Stat 311: HW 3, Chapter 3, Solutions

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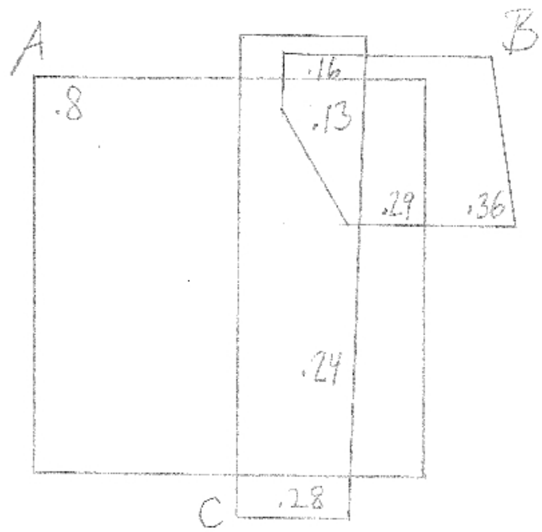
1. (a) My attempt at an appropriate Venn diagram is shown in the illustration below. While the remaining parts of this problem can be pieced together by appropriate overlaps of the diagram, we prefer and show the more or less mechanical approach that does not rely on a picture. Pictures can be deceiving.



(b) Since $P(A \cap B \cap C^c) + P(A \cap B \cap C) = P(A \cap B) \Longrightarrow P(A \cap B \cap C^c) = 0.29 - 0.13 = 0.16$.

(c) Since $P(A \cap (B \cup C)^c) + P(A \cap (B \cup C)) = P(A)$ we get

$$\begin{aligned} P(A \cap (B \cup C)^c) &= P(A) - P(A \cap (B \cup C)) = P(A) - P((A \cap B) \cup (A \cap C)) \\ &= P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C) \\ &= 0.8 - 0.29 - 0.24 + 0.13 = 0.40 \end{aligned}$$

(d) $P((A \cup B \cup C)^c) = 1 - P(A \cup B \cup C)$ and since

$$\begin{aligned} P(A \cup B \cup C) &= P(A \cup B) + P(C) - P((A \cup B) \cap C) = P(A) + P(B) - P(A \cap B) + P(C) - P((A \cap C) \cup (B \cap C)) \\ &= P(A) + P(B) - P(A \cap B) + P(C) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C) \\ &= 0.8 + 0.36 - 0.29 + 0.28 - 0.24 - 0.16 + 0.13 = 0.88 \quad \Longrightarrow \quad P((A \cup B \cup C)^c) = 0.12 \end{aligned}$$

(e) Since $P(A^c \cap (B \cup C)) + P(A \cap (B \cup C)) = P(B \cup C) = P(B) + P(C) - P(B \cap C) = 0.36 + 0.28 - 0.16 = 0.48$

and $P(A \cap (B \cup C)) = P((A \cap B) \cup (A \cap C)) = P(A \cap B) + P(A \cap C) - P(A \cap B \cap C) = 0.29 + 0.24 - 0.13 = 0.4$

we obtain $P(A^c \cap (B \cup C)) = 0.48 - 0.4 = 0.08$.

5. (a) 6^4 outcomes are possible. (b) $P(\text{all 4 \#'s different}) = (6 \cdot 5 \cdot 4 \cdot 3)/6^4 = 5/18$.

(c) $x_1 + x_2 + x_3 + x_4 = 5 \iff$ three 1's and one 2. Thus $P(X_1 + X_2 + X_3 + X_4 = 5) = 4/6^4 = 1/324$,

where the 4 in the numerator accounts for the number of ways a 2 can appear (while the other dice have a 1).

(d) $P(\text{at least one odd \#}) = 1 - P(\text{all \#'s even}) = 1 - 3^4/6^4 = 15/16$.

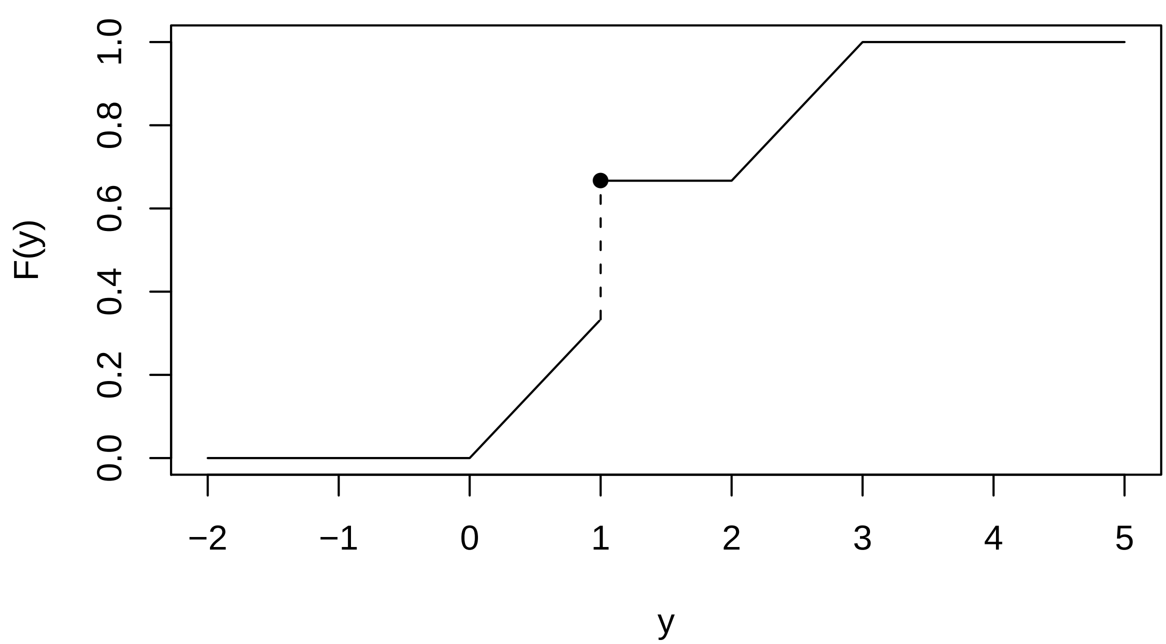
(e) $P(\text{3 dice with the same odd \# and an even \# on the fourth}) = (4 \cdot 3 \cdot 3)/6^4 = 1/36$, where we have 4 ways to pick the die with the even number, 3 ways to select which even number, and 3 ways to pick the odd number to be repeated by the other three dice.

13. We need to come up with an appropriate probability model. Since the winter break has some influence on the days that the checks will get written (note the proximity of the two days 16 and 18) we will consider the dates as fixed. It will make more sense to consider the current check sequence number as random, in particular it seems reasonable that the last two digits could be any of the 100 pairs $00, 01, \ldots, 99$ and that they are equally likely. It also seems reasonable that the last two digits of the sequence numbers for the two house mates are independent of each other. Since 00 does not correspond to a date we exclude it and consider the other 99 as equally likely. The question would not have been asked if 00 had shown up. In a sense, this is a conditional problem view, given that only $01, 02, \ldots, 99$ are possible. If X_1 and X_2 represent the respective numbers chosen randomly and independently from $01, 02, \ldots, 99$ we have

$$P(X_1 = 16 \cap X_2 = 18) = \frac{1}{99} \cdot \frac{1}{99} = \frac{1}{9801}$$

In any such chance calculations it should be understood that the question should be posed before such events are observed, i.e., $1/9801$ would be the chance that it might happen with a different set of house mates some time in the future. When considering or marveling about coincidences one tends to forget about the many cases that are not noteworthy.

14. The graph of the cdf is as follows:



(a) $P(X > 0.5) = 1 - P(X \le 5) = 1 - 0.5/3 = 1 - 1/6 = 5/6.$

(b) $P(2 < X \le 3) = P(X \le 3) - P(X \le 2) = 3/3 - 2/3 = 1/3$

(c) $P(0.5 < X \le 2.5) = P(X \le 2.5) - P(X \le 0.5) = 2.5/3 - 0.5/3 = 2/3$

(d) $P(X = 1) = F(1) - F(1-) = 2/3 - 1/3 = 1/3.$