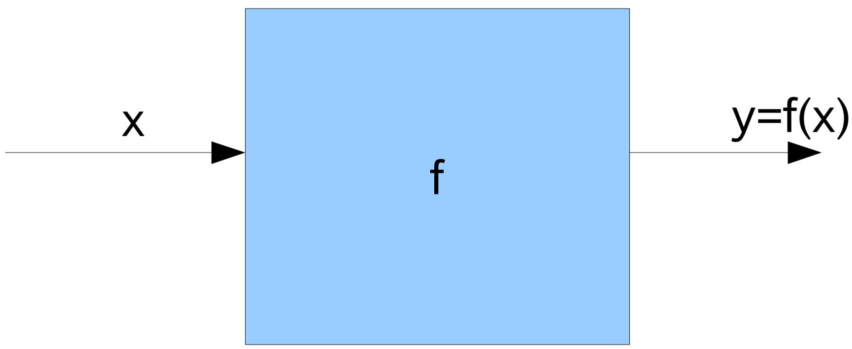
Combining Functions

We may think of a function as a sort of "meat-grinder" that takes in an input (a value for the "independent variable", grinds it in various ways, and spits out an output (the corresponding value for the "dependent variable"):



To help us work with *f*, we may want to break its action down into smaller pieces – after all, we do that all the time, when trying to grasp a complex item, analyzing it into components... Thus, it pays, sometimes, to think of a function *f* as the result of combinations involving simpler functions.

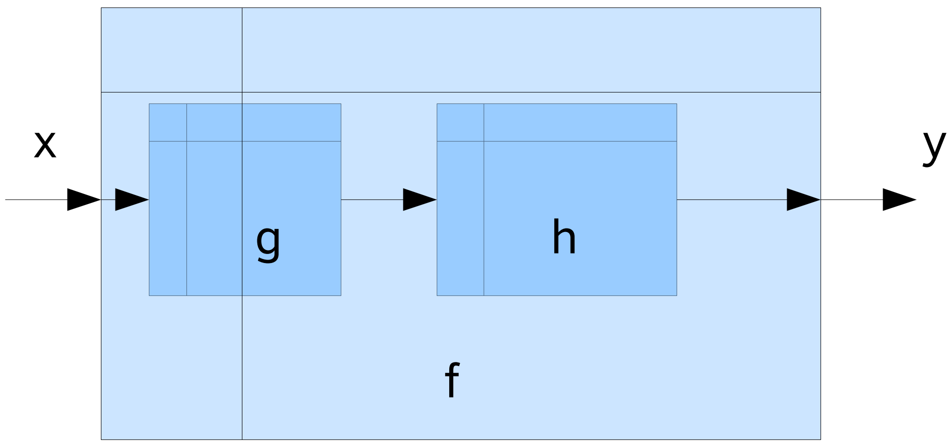
Examples are mentioned in the assignment sheet: we may take two functions, and combine them via arithmetic operations:

$$g(x)=1+x\,,h(x)=x^2$$

$$f_1(x)=g(x)+h(x)=1+x+x^2 \quad f_2(x)=g(x)\cdot h(x)=(1+x)\cdot x^2 \quad f_3(x)=\frac{h(x)}{g(x)}=\frac{x^2}{1+x}$$

and so on...

A slightly more complicated combination is when we decompose *f* into a "series" arrangement, where one function, say *h*, takes, as input, the output of the other:



Here, we think of *f* as working like this: take *x*; feed it into *g*; take the output, g*(x)*, and feed it into *h*, calling the result, *h(g(x))*, *f(x)*. With the functions above, we would have:

$$g(x)=1+x \quad h(x)=x^2$$
$$f(x)=h(g(x))=g(x)^2=(1+x)^2$$

The combination of functions in this way, is called “composition”, and it turns out to be a useful approach when trying to work with more complicated functions: after all, the great advances of science have all been based on the ability to reduce complex phenomena to a combination of simpler ones! A notation you will often see, for this way of combining *g* and *h* is

$$f(x)=h(g(x))=h\circ g(x)$$

The small circle stands for “composed with”, according to the method indicated above.