University of Washington

STATISTICS

Probability and Statistics Applications in Aviation and Space

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Great work if you can get it

The Associated Press

CHICAGO — Here is a list of 250 American jobs as rated by the editors of "The Jobs Rated Almanac." **(See story, Page A1.)**

1. Actuary
2. Computer programmer
3. Systems analyst
4. Mathematician
5. Statistician
6. Hospital administrator
7. Industrial engineer
8. Physicist
9. Astrologer
10. Paralegal (legal assistant)
11. Bank officer
12. Motion picture editor
13. Biologist
14. Technical-copy writer
15. Accountant
16. Civil engineer
17. Print editor
18. Pharmacist
19. Political scientist
20. Astronomer
21. Aerospace engineer
22. Broadcast newswriter
23. Physiologist

80. Optician
82. Typist-word processor
83. Attorney
84. Medical laboratory technician
85. Airplane pilot (commercial)
86. Barber
87. Rabbi
88. Physical therapist
89. Florist
90. Dental laboratory technician
90. Social worker
92. Architect
92. Computer operator
94. Dental hygienist
95. Newscaster
96. Medical secretary
96. Secretary
98. Buyer
99. Personnel recruiter
100. Flight attendant
100. Insurance agent-salesperson
100. Telephone operator
103. Carpet-tile installer
104. Telephone installer-re-

162. Cook
163. Home appliance repairer
164. Mail carrier
165. Communications equipment mechanic
166. Electrician
167. Child-care worker
168. Railroad conductor-engineer
169. Waiter-waitress
170. Tool-and-die maker
170. Travel agent
172. State police officer
173. Janitor
173. Machine tool operator
175. Office machine repairer
176. Precision assembler
177. Maid
177. Recreation worker
177. Retail salesperson
180. Aircraft mechanic
181. Fashion designer
182. Forklift operator
183. Dressmaker
184. Public relations specialist
185. Emergency medical

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Reliability and Risk

- reliability & risk analysis (high reliability through redundancy design)
- analysis of lifetime data, characterized by few failures, much censoring
- assess significance of incidents in proper context (constant vigilance)
- Probability of failure 10^{-9}, an industry standard
- Boeing was one of the cradles of Reliability as a separate discipline
  BSRL (Boeing Scientific Research Laboratory), demise 1971, Boeing bust

One Accident!

A Guiding Principle: Academia vs Industry

- the task is to find a path from **A** to **B**

- after considerable effort one finds that a path to **C** is easier & elegant

- this works fine for academic publishing since nobody insists on the goal **B**.

- In Industry you keep your eye on **B**,
  and achieve it by whatever hop, skip, and jump Method
  possibly not an airtight argument,
  but supported by simulations and other checks

- This is Usually not Elegant or Publishable, often Proprietary

Some of my Projects over Time (1)

- Detection of Electric Power Theft [Analyze consumption and other patterns or more effective screening for potential diverters] (Customer: EPRI)

- Detection of Nuclear Material Diversion [Measurement error, what if diverter during shipments of nuclear material takes out only small amounts, easily hidden behind individual measurement errors. Application of game theory: do the best against the best diversion strategy.] (Customer: NRC)

- Meteoroid and Space Debris Risk Assessment for ISS (Customer: NASA) More at end if time permits.

- Software Reliability, Technology Reliability Growth [Bug or Glitch Removal] (Customer: NASA) The object of study changes through data collection.

Some of my Projects over Time (2)

- Engine Shut-Downs in Relation to Flight Hours & Cycles [Rules for ETOPS (Extended-range Twin-engine Operational Performance Standards)]

- Aircraft Accidents in Relation to Crew Size (2 vs 3, Simpson's Paradox)
  Hidden or ignored factor was aircraft size.

- Lightning Risk Assessment, A6 Composite-Wing Program (Attachment Points)
  Limited lab tests $\rightarrow$ conceptual sphere: continents $\leftrightarrow$ attachment points.

- Setting Guarantees for Interior Aircraft Noise (Random Curves)
  Guarantees could vary along the fuselage, (quieter first class, etc.)
  80% chance of meeting guarantees with 95% confidence.

Simple Recipe for Random Curves

$Y_1, \ldots, Y_k$ iid $\sim \mathcal{N}(\mu, \sigma^2)$ (normal random sample)

How to generate a new $Y^\star$ using estimates $\bar{Y}$ and s^2 for μ and σ^2?

Generate $Y^\star$ from $\sim \mathcal{N}(\bar{Y}, s^2)$ or equivalently but more laboriously

$Y^\star = \bar{Y} + \sum_{i=1}^{k} w_i(Y_i - \bar{Y})$ with w_i iid $\sim \mathcal{N}(0, 1/(k-1))$ (only the w_i are random here)

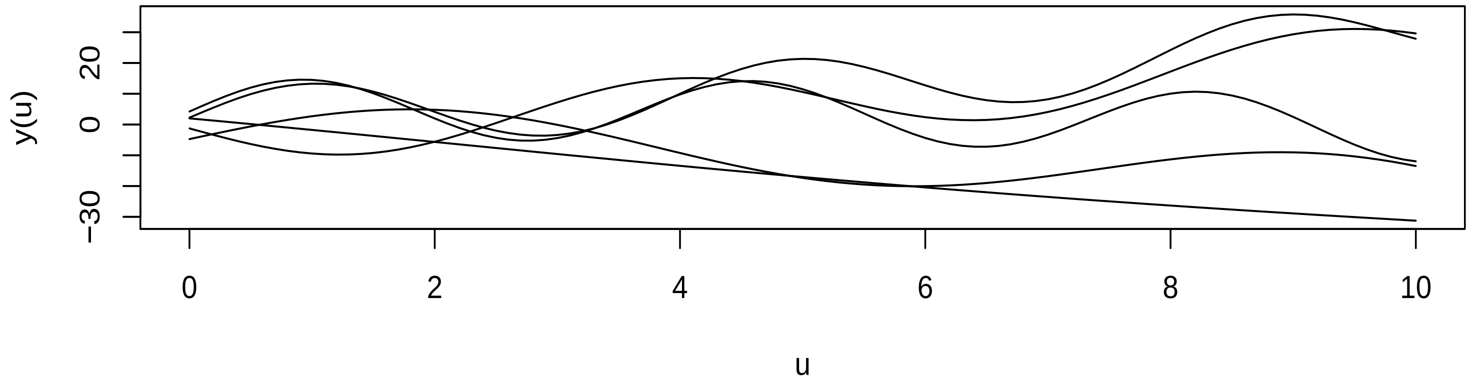
The second approach easily extends to curves $Y_i(u),\ i = 1, \ldots, k$ by generating

$$Y^\star(u) = \bar{Y}(u) + \sum_{i=1}^{k} w_i(Y_i(u) - \bar{Y}(u)) \quad \text{with } w_i \text{ iid} \sim \mathcal{N}(0, 1/(k-1))$$

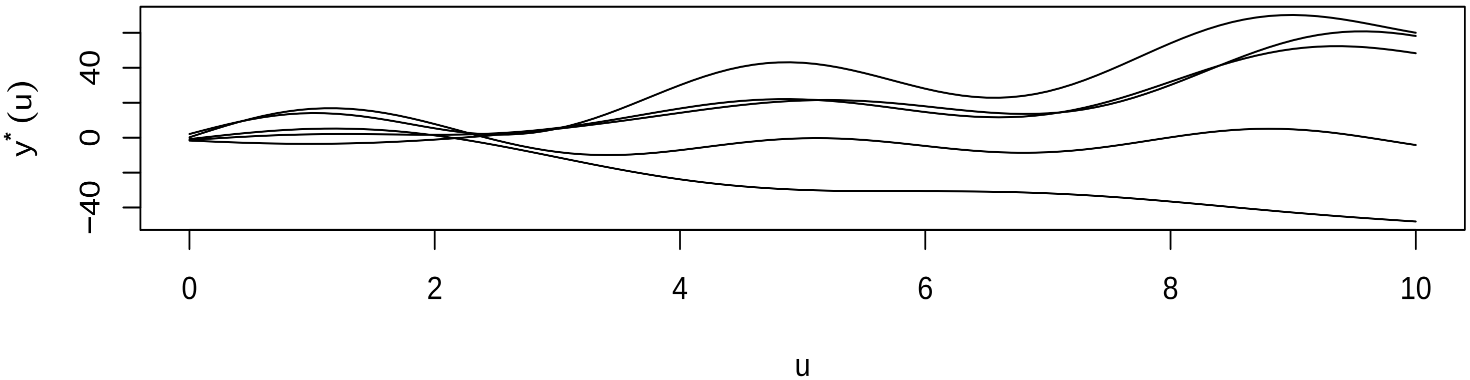
$$\begin{aligned} \text{cov}(Y^\star(u), Y^\star(u+h)) &= \frac{1}{k-1} \sum_{i=1}^{k} (Y_i(u) - \bar{Y}(u))(Y_i(u+h) - \bar{Y}(u+h)) \\ \text{generated autocovariance} &= \text{sample autocovariance} \end{aligned}$$

Random Curves

original sample of curves



bootstrapped curves



Some of my Projects over Time (3)

- AUTOLAND Sinkrate Risk Assessment (Nonparametric Tail Extrapolation)
  Simulated landings (costly even by computer), wanted extreme tail thresholds.

- Tolerance Analysis of Interchangeable Cargo Door Hinge Lines (Not RSS)
  RSS uses: $\text{var}(Y) = \text{var}(a_0 + a_1X_1 + \ldots + a_nX_n) = a_1^2\text{var}(X_1) + \ldots + a_n^2\text{var}(X_n)$,
  where $Y = f(X_1, \ldots, X_n)$ with $a_i = \partial f(\mu_1, \ldots, \mu_n)/\partial\mu_i$.

- Tolerance Analysis: Hole Positioning Requirements for Fuselage Assembly.
  Again not RSS.

- Quality Control Problems under Nonstandard Conditions
  Not a simple random sample, but with batch effects. See next slide.

Tolerance Bounds

Random sample $X_1, \ldots, X_N \sim \mathcal{N}(\mu, \sigma^2) \Longrightarrow$ average $\bar{X}$ and sample variance S^2
We can get a 95% lower confidence bound for the p-quantile $\mu + z_p\sigma$ for any p.
They are called tolerance bounds. For $p = 0.01$ we have $z_p = -2.326348$.

$P\left(\bar{X} - kS \leq \mu + z_p\sigma\right) = .95$ the factor k comes from the noncentral t-distribution

Often data come in batches: X_{bj}, $j = 1, \ldots, n_i$, $b = 1, \ldots, B$ with

$X_{bj} = \mu + X_b + X_{bj}$ with all X_b, X_{bj} independent, means 0 and variances σ_B^2 and σ^2

$\sigma_B = 0$: we have the original case with $N = n_1 + \ldots + n_B$ effective sample size.
$\sigma = 0$: we have effective sample size B. For either case we have a clean solution.

For intermediate situations we need a generalized version of effective sample size N_e based on estimates of $\sigma_B^2/(\sigma_B^2 + \sigma^2)$. We found it, but theoretically intractable.
N_e reduces to the previous two cases when $\sigma = 0$ or $\sigma_B = 0$.

We successfully validated its effectiveness through simulation.

The appeal was that it combined a known process with effective sample size, concepts that the customer was familiar with and could understand. Quick fix!

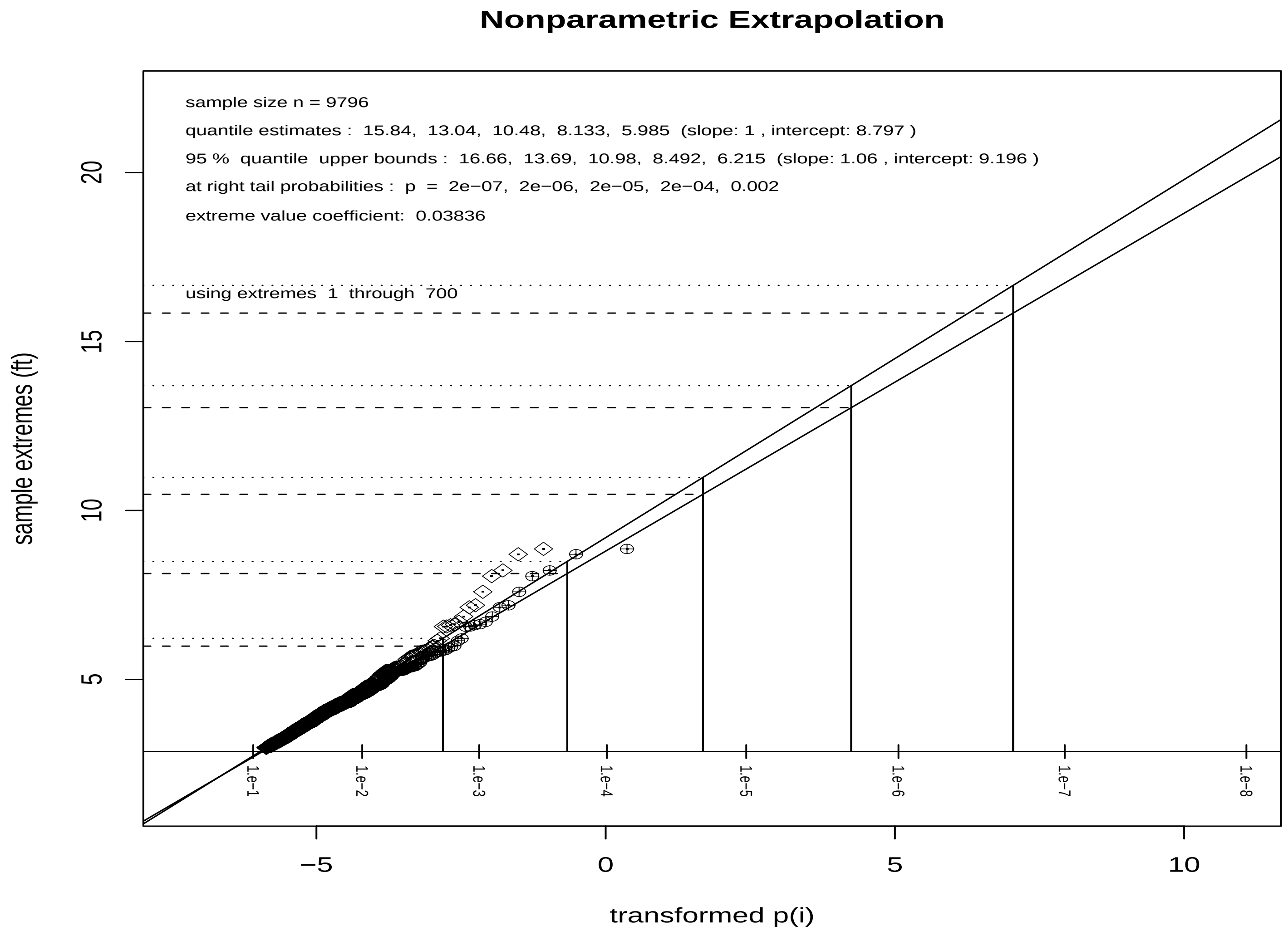
Some of my Projects over Time (4)

- Statistical Tolerancing for Fuselage Assembly Modeling,
  Combining RSS with systematic errors. Collaboration with IBM (Disk Drives)
  and Boeing Wichita ATA Program $\Longrightarrow$ Patent: Statistical Tolerancing

- Provide Statistical Expertise as Part of Air India Litigation (Vertigo, Horizon)
  Triply redundant artificial horizon, all indicating AC upside down.
  and Alaska 261 Crash (Jack Screw), override of limit wear.

- Taxiway Centerline Deviations for 747 (Joint Research with FAA)
  Extreme Value Behavior, Separation of Taxiways, Taxiway Widths,
  Implications for A-380. Typical deviation range $(-9\text{ft}, 9\text{ft})$
  3 same day instances with 18 ft deviation. What gives?

Taxiway Deviation Measurement by Laser ANC


Nonparametric Extrapolation
sample size n = 9796
quantile estimates : 15.84, 13.04, 10.48, 8.133, 5.985 (slope: 1 , intercept: 8.797)
95 % quantile upper bounds : 16.66, 13.69, 10.98, 8.492, 6.215 (slope: 1.06 , intercept: 9.196)
at right tail probabilities : p = 2e−07, 2e−06, 2e−05, 2e−04, 0.002
extreme value coefficient: 0.03836
using extremes 1 through 700
sample extremes (ft)
transformed p(i)
1.e−1
1.e−2
1.e−3
1.e−4
1.e−5
1.e−6
1.e−7
1.e−8
−5
0
5
10
15
20

Some of my Projects over Time (5)

- Assess Small Sample Properties of Bootstrap Methology
  The Bootstrap Gave Wings to Statistics, Handling Almost All Problems
  Very Intuitive and Appealing to Engineers

- Develop Monotone Confidence Bounds for Weibull Analysis (Web Page Tool)
  Engineers rightfully complained about nonmonotone bounds by classical method.

- Transfer Previous Results to Logistic Regression Analysis,
  Assessing Crack and Damage Detection

$$P(\text{detecting crack of length } \ell) = \frac{1}{1+\exp(-\ell)}$$

- Handled Hundreds of Hotline Calls

The International Space Station (ISS)

Space Rubble

Seattle Post-Intelligencer, Monday, June 27, 1994

Orbiting rubble could threaten space station

By William J. Broad, The New York Times

Dead satellites, shattered rocket stages and thousands of other pieces of man-made space junk speeding around Earth could destroy a planned international space station, and engineers are struggling to reduce the danger.

NASA estimates there is a 20 percent chance that debris could smash through the shield of the space station, an orbital outpost for the world's astronauts, during its construction and expected 10-year life.

The station will have gear to maximize safety and interior hatches to let astronauts seal themselves off from areas that would lose air if shattered. But the overall risk of a catastrophe that would result in death or destruction of the craft is still estimated at roughly 10 percent.

Space Rubble

NASA officials say they are confident that the risk of penetration can be reduced, perhaps to 10 percent, making the risk of catastrophe about 5 percent. Although the design already calls for much shielding, more may be added, they say, even while conceding that such a remedy adds cost and weight to an already heavily laden project.

”We'll do whatever is necessary to get adequate safety,” NASA Administrator Daniel S. Goldin said in an interview. ”If we need more shielding, we'll put more up.”

But Goldin also acknowledged that danger is inevitable in space exploration.

”We'll never be able to guarantee total safety,” he said. ”We could have loss of life with the shuttle, and the station as well. If you want to guarantee no loss of life, it's better not to go into space.”

Still, a station designer, who spoke' on the condition of anonymity, said the bureaucracy was playing down the problem and courting disaster.

”The traditional design philosophy says the mission's catastrophic risk should not exceed a few percent,” the designer said. ”Now, they've got it in the range of 10

Space Rubble

percent. That violates due diligence. If you're working in an uncertain environment, your bias should be on the side of safety."

Bigger than a football field at 361 feet in length and 290 feet in width, the station would have a six member international crew to study Earth, the heavens and human reactions to weightlessness in preparation for lengthy voyages to Mars and beyond.

The United States, Europe, Japan and Canada are longtime partners in the project, and Russia joined recently. The outpost would cost American taxpayers $43 billion, including $11 billion already spent on design studies.

Assembly flights are scheduled to begin in late 1997 and end in 2002, after which the completed outpost is to be used for a decade or more.

The U.S. military has found about 7,000 objects in orbit, ranging from the size of a school bus to the size of a baseball. Smaller objects cannot easily be tracked by radar. Because of the enormous speeds of everything in orbit, a tiny flake of metal can pack the punch of an exploding hand grenade.

ISS Wall Design

- The Modules of the ISS Have a Double Wall Design
  - How thick should the walls be?
  - How much space between the walls?
  - What material?
  - Other design factors.
- Risk of Penetration by Meteoroids and Space Debris
- Objective: Minimize Penetration Risk Subject to Economic Considerations
- Thicker Walls Lead to Heavier and More Costly Payloads.

The Poisson Process Probability Model

- The Poisson Process Provides a Very Useful and Appropriate Model for Describing the Probabilistic Behavior of Random Events over Time
- The Events are the Impacts by Space Debris and Meteoroids
- Flux or Intensity of Impacts per Surface Area per Year is a Driving Factor
- Penetration Factors of Impacting Object
  - Mass/Size, Velocity, Impact Angle of Objects
  - Wall Design

The Poisson Process

- The Poisson Process $N(t,A)$ Gives the Number of Random Impact Events on a Surface Area A during the Interval $[0,t]$, for any $t > 0$.

- $P(N(t,A) = k) = \exp(-\lambda t A)(\lambda t A)^k / k!$ for $k = 0, 1, 2, 3, \ldots$

- $\lambda > 0$ is the Event Intensity Rate per Unit Area (m^2) & per Unit Time (Year).

- $1/\lambda$ is the Average or Expected Time (Years) between Events per Unit Area, $\lambda = 10^{-3} \Longrightarrow$ on Average 1 Event per 10^3 Area $\times$ Time Units ($m^2 \times$ Years).

- The Probability of Seeing at Least One Event on Surface Area A during the Mission Interval $[0,T]$ is $P(N(T,A) > 0) = 1 - P(N(T) = 0) = 1 - \exp(-\lambda T A)$.

A Thinned Poisson Process

- For Each Event of a Poisson Process $N(t,A)$ an Independent Trial Determines whether it is a Penetration Event.

- If $p =$ Probability of a Penetration Event, then the Resulting Process $N^\star(t,A)$ of Penetration Events over Area A during $[0,t]$ is again a Poisson Process with Intensity Rate $\lambda^\star = p\lambda$.

- This is Called a Thinned Poisson Process because Events are Disregarded or Thinned out with Probability $1-p$.

- The Resulting Risk of Seeing at least one Penetration Event on Surface Area A during the Mission Interval $[0,T]$ is then $P(N(T,A)>0) = 1-\exp(-\lambda^\star TA)$

Finite Elements & Sums of Poisson Processes

- The ISS Surface was broken down into some $k = 5000$ Triangular Elements with Respective Areas $A_1, \ldots, A_k$

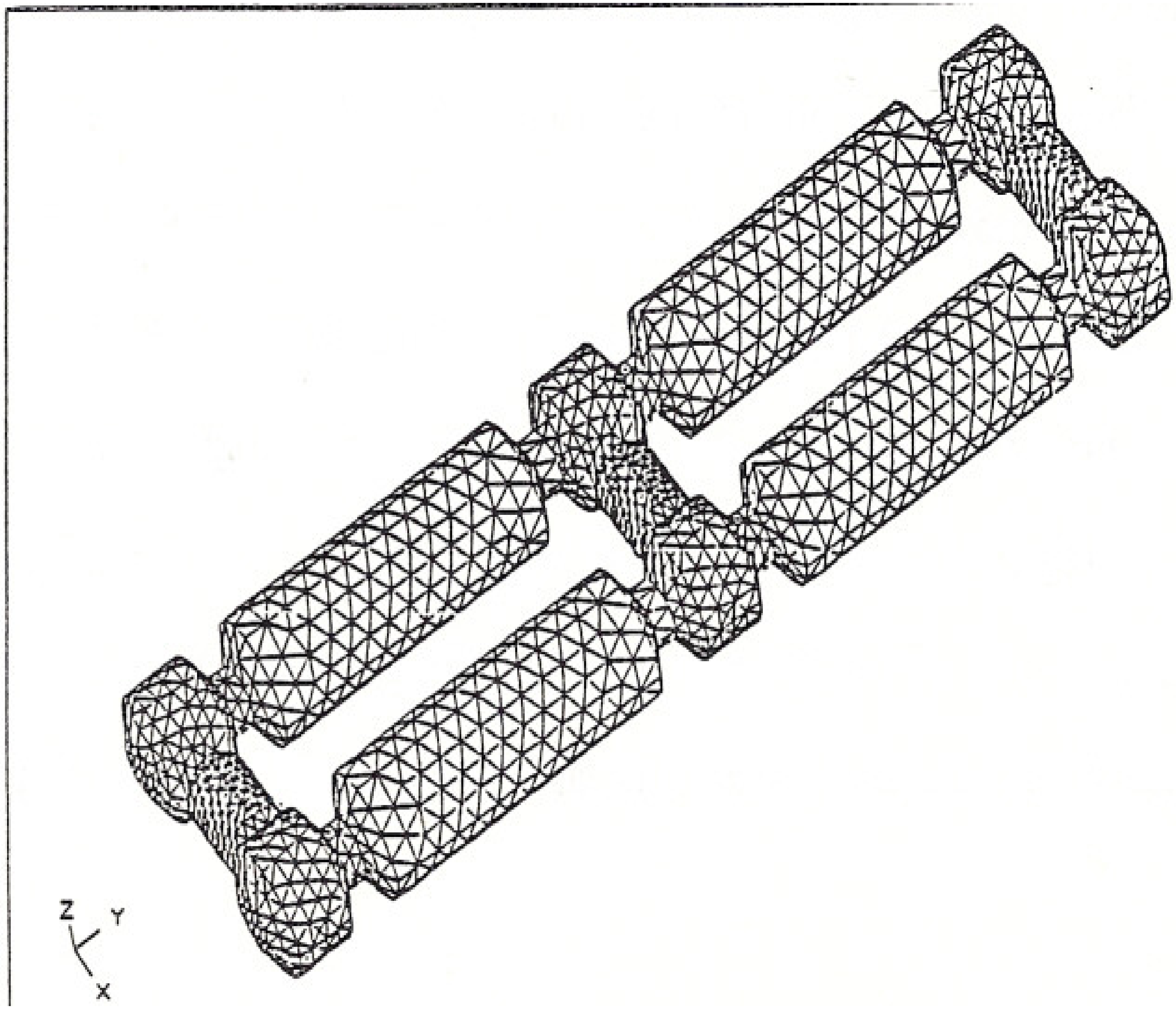
- Penetration Events for the k Surface Elements were Modeled by Independent Poisson Processes $N_i^\star(t)$, with Respective per Time Rates $\lambda_i^\star = p_i \lambda_i A_i$, $i = 1, \ldots, k$.

- $N_S(t) = N_1^\star(t) + \ldots + N_k^\star(t) =$ # of Penetration Events over Total Surface.

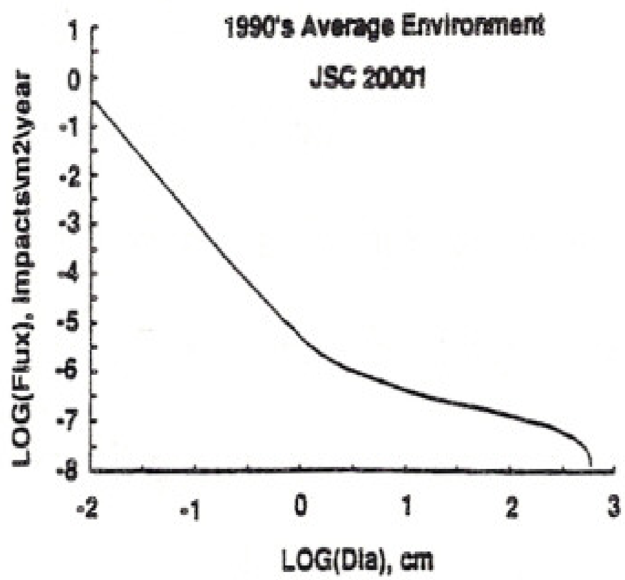
- $N_S(t)$ is a Poisson Process with Time Rate $\lambda_S = p_1 \lambda_1 A_1 + \ldots + p_k \lambda_k A_k$

  The Risk of at least one Penetration Event for the ISS during the Mission Interval $[0, T]$ is then $P(N_S(T) > 0) = 1 - \exp(-\lambda_S T)$

Finite Element ISS Surface Grid



Debris Size Flux Distribution



Flux Equation:

$\text{Log } F = -2.52 \text{ Log } D - 5.46$

D = diameter in centimeters; $D < 1.0$ cm

$\text{Log } F = -5.46 - 1.78 \text{ Log } D + 0.9889 (\text{Log } D)^2 - 0.194 (\text{Log } D)^3$

D = diameter in cemtimeters; $1.0 \text{ cm} \leq D \leq 200 \text{ cm}$

where:

F = Number of impacts of objects with diameter D or greater per square meter per year

Log = Logarithm base 10

Orbital Altitude = 500 km

Figure 2.1-1. Debris Flux Environment

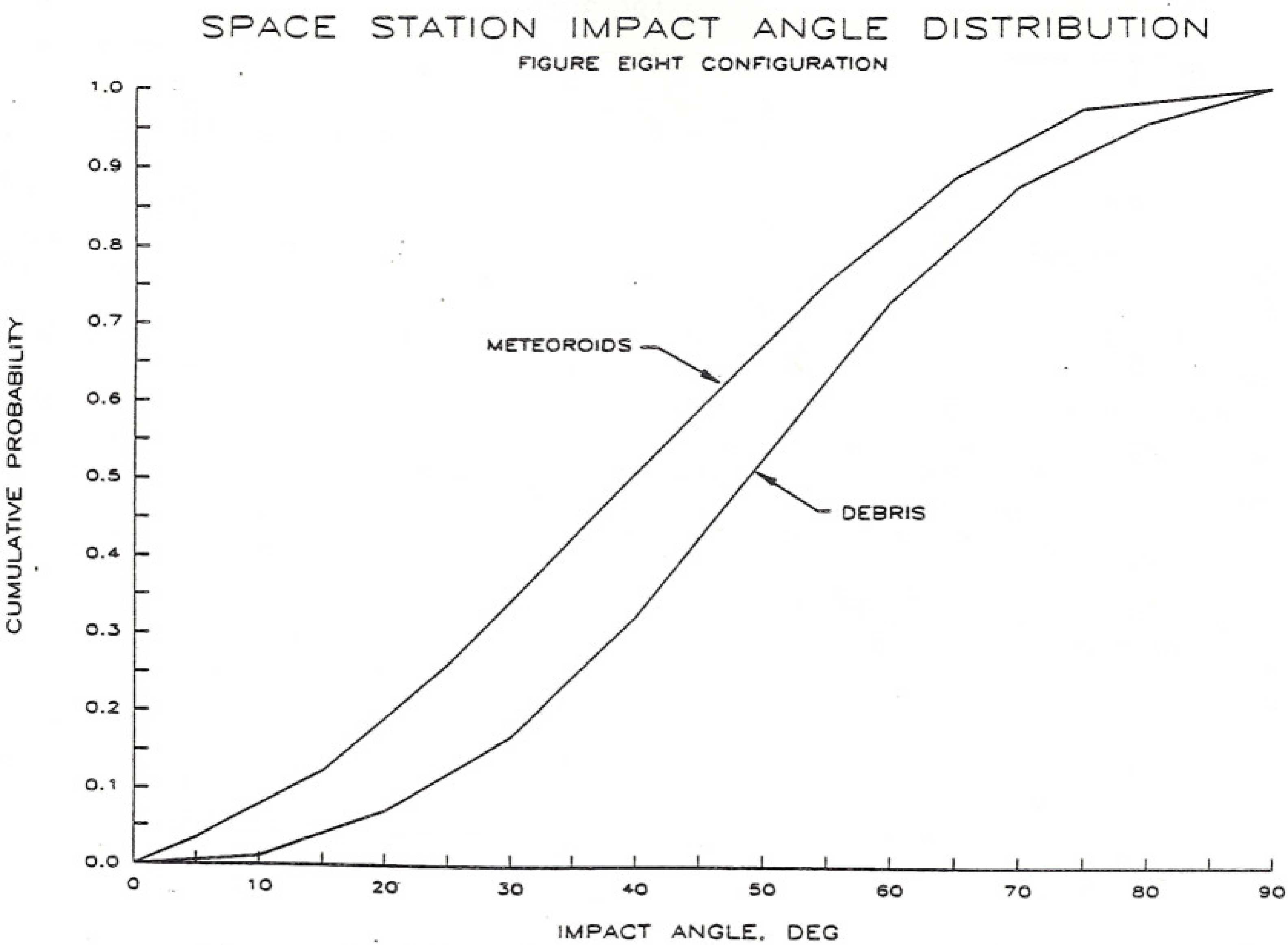


Figure 2.8-1. Impact Angle Probability Distribution

Meteoroid Velocity Distribution

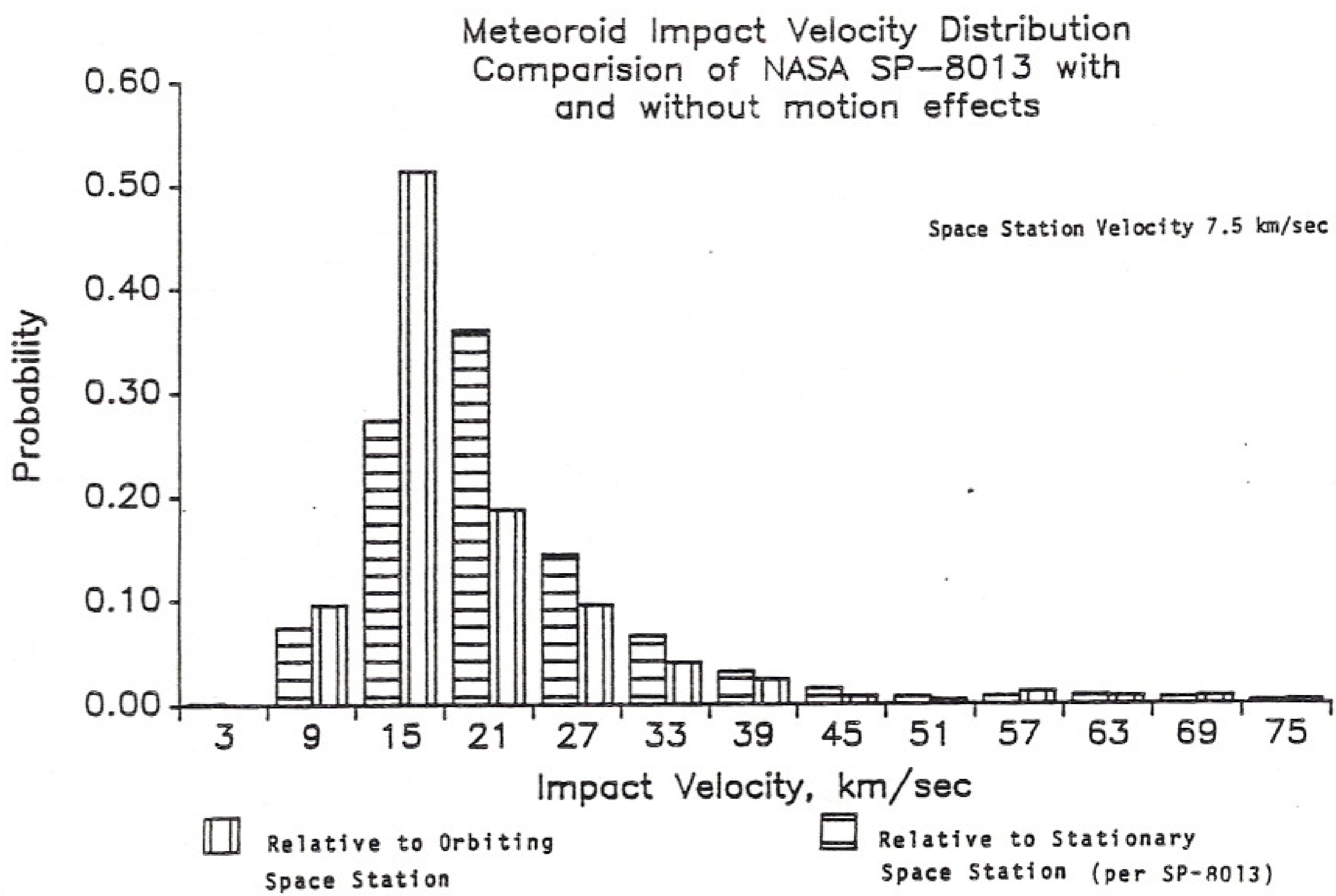


Figure 2.2-3. Meteoroid Impact Velocity Distribution Relative to Space Station

Debris Velocity Distribution

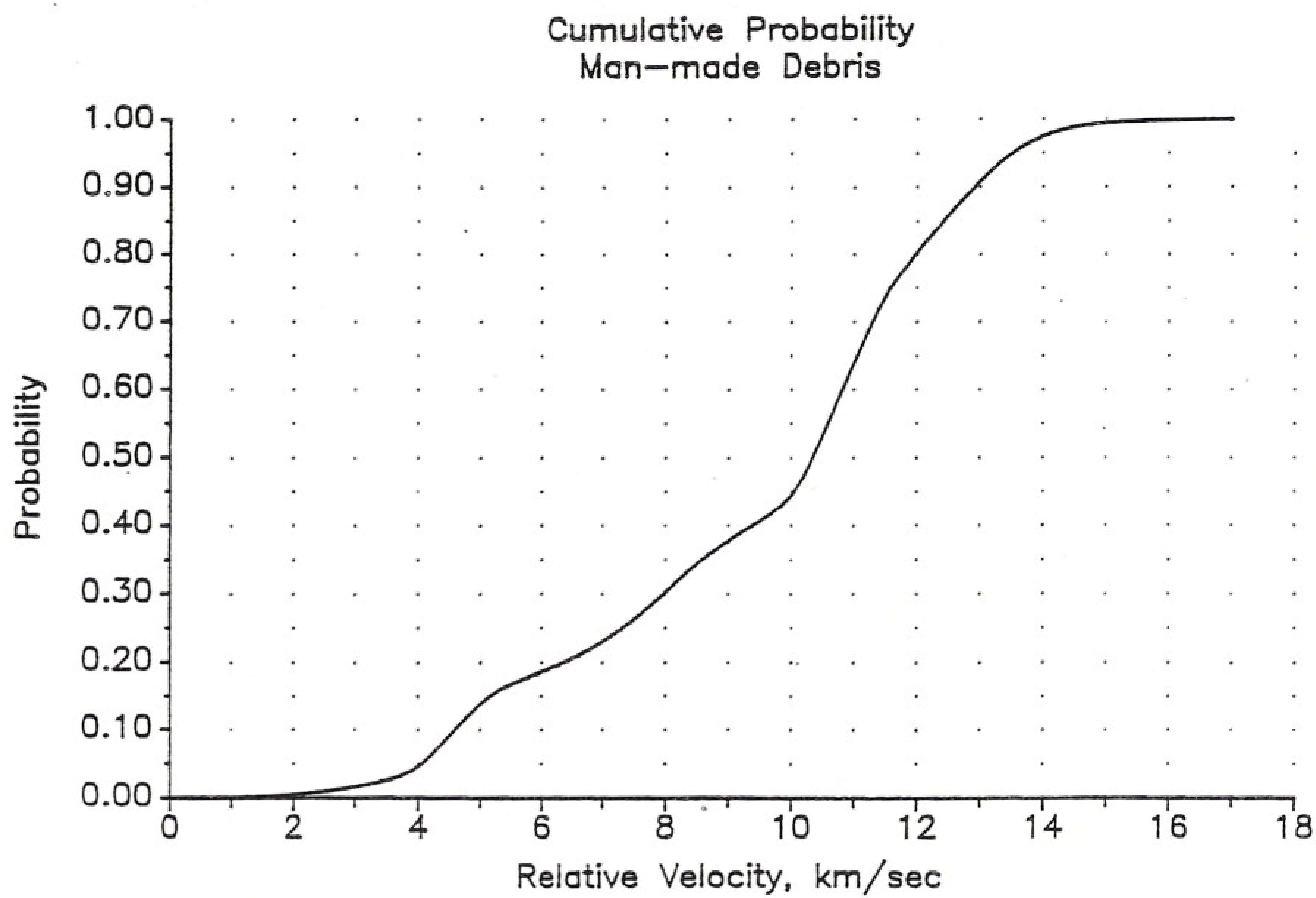


Figure 2.8-2. Impact Velocity Probability Distribution

Wall Filtering Probabilities

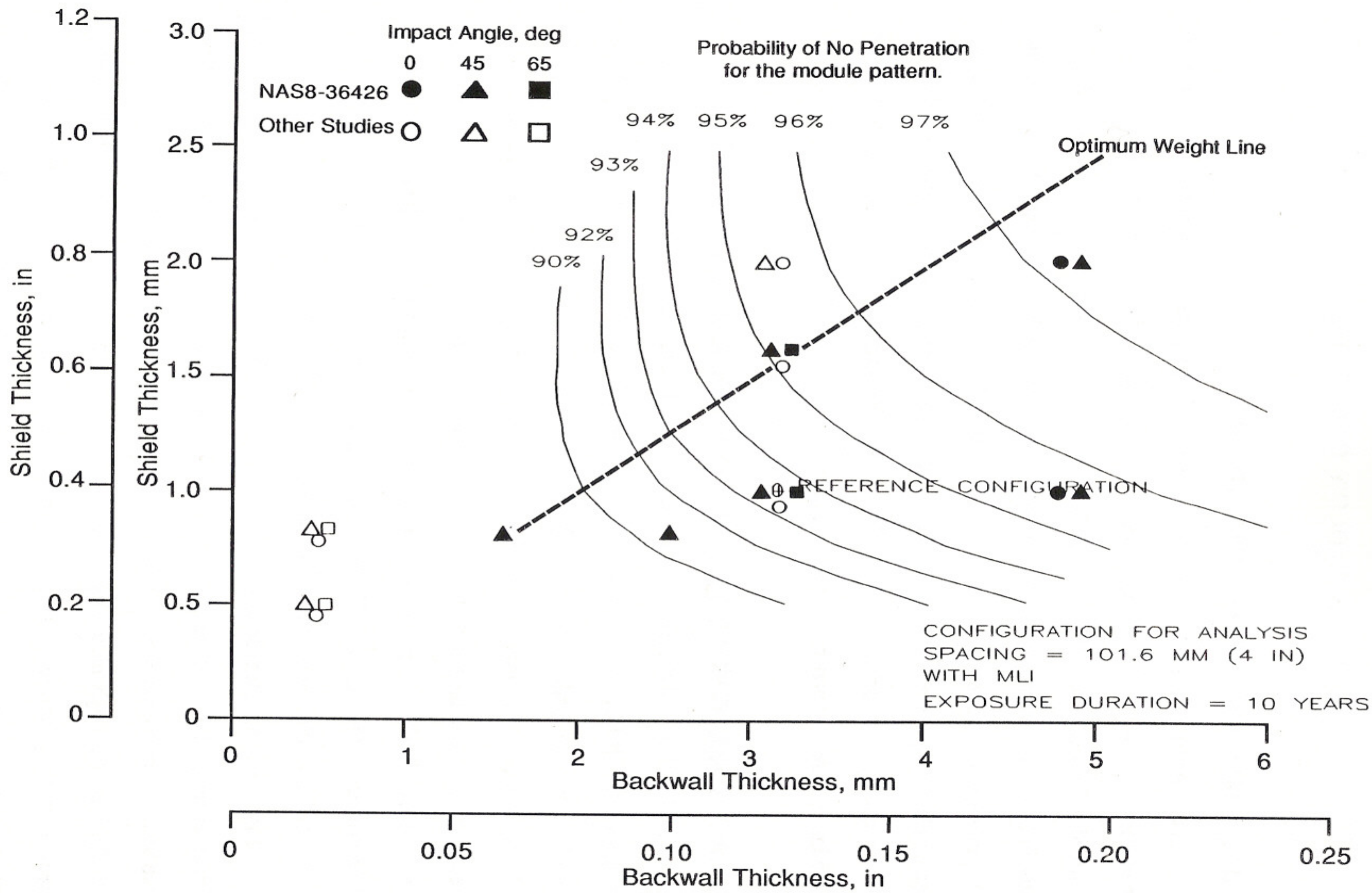


Figure 3.2-1. Module Design Data Comparison With Test Data.

ISS Penetration Risk Map