Stat 311: HW 3, Chapter 3, Comments

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1. It is possible to answer (b)-(e) by untangling the Venn diagram (provided you did it correctly) using the associated probabilities. However, that process is very prone to error. Thus I suggest that you work with probability formulas (note the notation AB for $A \cap B$) like

$$P(A \cup B) = P(A) + P(B) - P(AB) \;, \quad P(A^c) = 1 - P(A) \;, \quad \text{and} \quad P(A^cB) + P(AB) = P(B)$$

By choosing A and B to take various forms, e.g., $A = C \cup D^c$. The basic problem is that you are given probabilities where the events don't appear in complement form. Somehow, using the above formulas, you have to steer the calculation in the direction where you can use the given entities. I will provide one example where you are supposed to calculate $P((A^c \cup B^c) \cap C)$ based on the given event probabilities $P(A), P(B), P(C), P(AB), P(AC), P(BC), P(ABC)$.

$$P((A^c \cup B^c) \cap C) = P(CA^c \cup CB^c) = P(CA^c) + P(CB^c) - P(CA^cB^c) = P(C) - P(CA) + P(C) - P(CB) - P(CA^cB^c)$$

The first $=$ uses De Morgan's laws, the second $=$ uses the general formula for the probability of a union, the third $=$ uses $P(C) = P(CA^c \cup CA) = P(CA^c) + P(CA)$ and $P(C) = P(CB^c \cup CB) = P(CB^c) + P(CB)$. All we need now is to reduce $P(CA^cB^c)$ sufficiently, since all the other probabilities are known or given. Using the same method as in the third $=$ above we have

$$P(CA^cB^c) = P(CA^c) - P(CA^cB) = (P(C) - P(CA)) - (P(CB) - P(CBA)) = P(C) - P(AC) - P(BC) + P(ABC)$$

and we are done, after putting it all together.

$$P((A^c \cup B^c) \cap C) = P(C) - P(AC) + P(C) - P(BC) - [P(C) - P(AC) - P(BC) + P(ABC)] = P(C) - P(ABC)$$

There may be faster ways, but this process is very methodical. Just see how to get rid of a complement and do it. The simple form of the answer suggests this quick approach

$$P((A^c \cup B^c) \cap C) = P((AB)^cC) = P(C) - P(ABC)$$

13. We need to come up with an appropriate probability model. Since the winter break has some influence on the days that the checks will get written (note the proximity of the two days 16 and 18) we will consider the dates as fixed. It will make more sense to consider the current check sequence number as random, in particular it seems reasonable that the last two digits could be any of the 100 pairs $00, 01, \ldots, 99$ and that they are equally likely. It also seems reasonable that the last two digits of the sequence numbers for the two house mates are independent of each other. Since 00 does not correspond to a date we exclude it and consider the other 99 as equally likely. The question would not have been asked if 00 had shown up. Now the answer should be immediate.