Last name: First name:

Student ID #: Section:

STAT 311 Quiz 1 (**Show Your Work!**)

Fritz Scholz

1. **(20 points)** Let $A = \{2, 3, 4, 6, 9, 10\}$ and $B = \{2, 3, 7, 10\}$ be two sets, contained within the sample space S consisting of the integers from 1 to 100 inclusive. Describe the set $C = (A^c \cap B) \cup (A \cap B^c)$ in terms of its elements. How would this set C be affected if we let S consist of the interval $[0, 100]$?

$$B \cap A^c = \{7\}, \quad A \cap B^c = \{4, 6, 9\} \quad \Longrightarrow \quad C = (A^c \cap B) \cup (A \cap B^c) = \{4, 6, 7, 9\}$$

2. **(20 points)** We have two boxes. Box 1 contains 2 red balls and a blue ball. Box 2 also contains 3 balls, with respective colors blue, red and white. You select a box at random and from that box you select a ball at random. Find

$$\begin{aligned} P(\text{box 2 was chosen}|\text{a blue ball was selected}) &= P(Box_2|B) = \frac{P(Box_2 \cap B)}{P(B)} \\ = \frac{P(B|Box_2)P(Box_2)}{P(B|Box_2)P(Box_2) + P(B|Box_1)P(Box_1)} &= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{2}} = \frac{1}{2} \end{aligned}$$

$$\begin{aligned} P(\text{box 2 was chosen}|\text{a white ball was selected}) &= \frac{P(Box_2 \cap W)}{P(W)} \\ = \frac{P(W|Box_2)P(Box_2)}{P(W|Box_2)P(Box_2) + P(W|Box_1)P(Box_1)} &= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + 0 \cdot \frac{1}{2}} = 1 \end{aligned}$$

$$\begin{aligned} P(\text{box 2 was chosen}|\text{a red ball was selected}) &= \frac{P(Box_2 \cap R)}{P(R)} \\ = \frac{P(R|Box_2)P(Box_2)}{P(R|Box_2)P(Box_2) + P(R|Box_1)P(Box_1)} &= \frac{\frac{1}{3} \cdot \frac{1}{2}}{\frac{1}{3} \cdot \frac{1}{2} + \frac{2}{3} \cdot \frac{1}{2}} = \frac{1}{3} \end{aligned}$$

3. **(20 points)** The discrete random variable X takes on the possible values -1 and $+1$ with respective probabilities 0.6 and 0.4. Find EX and var X.

x	$p(x)$	$xp(x)$	x^2	$x^2p(x)$
-1	0.6	-0.6	1	0.6
1	0.4	0.4	1	0.4
		-0.2		1.0

$\Longrightarrow EX = -0.2$ and var $X = E(X^2) - (EX)^2 = 1.0 - (-0.2)^2 = 0.96$.

4. **(10 points)** Four female and two male rowers are randomly split into two groups of 4 and 2 to fill a quad and a double, respectively, without regard to seat order. What is the chance that the double (a 2 person boat) will consist of a man and a woman? Give the answer as a reduced fraction (e.g., 3/5 and not 6/10).

   There are $\binom{6}{2} = 6 \cdot 5/(1 \cdot 2) = 15$ ways to choose two persons for the double, and of those choices exactly $\binom{4}{1} \cdot \binom{2}{1} = 8$ consist of 1 woman and 1 man. Hence the desired chance is 8/15.

5. **(10 points)** In how many different orders can I present 5 distinct exam problems to my class?

$$5! = 1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 = 120 \quad \text{ways}$$

6. **(10 points)** What will the response in R be when given the command `choose(5,2)`?

$$\texttt{choose}(5, 2) = \binom{5}{2} = \frac{5 \cdot 4}{1 \cdot 2} = 10$$

7. **(10 points)** Describe in words what R computes when given the command

   `1-pbinom(10,100,.5)`.

   It computes the chance of observing at least 11 heads in 100 flips of a fair coin

$$1 - \texttt{pbinom}(10, 100, .5) = 1 - P(X \leq 10) = P(X \geq 11)$$

   where X is the number of successes in $n = 100$ trials with success probability $p = 1/2$.