Stretching, Reflecting, Shifting

Changing the constants that appear in an algebraic formula changes the graph in some predictable ways.

Principles:

- Changing the x (the input) changes the horizontal.
- Changing the y (the output) changes the vertical.
- Multiplying by a constant stretches (or squashes) the graph.
- Multiplying by –1 reflects the graph.
- Adding a constant shifts the graph.

Details: Start with the graph of $y = f(x)$. The graph of each of the following will have the same basic shape as $y = f(x)$, altered as noted. For all of these, a is a constant

- $y = af(x)$ is a times as tall.
- $y = -f(x)$ is reflected vertically across the x-axis (upside down).
- $y = f(x) + a$ is shifted up a units.

- $y = f(ax)$ is $\frac{1}{a}$ times as wide.
- $y = f(-x)$ is reflected horizontally across the y-axis.
- $y = f(x + a)$ is shifted to the left a units.
- Notice that for horizontal stretches or shifts, the effect is sort of the inverse of what you might expect at first. These can be confusing – check your answers by plotting a couple of points.

You can handle these all at once. You can do horizontal and vertical changes independently. For each of these, follow the order of operations – stretch and reflect first, then shift. Use the origin as your anchor point, even if it's not on your graph.

Example: The graph of $y = -4|x - 2| + 5$ has the same basic shape as $y = |x|$. It's shifted to the right 2 units. It's upside down, 4 times as tall, and has been shifted up 5 units. So the new graph has its vertex at (2, 5), opens down, and looks stretched vertically compared to the original. You can plug in a point or two to confirm your answer. The new graph goes through the point (0, –3), which makes sense. If you move 2 units to the left of the vertex, the unchanged graph goes up 2 units. Here, it goes down (because the graph is upside down) 8 units (because the graph is stretched by a factor of 4.)