Number Notation

As you will notice during this quarter, you will be strongly advised not to use "mixed numbers" in your math work. In fact, this notation (writing, for example, $2\frac{3}{4}$ instead of $\frac{11}{4}$) is never used in math, and is mostly unknown outside the USA. While perfectly usable in colloquial speech (or in highway signs), it does not work well in a math context, as it is inconsistent (juxtaposing symbols implies *multiplication* in math, not *addition*), and is awkward to use in further calculations where it lends itself to "silly" mistakes. The origin of this notation lies in the misguided notion of "improper" fraction, for fractions greater than 1. There is nothing "improper" in those rational numbers, and no reason to consider them apart from rational numbers less than 1. In fact, $\frac{11}{4}$ simply means that we are counting sub-units – in this case our unit is ¼ of the original one – and we got 11 of them. No reason why having 3 of them instead of 11 would be "different".

Some books may advise you to use this awkward notation when answering *modeling* questions, but I strongly urge you not to follow this direction. Aside from the inconsistency issue above, using this deprecated notation might suggest that your answer is "exact", while it really cannot be in an applied context where the data themselves have to be assumed to be approximate, as discussed extensively in our class. While it could conceivably seem the "natural" notation in a few very particular cases, you are instead invited to *complement* the exact answer ($\frac{11}{4}$, in our example) with its decimal representation, or approximation (2.75 in our case) in appropriate cases, as discussed below. Rarely, this might look strange, as when the answer represents a time span, but in these cases, we have used for a long time named sub-multiples as in "2 hours and 45 minutes", and "2:45", rather than " $2\frac{3}{4}$ or 2.75 hours"[1].

More about exact vs. decimal notation

In this, and any other, math class, we have to work on two types of problems.

First there are abstract math questions, used to hone our technical math skills – e.g., "solve the equation $x^2=2$ ". The proper answers to such questions are *exact expressions* – in our example, "there are two solutions, $x=\sqrt{2}$ and $x=-\sqrt{2}$ ". ±1.414 are **not solutions**: $1.414^2=1.999396$, not 2. 1.414 is only *approximately equal to* $\sqrt{2}$ *, but the catch is that there is no indication if the approximation is*

1 As a matter of fact, our use of dividing hours and minutes in sub-units of size 1/60th goes back to several thousands years ago, in Mesopotamia (dividing angles in 360 sub-units – degrees – also goes back to that time). The reason for this "strange" choice, is that the most common fractions (multiples of 1/2, 1/3, ¼, 1/6, 1/8, 1/10, 1/12, 1/15, 1/30, 1/60) are all simple (integer) multiples of 1/60 (360 adds multiples of 1/9, 1/18, 1/24 and so on), avoiding the need to work with fractions as representation of these rational numbers. This insight was lost as the positional notation used in Mesopotamia was forgotten in the West and eventually superseded by the positional notation in base 10 as probably developed in India and transmitted to the West through the Arabic empire in the late Middle Ages.

good enough, or too rough, since there is no context providing such information. So, 1.414, or any other decimal expression, will be flagged as an error. There is one "abstract" situation where it might make sense to compute a decimal approximation, and that's when plotting a point in a graph, where the coordinate is rational: it might be easier to find the point 1.59, or even better 1.6, on the number line rather than 43/27, which is approximately equal to 1.59259259259259 (you would usually be filling a table with these values, and you would have 43/27 in one column and 1.59 or 1.6 in the second column[2]).

Then there are what the book calls "modeling" or "authentic" problems (the rest of the world calls them "applications"). In our class, these are not *really* applications, since there is no real world case to which they will be applied, but they are examples of what *could be* a real world use of our math. In this case, **you are still required to produce an exact formula as your answer**, since the purpose of this class is to work on your math technique, and we need to make sure you applied it properly. However, it is all right to **add** a numerical approximation in your answer, since that would be the "solution" if it had been a real world situation (nobody asks for " $\sqrt{2}$ feet of fencing"). The difference with the abstract case is that the data you work with are implicitly approximate, since real world data is always the result of measurements, and measurements are limited in precision by your instrument. So, the (abstract) question "what is the length of the diagonal to a square of side 1" has (by a famous theorem, most likely improperly, called "Pythagorean Theorem") $\sqrt{2}$ as its answer (no decimal approximation makes sense here). On the other hand, the question: "a field has the shape of a square of side 1 mile; how long will be a fence that runs along its diagonal" has answer $\sqrt{2}\approx 1.4$ miles: we need the exact answer for our course's purpose, but we can add a decimal approximation for realism – here one decimal place might be OK, since the data is only given with precision to the mile, and we should limit to one (exceptionally two) decimal places more than the data's precision in the answer (1.4142135623730950488 would be wildly inappropriate, as the extra digits can't be but garbage, as the data was "1 mile", not "1.0000000000000000000 miles" – the extra 0's meaning that the measurement had been made to that level of precision). However, for our specific purposes, the decimal "realistic" answer is meant to be **in addition to, not in place of,** the exact form. So 1.4, without the preceding $\sqrt{2}\approx$, would still call for a flag, while writing $\sqrt{2}$ without the additional ≈ 1.4 would all right, at least in an exam, where you might not have a calculator handy.

We are talking about final answers: <u>intermediate calculations have always to be done with exact values</u> (very rarely, in fact, probably never, you might have to work with decimals in intermediate calculations – in such cases, you should work with as many decimal places as possible, rounding to the appropriate precision only at the end)**. It is a terrible habit to switch to decimals in the middle of a calculation: not only will this preclude you from producing an exact answer, but it will also result in snowballing rounding approximations which will throw off your final answer, possibly by a significant amount. That's why this practice too will be flagged as an error.**

2 Please, do not forget the rules on rounding: when discarding digits in the decimal representation of number, we look at the first discarded digit and round **up** the last remaining digit if the discarded one is 5 or higher, and round **down** if it is 4 or lower. So, in this example, 1.592… rounds **down** to 1.59, while 1.59… rounds **up** to 1.6. Failure to round properly is going to be flagged as an error.