There is more than one way

Two examples: equations with fractions, and products of polynomials

1

Suppose you want to solve

$$\frac{2}{3} + \frac{3}{4}x = \frac{1}{2} - \frac{x}{5}$$

1. You can find the LCD of all the fractions: 60, and rewrite this (it's the same equation!) as

   $$\frac{40}{60} + \frac{45}{60}x = \frac{30}{60} - \frac{12}{60}x$$

   Now, if you multiply both sides by 60 you end up with a *different*, but *equivalent* equation (solutions of the original are solutions of the new one, and vice-versa):

   $$40 + 45x = 30 - 12x \tag{1}$$

   and proceed as usual:

   $$(45 + 12)\, x = 30 - 40$$

   $$57x = -10$$

   $$x = -\frac{10}{57}$$

   Note that this is the same as writing the equation as

   $$\frac{40 + 45x}{60} = \frac{30 - 12x}{60}$$

   and observing that two fractions with the same denominator are equal if and only if the numerators are equal, resulting again in (1)

2. You can also treat the constants terms and the terms in x separately: the LCD for the constant terms is 6, and the one for the terms in x is 20:

$$\frac{4}{6}+\frac{15}{20}x=\frac{3}{6}-\frac{4}{20}x$$

Solving as usual

$$\left(\frac{15}{20}+\frac{4}{20}\right)x=\frac{3}{6}-\frac{4}{6}$$
$$\frac{19}{20}x=-\frac{1}{6}$$

We solve this as we always do: divide by $\frac{19}{20}$, that is multiply by $\frac{20}{19}$:

$$x=-\frac{20}{19}\cdot\frac{1}{6}=-\frac{10}{19}\cdot\frac{1}{3}=-\frac{10}{57}$$

Which one is better? Whichever you feel more natural, but it pays big time to be aware that you have a choice.

2

This is only an example: most problems will have several ways to be solved. A different good example is the product of two polynomials (you can see a more elaborate example in a file about operations on polynomials in the "Additional Materials" page). Look at

$$(3x+2)(5-2x)$$

In what order should you do the products? *It doesn't matter at all, provided each of the two terms in the first parenthesis is multiplied by each of the terms in the second* (it's the distributive and commutative rules at work). Hence you can go like

1. $3x\cdot 5+2\cdot 5+3x\cdot(-2x)+2\cdot(-2x)=15x+10-6x^2-4x=-6x^2+11x+10$
2. $2\cdot(-2x)+3x\cdot(-2x)+3x\cdot 5+2\cdot 5=-4x-6x^2+15x+10=-6x^2+11x+10$
3. ... you see how the only thing that changes is the order in which the individual terms appear after the first equality sign, so you can come up with the other possibilities. In fact you can do the 2×2 products separately, and adding them up in any order:

$$\begin{array}{cl} 3x\cdot 5 & =15x \\ 2\cdot 5 & =10 \\ 3x\cdot(-2x) & =-6x^2 \\ 2\cdot(-2x) & =-4x \end{array}$$