

STAT 311 Quiz 2 with Answers

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1. **(20 points)** Why is the sign test called sign test and which distribution is used as its null distribution? Which point estimator is motivated by the sign test?

It is called the sign test because we count the number of positive signs among $X_i - \theta_0$, $i = 1, \dots, n$, where θ_0 is the hypothesized median. The null distribution used is the Binomial distribution with parameters $(n, p = 0.5)$. The sample median is motivated as the estimator of the population median, by finding that value θ_0 that is least rejectable.

2. **(20 points)** Assuming a sample of size $n = 9$ from $\mathcal{N}(\mu, \sigma^2)$ with average $\bar{x}_n = 30$ and sample standard deviation $s_n = 3$, what is the 95% confidence interval for μ when given

$$\text{qt}(.95, 8) = 1.860 \quad \text{qt}(.95, 9) = 1.833 \quad \text{qt}(.95, 10) = 1.812$$

$$\text{qt}(.975, 9) = 2.262 \quad \text{qt}(.975, 8) = 2.306 \quad \text{qt}(.975, 10) = 2.228$$

Give the answer as the two decimal numbers that form the interval.

Do you have reason to reject the hypothesis $H_0 : \mu = 25$ at level $\alpha = 0.05$ when testing it against the alternative $H_1 : \mu \neq 25$? (**Explain your reasoning.**)

The confidence interval is $\bar{x} \pm \text{qt}(0.975, 9 - 1) \frac{s_n}{\sqrt{n}}$ i.e. $30 \pm 2.306 \frac{3}{3} = (27.694, 32.306)$

which does not contain 25. Hence $H_0 : \mu = 25$ should be rejected at level $\alpha = 0.05$.

3. **(20 points)** What form does the Welch test statistic take when testing $H_0 : \mu_1 = \mu_2$ versus $H_1 : \mu_1 \neq \mu_2$ in the normal 2-sample problem? (I am **not** asking for the null distribution!)

$$T_W = \frac{\hat{\Delta} - \Delta_0}{\sqrt{S_1^2/n_1 + S_2^2/n_2}} = \frac{\bar{X} - \bar{Y} - \Delta_0}{\sqrt{S_1^2/n_1 + S_2^2/n_2}}$$

4. **(20 points)** Assume independent random samples of respective sizes $n_1 = 4, n_2 = n_3 = n_4 = 5$, and $n_5 = 9$ from $\mathcal{N}(\mu_i, \sigma^2)$, $i = 1, \dots, 5$ with σ unknown. When testing $H_0 : \mu_1 = \mu_2 = \mu_3 = \mu_4 = \mu_5$ against the alternative that the means are not all the same, which R command with which exact numerical arguments would you use to calculate the significance probability of an observed value of 12 for the F -ratio test statistic?

$$1 - \text{qf}(12, 5 - 1, 4 + 5 + 5 + 5 + 9 - 5) = 1 - \text{qf}(12, 4, 23)$$

5. **(20 points)** Explain the notion of sampling variability for estimators and based on that explain the distinction between **sampled distribution** and **sampling distribution**.

Estimators evaluated for randomly varying samples will exhibit random variation or sampling variability. Here the samples come from the sampled population or sampled distribution and the randomly varying nature of these estimator values is described by the sampling distribution of the estimator.