)^n$ and $(q-p)^n$, where $q = 1-p$. In these expansion view the individual summands as $P(X=i)$ and see what you get when you look at $(p+q)^n + (q-p)^n$. Discuss the effect of $p <, =, > .5$ and $n$.

**Problem 2:** Let $S_n \sim \text{Bin}(n, p=.7)$. Approximately, how large would $n$ have to be in order to get

$$P\left(\left|\frac{S_n}{n} - .7\right| \le .001\right) \ge .999$$

**Problem 3:** Find the probability of getting a single pair in a random poker hand of 5 cards from a regular deck of 52 cards. For example, 2 kings, an ace, a 7 and a jack would be a single pair. Using this, find the approximate probability that you get at least 450 hands with a single pair in 1000 independent poker hands.

**Problem 4:** If I park on campus it will cost me \$6 for a parking voucher per day that I park. If I don't use a voucher and I get caught for not displaying it the fine is \$35. From past experience it is known that the chance of being caught out during the period when I park on campus is $p = .2$. Find the approximate chance that my net loss $L_n$ due to being caught out during $n = 100$ visits on campus (while parking illegally and thus giving me a \$6 $n =$ \$600 gain) is at most \$150. Discuss the impact on my flagrant behavior if a) they increase the fine, b) they increase the price of the parking voucher, c) they increase the surveillance on illegal parking. Explain the effect in each case on the probability $P(L_n \le 150)$.
You can treat the number of ticketed illegal parking events as a binomial random variable.
Since the normal approximation for a binomial random variable tends to fall apart for $p \approx 0$ and $p \approx 1$ it is better to answer question c) by looking at $P(L_n \le 150)$ directly. To that end show that for a binomial random variable $X_n \sim \text{Bin}(n,p)$ the probability $P(X_n \le k)$ is a decreasing function of $p$. To show this take the derivative of $P(X_n \le k)$ with respect to $p$, make use of the identities $i\binom{n}{i} = n\binom{n-1}{i-1}$ and $(n-i)\binom{n}{i} = n\binom{n-1}{i}$ and cancel out equal terms until you are left with an expression involving a simple multiple of $P(X_{n-1} = k)$.

**Problem 5:** A survey firm is tasked to estimate the proportion $p$ of a large voting population which would vote yes on an important proposal. Since the population is large and we take a random sample of size $n$ we can view such a survey count $X_n$ of "yes" answers as a binomial random variable, i.e., $X_n \sim \text{Bin}(n,p)$. Regardless of the unknown $p$ how large a sample size $n$ should be taken so that $P(|X_n/n - p| \le .01) \ge .95$. For that choice of $n$ determine for how small a $p$ a normal approximation makes sense according to the traditional rule of thumb $np(1-p) > 10$.