Stat 311: HW 2, Chapter 2, Solutions

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1. (a) $A \cup B \cup C = \{1,2,3,4,5,6,7,8,9,10,11\}$. (b) $A \cap B \cap C = \emptyset$

   (c) No, since $A \cap B = \{4,5\} \neq \emptyset$ and also because $B \cap C = \{7,8\} \neq \emptyset$.

You can do these set operations also in R using `union` and `intersect`.

```
> A <-1:5
> B <- 4:8
> C <- 7:11
> union(union(A,B),C)
 [1]  1  2  3  4  5  6  7  8  9 10 11
> intersect(intersect(A,B),C)
integer(0)
> intersect(A,B)
[1] 4 5
> intersect(A,C)
integer(0)
> intersect(B,C)
[1] 7 8
```

2. (a) $A \cap (B \cup C) = \{4,5\}$ (b) $A \cup (B \cap C) = \{1,2,3,4,5,7,8\}$

   (c) $B \cap (A \cup C) = \{4,5,7,8\}$ (d) $B \cup (A \cap C) = \{4,5,6,7,8\}$

   (e) $(C \cap A) \cup (C \cap B) = \{7,8\}$ (f) $(C \cup A) \cap (C \cup B) = \{4,5,7,8,9,10,11\}$

In R

```
> intersect(A,union(B,C))
[1] 4 5
> union(A,intersect(B,C))
[1] 1 2 3 4 5 7 8
> intersect(B,union(A,C))
[1] 4 5 7 8
> union(B,intersect(A,C))
[1] 4 5 6 7 8
> union(intersect(C,A),intersect(C,B))
[1] 7 8
> intersect(union(C,A),union(C,B))
[1]  7  8  9 10 11  4  5
```

3. (a) $A^c = \{2,4,6,8,10\}$ (b) $B^c = \{4,6,8,9,10\}$ (c) $(A \cup B)^c = \{4,6,8,10\}$ (d) $(A \cap B)^c = \{2,4,6,8,9,10\}$

   In R the complement of A is obtained via $A^c =$ `setdiff(S,A)`

```
> S <- 1:10
> A <- c(1,3,5,7,9)
> B <- c(1,2,3,5,7)
> setdiff(S,A)
```

```
[1]  2  4  6  8 10
> setdiff(S,B)
[1]  4  6  8  9 10
> setdiff(S,union(A,B))
[1]  4  6  8 10
> setdiff(S,intersect(A,B))
[1]  2  4  6  8  9 10
```

14. (a) $\phi(6) = 2^6 = 64$, (b) $\phi(-3) = 2^{-3} = 1/8 = .125$, (c) $\phi(\mathcal{R}) = (0,\infty)$, (d) $\phi^{-1}(16) = \phi^{-1}(2^4) = 4$

(e) $\phi^{-1}(1/4) = \phi^{-1}(2^{-2}) = -2$, (f) $\phi^{-1}([2,32]) = \phi^{-1}([2^1,2^5]) = [1,5]$

15. (a) C is denumerable since there is a $1-1$ correspondence between $C = \{10^0, 10^1, 10^2, 10^3, \ldots\}$ and $\mathcal{N} = \{1,2,3,\ldots\}$ by the rule 10^{i-1} for $i = 1,2,3\ldots$.

(b) y_n does not converge since it is constant $= 1$ at every power of 10 (which happens infinitely often), but it gets arbitrarily small for any other n. y_n can't be within $\varepsilon = 1/10$ of both 0 and 1 as n gets large.

16. (a)

$$y_0 = \sum_{k=0}^{0} 2^{-k} = 2^{-0} = 1 = 2-1, \quad y_1 = \sum_{k=0}^{1} 2^{-k} = 2^{-0} + 2^{-1} = 1 + \frac{1}{2} = 2 - \frac{1}{2}, \quad y_2 = \sum_{k=0}^{2} 2^{-k} = 1 + \frac{1}{2} + \frac{1}{4} = 2 - \frac{1}{4}$$

$$y_3 = \sum_{k=0}^{3} 2^{-k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} = 2 - \frac{1}{8}, \qquad y_4 = \sum_{k=0}^{4} 2^{-k} = 1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \frac{1}{16} = 2 - \frac{1}{16}$$

In general

$$y_n = \sum_{k=0}^{n} 2^{-k} = 2 - \frac{1}{2^n} \longrightarrow 2 = \sum_{k=0}^{\infty} 2^{-k} \quad \text{as} \quad n \longrightarrow \infty$$