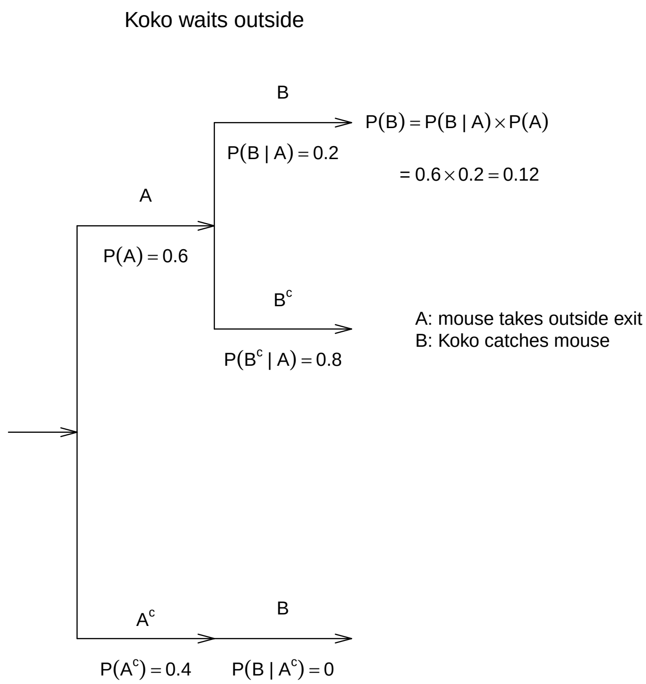
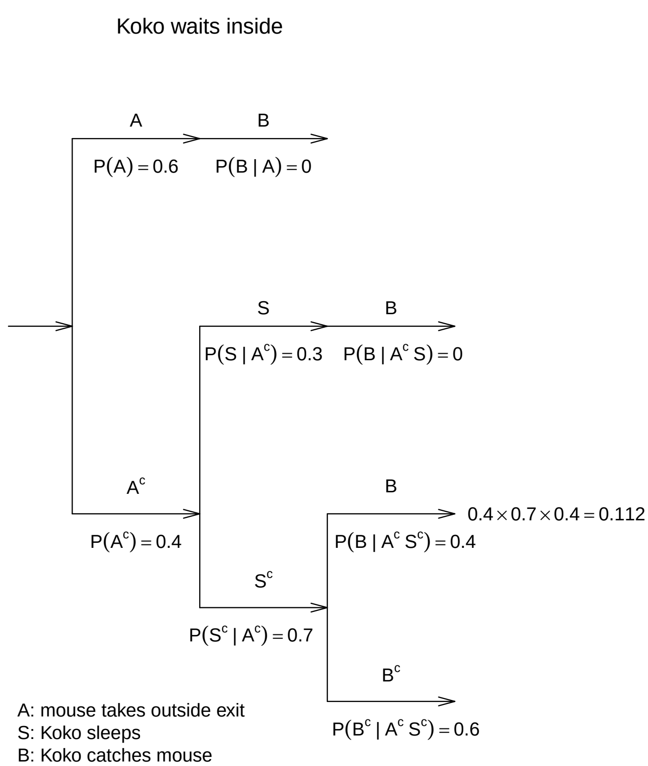
Stat 311: HW 4, Chapters 4 & 5, Solutions

Fritz Scholz

Ch. 4, Problem 7. (a) Koko is more likely to catch the mouse outside than inside, since $0.12 > 0.112$, as the two tree diagrams show.





(b) Let C_7 be the event that the mouse is caught within the next 7 days when Koko waits outside each night. It is easier to contemplate the complementary event of missing the mouse 7 nights in a row, the misses each night being independent events. Then

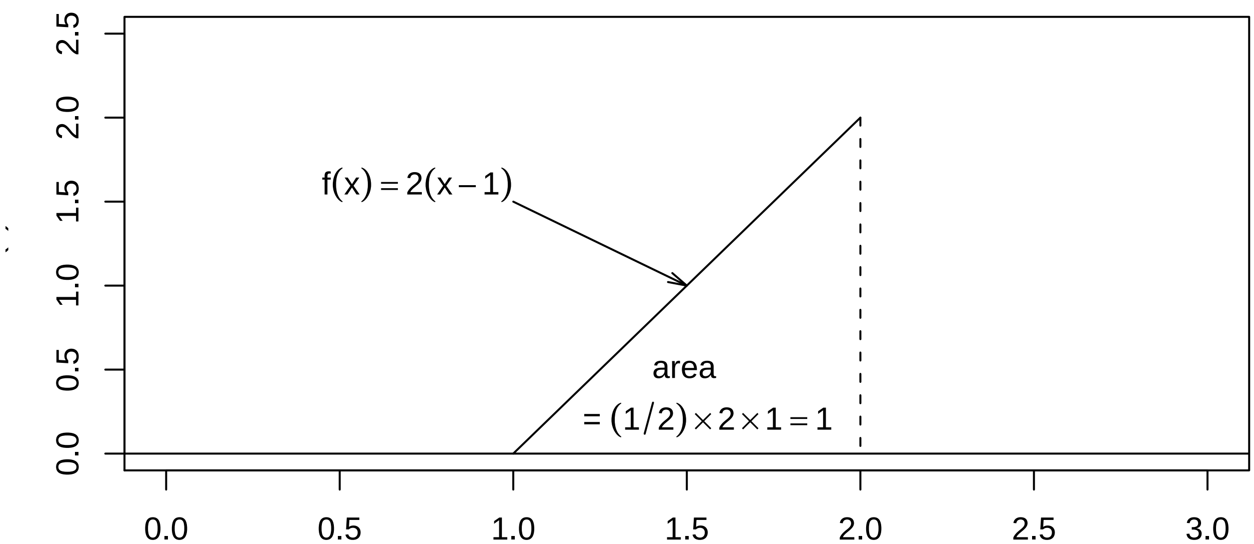
$$P(C_7) = 1 - P(C_7^c) = 1 - (1 - 0.12)^7 = .5913$$

Ch. 4, Problem 11. View the $n = 110$ room reservations as so many independent Bernoulli trials. Then the number Y of people actually showing up is binomial with parameters $n = 110$ and $p = .96$. Then

$$P(Y > 100) = 1 - P(Y \leq 100) = 1 - \texttt{pbinom}(100, 110, .96) = 0.9870095$$

i.e., there is a 98.7% chance that the hotel will have to turn someone away.

Ch. 2, Problem 2. (a) see plot. (b) Clearly $f(x) \geq 0$ for all x and the triangle area $(1/2)\times$ height $\times$ base width $= 1$.



$$(c) \qquad P(1.50 < X < 1.75) = P(X \leq 1.75) - P(X \leq 1.5) = 0.5625 - 0.25 = 0.3125 \quad \text{since}$$

$$P(X \leq 1.5) = \frac{1}{2} \cdot 2 \cdot (1.5-1) \cdot (1.5-1) = 0.25 \quad \text{and} \quad P(X \leq 1.75) = \frac{1}{2} \cdot 2 \cdot (1.75-1) \cdot (1.75-1) = 0.75^2 = 0.5625$$

Ch. 2, Problem 7. $X \sim \mathcal{N}(-5, 10^2)$, then

$$(a) \qquad P(X < 0) = P\left(\frac{X-\mu}{\sigma} \leq \frac{0-(-5)}{10}\right) = \texttt{pnorm}(0.5) = 0.6914625$$
$$= \texttt{pnorm}(0, -5, 10) = 0.6914625$$

$$(b) \quad P(X > 5) = 1 - P(X \leq 5) = 1 - P\left(\frac{X-\mu}{\sigma} \leq \frac{5-(-5)}{10}\right) = 1 - \texttt{pnorm}(1) = 0.1586553$$
$$= 1 - \texttt{pnorm}(5, -5, 10) = 0.1586553$$

$$(c) \quad P(-3 < X < 7) = P(X \leq 7) - P(X \leq -3) = \texttt{pnorm}(1.2) - \texttt{pnorm}(0.2) = 0.3056706$$
$$= \texttt{pnorm}(7, -5, 10) - \texttt{pnorm}(-3, -5, 10) = 0.3056706$$

$$(d) \quad P(|X+5| < 10) = P(-10 < X+5 < 10) = P(-10 \leq X+5 \leq 10) = P(-15 \leq X \leq 5)$$
$$= P(X \leq 5) - P(X \leq -15) = \texttt{pnorm}(1) - \texttt{pnorm}(-1) = 0.6826895$$
$$= \texttt{pnorm}(5, -5, 10) - \texttt{pnorm}(-15, -5, 10) = 0.6826895$$

$$
\begin{aligned}
(e) \quad P(|X-3|>2) &= P(\{X>5\}\cup\{X<1\}) = P(X<1)+P(X>5) \\
&= P(X<1)+1-P(X\le 5) = \texttt{pnorm}(0.6)+1-\texttt{pnorm}(1) = 0.8844021 \\
&= \texttt{pnorm}(1,-5,10)+1-\texttt{pnorm}(5,-5,10) = 0.8844021
\end{aligned}
$$

Ch. 2, Problem 8. (a) Since $X_1 \sim \mathcal{N}(1,9)$ and $X_2 \sim \mathcal{N}(3,16)$ are independent, we have

$$E(X_1+X_2) = 1+3 = 4 \quad \text{and} \quad \text{var}(X_1+X_2) = 9+16 = 25$$

(b) $E(-X_2) = -3$ and $\text{var}(-X_2) = (-1)^2\text{var}(X_2) = 16$.

(c) $E(X_1-X_2) = 1-3 = -2$ and $\text{var}(X_1-X_2) = \text{var}(X_1+(-1)X_2) = \text{var}X_1 + (-1)^2\text{var}X_2 = 9+16 = 25$

(d) $E(2X_1) = 2EX_1 = 2\cdot 1 = 2$ and $\text{var}(2X_1) = 2^2\text{var}(X_1) = 4\cdot 9 = 36$

(e) $E(2X_1-2X_2) = E(2X_1)-E(2X_2) = 2EX_1 - 2EX_2 = 2\cdot 1 - 2\cdot 3 = -4$ and

$$\text{var}(2X_1-2X_2) = \text{var}(2X_1)+\text{var}(2X_2) = 2^2\text{var}(X_1)+2^2\text{var}(X_2) = 4\cdot(9+16) = 100$$