8.4 Exercises

1. Suppose that I toss a fair coin 100 times and observe 60 `Heads`. Now I decide to toss the same coin another 100 times. Does the Law of Averages imply that I should expect to observe another 40 `Heads`?

2. In Example 7.7, we observed a sample of size $n = 100$. A normal probability plot and kernel density estimate constructed from this sample suggested that the observations had been drawn from a nonnormal distribution. True or False: *It follows from the Central Limit Theorem that a kernel density estimate constructed from a much larger sample would more closely resemble a normal distribution.*

3. Suppose that an astragalus has the following probabilities of producing the four possible uppermost faces: $P(1) = P(6) = 0.1$, $P(3) = P(4) = 0.4$. This astragalus is to be thrown 100 times. Let X_i denote the value of the uppermost face that results from throw i.

   (a) Compute the expected value and the variance of X_i.

   (b) Compute the probability that the average value of the 100 throws will exceed 3.6.

4. Chris owns a laser pointer that is powered by two `AAAA` batteries. A pair of batteries will power the pointer for an average of five hours use, with a standard deviation of 30 minutes. Chris decides to take advantage of a sale and buys 20 2-packs of `AAAA` batteries. What is the probability that he will get to use his laser pointer for at least 105 hours before he needs to buy more batteries?

5. Consider an urn that contains 10 tickets, labelled

   $$\{1, 1, 1, 1, 2, 5, 5, 10, 10, 10\}.$$

   From this urn, I propose to draw (with replacement) $n = 40$ tickets. Let Y denote the sum of the values on the tickets that are drawn.

   (a) To approximate $P(170.5 < Y < 199.5)$, one Math 351 student writes an `R` function `urn.model` that simulates the proposed experiment. Evaluating `urn.model` is like observing a value, y, of the random variable Y. Then she writes a loop that repeatedly evaluates `urn.model` and computes the proportion of times that `urn.model` produces $y \in (170.5, 199.5)$. She reasons that, if she evaluates `urn.model` a large number of times, then the observed proportion of $y \in (170.5, 199.5)$ should approximate $P(170.5 < Y < 199.5)$. Is her reasoning justified? Why or why not?

(b) Another student suggests that $P(170.5 < Y < 199.5)$ can be approximated by performing the following R commands:

```
> se <- sqrt(585.6)
> pnorm(199.5,mean=184,sd=se)-
+ pnorm(170.5,mean=184,sd=se)
```

Do you agree? Why or why not?

(c) Which approach will produce the more accurate approximation of $P(170.5 < Y < 199.5)$? Explain your reasoning.

6. A certain financial theory posits that daily fluctuations in stock prices are independent random variables. Suppose that the daily price fluctuations (in dollars) of a certain value stock are independent and identically distributed random variables $X_1, X_2, X_3, \ldots$, with $EX_i = 0.01$ and $\operatorname{Var} X_i = 0.01$. (Thus, if today's price of this stock is \$50, then tomorrow's price is $\$50 + X_1$, etc.) Suppose that the daily price fluctuations (in dollars) of a certain growth stock are independent and identically distributed random variables $Y_1, Y_2, Y_3, \ldots$, with $EY_j = 0$ and $\operatorname{Var} Y_j = 0.25$.

Now suppose that both stocks are currently selling for \$50 per share and you wish to invest \$50 in one of these two stocks for a period of 400 market days. Assume that the costs of purchasing and selling a share of either stock are zero.

(a) Approximate the probability that you will make a profit on your investment if you purchase a share of the value stock.

(b) Approximate the probability that you will make a profit on your investment if you purchase a share of the growth stock.

(c) Approximate the probability that you will make a profit of at least \$20 if you purchase a share of the value stock.

(d) Approximate the probability that you will make a profit of at least \$20 if you purchase a share of the growth stock.

(e) Assuming that the growth stock fluctuations and the value stock fluctuations are independent, approximate the probability that, after 400 days, the price of the growth stock will exceed the price of the value stock.