How Do I Choose the Precision Level

When calculating (with a computer, a calculator, by hand...) a numerical value for a formula, most of the time you will have to decide how many digits you will keep, and round the answer correspondingly.

Note. If your formula involves only fractions whose denominators (when reduced to lowest terms) only involve 2 and 5 as prime factors, the corresponding decimal expansion will be exact. Depending on the context, you might still want to round to a smaller number of digits, but be prepared to skip the decimal form, if appropriate, or keeping all digits, if that would make sense

1 Sometimes it makes no sense

If the problem you are solving is without context, like an abstract equation, or some other purely mathematical question, any number involved in the statement is supposed to be exact, by default. This implies that your answer should be just as exact, that is it should not involve any rounding approximation. In fact, if you were to evaluate your answer numerically, any choice of precision would be completely arbitrary. Suggestions you may have seen in other classes to "round to two (or three, or...) decimals" are intended to give you a way to use your calculator in a situation where it can't really work properly, but they are essentially a random choice. In such an abstract situation, **your answer has to be a formula or similar exact expression**. Any numerical approximation should be an add-on, if you really feel the need to add it, but cannot be your "true" answer. The point is that, lacking an objective measure, it would be up to the reader to decide what kind of approximation s/he would like to apply, if any, and it's not up to you to pre-empt her/his preference.

2 Applied problems

The situation is different in an "applied" problem. Here, the numbers that appear in the question are, by definition, approximate. For example, here is a problem involving a ball being thrown in a vacuum (sorry, but that's the simple, if unrealistic, situation). It would have its height satisfy an equation of the form

$$h(t) = -16t^2 + 10t + 5$$

where 10 (feet/second) is the initial vertical velocity, and 5 (feet) is the initial height, using feet and seconds as units. In this formula, 16 (feet/seconds squared) is a rough rounding of the effect of gravity (a slightly more precise form would be 16.1, but, in theory, one could go further). Since the remaining data are given with similar precision, your answer to a question like "how long will it take for the ball to hit the ground", should take this into account. The answer is the value of t such that $h(t) = 0$, which is the solution to a quadratic equation, and is, in our case,

$$t = \frac{-10 \pm \sqrt{100 + 4 \cdot 16 \cdot 5}}{-32} = \frac{5}{16} \pm \frac{\sqrt{420}}{32}$$

The negative solution has to be discarded (that would be before the ball was thrown), and the positive solution is approximated by 0.9529344228724748989... Now, all these decimal places make no sense, since we have such rough data. A reasonable answer is, seeing that our data comes with two *significant digits* at most, 0.953, rounding to one more significant digit (this is actually a bit aggressive, and 0.95 is probably better).

2.1 Rounding Rules

Recall the rules for rounding: they are chosen to minimize the error involved in the operation. When rounding to, say, n digits, we have to look at the first dropped digit, in position $n + 1$.

- If the first dropped dgit is 0, 1, 2, 3, or 4, **we round down**. So rounding 2.453 to two decimal digits results in 2.45. This way, you are making an error of 0.003. If you rounded up to 2.46 you would be making an error of 0.007

- If the first dropped digit is 5, 6, 7, 8, or 9, **we round up**. So orunding 2.456 yo two decimal digits results in 2.46, resulting in an error of 0.004, while rounding down would result in an error of 2.006

But why round up when the first ignored digit is 5, which generates, apparently, the same error as rounding down? The logic is that, in fact, any number is actually a truncated or rounded number (we are talking applications, hence, numbers are the result of measurements, and measurements are limited by the precision of the measuring instrument). Hence, 2.455 is a truncation (unless it was the result of a previous rounding up – but why are you rounding in two steps?), and there are unknown further digits that would be available if we had a more precise instrument. So, we have to presume that the "true" number is greater than that (it could be, say, 2.4550000001), hence, we round up.

2.2 Significant Digits

The phrase "significant digits" is not necessarily obvious. It is mostly elaborated in *scientific* contests (physics, chemistry, etc.). You might want to check a good website, like https://www.physics.uoguelph.ca/tutorials/sig_fig/SIG_dig.htm for the nitty-gritty details (it's not prefect, but it pushes all the right buttons). As you can see, when you switch to decimal representation **in applied problems** (in abstract problems, you should not consider using decimals at all), there are a number of things you need to keep track of: don't think you can rush and chop off decimals on a whim.