A Quick Comment On Precedence of Operators

Preliminary Material

We need to be clear on how you are supposed to read expressions without parentheses. The accepted rules follow: they are the same that you find spelled out in the instructions of your scientific calculator, or in the instructions of your favorite computer language. Though they are, in a sense, arbitrary, they are fully agreed upon, without mention, in any text you might encounter, so it is very well worth while to learn them, to accept them, and to implement them in your work

1 Precedence

Operations are called "unary" when they involve a single number. E.g., $-a$ means take number a and find its opposite. They are "binary" when they involve two numbers (at least), e.g. sum, subtract, multiply, divide. Powers (of any type, thus including radicals) are not "operations", but rather "functions".

In many cases, it doesn't matter in what order you perform a sequence of operations: in these cases there is no need to write out parentheses. For instance,

$$2 + 3 - 1 + 5 - 10 = (((2 + 3) - 1) + 5) - 10 = (((2 - 10) + 5) + 3) - 1 = \dots$$

or any other sequence you might choose.

On the other hand, it makes a lot of difference whether you do first a multiplication and next a sum or vice-versa:

$$2 + 3 \cdot 4 = 2 + 12 = 14$$

if you first do the multiplications, but $(2 + 3) \cdot 4 = 24$ if you first do the sum!To make sure you do operation sin the intended order, you can use parentheses, so we could write $2 + (3 \cdot 4)$, and $(2 + 3) \cdot 4$ in the two cases just mentioned.

People get weary of writing parentheses all the time, so they have agreed on a convention, and *imply* a certain order of operations unless parentheses tell them otherwise. The convention is

> Confronted with a sequence of operations, with unary, binary operations, and functions, proceed as follows, unless directed otherwise by explicit parentheses:

1. Evaluate all functions (the order in which they have to be evaluated will be either expressed by parentheses or implied to be "left-to-right")

2. Next, perform multiplications and divisions (by commutativity, it doesn't matter what order you follow here).

3. Next perform sums and differences (again, by commutativity, it doesn't matter what you do first)

4. Finally, apply unary operations.

2 Examples

Applying the rules is easy, once you get the knack.

1. $2 \cdot x + x^2 - \frac{2}{x^{\frac{1}{2}}}$: first evaluate the functions – here x^2, and $x^{\frac{1}{2}}$– then multiply/divide - here $2 \cdot x$, and $\frac{2}{x^{\frac{1}{2}}}$ - finally sum and subtract (there are no unary operators here). Wrapping up, with parentheses in the right spot this means:
$$(2 \cdot x) + \left(x^2\right) - \left(\frac{2}{\left(x^{\frac{1}{2}}\right)}\right)$$

2. $-x \cdot y^3$: first the functions - y^3 - next multiply - $x \cdot y^3$ - finally (there are no sums) apply the unary operator "negative". With parentheses, this would be written
$$-\left(x \cdot \left(y^3\right)\right)$$

3. $\sqrt{-3^2}$ is meaningless, because it means
$$\sqrt{-\left(3^2\right)}$$
i.e., the function $\surd$ is evaluated on whatever is inside, and this is the function square (3^2), followed by the unary $-$.

4. $\sqrt{(-3)^2}$ is 3, because now we are instructed to first apply the unary $-$ and then square.